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LISTE DES ABRÉVIATIONS, SIGLES ET ACRONYMES

$u_1(\cdot)$:	taux de production de la machine de *manufacturing* (produit/UT)
$u_2(\cdot)$:	taux de production de la machine de *remanufacturing* (produit/UT)
$u_{1\max}$:	taux de production maximal de la machine de *manufacturing* (produit/UT)
U :	taux de production économique de la machine de *manufacturing* (produit/UT)
$u_{2\max}$:	taux de production maximal de la machine de *remanufacturing* (produit/UT)
x_1^+ :	inventaire des produits finis (produit)
x_1^- :	pénuries (quantité de produit manquante)
x_1 :	stock de produits finis (produit)
x_2 :	stock de retours (produit)
d :	taux de demande des clients (produit/UT)
r :	taux de retour (produit/UT)
$Disp$:	taux de destruction (disposal)
c_1^+ :	coût d'inventaire des produits finis (\$/produit/UT)
c_1^- :	coût de pénuries (\$/produit/UT)
c_2 :	coût d'inventaire des produits retournés (\$/produit/UT)
k :	nombre pannes
c_r :	coût de maintenance corrective (\$/UT)
c_m :	coût de maintenance préventive (\$/UT)
$\omega_r^{\min}$:	taux de maintenance corrective minimal (1/UT)
$\omega_r^{\max}$:	taux de maintenance corrective maximal (1/UT)
$\omega_m^{\min}$:	taux de maintenance préventive minimal (1/UT)
$\omega_m^{\max}$:	taux de maintenance préventive maximal (1/UT)
$\lambda_{\alpha\beta}$:	taux de transition du mode α au mode β (1/UT)
π :	vecteur des probabilités limites

$g(\cdot)$:	fonction coût instantanée ($/UT)
$J(\cdot)$:	coût total du système
$v(\cdot)$:	fonction valeur
ρ :	taux d'actualisation
UT:	Unité de Temps
CCSCA:	Conseil Canadien Sectoriel de la Chaîne d'Approvisionnement
TI:	Technologie de l'Information
HPP:	Hedging Point Policy (Politique à Seuil Critique)
HJB:	Hamilton-Jacobi-Bellman
MTTF:	Mean Time To Failure (Temps moyen entre les pannes)

INTRODUCTION GÉNÉRALE

Les industries manufacturières, face à la globalisation des marchés et à l'avancement technologique, sont confrontées aux problèmes d'optimisation de leur chaîne logistique globale de production. Les problèmes de planification de la production deviennent plus complexes lorsque les contraintes environnementales requièrent l'optimisation des procédés de fabrication et la réutilisation, en fabrication, des pièces retournées par les consommateurs après utilisation (logistique inverse). Comparer à une situation où la demande des clients est seulement satisfaite par les pièces de la ligne directe (production à partir des matières premières), le contrôle simultané de la production et de la réutilisation (*remanufacturing*) est très complexe (Kiesmüller, 2003).

Des exemples de réutilisation se trouvent dans les domaines de fabrication des pièces mécaniques, de transformation de l'aluminium, de lignes d'assemblage de véhicules et des aéronefs, des ordinateurs, des photocopieurs, *etc*. Le problème qui se pose est de savoir comment planifier la production de manière à satisfaire la demande et minimiser le coût total du système dans une chaîne d'approvisionnement caractérisée par des incertitudes (incertitudes sur la demande, les retours et les matières premières, les pannes et réparations des machines) et la dégradation des machines en fonction de leur taux d'utilisation. En effet, une des caractéristiques fondamentales de la chaîne d'approvisionnement est qu'elle se comporte justement comme une chaîne, c'est-à-dire que chacun des maillons a un impact sur le reste des intervenants, positivement ou négativement. Ainsi, toute rupture de matières premières ou toute panne de machines au niveau de la production pourrait se répercuter jusqu'au client final. De ce fait, une gestion efficace de chaque élément ainsi que ses interactions avec les autres intervenants de la chaîne sont indispensables afin de rallier les visions souvent disparates des différents intervenants; sans oublier la nature dynamique et stochastique de l'environnement à laquelle la chaîne d'approvisionnement est assujettie.

