Opportunistic Spectrum Access for Cognitive Radio Networks: A Queueing Analysis

Contents

3.1 Introduction

Since the FCC has proposed, in November 2002, to open the use of many bands that has already been assigned but not sufficiently utilized, CR-based wireless network architectures have been proposed in order to allow SUs to access licensed channels. Indeed, the FCC report reveals that the electromagnetic spectrum has gaps, i.e. frequency bands that are assigned to licensed users, at a particular time and specific geographic location, are not being utilized. Note that locating unused frequencies, accounting for the energy spent in sensing, represents a big challenge for SUs. Moreover, the proposed CR architectures do not guarantee some QoS levels for SUs, which are mainly impacted by the PUs' activity and the interaction between SUs.

The operation model, described in [20], introduces a new set of theoretical problems involving game theory, queueing theory, and decision theory. Specifically, we focus, in this chapter, on SUs having the faculty to sense licensed bands and access them if idle, or to access a dedicated channel. We are interested in designing an optimal OSA policy for unlicensed users. In the first part of this chapter, we consider slotted communications for PUs and SUs. Indeed, we consider that the system is perfectly synchronized, and we assume that PUs and SUs have the same slot duration. Moreover, we ignore the sensing errors, i.e. the false alarm and missing probability of sensing are zero. Thus, if the SU senses a licensed channel as idle, it is still idle during the whole time slot. Most of previous works in the OSA area for CR networks have already taken these assumptions (see [50], [51], [52], and [53]). In the second part of this chapter, we consider a more realistic scenario, where PUs operate in a non-slotted mode. Due to the agreement between the service provider and PUs, the number of licensed channels should be higher than the number of PUs transmitting simultaneously. We further assume that PUs are able to determine whether there is a free licensed channel or not. As PUs have the highest priority to access their own licensed channels, if all the licensed channels are occupied, a new PU preempts a SU that is using a licensed channel. The rejected SU aborts the transmission and tries to transmit the whole packet at the next time slot. As the access to licensed channels is opportunistic, successful SUs' transmissions are highly dependent of the presence of PUs. Note that the dedicated channel represents a guarantee of a QoS level for SUs.

Lots of recent works dealt with CR technologies and their performances. The survey paper [2] presented some interesting problems for evaluating the performance of CR systems. In [54], authors considered an energy efficient spectrum access policy. Each SU senses the spectrum and selects subcarriers taking into account data rate requirements and maximum power limit. This work is close to ours as authors studied the problem by considering a non-cooperative behavior of SUs. Moreover, they considered energy efficient allocation scheme. Note that authors considered that each SU that has traffic to transmit systematically senses the spectrum and locates the available subcarrier set. In fact, authors decoupled the sensing and the access decisions, and the OSA problem is resumed to a decision about which channel to access from the set of available subcarriers. However, in our model, we consider that SUs may decide to access the dedicated channel without sensing the licensed spectrum.

Authors of [55] proposed an OSA algorithm for SUs composed of two parts: first, a SU decides whether the licensed channel is idle or not. Second, it determines whether this channel is a good opportunity or not. However, authors did not consider the impact of multiple SUs. In fact, they have focused on the model of one SU accessing opportunistically a channel licensed for a PU. In [56], [57] and [58], authors considered the non-cooperative behavior of CR users accessing multiple licensed channels.

Unlike most of previous works in the DSA area, we study decision-making methods and the corresponding equilibrium analysis using the queueing theory. Jagannathan et al. considered in [59] a model similar to ours, where SUs choose either to acquire dedicated spectrum or to use spectrum holes. They considered a pricing model and studied the interaction between SUs as a non-cooperative game. There are several differences between their work and ours. Firstly, they considered that SUs sense systematically the licensed spectrum and make the decision about transmitting over licensed channels or through the dedicated spectrum after the sensing outcome. However we consider that SUs choose the transmission medium before sensing in order to economize the energy spent for sensing when accessing dedicated bands. Secondly, they considered that there is a centralized component that schedule SUs trying to access licensed channels, whereas we consider that SUs are in competition, and collisions occur when several SUs access the same licensed channels. Moreover, authors did not consider the energy spent for sensing licensed channels.

In [60], authors considered a model where there are several channels available to choose from. The transmitter has to probe the channels to learn their quality. Probing many channels may yield one with a good gain but reduces the effective time for transmission within the channel coherence period. The problem is to obtain optimal strategies to decide when to stop probing and start transmitting.

Author of [61] proposed a cross-layer queueing model that considers multiple CR users competing for spectrum opportunities. They considered an infrastructure-based CR system consisting a CR base station and multiple CR users. The base station controls transmissions to/from CR users. In this chapter, we consider an infrastructure-less CR network, where CR users access, solely, licensed channels.

In [62], authors considered a scheduling algorithm that estimates the number of packet which can be transmitted over a frame by each SUs in each licensed channel. In contrast to this work, where a central scheduler performs the spectrum scheduling, we consider that SUs contend to access licensed channels, without the need of a central controller.

Authors of [63] applied the queueing analysis to characterize the relationship between the arrival rate of the cognitive traffic and the queue distribution of CR user. The design of cooperative CR for SUs was depicted, using a queueing analysis, in [64] and [65].

The remainder of this chapter is as follows. In the next section, we present the system model. Section 3.3 focuses on the model where PUs' transmissions are slotted. In Section 3.4, we present the non-slotted model, and we conclude the chapter in Section 3.5.

3.2 The system model

In this chapter, we consider a system composed of $K + 1$ channels, where PUs are licensed to use K channels, and one dedicated channel is shared between all SUs. PUs (resp. SUs) arrive following a Poisson process with rate λ_p (resp. λ_s). Note that each SU decides whether to sense the licensed channels or not. If it senses the spectrum and finds one free channel, it transmits its packets. We denote by p the probability that a SU senses licensed channels. This probability may be considered as the proportion of SUs that chooses to sense the spectrum. This repartition of SUs can be set by a central controller, or determined individually by SUs in a decentralized manner. Moreover, we consider that SUs are operating via a limited battery, and have to be energy efficient. We assume that there is a cost α for sensing one licensed channel, and if a SU decides to sense, it senses all the K licensed channels. Note that SUs may sense licensed channel and stop sensing once they find a free channel. However, this strategy will increase the collision between SUs. Many works, such as [54], [59] and [66], considered that SUs sense all the licensed channels. Some other works considered periodic sensing (see [67] and [68]), whereas authors of [69] and [70] considered that the SU selects and senses randomly one licensed channel. None of these strategy was shown to be better than the others since it is highly dependent to the studied model. For example, if SU do not care about energy consumption, total sensing is the best strategy. However, if SUs do not care about the transmission delay, sensing one licensed channel (either random or periodic) may be the best strategy. The service rates are denoted by μ_p (resp. μ_s) for the licensed channels (resp. the dedicated channel), and are supposed to have an exponential distribution. The system model is depicted in Figure 3.1, and is composed of two sub-systems. The first one, namely subsystem S_1, represents the secondary subsystem, and the primary subsystem, denoted by S_2, is licensed for PUs and open for SUs' opportunistic access.

We give, in the following, some intuitions about the optimal OSA strategy for SUs in our model. Because of the cost of sensing, when the blocking probability in the primary subsystem S_2 increases, SU have less incentive to sense the spectrum. In fact, even if SUs do not find a free licensed channel, they also pay the sensing cost. However, if they decide to use the dedicated channel without sensing, they do not pay the sensing cost but transmit their packet with higher delay than using the licensed channels. Moreover, the more there are SUs in the subsystem S_1, the higher is the transmission delay for all the SUs using that subsystem. Thus, there is a tradeoff for SUs whether to sense or not licensed channels. Table 3.1 summarizes the parameters of the model.

Obviously, SUs have to deal with the two following performance metrics: the packet delay and the energy spent for transmission. In fact, if the SU senses licensed channels and finds one free channel, it transmits the held packet with a lower delay than transmitting

Table 3.1: Description of system parameters

Parameter	Description
λ_p	arrival rate of PUs
λ_s	arrival rate of SUs
μ_p	service rate in a licensed channel
μ_s	service rate for a SU in the dedicated channel
K	the number of channels allocated for PUs
p	probability of sensing licensed channels
α	the cost of sensing one channel for a SU
$\rho(p)$	$\frac{(\lambda_p+p\lambda_s)}{\mu_p}$

over the dedicated channel. However, it spends energy for sensing licensed channels. We define the main global metric of the system, which is the average total cost U_S, as a composition of the two following parts: the average sojourn time of a SU inside the system and the cost of sensing:

- The average sojourn time, denoted by T_S, depends on several parameters: arrival rates of PUs and SUs, service rates, the number of licensed channels and the sensing probability.
- The sensing cost c_s depends on the number of licensed channels, and on the probability of sensing. We assume that this cost is linear with the number of licensed channels, i.e. $c_s(p, K) = \alpha K p$. In fact, the cost of sensing represents the energy spent in sensing. Note that SUs are supposed to sense all the licensed channels.

