# State-of-the-Art Review

State-of-the-art review of the co-optimization of design and control for advanced vehicle propulsion systems is concisely synthesized from the latest literatures on powertrain components and vehicle propulsion systems.

# **1.1** Need for Better Energy Efficiency

In response to the global climate change caused by Green-House Gas (GHG)s,  $CO_2$  emissions are stringently regulated on the new vehicle models in the worldwide automotive industry. Considering  $CO_2$  emissions of a Light-Duty Vehicle (LDV), regulations on passenger cars and light-duty commercial vehicles have been adopted globally. Details of three major automotive markets around the world are exemplified in Table 1.1. In the near future,  $CO_2$  emissions of 95 g/km and 143 g/mi are obliged to achieve in EU and US, receptively; whereas, fuel consumption of 5 L/hkm (hkm: hundred kilometers) is mandatory in China. Nevertheless, more stringent energy efficiency targets in terms of  $CO_2$  emissions or fuel consumption are under development. The continuously improved energy efficiency targets advance powertrain technologies and innovations, for example, market penetration of battery-electric and hybrid-electric vehicles.

Compared with the obliged energy efficiency targets of LDVs, the one of a Heavy-Duty Vehicle (HDV) is less widely controlled by regulations around the world. The main reason is due to the European countries, where a market-driven policy of the energy efficiency is adopted rather than mandatory  $CO_2$  emission targets. However, targets of the energy efficiency of HDVs have been regulated in China and US by Phase 2 and Phase 1 (2014-2018), respectively. In the future, HDVs in all three major markets

Region	Target Year	Standard Type	Fleet Target
European Union	2021	CO <sub>2</sub>	95 gCO <sub>2</sub> /km
China	2020	fuel consumption	5 L/100km
United States 2025		fuel economy/	56.2 mpg or
		CO <sub>2</sub> +other GHGs	143 gCO <sub>2</sub> /mi

Table 1.1 – Energy efficiency targets of three major automotive markets in the near future.

must meet more stringent energy efficiency targets because of the development of CO<sub>2</sub> emission certification, monitoring, reporting, and standards in EU.

# 1.2 Single-Source Vehicle

# 1.2.1 Conventional Vehicle

As a leading player in the automotive market, conventional vehicle must meet the stringent regulations on fuel consumption and pollutant emissions. Technologies to improve fuel efficiency concentrate on the continuous improvement of powertrain components, which are essentially composed of an internal combustion engine, a transmission, and a final drive (see Fig. 1.1).



Figure 1.1 – Propulsion system of a four-wheel drive conventional vehicle.

Considering internal combustion engines, their efficiencies are always under improvement by advanced technologies, which consist of engine downsizing technology [8], turbocharger technology [9], friction reduction technology [10], variable compression ratio technology [11], alternative fuels [12], and the advanced combustion technology [13]. However, details of these technologies are out of the scope of this thesis work. Nonetheless, some of them are used as dimensioning parameters to develop the predictive analytic models for internal combustions. For instance, the implementation of turbocharger affects the descriptive analytic models.

As for transmissions, current technologies include advanced gear ratio design [14], implementation of higher gear number [15], sophisticated shift strategy [16], highly efficient transmission [15], and advanced automatic transmissions [17, 18], which directly influence the fuel consumption of a conventional vehicle. Therefore, the optimization of transmission dimensioning parameters is capable of further improving the energy efficiency.

In addition to technologies of engines and transmissions, stop-start systems are implemented to conventional vehicles so that the idling fuel consumption is eliminated [19]. Throughout this thesis, the idling fuel consumption is not considered in conventional and hybrid-electric vehicles, due to the wide application of stop-start systems.

### 1.2.2 Battery-Electric Vehicle

Battery electric vehicle, as a technological endpoint to achieve tank-to-wheel zero emission, is continuously penetrating the automotive market around the world. As shown in Fig. 1.2, key powertrain components of a battery-electric vehicle consist of an electric motor/generator, a battery, a power electronics, and a transmission.



Figure 1.2 – Propulsion system of a battery-electric vehicle.

The main technological concerns on battery-electric vehicles are over the electric vehicle range, battery cost and lifespan, performance in cold weather, maintenance, available charging infrastructures. Nonetheless, the energy consumption of a battery-electric vehicle can be further reduced by improvements on powertrain efficiency, power electronics, aerodynamics, and light-weighting technologies, which enlarges electric vehicle range in turn [20].