Dans ce contexte, et en réponse à l'accroissement de l'incertitude et de la complexité de l'environnement industriel, les gestionnaires doivent détenir les outils nécessaires afin de garantir une meilleure intégration de tous les intervenants et atteindre leurs objectifs communs. Une des étapes cruciales pour y arriver consiste à faire un bon choix des outils de modélisation et d'analyse. L'objectif de cette thèse est de proposer des modèles d'optimisation conjointe des stratégies de production, des stratégies de réutilisation et les politiques de maintenance des équipements. L'approche proposée vise non seulement à minimiser les coûts d'opération des systèmes manufacturiers (rentabiliser les investissements), mais aussi à améliorer la fiabilité ou la disponibilité des équipements. Les contributions de ce travail permettront de proposer une structure générale pour les systèmes dynamiques de production en logistique inverse intégrant les opérations de production, de réutilisation et de maintenance des machines.

Les résultats seront obtenus à travers des modèles mathématiques décrivant la dynamique des systèmes de production étudiés. La résolution numérique du problème d'optimisation et une analyse de sensibilité des stratégies optimales (seuils critiques du stock des produits finis) permettront aux entreprises manufacturières de maximiser le profit de leur chaîne d'approvisionnement, d'optimiser la production et les stratégies de maintenance des machines, tout en récupérant les produits déjà utilisés sur le marché.

Le prochain chapitre décrit la problématique de notre recherche. Ce chapitre fait aussi une revue critique de la littérature liée à notre domaine d'étude, présente la méthodologie adoptée et les principales contributions de la thèse.

CHAPITRE 1

PROBLÉMATIQUE ET REVUE DE LITTÉRATURE

1.1 Introduction

Nous présentons dans la première partie de ce chapitre la problématique de cette thèse. La deuxième partie est une revue critique de la littérature qui touche les aspects d'ordre général de notre problématique. La troisième partie présente la méthodologie adoptée pour réaliser ce travail. Dans la quatrième partie du chapitre, les contributions et la structure de la thèse sont présentées. Le chapitre se termine par les retombées et l'impact industriel de notre travail.

1.2 Problématique de recherche

Depuis plus d'une décennie, plusieurs modèles intégrant la production et la logistique inverse ont été publiés. Tous ces modèles ne traitent pas le problème en continu et ne tiennent pas compte des aspects stochastiques reliés à la dynamique des machines, la dégradation du système de production en fonction de son taux d'utilisation et la maintenance des équipements de production. Pour intégrer tous ces aspects dans un modèle continu d'optimisation conjointe de production, de réutilisation et de maintenance des machines en contexte dynamique et stochastique, il est nécessaire de connaître la signification de certains mots clés utilisés en planification de la production des systèmes hybrides de production/réutilisation (*manufacturing/remanufacturing*).

1.2.1 Définition des mots clés – Terminologie

a. Dynamique stochastique

Un système manufacturier a une dynamique stochastique si au moins une de ses sorties ou un de ses paramètres est aléatoire (Sader et Sorensen (2003)).

b. Chaîne de Markov homogène

Une chaîne de Markov est homogène quand ses taux de transition d'un état à un autre sont considérés constants. En système manufacturier, cela suppose que les machines peuvent tomber en panne même durant l'arrêt de production.

c. Chaîne de Markov non-homogène

Un système manufacturier est modélisé par un processus de Markov non-homogène lorsque la probabilité qu'un équipement tombe en panne change avec son âge ainsi que son cycle de fonctionnement. La fiabilité de la machine est liée à son âge (Boukas et Haurie, 1990).