The average total cost, for a SU that chooses to sense licensed channels with probability p, is given by:

$$U_S(p, K) \quad = \quad T_S(p, K) + c_s(p, K) = T_S(p, K) + \alpha p K. \tag{3.1}$$

The average sojourn time T_S of a SU inside the system depends on the decision taken by the SU: to use licensed channels or the dedicated one. We denote by T_{S_1} (resp. T_{S_2}) the sojourn time if the SU that decides to transmit over the dedicated channel (resp. licensed channels). We assume that the sensing period is negligible compared to the sojourn time in both subsystems. Thus, the average sojourn time T_S is expressed by:

$$T_S(p, K) = (1 - p)T_{S_1}(p, K) + pT_{S_2}(p, K). \tag{3.2}$$

3.3 The slotted model

In this section, we consider that SUs and PUs evolve in a slotted model, and that they have the same time slots' durations. Moreover, we consider a perfect sensing, i.e. the false alarm and the missing probability equal zero. The secondary subsystem S_1 is composed of SUs that have not sensed the licensed channels (see Figure 3.1). Note that SUs that sensed licensed channels and do not find a free one are rejected from the system. As the dedicated channel is equally shared between all SUs, the subsystem S_1 can be modeled using an M/M/1 queue. In fact, SUs are sharing one dedicated channel during the time slot.

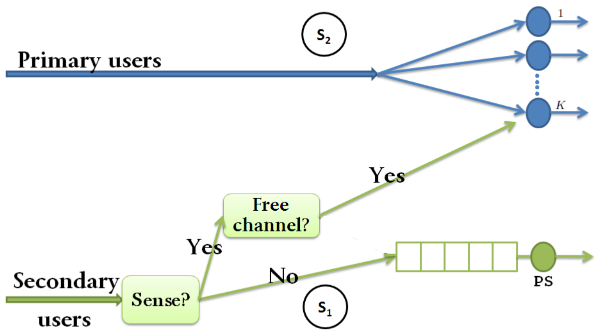

FIGURE 3.1: The OSA model for CR networks

The primary subsystem, namely S_2, is composed of the two following types of users:

- PUs,
- SUs that have sensed the licensed channels and have found, at least, one free channel.

The subsystem S_2 can be modeled using an M/M/K/K queue, known as the Erlang-B model, with arrival rate $\lambda_p + p\lambda_s$. Note that the Erlang-B model (M/M/K/K) was used in order to model CR networks in [71]. The blocking probability, which is the probability that any mobile finds all channels occupied, is given by the following Erlang-B formula:

$$\Pi(p, K) = \frac{\frac{\rho(p)^K}{K!}}{\sum_{n=0}^{K}(\frac{\rho(p)^n}{n!})}, \tag{3.3}$$

where $\rho(p) = \frac{(\lambda_p + p\lambda_s)}{\mu_p}$. This blocking probability depends not only on the number of licensed channels K, but also on the probability p of sensing. In fact, if the sensing probability increases, the input rate in the subsystem S_2 increases, and the blocking probability $\Pi(p, k)$ increases. Note that, for simplicity reasons, we have considered in this section that PUs and SUs have the same priority to access licensed channels. The paper [72] extended our model taking into account the priority of PUs in the expression of the blocking probability. However, They did not consider the possibility for a PU to reject a SU in service if it does not find a free channel. In the next section of this chapter, we consider a more general system taking into account the priority of PUs, where a PU that does not find a free channel may reject a SU in service.

In the next section, we focus on the optimal sensing probability or the optimal proportion of SUs that sense licensed channels, which minimizes two important metrics: the average sojourn time and the average total cost.

3.3.1 Optimization of global performances

The global analysis is well-suited for models where a CR base station transmits the traffic of SUs over multiple licensed channels in the wireless spectrum. We focus, in this section, on the average cost function of SUs. The arrival rate in the dedicated channel (subsystem S_1) is composed of SUs that have not sensed licensed channels. Then, the arrival rate of SUs for that dedicated channel is $\lambda_s(1-p)$. We assume that the maximum arrival rate, that is λ_s, which corresponds to the case where all SUs do not decide to sense, is lower than the service rate μ_s. Thus, we have a sufficient stability condition for the M/M/1 queue with a PS policy, which models the subsystem S_1. As the dedicated channel is shared between all SUs, the more there are SUs transmitting over the dedicated channel, the higher is the sojourn time in the system (higher is the transmission delay). Note that QoS requirements for SUs may be achieved by using an admission control mechanism by the Service Provider (SP).

The average sojourn time T_{S_1} for a SU, depending on the probability p that SUs sense the licensed channels and the number of licensed channels, is expressed as follows:

$$T_{S_1}(p,K) = \frac{1}{\mu_s - \lambda_s(1-p)}. \tag{3.4}$$

If a SU decides to sense licensed channels, its average sojourn time depends on the arrival rate of the PUs λ_p, and the proportion of SUs $p\lambda_s$ that have decided to sense licensed channels. Then, we determine explicitly, in the following, the average sojourn time T_{S_2} for a SU that decide to sense the licensed channels:

$$T_{S_2}(p,K) = \frac{1-\Pi(p,K)}{\mu_p}. \tag{3.5}$$

Note that a SU that decides to sense and does not find a free licensed channel is rejected from the system and try to retransmit at next time slot. Thus, the average sojourn time of a SU in the system is expressed as follows:

$$T_S(p, K) = \frac{1-p}{\mu_s - \lambda_s(1-p)} + \frac{p(1-\Pi(p,k))}{\mu_p}. \tag{3.6}$$

For notation convenience, let us consider the following function: $X(p, K) = p(1 - \Pi(p, K))$. By introducing the function $X(p, K)$ in the expression of the average sojourn time, we obtain the following simpler expression of the average sojourn time:

$$T_S(X(p, K)) = \frac{1-p}{\mu_s - \lambda_s + \lambda_s p} + \frac{X(p, K)}{\mu_p}. \tag{3.7}$$

In order to avoid the interference with PUs, SUs have to sense licensed channels before accessing them, and pay a cost for sensing. Note that SUs spend energy for sensing the spectrum. In fact, we model by the sensing cost, the energy spent for sensing licensed channels. The average cost function $U_S(p, K)$ for a SU that senses licensed channels with a probability p, is expressed as follows:

$$\begin{aligned} U_S(p, K) &= T_S(p, K) + \alpha p K. \\ &= \frac{1-p}{\mu_s - \lambda_s + \lambda_s p} + \frac{p(1-\Pi(p,K)}{\mu_p} + \alpha K p. \end{aligned} \tag{3.8}$$

We denote by $\Pi'(p, K)$ the derivative of the blocking probability with respect to the sensing probability p. The following proposition states the sensing probability that minimizes the average cost function.

Proposition 3.1. *For all values of α and K, the average cost function $U_S(p, K)$, defined in Equation (3.9), is minimized when the sensing probability is equal to:*

$$p = min(1, max(p_0, 0)) := p^*,$$

where p_0 is the solution of the following equation:

$$1 - \Pi(p, K) - p\Pi'(p, K) = -\alpha K \mu_p + \frac{\mu_p \mu_s}{(\mu_s - \lambda_s(1-p))^2}. \tag{3.9}$$

Proof. By replacing the function $X(p, K)$ in Equation (3.9), the average cost function can be rewritten as follows:

$$U_S(p, K) = \frac{1-p}{\mu_s - \lambda_s + \lambda_s p} + \frac{X(p, K)}{\mu_p} + \alpha p K.$$

After some algebra, the derivative of the average cost function, with respect to the sensing probability p, is expressed as follows:

$$\frac{\partial U_S}{\partial p}(p,K) = \frac{-\mu_s + \frac{\partial X}{\partial p}(p,K)(\mu_s - \lambda_s(1-p))^2 + \alpha K \mu_p (\mu_s - \lambda_s(1-p))^2}{\mu_p(\mu_s - \lambda_s(1-p))^2}.$$

Note that $\frac{\partial X(p,K)}{\partial p} = 1 - \Pi(p,K) - p\Pi'(p,K)$. Thus, the derivative of the cost function $U_S(p,K)$ equals 0 if and only if:

$$1 - \Pi(p,K) - p\Pi'(p,K) = -\alpha K \mu_p + \frac{\mu_p \mu_s}{(\mu_s - \lambda_s(1-p))^2}.$$

Therefore, the derivative of the average cost function with respect to the sensing probability equals 0 if and only if $p = min(1, max(p_0, 0)) := p^*$, where p_0 is the solution of Equation (3.9). □

The main drawback of the optimal sensing probability p^*, the solution of the global optimization, is that it needs a central controller, in order to develop an optimal OSA mechanism. Indeed, the SP has to design the network such that a proportion p^* of SUs senses the licensed channels. In practice, it would be difficult to control and to design such centralized control. To overcome this hurdle, we look in the next section for a distributed mechanism, based on individual decisions of SUs about the OSA.

3.3.2 Individual opportunistic sensing policy

The main characteristic of the next generation networks is the transition from well-structured networks to infrastructure-less networks, and from centralized to decentralized networks. Recently, several researches focused on self-adaptive networks and autonomous devices. In this section, we consider that SUs decide individually whether to sense or not licensed channels. In fact, SUs try to minimize, solely, their average cost functions. Specifically, we model this system using a non-cooperative game with an infinite number of players (as we do not restrict neither the time horizon of the system nor the number of SUs). Note that game theory principle may be applied for resource allocation problems in a decentralized manner for wireless communications (see the survey paper [73] and [74] for some examples). Thus, we consider a game theoretical approach in order to design a decentralized OSA mechanism.