Recent studies show that the optimized design of power electronics [21], suitable topology of transmission [22], and intelligent control technologies – such as gear shift schedule design [23] and eco-driving technique [24] – are capable of enhancing the powertrain efficiency of a battery-electric vehicle.

## 1.2.3 Vehicle Propulsion System Design

To meet the desired vehicle performance, single-source vehicle propulsion systems for conventional and battery-electric vehicles are often designed through heuristic methods, such as an iterative process to find suitable powertrain components that meet the requirement. Despite lack of systematic design optimization approach, the design of a battery-electric vehicle can be optimized by finding the best dimensioning parameters of powertrain components such that the energy consumption is minimized. A battery-electric vehicle is optimally designed through multi-objective optimization method by optimizing dimensioning parameters of electric motor and battery size to meet the design targets defined by drivability parameters [25]. Alternatively, genetic algorithm method is also used to optimize the design of an battery-electric vehicle with two-speed dual-clutch transmission at system level [26].

# 1.3 Hybrid-Electric Vehicle: Architecture and Control

The propulsion system of a Hybrid Electric Vehicle (HEV) is characterized by multiple energy sources, which are internal combustion engine and battery. A Vehicle Propulsion System (VPS) of a hybrid-electric vehicle consists of powertrain components of conventional and battery-electric vehicles. Moreover, several aspects of hybrid-electric vehicles are of essence, including powertrain architecture, powertrain control, and VPS design, of which the first two aspects are introduced in this section.

#### **1.3.1** Powertrain Architecture

Hybridization of conventional vehicles can be realized in three different basic architectures, including series, parallel, and power-split architecture. Sub-configurations of each basic architecture may exist, such as pre-transmission and through-the-road configuration in the parallel architecture.

#### Series HEV

A series hybrid-electric vehicle consists of a propulsion system in which two electrical power sources feed a single electric traction motor that propels the vehicle. A simplified configuration with the major powertrain components is sketched in Fig. 1.3. The unidirectional energy converter, which is an internal combustion engine, is mechanically coupled to an electric generator through a simple gear train or rigid connection, which are usually referred to as Auxiliary Power Unit (APU). The bidirectional energy source is a battery pack that provides and stores electrical energy during different operating phase of a hybrid-electric VPS. Power electronics manages all of the electrical power flows in the propulsion system.



Figure 1.3 – Propulsion system of a series hybrid-electric vehicle.

In a series HEV, the full electrical connection between power sources and driven wheels is through an electric traction motor, instead of a mechanical transmission. This substitution allows the internal combustion engine to potentially operate at the desired region, such as maximum efficiency zone, according to the control objectives. Therefore, the performance of the internal combustion engine, such as efficiency and emissions, may be further improved by design and calibration. On the other hand, the absence of transmission results in a simple powertrain structure. Furthermore, the energy management of this hybrid architecture is simple, since internal combustion engine is often controlled to be more efficient.

However, disadvantages of series HEV are obvious. One is the poor efficiency of whole propulsion system resulting from multiple conversions of energy between electrical and mechanical form. Another one is the additional cost and weight by adding the electric motor/generator in APU. Additionally, traction motors are not so competitive as internal combustion engines in the heavy-duty application.

#### Parallel HEV

A parallel hybrid-electric vehicle consists of a propulsion system in which one mechanical power and one electrical power source propel the vehicle through a transmission or directly. Simplified configurations of parallel HEVs with the major powertrain components are sketched in Fig. 1.4. The unidirectional energy converter (internal combustion engine), is mechanically coupled to the driven wheels through a transmission; whereas the bidirectional energy source (battery pack) provides and stores electrical energy in the propelling and braking phase, respectively. Power electronics manages the electrical power flows in the propulsion system.



Figure 1.4 – Propulsion system of a parallel hybrid-electric vehicle.

In a parallel HEV, the mechanical connection between internal combustion engine and driven wheels remains the same as that in a conventional VPS. However, electric motor/generator propels the driven wheels either through the transmission or directly according to the coupling position between the mechanical drivetrain and electric motor/generator. Furthermore, different coupling positions result in several configurations, which are composed of P0 (belt-driven stator generator), P1 (crankshaft-mounted stator generator), P2 (pre-transmission), P3 (post-transmission), and P4 (axle drive) as depicted in Fig. 1.4.