Plusieurs termes sont utilisés comme des synonymes en logistique inverse, bien qu'il ait des similarités entre les termes, ils ne veulent pas tous dire exactement la même chose (Lambert et Riopel, 2003). Dans les lignes qui suivent, nous allons définir quelques termes rencontrés dans les flux inverses.

d. Logistique inverse

Stock (1992) utilise les termes comme le recyclage, la destruction des déchets et la gestion des matières dangereuses pour définir la logistique inverse. Une perspective plus large inclut la substitution, la réutilisation de matières et des déchets. Plus tard, Fleischmann (2001) propose une nouvelle définition de la logistique inverse comme étant le processus de planification, l'exécution et le contrôle du flux des produits collectés (retours) dans une chaine d'approvisionnement (production) dans le but de les remettre sur le marché. Lambert et Riopel (2003) mentionnent que la logistique inverse est le processus de planification, d'implantation, et de contrôle de l'efficience, de la rentabilité des matières premières, des en-cours de production, des produits finis, et l'information pertinente du point d'utilisation jusqu'au point d'origine dans le but de reprendre ou générer de la valeur ou pour en disposer de la bonne façon tout en assurant une utilisation efficace et environnementale des ressources mises en œuvre. Alors que pour Bennekrouf *et al.* (2010), la logistique inverse regroupe plusieurs profils à savoir : le retour des produits (suite à la non satisfaction d'un critère), la réutilisation de certains produits (comme l'emballage et les containers), le retraitement (*remanufacture*) et le cannibalisation (démontage d'un produit pour réutiliser ses pièces). Ce dernier dépend de l'indice de qualité des produits récupérés.

e. Distribution inverse

Carter et Ellram (1998) présentent la distribution inverse comme le retour, mouvement à contre-courant d'un produit ou de matière découlant de la réutilisation, du recyclage ou de la disposition. Selon ces auteurs, le mouvement à contre-courant peut être associé aux problèmes environnementaux, tout comme à la qualité et l'usure (dégradation dans le temps) et qui sont souvent effectués par des nouveaux membres auxiliaires au système.

f. Réutilisation

Selon Abdessalem *et al.* (2007), réutiliser un produit signifie que le produit est utilisé immédiatement dans le même contexte ou un autre, suite à une opération additionnelle mineure telle que le nettoyage, la maintenance. Réutiliser un produit peut aussi signifier la réutilisation des pièces qui le composent comme pièces de rechange ou matières premières.

g. Recyclage

Le recyclage consiste à collecter et désassembler un produit à la fin de son cycle de vie en vue de la récupération des matériaux (Abdessalem *et al.*, 2007).

h. *Refabrication* (*remanufacturing*)

Abdessalem *et al.* (2007) définissent la *refabrication* comme un processus de désassemblage des produits utilisés, d'inspection, de réparation/remplacement des composants et leur utilisation pour fabriquer un nouveau produit.

1.2.2 Structure du système étudié

Le modèle des systèmes hydrides de *manufacturing/remanufacturing* tel que défini à la figure 1.1 est constitué des unités de production. La première série d'unités de production est utilisée pour la production de la ligne directe (*manufacturing*) et la deuxième série permet d'intégrer les retours dans le système (*remanufacturing*) de production. Les unités de production sont sujettes à des pannes et des réparations aléatoires. Nous considérons un produit dont la demande est satisfaite soit par *manufacturing* à partir de la matière première, soit par *remanufacturing* des produits usagers.