We consider that each SU decides on its probability p to sense or not licensed channels. It looks for minimizing its average cost function $U(p, p', K)$, which depends on its probability p, and the probability p' of all other SUs. The individual average cost function

$U(p, p', K)$ is expressed as follows:

$$U(p, p', K) \quad = \quad (1-p)T_{S_1}(p', K) + pT_{S_2}(p', K) + \alpha pK. \tag{3.10}$$

Note that the contribution to the cost by any individual SU is zero as we are not limited to a fixed number of SUs. Then, the equilibrium of this game is a Wardrop equilibrium, which was first studied in the context of road traffic since the 1950s in [75]. For notation convenience, we denote by $U_S(p, K) = U(p, p, K)$. Let us define, in the following theorem, the equilibrium for our non-cooperative game as a strategy that minimizes the cost function U, against others using the NE strategy.

Theorem 3.2. *The sensing probability p^E is a NE policy for the OSA problem between SUs if and only if:*

$$p^E = \arg\min_p U(p, p^E, K), \ \forall p \in [0, 1].$$

The following proposition proves the existence of a NE strategy for our non-cooperative game between SUs.

Proposition 3.3. *For all values of α and K, the NE policy for the OSA problem between SUs exists. Moreover, the sensing probability at the NE is expressed as follows:*

- *if* $\frac{1}{\mu_s - \lambda_s} > \alpha K + \frac{1-\Pi(0,k)}{\mu_p}$;
  - *if* $\frac{1}{\mu_s} < \alpha K + \frac{1-\Pi(1,k)}{\mu_p}$ *then* $p^E = \{0, p', 1\}$.
  - *else* $p^E = 0$;
- *else*
  - *if* $\frac{1}{\mu_s} > \alpha K + \frac{1-\Pi(1,k)}{\mu_p}$ *then* $p^E = p'$;
  - *else* $p^E = 1$.

where p' is the solution of the following equation:

$$\frac{1}{\mu_s - \lambda_s(1-p)} = \alpha K + \frac{1 - \Pi(p, K)}{\mu_p}. \tag{3.11}$$

Proof. From Equation (3.10), the first argument derivative of the average cost function is expressed as follows:

$$\frac{\partial U}{\partial p}(p, p') = T_{S_2}(p', K) - T_{S_1}(p', K) + \alpha K.$$

The probability p' is a NE strategy for the OSA problem if and only if the first argument derivative of the average cost function equals 0:

$$\alpha K + T_{S_2}(p', K) = T_{S_1}(p', K).$$

This equation characterizes a NE strategy for SUs. After some algebra, this expression may be expressed as follows:

$$T_{S_1}(p', K) = \alpha K + \frac{1 - \Pi(p', K)}{\mu_p}.$$

Thus, the necessary and sufficient condition for the existence of a NE strategy for the OSA problem between SUs is:

$$\frac{1}{\mu_s - \lambda_s(1 - p^E)} = \alpha K + \frac{1 - \Pi(p^E, K)}{\mu_p}.$$

Let us prove that $\frac{1}{\mu_s - \lambda_s(1-p)}$ and $\alpha K + \frac{1-\Pi(p,K)}{\mu_p}$ intersect once in $[0, 1]$. Suppose that $\exists p_1 < p_2 \in [0, 1]$ such that $\frac{1}{\mu_s - \lambda_s(1-p_1)} = \alpha K + \frac{1-\Pi(p_1,K)}{\mu_p}$ and $\frac{1}{\mu_s-\lambda_s(1-p_2)} = \alpha K + \frac{1-\Pi(p_2,K)}{\mu_p}$. Therefore, we obtain:

$$\frac{\Pi(p_2, K) - \Pi(p_1, K)}{\mu_p} = \frac{\lambda_s(p_2 - p_1)}{(\mu_s - \lambda_s(1 - p_1))(\mu_s - \lambda_s(1 - p_2))}.$$

After some algebra, we obtain:

$$\frac{\mu_p \lambda_s(p_2 - p_1)}{\Pi(p_2, K) - \Pi(p_1, K)} \leq \mu_s^2 - 2\mu_s\lambda_s - \lambda_s^2 - \lambda_s p_1 p_2,$$

which leads to a contradiction as $\frac{\mu_p\lambda_s(p_2-p_1)}{\Pi(p_2,K)-\Pi(p_1,K)} > 0$ and $\mu_s^2 - 2\mu_s\lambda_s - \lambda_s^2 - \lambda_s p_1 p_2 < 0$. Note that we have assumed that $\mu_s \leq 2\lambda_s$ in order to give SUs incentive to sense and access licensed channels.

Consider that $\alpha K + \frac{1-\Pi(0,K)}{\mu_p} < \frac{1}{\mu_s-\lambda_s}$ and $\alpha K + \frac{1-\Pi(1,K)}{\mu_p} > \frac{1}{\mu_s}$. Thus, we have two equilibriums $p^E = 0$ and $p^E = 1$. These equilibriums represent a Follow The Crowd (FTC) phenomenon (see [74]). In fact, there is an FTC behavior when the individual's tendency to choose an action increases with the probability of choosing this action by other individuals. For instance, when all SUs choose to sense licensed channels ($p' = 1$) the best response of a SU is to sense licensed channels, and therefore the equilibrium $p^E = 1$ exhibits an FTC characteristic. Moreover, there exists a unique equilibrium $p' = p' \in]0, 1[$, where p' is the unique solution of Equation 3.11. Therefore, $p^E = \{0, p', 1\}$.

Consider that $\alpha K + \frac{1-\Pi(0,K)}{\mu_p} > \frac{1}{\mu_s-\lambda_s}$ and $\alpha K + \frac{1-\Pi(1,K)}{\mu_p} > \frac{1}{\mu_s}$. It follows that $p^E = 1$ is the unique Nash equilibrium for the OSA game between SUs.

Consider that $\alpha K + \frac{1-\Pi(0,K)}{\mu_p} > \frac{1}{\mu_s-\lambda_s}$ and $\alpha K + \frac{1-\Pi(1,K)}{\mu_p} < \frac{1}{\mu_s}$. Therefore, $p^E = p'$ is the equilibrium strategy for our OSA game, where p' is the solution of Equation 3.11.

Consider that $\alpha K + \frac{1-\Pi(0,K)}{\mu_p} < \frac{1}{\mu_s-\lambda_s}$ and $\alpha K + \frac{1-\Pi(1,K)}{\mu_p} < \frac{1}{\mu_s}$. It follows that $p^E = 0$ is the unique Nash equilibrium for the OSA game between SUs. □

Given the existence of a NE strategy for the OSA problem between SUs, the following proposition compares the sensing probability at the NE and the optimal sensing probability.

Proposition 3.4. *For all values of α and K, the optimal sensing probability is higher than the sensing probability at the NE, i.e. $p^E \leq p^*$.*

Proof. We prove this proposition by contradiction. Assume that there exists a sensing cost $\alpha_0 > 0$ and a number of licensed channels K_0 such that $p^E > p^*$. As p^* minimizes the average cost function, we have:

$$T_S(p^*, K_0) + \alpha_0 p^* K_0 \leq T_S(p^E, K_0) + \alpha_0 p^E K_0.$$

However, p^E is the sensing probability at the NE. Therefore, we have the following inequality:

$$T_S(p^*, p^E) + \alpha_0 p^* K_0 \geq T_S(p^E, K) + \alpha_0 p^E K_0.$$

After some algebra, combining the two previous inequalities, we obtain:

$$(1-p^*)T_{S_1}(p^*) + \frac{p^*(1-\Pi(p^*, K_0))}{\mu_p} \leq (1-p^*)T_{S_1}(p^E) + \frac{p^*(1-\Pi(p^E, K_0))}{\mu_p}.$$

It follows that:

$$(1-p^*)(T_{S_1}(p^*) - T_{S_1}(p^E)) \leq \frac{p^*}{\mu_p}(\Pi(p^*, K_0) - \Pi(p^E, K_0)).$$

Note that T_{S_1} is decreasing with p and Π is increasing with p, then for $p^E > p^*$, the left hand side is positive and right hand one is negative which leads to a contradiction.

Finally, for all α and all K, the optimal sensing probability is higher than the sensing probability at the NE, i.e. $p^E \leq p^*$. □

This result is somehow intuitive. In fact, there is a lack of performance due to the selfishness of SUs in the decentralized system. In fact, SUs have less incentive to sense licensed channels in a self-adaptive context than in a centralized network. Furthermore, the following proposition gives us a higher bound of the average cost function at the NE.

Proposition 3.5. *For all values of α and K, we have the following higher bound of the average cost function when using a NE policy:*

$$U_S(p^E, K) \leq \frac{1}{\mu_s - \lambda_s}.$$

Proof. Consider that $\frac{1}{\mu_s} < \alpha K + \frac{1-\Pi(1,k)}{\mu_p}$ and $\frac{1}{\mu_s-\lambda_s} < \alpha K + \frac{1-\Pi(0,k)}{\mu_p}$. Therefore, the average cost function is expressed as follows:

$$U_S(p^E, K) = \frac{1}{\mu_s - \lambda_s}.$$

Second, Consider that $\frac{1}{\mu_s} > \alpha K + \frac{1-\Pi(1,k)}{\mu_p}$ and $\frac{1}{\mu_s-\lambda_s} > \alpha K + \frac{1-\Pi(0,k)}{\mu_p}$. Thus, the average cost function verifies:

$$U_S(p^E, K) = \frac{1 - \Pi(1, K)}{\mu_p} + \alpha K \leq \frac{1}{\mu_s} \leq \frac{1}{\mu_s - \lambda_s}.$$

Otherwise, the average cost function can be bounded as follows:

$$U_S(p^E, K) = \alpha K + \frac{1 - \Pi(p^E, K)}{\mu_p} = \frac{1}{\mu_s - \lambda_s(1 - p^E)} \leq \frac{1}{\mu_s - \lambda_s}.$$

Finally, the higher bound of the average cost function is $U_S(p^E, K) \leq \frac{1}{\mu_s-\lambda_s}$. □

It is well known that the utility of the global optimization is higher than the utility when using NE strategies. Giving the existence of the NE strategy for SUs, we focus in the next section on the lack of performance (utility) induced by the competition between SUs. In order to measure this gap of performance, we introduce the metric of the PoA.