As many attributes of a conventional VPS are preserved, parallel HEV allows direct torque supply from both engine and electric motor/generator to the driven wheels, which makes the energy losses possibly less. The vehicle propulsion system of a parallel hybrid is compact since it is unnecessary for an additional electric generator and smaller dimensions of the electric traction motor than that in series HEV.

However, the mechanical coupling between the engine and driven wheels with an additional electric motor/generator causes the complex problems, such as energy management, and drivability issues.

#### **Power-Split HEV**

A power-split hybrid-electric vehicle consists of one mechanical and one electrical power source propelling the vehicle through a planetary gear set. Simplified power-split HEV with the major powertrain components is sketched in Fig. 1.5. The unidirectional energy converter (internal combustion engine) is mechanically coupled to the driven wheels and an electric motor/generator (denoted by EMG1) through a planetary gear set; whereas the bidirectional energy source (battery pack) provides and stores electrical energy via both electric motor/generators depending on vehicle's operating modes. Power electronics manages all of the electrical power flows in the propulsion system.



Figure 1.5 – Propulsion system of a power-split hybrid-electric vehicle.

In a power-split HEV, the mechanical connection between internal combustion engine and driven wheels is realized through a planetary gear set. Thanks to an additional mechanical connection of the planetary gear set and another mechanical coupler, two electric motor/generators are implemented in this architecture. Both internal combustion engine and electric motor/generator EMG2 can propel the driven wheels; and both EMG1 and EMG2 are capable of recharging the battery.

Compared with parallel HEVs, power-split HEVs benefit better fuel economy, drivability, and electric drive efficiency; however, the maximum vehicle speed and grade capability are not as good as parallel HEVs [27].

#### 1.3.2 Powertrain Control

In an HEV, powertrain control manages power flows to meet the desired operation. Particularly, optimal control has been investigated for almost forty years to achieve the minimum fuel consumption since dynamic programming was firstly introduced in [28]. To homogeneously benchmark the optimal design of a hybrid-electric vehicle propulsion system, optimal control techniques are implemented that consist of Dynamic Programming (DP), Pontryagin's Minimum Principle (PMP), Convex Optimization (CVX), and their variants.

The optimal control problem of a hybrid-electric vehicle consists of finding the optimal signal of control variables, for instance battery power in a series HEV and motor power in a parallel HEV, such that the fuel consumption is minimized and the final state of charge of the battery meets the desired value, such as maintaining the same as its initial one. Assuming that battery electrochemical power is independent from the state of charge, the optimal control problem is summarized as

$$\min_{u \in U} \int_{t_0}^{t_f} P_{ef}(u(t), t) dt,$$
(1.1)

s.t. 
$$\dot{x}(t) = P_{be}(u(t), t),$$
 (1.2)

$$x(t_0) = E_{be0}, x(t_f) = E_{be0},$$
(1.3)

$$h(u(t), t) = 0,$$
 (1.4)

$$g_i(u(t), t) \le 0$$
, for each  $i \in \{1, \dots, m\}$ , (1.5)

where the optimal control problem is over time horizon  $[t_0, t_f]$ ; the control variable uin its admissible set U is defined depending on the hybrid powertrain architecture; the burned fuel power  $P_{ef}$  estimates the fuel consumption over an investigated mission; the system dynamics  $\dot{x}$  is defined by the electrochemical power of battery; the initial and final state x are equal to the battery energy  $E_{be0}$ ; the equality constraint h(u(t), t) refers to the power balance; and the in-equality constraints  $g_i(u(t), t)$  represent the operating constraints due to physical limits of powertrain components. For example, the operating power of an electric motor must be always constrained within its limits ( $P_m \in [\underline{P}_m, \overline{P}_m]$ ). The constraint of system dynamics is not considered throughout this thesis.

#### Pontryagin's Minimum Principle

Embodied by variational methods, Pontryagin's Minimum Principle (PMP) states a necessary condition that must hold on an optimal trajectory. For the optimal control problem in Eq. 1.1, the Hamiltonian function is defined as

$$H(u(t), s(t), t) = P_{ef}(u(t), t) + s(t)P_{be}(u(t), t),$$
(1.6)

where *s* is a scalar adjoint variable.