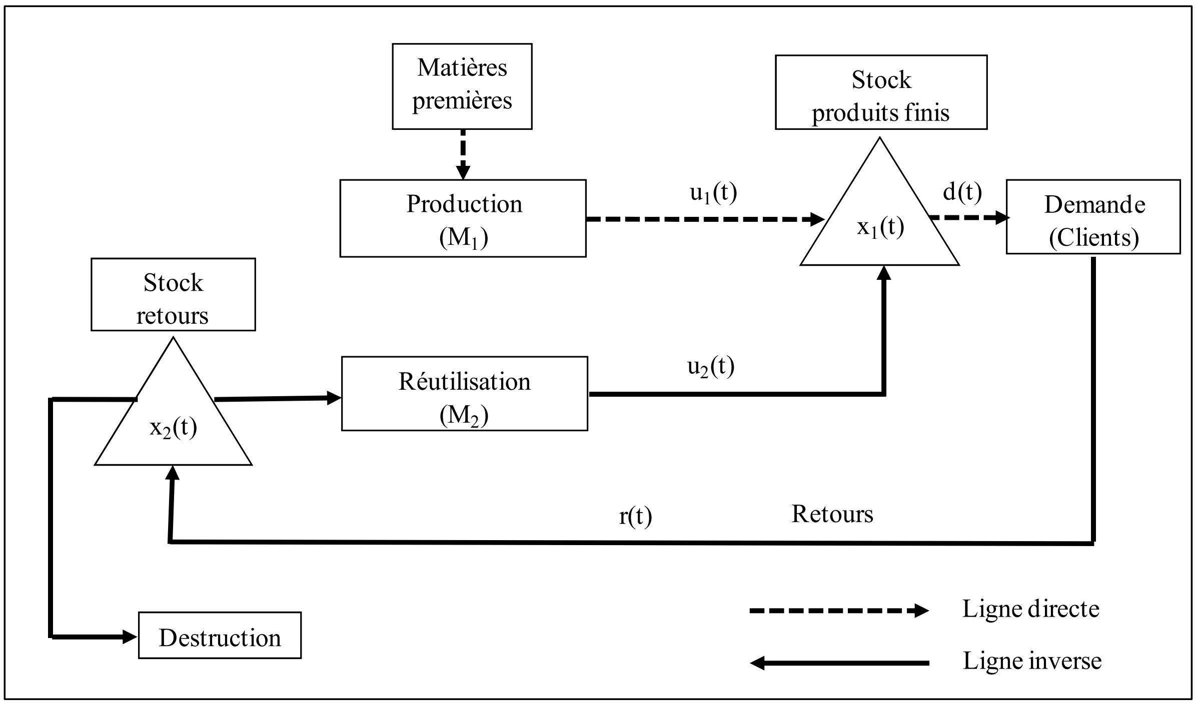

Figure 1.1 Structure d'un système hybride de production/réutilisation

Le comportement du système est décrit par une composante continue (stocks des pièces) et une composante discrète (modes des unités de production). La composante continue est constituée des variables continues qui sont le stock des produits finis et le stock des retours. Les retours qui ne respectent pas les normes de *remanufacturing* sont détruits (*disposal*). Les unités de production peuvent être soit en opération, soit en maintenance (préventive et corrective). Les pièces refabriquées sont considérées comme neuves.

Le problème qui se pose est de savoir comment planifier la production et les opérations de maintenance, de manière à satisfaire le client tout en maximisant le profit de l'entreprise, et en récupérant les produits déjà utilisés qui sont réintroduits dans le système de production.

Pour résoudre ce problème, nous allons tout au long de notre thèse essayer de répondre aux questions suivantes:

- À quel rythme (taux) manufacturer ou refabriquer les pièces en fonction de l'état du système?
- Comment optimiser les stratégies de maintenance des machines (maintenance corrective, préventive)?
- Quelles sont les politiques de production et les stratégies de maintenance qui permettent de minimiser le coût total du système de production sur un horizon infini?
- Quelle est la quantité de retours et des produits finis à stocker pour minimiser les coûts de stockage?
- Quels sont les retombées et l'impact industriel de notre recherche?