3.3.3 Price of anarchy

Koutsoupias and Papadimitriou [76] introduced the concept of *Price of Anarchy*, which captures the deterioration of the performance of a decentralized system, due to the selfishness of its agents. This metric is well studied in routing games [77], where the PoA describes the worst possible ratio between the total latency of a NE strategy and the latency of an optimal routing of the traffic. This metric describes the gap of performance in terms of individual utility between an optimal centralized system and a totally decentralized system.

The PoA is expressed as the ratio between the optimal utility (obtained with a centralized system) and the utility at the NE (obtained with a decentralized system when using a NE policy). In our context, we define the PoA as follows:

$$PoA = \frac{U_S(p^*, K)}{\max_{p \in p^E} U_S(p, K)} \leq 1. \tag{3.12}$$

Our aim is to determine an expression of the minimal value of the PoA or to bound it, in order to measure the worst performance of the decentralized system. The following proposition gives us the worst-case lack of performance when upgrading from centralized networks to self-adaptive networks

Proposition 3.6. *For all values of α and K, we have the following lower bound of the PoA:*

$$PoA(\alpha, K) \geq \frac{2(\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)})}{\lambda_s} := \underline{PoA}. \tag{3.13}$$

Proof. The price of anarchy is expressed by the following ratio:

$$PoA(\alpha, K) = \frac{U_S(p^*, K)}{\max_{p \in p^E} U_S(p, K)}.$$

Suppose, first, that $\frac{1}{\mu_s} > \alpha K + \frac{1-\Pi(1,k)}{\mu_p}$ and $\frac{1}{\mu_s - \lambda_s} > \alpha K + \frac{1-\Pi(0,k)}{\mu_p}$. Therefore, we have $p^E = 1$. As we have proved in Proposition 3.4, $p^* \geq p^E$, then $p^* = 1$. Thus, we have $PoA(\alpha, K) = 1$. Let us focus on the gap between the utility function at the equilibrium and the optimal utility function. We have for all p^*, α and K

$$\begin{aligned} U_S(p^E, K) - U_S(p^*, K) &= \frac{1}{\mu_s - \lambda_s(1-p^E)} - p^* \frac{1-\Pi(p^*, K)}{\mu_p} - \alpha K p^* - \frac{1-p^*}{\mu_s - \lambda_s(1-p^*)} \\ &= -p^* \frac{1-\Pi(p^*, K)}{\mu_p} - \alpha K p^* + \frac{p^* \mu_s - \lambda_s p^E (1-p^*)}{(\mu_s - \lambda_s(1-p^*))(\mu_s - \lambda_s(1-p^E))} \end{aligned}$$

It's clear that the difference between the utility function at the equilibrium and the optimal utility function is maximal when $p^E = 0$. Note that the price of anarchy is minimal when $U_S(p^E) - U_S(p^*)$ is maximized. Then, the PoA is minimized when $p^E = 0$. We focus on the analysis of the PoA in this particular case. Suppose that $\frac{1-\Pi(p^*,K)}{\mu_p} + \alpha K < \frac{1}{\mu_s - \lambda_s}$. Then, we have for all p^*, α and K

$$U(p^*, p^*) < \frac{p^*}{\mu_s - \lambda_s} + \frac{1-p^*}{\mu_s - \lambda_s(1-p^*)}.$$

Thus, we obtain

$$U(p^*,0) < \frac{p^*}{\mu_s-\lambda_s} + \frac{1-p^*}{\mu_s-\lambda_s} = \frac{1}{\mu_s-\lambda_s},$$

which leads to a contradiction. In fact $U(0,0) = \frac{1}{\mu_s-\lambda_s} > U(p^*,0)$, and if $p^E = 0$ is an equilibrium, then $U(0,0) < U(p',0)$ for all p'. Finally, we have when $p^E = 0$, $\frac{1-\Pi(p^*,K)}{\mu_p} + \alpha K \geq \frac{1}{\mu_s-\lambda_s}$.

Moreover, when $p^E = 0$, we have the following expression of the price of anarchy:

$$PoA(\alpha,K) = \frac{\frac{p^*(1-\Pi(p^*,K))}{\mu_p} + \alpha p^* K + \frac{1-p^*}{\mu_s-\lambda_s(1-p^*)}}{\frac{1}{\mu_s-\lambda_s}}.$$

Thus, combining previous results, the price of anarchy is bounded by:

$$PoA(\alpha,K) \geq p^* + \frac{\frac{1-p^*}{\mu_s-\lambda_s(1-p^*)}}{\frac{1}{\mu_s-\lambda_s}}.$$

After some algebra, we obtain the following lower bound of the PoA:

$$PoA(\alpha,K) \geq p^* + \frac{(\mu_s-\lambda_s)(1-p^*)}{\mu_s-\lambda_s(1-p^*)} = \frac{\mu_s-\lambda_s(1-(p^*)^2)}{\mu_s-\lambda_s(1-p^*)}.$$

We denote the following function $F(X) = \frac{\mu_s-\lambda_s(1-X^2)}{\mu_s-\lambda_s(1-X)}$. The derivative of $F(X)$ with respect to X is expressed as follows:

$$F'(X) = \frac{\lambda_s^2 X^2 + (2\mu_s\lambda_s - 2\lambda_s^2)X + \lambda_s^2 - \lambda_s\mu_s}{(\mu_s-\lambda_s(1-X))^2}.$$

Note that $F'(X) = 0$ for $X = \frac{\lambda_s-\mu_s\pm\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$. Moreover, we have $F(0) = 1$. Then, the function $F(X)$ is minimized when $X = \frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$, and its minimum is $F(X) = \frac{\mu_s-\lambda_s(1-(\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})^2)}{\mu_s-\lambda_s(1-\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})}$.

Finally, for all α and K, we obtain the lower bound of the price of anarchy:

$$PoA(\alpha,K) \geq 2(\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}).$$

□

This closed-form of the lower bound of the PoA is very interesting as it depends neither on the sensing cost α nor on the number of licensed channels K. Therefore, the SP may tune the service rate of the dedicated channel, μ_s, and the arrival rate of SUs, λ_s, by using some admission control for example, in order to minimize the gap between the NE and the global optimization's performance. In the following section, we present some numerical illustrations.

3.3.4 Numerical illustrations

This section presents the performance analysis of the proposed OSA mechanism. For this end, we have performed extensive numerical computations with different configurations of the system. Furthermore, two performance metrics are considered: the sensing cost and the capacity of the system (number of licensed channels). We fix the arrival rate for PUs (reps. SUs) at 0.6 (reps. 0.8), and we consider different service rates for the licensed channels ($\mu_p = 0.8$) and the dedicated channel ($\mu_s = 1.1$). Under these setting, the PoA is analytically evaluated to $PoA \geq 0.7524$ from Proposition 3.6.

We focus, first, on the case of one licensed channel, and we set the sensing cost to 0.1. Figure 3.2 illustrates the average total cost depending on the sensing probability of SUs. We observe that the average total cost is minimized when the SUs sense licensed channels with a probability $p = 1$, i.e. all SUs sense licensed channels. In fact, since the sensing cost is relatively low ($c_s = 0.1$), all SUs have incentive to sense licensed channels.

Secondly, we consider multiple licensed channels and we set K to 10. As we have already assumed that the sensing cost is linear with the number of licensed channels, choosing to sense licensed channel become costly for SUs with the increase of the number of licensed channels ($c_s = 1$). We plot, in Figure 3.2, the average total cost, with $K = 10$ licensed channels, and we observe that SUs have less incentive to sense the licensed channels compared to the first scenario ($K = 1$). In fact, the average cost is minimal when SUs sense licensed channels with a probability of 0.427.

3.3.4.1 Sensing cost

We evaluate, in the present section, the impact of the sensing cost parameter α on the performance of the proposed OSA mechanism, given a fixed number of licensed channels ($K = 10$). Mobile devices equipped with a CR have usually a limited battery, and have to be energy efficient. The main challenge of designing an energy-aware CR is to determine the appropriate OSA strategy, as SUs spend energy for sensing licensed channels. We plot, in Figure 3.4, the optimal probability of sensing for SUs p^* and the sensing probability of SUs at the NE p^E. We remark that both probabilities are decreasing with the sensing cost α. This result is intuitive, as increasing the sensing cost decreases the incentive of SUs to sense licensed channels. Furthermore, this observation validates the analytical result obtained in Proposition 3.4. In fact, the optimal sensing probability p^* (obtained from the global optimization of the centralized system) is always higher than p^E (the sensing probability obtained at the NE).