PMP states that if  $u^*(t)$  is the optimal control law for problem in Eq. 1.1, the following conditions are satisfied: (1) the state and adjoint state must satisfy the following conditions:

$$\dot{x}^*(t) = \left. \frac{\partial H}{\partial s} \right|_{u^*(t)} = P_{be}(u^*(t), t), \tag{1.7}$$

$$\dot{s}^{*}(t) = \frac{\partial H}{\partial x}\Big|_{u^{*}(t)} = 0, \qquad (1.8)$$

$$x^*(t_0) = E_{be0}, (1.9)$$

$$x^*(t_f) = E_{be0}, (1.10)$$

(2) for all  $t \in [t_0, t_f]$ ,  $u^*(t)$  globally minimizes the Hamiltonian:

$$H(u(t), s^{*}(t), t) \geq H(u^{*}(t), s^{*}(t), t), \ \forall u \in U, \forall t \in [t_{0}, t_{f}],$$

i.e., the optimal solution  $u^*(t)$  is such that

$$u^{*}(t) = \underset{\substack{h(u,t)=0\\g(u,t)\leq 0}}{\arg\min} H(u, s^{*}, t),$$
(1.11)

where *s* is a constant adjoint state, the minimization of Hamiltonian function can be solved either through numeric computation or by analytic solution.

#### **Dynamic Programming**

Dynamic programming, as an alternative optimal control technique to to solve the optimal control problem of HEVs, is based on the Bellman's Principle of Optimality [29]:

An optimal policy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions. Alternatively, from any point on an optimal state space trajectory, the remaining trajectory is optimal for the corresponding problem initiated at that point.

As the Principle of Optimality implies, a complex optimal control problem is solved by breaking the problem down into a collection of simpler subproblems, and then computed "backwards". Accordingly, the formulation is discretized by sampling period  $\Delta t$  to x(k) and u(k),  $k = 0, \dots, N-1$ . The system dynamics is expressed by a difference equation,

$$x(k+1) = f(x(k), u(k), k).$$
(1.12)

The objective function in Eq. 1.1 is replaced by

$$J(u(k)) = \sum_{0}^{N-1} P_{ef}(x(k), u(k)).$$
(1.13)

In practice, implementation of DP requires the state variable to be quantified. Hence, the curse of dimensionality is induced. As a result, computational load of DP is not negligible, especially in the massive evaluations.

#### **Convex Optimization**

Convex Optimization (CVX) is also implemented to solve optimal control problems of HEVs. As indicated, CVX minimizes objective function which is convex over convex sets. Detailed theory of CVX is introduced in [30].

To implement CVX in the energy management problem of an HEV, the core is the convexification of the optimal control problem. The objective functional and constraints must be adapted to be convex. Therefore, the objective function in Eq. 1.1 and inequality constraints in Eq. 1.5 must be convex functions, and the equality constraints Eq. 1.4 are affine. With the convex formulation, the optimal control problem can be solved through convex optimization. However, convexification is always challenging due to inevitable non-convex models and signals during the formulation of the optimal control problem.

#### Nested Optimal Control Techniques

Recent studies present a novel method to solve the optimal control problem, which is the nested optimal control technique. The nested technique targets to solve the drawbacks of single optimal control techniques, such as restrictions of system dimensions in DP, non-convex models and signals in CVX. Two representatives are summarized as follows, which are dynamic-programming-based and convex-optimization-based nested optimal control technique.

DP-based nested optimal control technique, as the name implies, solves the optimal control problem directly by dynamic programming that determines the whole set of control variables. A second optimal control technique is implemented to find the optimal value of a few limited number of control variables. DP-based nested optimal control technique helps to cope with the dimensionality curse of DP, which is the exponential increment of computation time as state variables augment.

DP-based nested optimal control technique solves the optimal control of a parallel HEV by taking three control variables and three state variables into account [6, 31]. Through the nested optimal control technique of DP-PMP, the multi-variable mixed-integer non-linear problem is solved with significantly reduced computation time.

A scheme of this complex optimal control technique is illustrated in Fig. 1.6a. The battery power  $u_3^*$  optimized by PMP is transferred to DP for further optimization. When determining control variable  $u_3^*$ , the adjoint state variable in PMP is mildly tuned by a proportional controller. The whole control variables, including gear shift command  $u_1$  and engine on/off command  $u_2$ , are concurrently optimized by DP. Compared with single DP approach, DP–PMP nested optimal control technique presents 0.4% difference of the fuel consumption but an average 420 times reduction of the computation time [6].

To solve a similar problem, another nested methodology based on DP is proposed to optimally determine the adjoint state variable in PMP. In [32], the objective function is rewritten as a function of Hamiltonian function for easy implementation of DP. The scheme of this approach is depicted in Fig. 1.6b. The objective function is minimized by DP, in which the optimal adjoint state  $u_3^*$  is solved by convex optimization. Results of fuel consumption obtained by DP(PMP)-CVX nested optimal control approach are almost the same as that of DP. In addition, the computation time is reduced significantly.



Figure 1.6 – DP-based nested optimal control techniques.

CVX-based nested optimal control technique, solves the optimal control problem of HEVs directly through convex optimization. To cope with the non-convex signals and models in the optimal control problem, extra optimal control techniques can be of great help to solve the mixed-integer problem first (such as engine on/off decision or gear selection). Albeit CVX-DP nested optimal control technique for series HEV application is proposed in [33], in fact, it is a DP-based nested optimal control technique because the philosophy is the same as DP(PMP)-CVX one. The CVX-based nested optimal control technique is implemented as reported in [34, 35].

As illustrated in Fig. 1.7, the mechanism of CVX-PMP nested optimal control technique solves the optimal engine on/off strategy  $B_e^*$  by PMP, and the optimal adjoint state  $s^*$  is numerically determined.



Figure 1.7 – CVX-based nested optimal control technique.

# 1.4 Hybrid-Electric Vehicle: Powertrain Design

The optimization of a vehicle propulsion system consists of the design and control optimizations. Depending on different optimization methods, optimal control techniques are applied in terms of various combinations, or only one optimization method. Commonly applied powertrain design methods are summarized as heuristic and optimal design approach.

## 1.4.1 Heuristic Design Approach

Fundamentals of vehicle design are embedded in the basic mechanics, particularly in Newton's second law of motion relating force and acceleration [36]. The power and energy requirements to internal combustion engine and electric motor/generator are estimated by analyzing the vehicle longitudinal dynamics [37]. The power and energy characteristics of powertrain components strongly depend on the experience of design engineers due to the development of energy management strategy.

Heuristic design approach determines dimensioning parameters of powertrain components to meet the technical targets. Iterative simulation is a often used in the heuristic design approach [38, 39]. The dimensions of main powertrain components are firstly estimated according to the technical targets. If the first estimation fails, a second one is performed in the next iteration. The dimensions of mechanical and electrical powertrain components are required to account for powertrain architectures [40, 41].

Evidently, the heuristic design approach is only a primary solution that needs to optimize. Further improvement of the hybrid powertrain design can be achieved by

considering more factors, such as fuel consumption.

# 1.4.2 Optimal Design Approach

Three-layer optimization problems exist in the design problem of vehicle propulsion systems, which consists of the structural optimization, parametric optimization, and control system optimization [42]. Moreover, the structural optimization can be aggregated into the parametric optimization when the structure is parameterized in the design problem.

Optimal design of hybrid propulsion systems faces grave inherent complexity because of the necessity of control optimization to benchmark the minimal energy consumption. Basically, two types of optimizations reside in the optimal design of vehicle propulsion systems, which are design and control optimization. The design optimization finds the best dimensioning parameters of powertrain components such that the energy consumption is minimized, whereas the control optimization minimizes the energy consumption of an investigated vehicle propulsion system by identifying optimal control laws. However, optimal control laws are developed based on optimal control techniques and affected by dimensioning parameters of powertrain components.

Recent investigations on the optimal design of vehicle propulsion systems are classified into three categories as shown in Fig. 1.8, where  $\mathcal{D}$  indicates the dimension-related parameters. Three categories of optimization methods are composed of bi-level design optimization (see Fig.1.8a), bi-level co-optimization (see Fig.1.8b), and simultaneous co-optimization (see Fig.1.8c). Bi-level indicates that powertrain design and powertrain control are performed at separate levels with different optimization techniques; whereas co-optimization means that both powertrain design and powertrain control are optimized to achieve the minimum energy consumption. For example, the powertrain dimensioning parameters are optimized in the outer level through an optimization technique in the bi-level co-optimization method. Meanwhile, the powertrain control is optimized with another technique in the inner level so that the minimum fuel consumption is achieved. Both optimizations find the optimal dimensioning parameters such that the fuel consumption is minimized over an investigated mission.