It is straightforward that the non-cooperative behavior of SUs induces a worse performance compared to the centralized system. We focus on the gap of performance induced when migrating from centralized to decentralized networks. We illustrate the PoA, defined by Equation (3.12), in Figure 3.5. We observe that the minimum of the PoA equals 0.7559. Note that theoretically, the PoA is higher than 0.7524. Thus, the performances obtained by simulations are slightly better than the lower bound obtained analytically from Proposition 3.6. Given this result, we are able to design a decentralized OSA mechanism for energy-efficient SUs in self-adaptive CR networks, which is at worst 75% far from the optimal.

Note that the energy spent for sensing licensed channels depends not only on the cost of sensing α, but also on the number of licensed channels K, as the sensing cost c_s is assumed to be linear with K. We evaluate, in the following section, the impact of the capacity on the performance of the proposed OSA mechanism.

3.3.4.2 Capacity

In this section, we are interested in the impact of the number of licensed channels on the proposed OSA mechanism. We fix the sensing cost α at 0.3, and we vary the number of licensed channel from 1 to 20. An interesting analysis of [78] shows that the average number of available licensed channels in TV white-bands is about 15. Note that under these settings, the blocking probability decreases with the number of licensed channels whereas the sensing cost increases.

Figure 3.3 depicts the impact of the number of licensed channels on both the optimal sensing probability and the sensing probability at the NE for SUs. We observe that both p^* and p^E are decreasing, and that p^* is always higher than p^E. This result has already been proved analytically in Proposition 3.4. We plot, in Figure 3.6, the average total cost with the number of licensed channels. We remark that the average cost is minimal for $K = 2$. Note that increasing the capacity of the system increases the opportunities in the primary subsystem S_1, but also increases the sensing cost c_s.

Similarly to the sensing cost analysis, we measure the gap of performance between the global system and the decentralized system through the PoA. Figure 3.7 illustrates the PoA depending on the number of licensed channels K. The worst-case performance gap is 0.7619 obtained with 4 licensed channels. This result is slightly higher than the

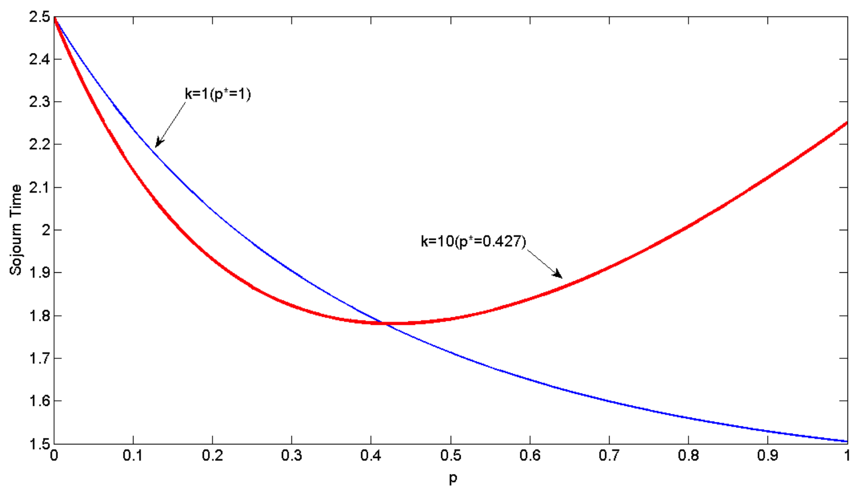

FIGURE 3.2: The average total cost function $U_S(p)$ for $\alpha = 0.1$, with one licensed channel, $K = 1$, and ten licensed channels, $K = 10$.

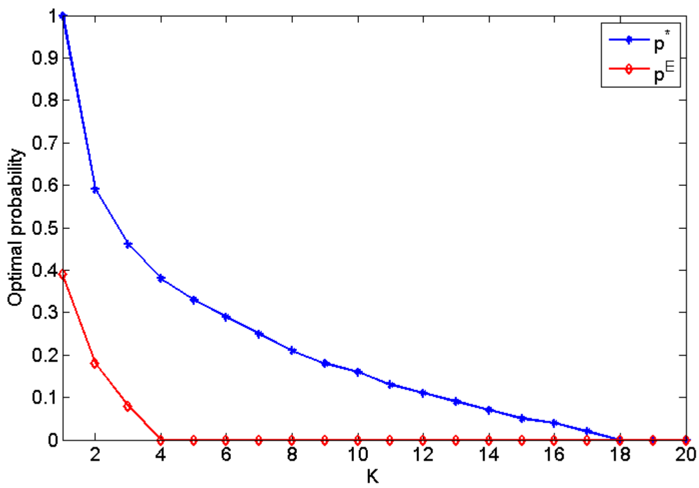

FIGURE 3.3: The probability of sensing depending on the number of licensed channels in both the centralized and the decentralized systems.

analytical result of the Proposition 3.6, which says that the lower bound of the PoA equals 0.7524.

3.3.5 Summary

In this section, we have defined an optimal OSA policy for SUs. Moreover, we have proposed a decentralized policy for self-interested SUs and we have evaluated the gap of performance between both approaches (global optimization and decentralized optimization) through the PoA metric. Nonetheless, we have taken the assumptions that PUs operate in a slotted model, and that they are perfectly synchronized with SUs. We

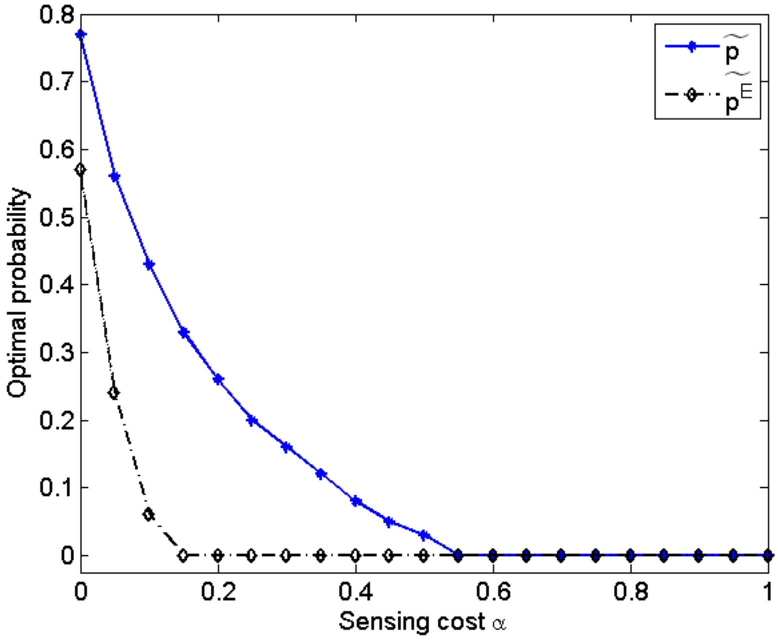

FIGURE 3.4: The optimal probability of sensing depending on the sensing cost α.

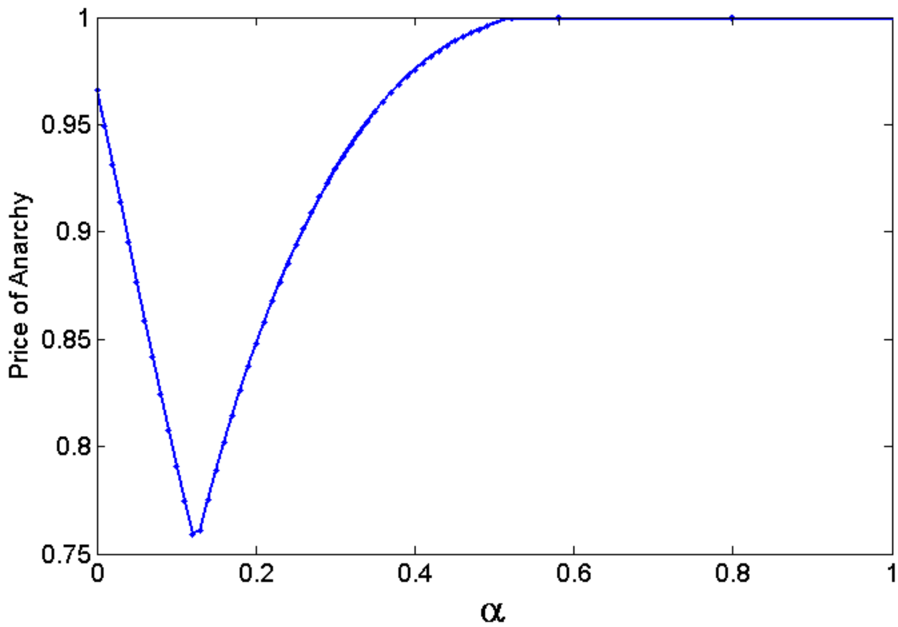

FIGURE 3.5: The price of anarchy depending on the sensing cost α.

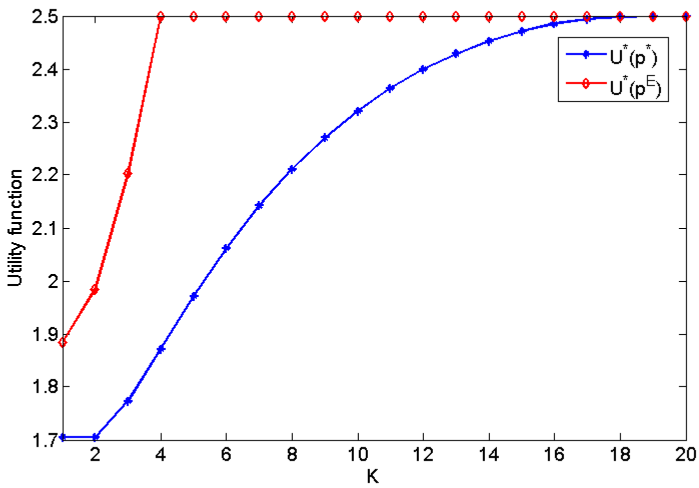

FIGURE 3.6: The average total cost depending on the number of licensed channels in both the centralized and the decentralized system for the slotted model.