Furthermore, details of recent investigations are summarized and listed in Table 1.2, including reference paper, published year, design optimizer and parameter, control optimizer, and powertrain architecture. Design parameters are summarized into the overall set of design parameters S, which consists of internal combustion engine  $S_e$ , drivetrain (including transmission and differential)  $S_d$ , battery  $S_b$ , electric motor  $S_m$ , and



Figure 1.8 – Optimal design methods for vehicle propulsion systems.

electric generator  $S_g$ . In addition, design parameters  $S_h$  and  $S_u$  refer to hybridization ratio and control variables, respectively.

Various design optimizers are applied to optimize the dimensioning parameters. The design optimizer consists of sequential quadratic programming (SQP), bundle method, Dividing Rectangles Optimization (DIRECT), Simulated Annealing (SA), Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Non-dominated Sorting Genetic Algorithm (NSGA-II), Feature-Based Generic Algorithm (FBGA), Non-Linear Programming by Quadratic Lagrangian (NLPQL), General Purpose Solver (GPS), Constraint Programming (CP), Spearman Rank Correlation Coefficient Method (SRCCM), and Requirements engineering, Functional analysis, Logical design and Physical design (RFLP). Particularly, the short line indicates that no specific nonlinear solver is applied.

Considering the control optimizer, it includes heuristic one, Hamilton–Jacobi–Bellman method (HJB), dynamic programming (DP), combined convex optimization and Pontryagin's Minimum Principle (CVX-PMP), rule-based one (RB), combine convex optimization and dynamic programming (CVX-DP), equivalent fuel consumption minimization strategy (ECMS), vectorized hybrid optimization tool (VHOT), selective Hamiltonian minimization (SHM), graphical-analysis-based energy consumption optimization (GRAB-ECO), and fully-analytic energy consumption estimation (FACE).

#### **Bi-Level Design Optimization**

Compared with bi-level co-optimization, the bi-level design optimization refers to only one optimization technique that is implemented to solve the optimal design problem. However, only one problem of powertrain design and control is optimized. Therefore,

Ref.	Year	Design Optimizer	Parameter	Control Optimizer	Architecture
[43]	1999	SQP	$S_i(i = e, b, m)$	Heuristic	parallel
[44]	2004	Bundle Method	$\mathcal{S}_d$	HJB	parallel
[4]	2005	DIRECT/SA/GA	$S_i(i = e, b, m, d)$	Heuristic	power-split
[45]	2007	-	$S_i(i = b, m, g)$	DP	power-split
[46]	2015	-	$S_t$	DP	power-split
[47]	2009	-	$S_h$	DP	parallel
[48]	2012	PSO	$S_i(i = e, b, m)$	DP	parallel
[34]	2014	CVX-PMP	$S_b$	CVX-PMP	series
[49]	2010	-	$S_i(i = e, b, m)$	PMP	parallel
[50]	2011	-	$S_i(i=b,m)$	DP	parallel
[51]	2011	NSGA-II/DIRECT	$S_i(i = e, b, m, u)$	Heuristic	series/
		/SA/GA/PSO	$S_i(i = e, b, m, d)$		parallel
[52]	2011	GA/FBGA	$S_i(i = e, b, m, u)$	GA/FBGA	series
[53]	2012	NLPQL	$S_i(i = e, b, m, d)$	DP	parallel
[54]	2011	CVX	$S_b$	CVX+RB	series/parallel
[55]	2013	CVX	$S_b$	CVX+RB	series
[56]	2013	CVX	$S_i(i = e, b, m)$	CVX+RB	parallel
[33]	2015	CVX-DP	$S_i(i = b, e, g)$	CVX+DP	series
[5]	2015	GA	$S_i(i = a, e, b, m, u)$	ECMS	series/parallel
[6]	2014	GPS (SQP)	$S_i(i = a, m)$	DP	parallel
[57]	2015	CP/SQP/PSO/	$S_i(i = e, b, m)$	DP	parallel
		GA/DIRECT			
[25]	2015	SRCCM	$S_i(i=b,m,d)$	-	electric vehicle
[58]	2015	RFLP (NSGA-II)	$S_i(i=b)$	-	electric vehicle
[59]	2016	DIRECT	$S_i(i=e)$	SHM	parallel
[60]	2017	-	$S_i(i = e, b, m, g, d)$	VHOT, FACE	series
				SHM, GRAB-ECO	