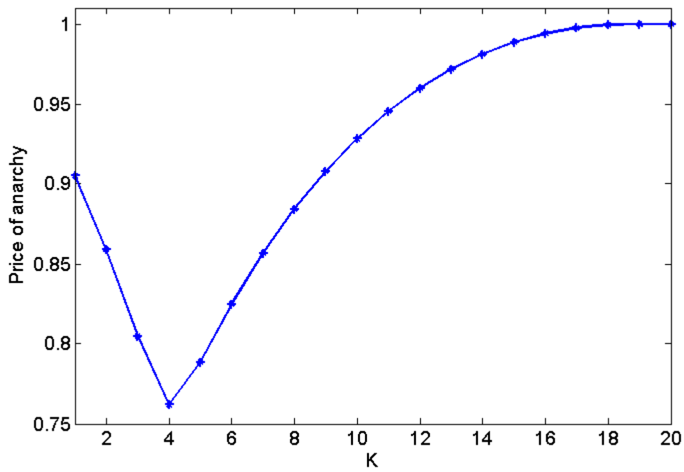

FIGURE 3.7: The price of anarchy depending on the number of licensed channels in the slotted model.

release theses assumptions in the next section by considering that the PUs evolve in a non-slotted regime, and that they may preempt a SU using licensed channels at their arrival. Releasing these assumptions significantly complicates the problem, as SUs have to face the reject form licensed channels by PUs, as well as the competition with each other.

3.4 The non-slotted model

In the present section, we relax some assumptions that were taken in order to simplify the study of the system. Indeed, we consider a more realistic model in which PUs evolve in a non-slotted mode, and have the highest priority to access licensed channels. Thus, if a PU does not find a free licensed channel, it rejects one SU (if there is one SU using licensed channels) and start transmission. We consider that SUs can detect that a PU is present and free immediately the channel. We further assume that if the SU is rejected, it gets no reward and is rejected from the system. When there are several SUs using licensed channels, a PU chooses randomly one SU to reject. Note that interruption from PUs is a key factor impacting the performance of SUs in CR networks. This assumption was also considered in [79]. We model, in the following section, the reject probability of SU in the primary subsystem.

3.4.1 Reject probability

We denote by $W_p(t)$ (resp. $W_s(t)$) the number of PUs (resp. SUs) using the licensed channels at the time slot t, where $W_p(t)+W_s(t) \leq K$. Specifically, the primary subsystem can be modeled using a bi-dimensional Markov process, $Z(t) = \{W_p(t), W_s(t)\}$. The probability that a SU will be rejected, when using a licensed channel, is denoted by $P_r(p, K)$. This probability depends on the proportion p of SUs that senses licensed channels, and the number of licensed channels. Note that each SU that joins the system with a Poisson process observes the system in its stationary regime, according to the PASTA property (see Definition 2.6).

We denote by $P_0(n, m)$ the probability that a SU will be rejected, when it joins a licensed channel and the primary subsystem has already n PUs and m SUs. Note that we have necessary $n + m < K$, and the reject probability is expressed as follows:

$$P_r(p, K) = \sum_{n,m/n+m=0}^{n+m=K-1} P_0(n, m)\pi(n, m), \tag{3.14}$$

where $\pi(n, m)$ is the stationary probability of the Markov process $Z(t)$, described in Figure 3.8. The stationary probabilities $\pi(n, m)$ can be computed using standard tools of Markov theory. Let us focus on the reject probabilities $P_0(n, m)$, it is possible to express the relation between probabilities $P_0(n, m)$ as a linear system. Note that for all states $(W_p(t), W_s(t)) = (n, m)$, such that $n + m = K - 1$, $P_0(n, m)$ is expressed as

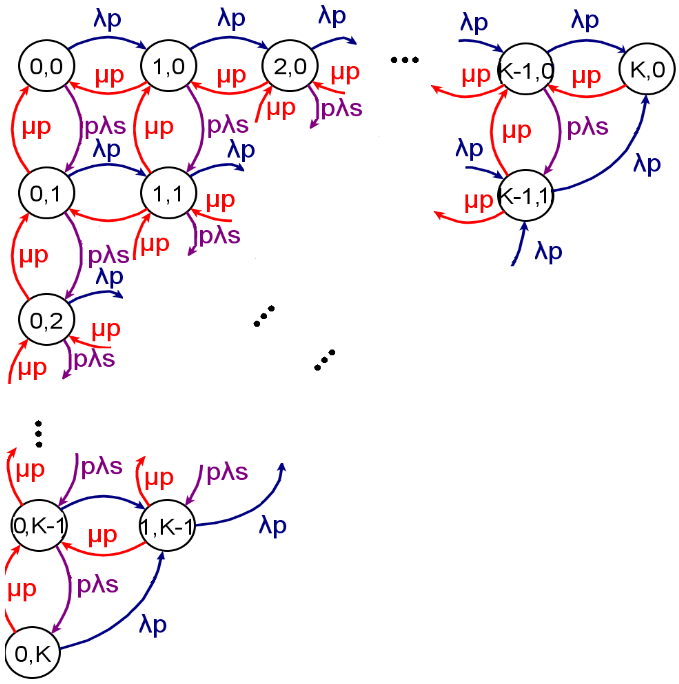

FIGURE 3.8: The bi-dimensional Markov chain of $Z(t)$.

follows:

$$P_0(n,m) = \begin{cases} \frac{1}{K}\frac{\lambda_p}{\lambda_p+\mu_p} + \frac{K-1}{K}\frac{\lambda_p}{\lambda_p+\mu_p}P_0(1,K-2) + \frac{\mu_p}{\lambda_p+\mu_p}P_0(0,K-2) & \text{if } \quad n=0, \\ \frac{\lambda_p}{\lambda_p+2\mu_p} + \frac{\mu_p}{\lambda_p+2\mu_p}P_0(K-2,0) & \text{if } \quad m=0, \\ \frac{1}{m+1}\frac{\lambda_p}{\lambda_p+2\mu_p} + \frac{m}{m+1}\frac{\lambda_p}{\lambda_p+2\mu_p}P_0(n+1,m-1) & \\ +\frac{\mu_p}{\lambda_p+2\mu_p}(P_0(n-1,m)+P_0(n,m-1)) & \text{otherwise.} \end{cases}$$

Otherwise, for $n+m < K-1$, the probability $P_0(n,m)$ is expressed as follows:

$$P_0(n,m) = \begin{cases} \frac{p\lambda_s}{p\lambda_s+\lambda_p+\mu_p}P_0(n,m+1) + \frac{\lambda_p}{p\lambda_s+\lambda_p+\mu_p}P_0(n+1,m) & \\ +\frac{\mu_p}{p\lambda_s+\lambda_p+\mu_p}P_0(n,m-1) & \text{if } \quad n=0, \\ \frac{p\lambda_s}{p\lambda_s+\lambda_p+2\mu_p}P_0(n,m+1) + \frac{\lambda_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n+1,m) & \\ +\frac{\mu_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n-1,m) & \text{if } \quad m=0, \\ \frac{p\lambda_s}{p\lambda_s+\lambda_p+2\mu_p}P_0(n,m+1) + \frac{\lambda_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n+1,m) & \\ +\frac{\mu_p}{p\lambda_s+\lambda_p+2\mu_p}(P_0(n-1,m)+P_0(n,m-1)) & \text{otherwise.} \end{cases}$$

We assume that the reject probability $P_r(p,k)$ is increasing with the sensing probability p. This assumption is somehow realistic. Indeed, the greater is the number of SUs that choose to sense, the higher is the probability to be rejected by PUs. In the following section, we study the impact of the reject probability on the average cost function, and we determine the optimal OSA policies for SUs.

3.4.2 Average total cost

The average sojourn time $T_{S_1}^r$ for a SU that chooses to join the dedicated channel without sensing licensed channels is given by:

$$T_{S_1}^r(p, K) = \frac{1}{\mu_s - \lambda_s(1-p)}. \tag{3.15}$$

Moreover, the average sojourn time of a SU that chooses to sense licensed channels is defined by:

$$T_{S_2}^r(p, K) = \frac{(1 - \Pi(p, K))(1 - P_r(p, K))}{\mu_p}. \tag{3.16}$$

Therefore, the average sojourn time of a SU in the non-slotted model is expressed as follows:

$$T_S^r(p, K) \quad = \quad \frac{1-p}{\mu_s - \lambda_s(1-p)} + \frac{p(1 - \Pi(p, K))(1 - P_r(p, K))}{\mu_p}. \tag{3.17}$$

The average cost function is expressed as follows:

$$U_S^r(p, K) = \frac{1-p}{\mu_s - \lambda_s(1-p)} + \frac{p(1 - \Pi(p, K))(1 - P_r(p, K))}{\mu_p} + \alpha p K.$$

For notation convenience, we define $Y(p, K) = p(1 - \Pi(p, K))(1 - P_r(p, K))$. By substituting $Y(p, K)$ in the expression of the average cost function, we obtain the following expression:

$$U_S^r(p, K) = \frac{1-p}{\mu_s - \lambda_s(1-p)} + \frac{Y(p, K)}{\mu_p} + \alpha p K.$$

The first intuition one can make is that releasing the assumption that PUs evolve in a slotted model induces a loss of performance. Let us denote by p_r^* the optimal sensing probability of a SU in the non-slotted model.