Table 1.2 – Summary of powertrain design optimization for hybrid- and battery-electric vehicles.

the bi-level design optimization is regarded as a partial optimization method. The partial optimality could be achieved either at the outer loop that determines the optimal dimensioning parameters or at the inner loop that minimizes the fuel consumption. In [4, 43, 51], the design parameter set of hybrid-electric vehicles are optimized only at the outer loop, yet the control laws are heuristic.

On the contrary, the bi-level design optimization solely occurs at the inner loop in [47, 49, 50], where optimal control laws are realized by DP mainly due to the global optimality without considering the heavy computational load. The outer loops are performed iteratively or manually. As for power-split HEVs [45, 46], only the possible topologies of the planetary gear sets are screened in the exhaustive research method since the presence and absence of clutches significantly impact the operating modes of

power-split HEV.

#### **Bi-Level Co-Optimization**

As listed in Table 1.2, the bi-level co-optimization method for the optimal design of vehicle propulsion systems has been widely used by optimizing various objectives, for example, the cost of hybridization and operation, the fuel economy, and the pollutant emissions.

The bi-level co-optimization refers to the design optimization in the outer loop and the control optimization in the inner loop. Design optimization selects the best dimensioning parameters, whereas control optimization derives the optimal control laws discussed in previous section. The optimal design problem that globally optimizes the dimensioning parameters are solved by various optimization techniques, such as DIRECT, GA, and PSO.

#### DIRECT

Dividing RECTangles (DIRECT) optimization algorithm is motivated by a modification to Lipschitzian optimization that eliminates the need to specify the Lipschitz constant [61]. It is created in order to solve difficult global optimization problems with bound constraints and a real-valued objective function. Unfortunately, this global optimal convergence may come at the expense of a large and exhaustive search over the domain. In [4], a parallel hybrid-electric vehicle is optimally designed with the implementation of heuristic control laws. Heavy computational load eventually leads to hundred hours for the complete optimization process. Possible improvements is proposed as well in order to overcome the slow convergence.

## **Genetic Algorithms**

Genetic Algorithms (GA) are adaptive heuristic search methods that mimic the natural biological evolutionary idea of natural selection and genetics. They present an intelligent exploitation of a random search to solve optimization problems. Despite randomized, GA use historical knowledge to direct the search into the region of better performance within the search space. Being a global search method, GA are capable to optimize the hybrid powertrain design once the control system optimization is achieved. In [5], the hybrid powertrain design is optimally designed through the combination of GA and equivalent fuel consumption minimization strategy. The fuel consumption is minimized in the condition that the final state of charge of battery is maintained the same as the initial one. The investigated dimensioning parameters associate with powertrain architecture, internal combustion engine, electric motor/generator, battery, and control

variable. Results proved the effectiveness of bi-level co-optimization approach in the optimal design of a hybrid powertrain.

## Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a stochastic search method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. The change in direction and velocity of each individual particle is the effect of cognitive, social and stochastic influences. The common goal of all group members is to find the favourable location within a specified search space. In [48], the primary optimization problem is to find the design parameter set  $S = \{V_e, P_m, Q_b\}$  that minimizes the objective function subject to inequality constraints. Both objective function and constraints are non-convex functions with respect to S. An efficient tuning methodology of the intrinsic parameters are established by exploiting the results of exhaustive search as a look-up table for PSO algorithm.

## Simultaneous Co-Optimization

Simultaneous co-optimization means that both powertrain design and control are optimized through the only one optimization technique. Due to the application of one optimization technique, powertrain design and optimal control are merged into the same level. Thus, powertrain design and control are simultaneously optimized. The simultaneous co-optimization is currently realized by convex optimization (CVX), which is elaborated in [33, 55, 62]. The essence of convex optimization is to construct convex objective function and constraints.