Proposition 3.7. *For all values of α and K, the average cost function $U_S^r(p, K)$ is minimized when the sensing probability is equal to:*

$$p = min(1, max(p_0^r, 0)) := p_r^*,$$

where p_0^r is the solution of the following equation:

$$\frac{\partial Y}{\partial p}(p, K) = -\alpha K \mu_p + \frac{\mu_p \mu_s}{(\mu_s - \lambda_s(1-p))^2}. \tag{3.18}$$

Proof. The proof of this proposition is analogous to the proof of Proposition 3.1 by replacing $X(p, K)$ by $Y(p, K)$. □

Furthermore, the following proposition gives us a relation between the average cost obtained with the slotted system and the average cost obtained with the non-slotted model.

Proposition 3.8. *For all values of α and K, the optimal value of the average cost function is higher in the non-slotted model than in the slotted one:*

$$U_S(p^*, K) \leq U_S^r(p_r^*, K).$$

Proof. Suppose, first, that $\mu_s - \alpha K(\mu_s - \lambda_s(1-p))^2 \leq 0$. Then, it follows from Proposition 3.7 that $p^* = 0$. Therefore, the average cost function is expressed as follows:

$$U_S(p^*, K) = \frac{1}{\mu_s - \lambda_s}.$$

Let us derive the average cost function with respect to the reject probability. After some algebra, we obtain the following expression of the derivative of the average cost function with respect to the reject probability:

$$\frac{\partial U_S^r}{\partial P_r}(P_r) = -\frac{p(1 - \Pi(p, K))}{\mu_p} \leq 0.$$

We remark that $U_S^r(P_r)$ is decreasing with P_r. Thus, we have the following lower bound of the average cost function:

$$U_S^r(P_r) \geq U_S^r(1) = \frac{1}{\mu_s - \lambda_s} + \alpha p_r^* K \geq \frac{1}{\mu_s - \lambda_s},$$

which leads to:

$$U_S(p^*, K) \leq U_S^r(p_r^*, K).$$

Second, suppose that $\mu_s - \alpha K(\mu_s - \lambda_s(1-p))^2 > 0$. Therefore, we prove analogously that $U_S^r(P_r)$ is increasing with P_r, and we obtain that $U_S^r(P_r) \geq U_S^r(0)$.

Finally, the average cost function in the non-slotted model is higher than the average cost function in the slotted one, i.e. $U_S(p^*, K) \leq U_S^r(p_r^*, K)$. □

This result is somehow intuitive as the reject of a SU introduces a lack of performance to the system. We focus, in the next section, on the study of the non-slotted self-adaptive CR network model.

3.4.3 Individual optimization

We consider a distributed system in which each SU decides individually whether to sense or not licensed channels. In fact, each SU decides on its probability p of sensing licensed channels. Note that a SU aims to minimize its average cost function $U_r(p, p', K)$, which depends on its probability p and the probability p' of other SUs. Thus, the average cost function is expressed as follows:

$$U_r(p, p', K) = (1-p)T^r_{S_1}(p', K) + pT^r_{S_2}(p', K) + \alpha p K. \tag{3.19}$$

We prove, in the following proposition, that the non-cooperative OSA for SUs has a NE.

Proposition 3.9. *For all values of α and K, the NE strategy for the OSA problem exists. Moreover the sensing probability at the NE is given by:*

- *if* $\frac{1}{\mu_s-\lambda_s} > \alpha K + \frac{(1-\Pi(0,k))(1-P_r(0,K))}{\mu_p}$;
  - *if* $\frac{1}{\mu_s} < \alpha K + \frac{(1-\Pi(1,k))(1-P_r(1,K))}{\mu_p}$ *then* $p_r^E = \{0, p_r', 1\}$.
  - *else* $p_r^E = 0$;
- *else*
  - *if* $\frac{1}{\mu_s} > \alpha K + \frac{(1-\Pi(1,k))(1-P_r(1,K))}{\mu_p}$ *then* $p_r^E = p_r'$;
  - *else* $p_r^E = 1$.

where p_r' is the solution of the following equation:

$$\frac{1}{\mu_s - \lambda_s(1-p)} = \alpha K + \frac{(1-\Pi(p,K))(1-P_r(p,K))}{\mu_p}. \tag{3.20}$$

Proof. The proof of this proposition is analogous to the proof of Proposition 3.3 by replacing $X(p, K)$ by $Y(p, K)$. □

For notation convenience, we denote for all p and K, $U_r(p, p, K)$ by $U^r_S(p, K)$. Furthermore, the following proposition gives us a higher bound of the average total cost at the NE.

Proposition 3.10. *For all values of α and K, we have the following higher bound of the average cost function at the NE:*

$$U^r_S(p_r^E, K) \leq \frac{1}{\mu_s - \lambda_s}.$$

Proof. The proof of this proposition is analogous to the proof of Proposition 3.5 by replacing $X(p, K)$ by $Y(p, K)$. □

Given the existence of the NE for the proposed OSA mechanism in the non-slotted model, we study the gap of performance between the average cost at the NE and the average cost of the centralized system.

3.4.4 Price of anarchy

The PoA models the lack of performance between the utility at the NE and the optimal utility, and is defined by the following ratio:

$$PoA_r(\alpha, K) = \frac{U_s^r(p_r^*, K)}{\max_{p \in p_r^E} U_s^r(p, K)} \leq 1. \tag{3.21}$$

Let us focus on the expression of PoA. Similarly to the slotted model, our aim is to determine a lower value of the PoA or to bound it, in order to define the worst-possible lack of performance of the decentralized system. The following proposition gives us a lower bound of the price of anarchy, called $\underline{PoA_r}$.

Proposition 3.11. *For all values of α and K, we have the following lower bound of the PoA:*

$$PoA_r(\alpha, K) \geq \frac{2(\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)})}{\lambda_s} := \underline{PoA_r}.$$

Proof. The proof of this proposition is analogous to the proof of Proposition 3.6 by replacing $X(p, K)$ by $Y(p, K)$. □

This closed-form lower bound of the PoA is interesting, as it depends neither on the sensing cost α, nor on the number of licensed channel k. Thus, the SP has only to tune μ_s and λ_s in order to maximize the performance of the decentralized system.

In the following section, we present some numerical illustrations that validate our theoretical findings.

3.4.5 Numerical illustrations

This section presents the performance analysis of the proposed OSA mechanism. For this end, we have performed extensive Matlab simulations with different configurations of the system. Furthermore, two performance metrics are considered: the sensing cost and the capacity of the system. We consider the same values of the system model parameters defined in Section 3.3.4. Moreover, we assume that PUs may preempt SUs in service. Firstly, we focus on the sensing cost α. Thereafter, we study the impact of the capacity (number of licensed channels) on the OSA mechanism.

3.4.5.1 Sensing cost

We evaluate, in this section, the impact of the sensing cost α on the performance of the proposed OSA mechanism. Figure 3.9 illustrates the average cost function in both the slotted PUs transmissions and the non-slotted model. We observe that the average cost of SUs is always higher in the non-slotted model than in the slotted one, which validates the results of Proposition 3.8.

We observe, in Figure 3.10, that the optimal probability of sensing licensed channels is decreasing with α in both models. However, we remark that the optimal probability of sensing in the non-slotted model p_r^* is more sensitive to the sensing cost α than the optimal probability of sensing in the slotted model p^*. In fact, in the non-slotted model, the reject probability decreases the benefit of sensing in term of utility.

Let us focus on the lack of performance induced by the non-cooperative behavior of SUs in the decentralized model. We obtain from Proposition 3.11 a lower bound of the price of anarchy $\underline{PoA_r} = 75.24\%$. This result is lower than the minimum value of the PoA obtained from Figure 3.11, which is 0.8289.

The number of licensed channels has a major leverage on the behavior of SUs and impacts not only the average sojourn time, but also the energy consumption, as the sensing cost grow linearly with the number of licensed channels. We depict, in the next section, the impact of the capacity on the performance of the proposed OSA policy.

3.4.5.2 Capacity

In the present section, we are interested in the impact of the number of licensed channels on the performance of the proposed OSA mechanism for SUs. We set the sensing cost α to 0.3 and we vary the number of licensed channel from 1 to 20. Note that under these settings, the blocking probability decreases with the number of licensed channels, whereas the sensing cost increases.

Firstly, we observe, in Figure 3.12, that both the optimal sensing probability p_r^* and the sensing probability at the NE p_r^E are decreasing with number of licensed channel K. Moreover, we remark that the sensing probability at the NE is lower or equal than the optimal sensing probability. In fact, the non-slotted system is more sensitive to the number of licensed channels than the slotted one. Second, we obtain from Figure 3.13

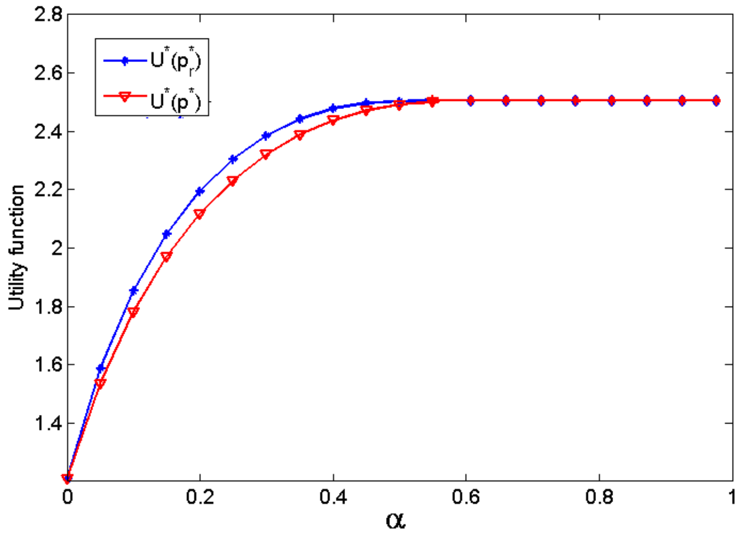

FIGURE 3.9: The global optimum depending on the sensing cost α in both the slotted and the non-slotted models.

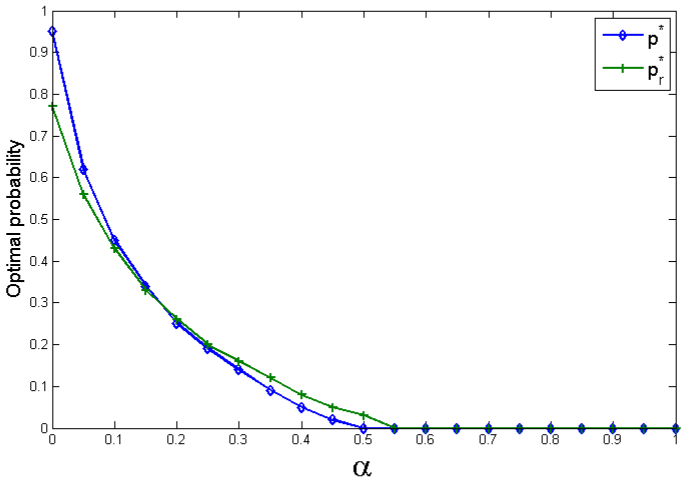

FIGURE 3.10: The optimal sensing probability depending on the sensing cost α.

that the non-slotted model induces a higher average cost for SUs compared to the slotted model. Finally, we conclude with the analysis of the price of anarchy depending on the number of licensed channels K. In Figure 3.14, we observe that the minimal value of the price of anarchy is 0.8672, which is not so far from the lower bound given by Proposition 3.11, which is 75.24%.

Both the sensing cost and the capacity of the system are important factors in the performance of CR users. The SP may tune the system parameters in order to optimize the QoS for its SUs without the need for a centralized controller.

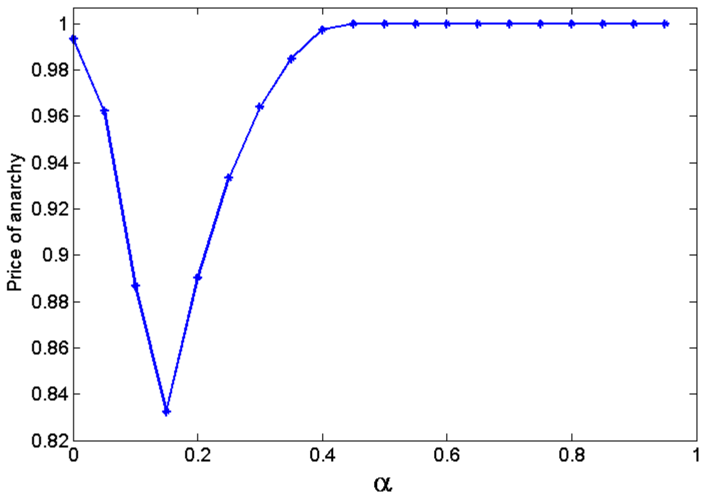

FIGURE 3.11: The price of anarchy depending on α.

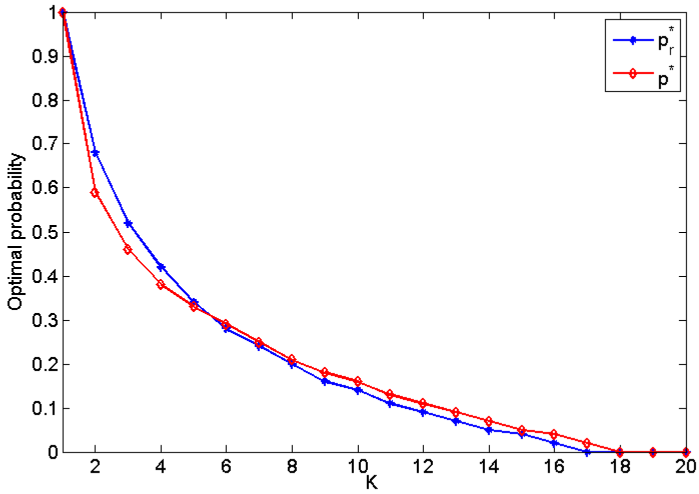

FIGURE 3.12: The probability of sensing depending on the number of licensed channels in non-slotted model.

3.4.6 Summary

As like as the slotted model, we have studied, in this section, the non-slotted OSA in both the centralized and the decentralized manners. We have proved the existence of a NE strategy, and we have evaluated the gap of performance in the decentralized system through the POA.

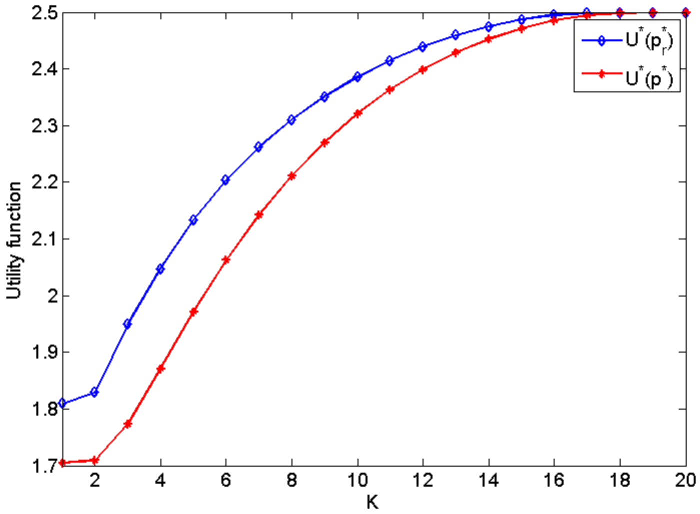

FIGURE 3.13: The average cost function with the number of licensed channels in both the slotted and the non-slotted models.

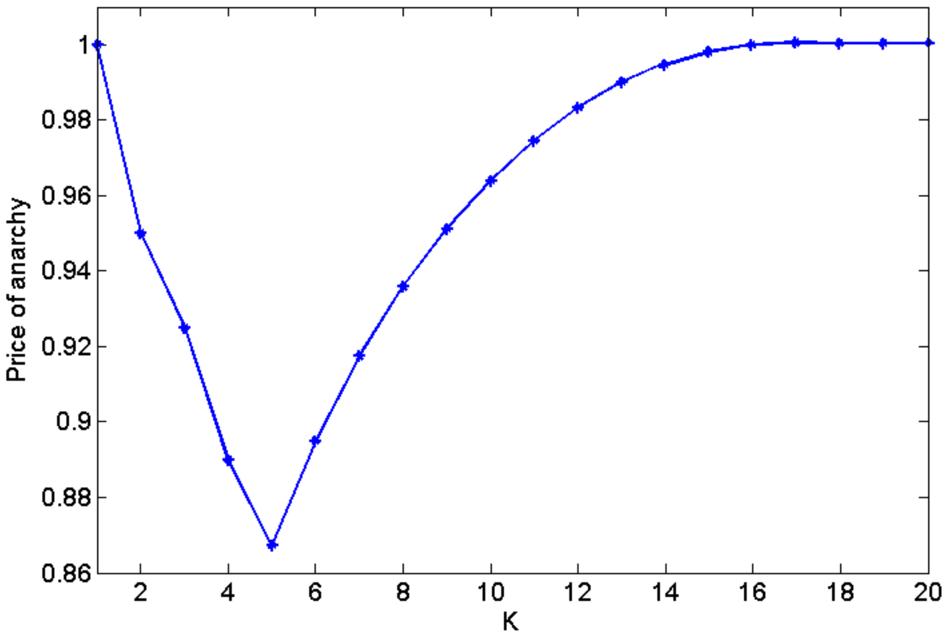

FIGURE 3.14: The price of anarchy with the number of licensed channels K in the non-slotted model.

3.5 Conclusion

In this chapter, we have studied the performance of OSA in CR networks. We have considered both the slotted and the non-slotted models. We have considered the global optimization of the centralized system, and we have determined the optimal sensing probability. Furthermore, we have considered the the individual optimization in a decentralized manner, and we have proved the existence of a NE equilibrium between SUs. We have studied the performance of these approaches and we have evaluated the gap of performance between them using the well-studied metric: the PoA. Simulation results have validated our theoretical findings.

In the next chapter, we study the OSA for CR under energy and QoS constraints. Specifically, we formulate the model using a POMDP framework, and we present an optimal threshold-based OSA policy.