1.2.3 Hypothèses de travail

Les hypothèses suivantes seront considérées:

- Les sites de localisation des fournisseurs, usines, entrepôts et centres de distribution existent et sont connus. Donc, nous ne ferons pas la conception et la localisation des sites;
- Les taux de demandes et de retours sont constants;
- Les produits refabriqués sont identiques (en termes de qualité) aux produits manufacturés;
- Les coûts de stockage, de pénuries et de maintenance des machines sont connus;
- Le coût des stocks dépend de la quantité de produits stockés;
- Le coût des pénuries dépend de la quantité de produits manquants et est supérieur au coût des stocks;
- Le coût de stockage des retours est inférieur au coût de stockage des produits finis. Ce qui encourage la récupération des produits usagés sur le marché;
- Les taux maximums de production et de réutilisation sont connus;

- Les unités de production et de réutilisation sont en parallèle et flexibles;
- Les pannes et réparations des machines sont aléatoires.

Les hypothèses précédentes sont détaillées et utilisées dans les modèles développés aux chapitres suivants.

1.2.4 Objectifs de la recherche

L'objectif principal de cette thèse est de proposer des modèles pour améliorer la production, la maintenance des unités de production et la réutilisation de produits déjà utilisés (logistique inverse) aux entreprises manufacturières (exemples: les lignes d'assemblage d'automobile et des aéronefs, les usines de fabrication des pièces, les usines de production et de réutilisation des cartouches d'imprimantes.).

Nous visons spécifiquement les trois (3) objectifs suivants:

1. Proposer le schéma des systèmes hybrides de production/réutilisation des produits, qui intègre la production, la réutilisation et la maintenance des unités de production;
2. Modéliser le système dans le but d'étudier l'impact de ses paramètres sur l'objectif de la chaîne de production de l'entreprise;
3. Analyser les performances (exemple la minimisation des coûts) système soumis aux stratégies obtenues.

1.3 Revue critique de la littérature

Dans les lignes qui suivent, nous ferons une revue critique de la littérature sur

- les chaînes d'approvisionnement;
- les stratégies de maintenance des machines;
- l'optimisation de la production des systèmes manufacturiers;
- la gestion simultanée de la production et de la maintenance des machines;
- la dégradation des équipements de production;

- les systèmes hybrides de production/réutilisation (*manufacturing*/*remanufacturing*) des produits;
- les systèmes de production/réutilisation dont les unités de production sont soumises aux pannes et réparations aléatoires;
- les modèles mathématiques utilisés en logistique inverse.

1.3.1 Chaînes d'approvisionnement

De nos jours, la compétition des marchés et les exigences des consommateurs obligent les entreprises à porter beaucoup plus d'attention sur leurs relations avec les fournisseurs et les clients tout en optimisant leur production et la disponibilité des équipements. Ainsi, la gestion des chaînes d'approvisionnement devient de plus en plus importante.

D'après le Conseil Canadien Sectoriel de la Chaîne d'Approvisionnement (CCSCA), la définition des chaînes d'approvisionnement tel que défini à la figure 1.2 englobe les trois fonctions suivantes:

- la fourniture des produits à un fabricant;
- le processus de fabrication (notre zone de travail);
- la distribution des produits finis aux consommateurs par un réseau de distributeurs et de détaillants.

La diversité des paramètres, le volume des données et les différents niveaux de décisions impliqués dans une chaîne d'approvisionnement font qu'il n'existe pas une approche universelle de modélisation (Hajji, 2007). Selon Min et Zhou (2002), il existe quatre approches pour modéliser les chaînes d'approvisionnement; déterministe, stochastique, hybride et les modèles basés sur la technologie de l'information (TI).

Les modèles déterministes supposent que les paramètres de la chaîne sont connus et fixés avec certitude. Les modèles stochastiques permettent de se rapprocher plus des cas réels caractérisés par la présence des phénomènes aléatoires (exemple: les pannes et réparations des unités de production). De part la nature même des chaînes d'approvisionnement, plusieurs modèles incluent simultanément des aspects déterministes et stochastiques; ce sont les modèles hydrides. Les modèles basés sur la TI visent l'intégration et la coordination de plusieurs phases de planification dans une chaîne d'approvisionnement, avec une vision de commande en temps réel et ce, en utilisant des mécanismes de partage d'information entre les différents partenaires de la chaîne.

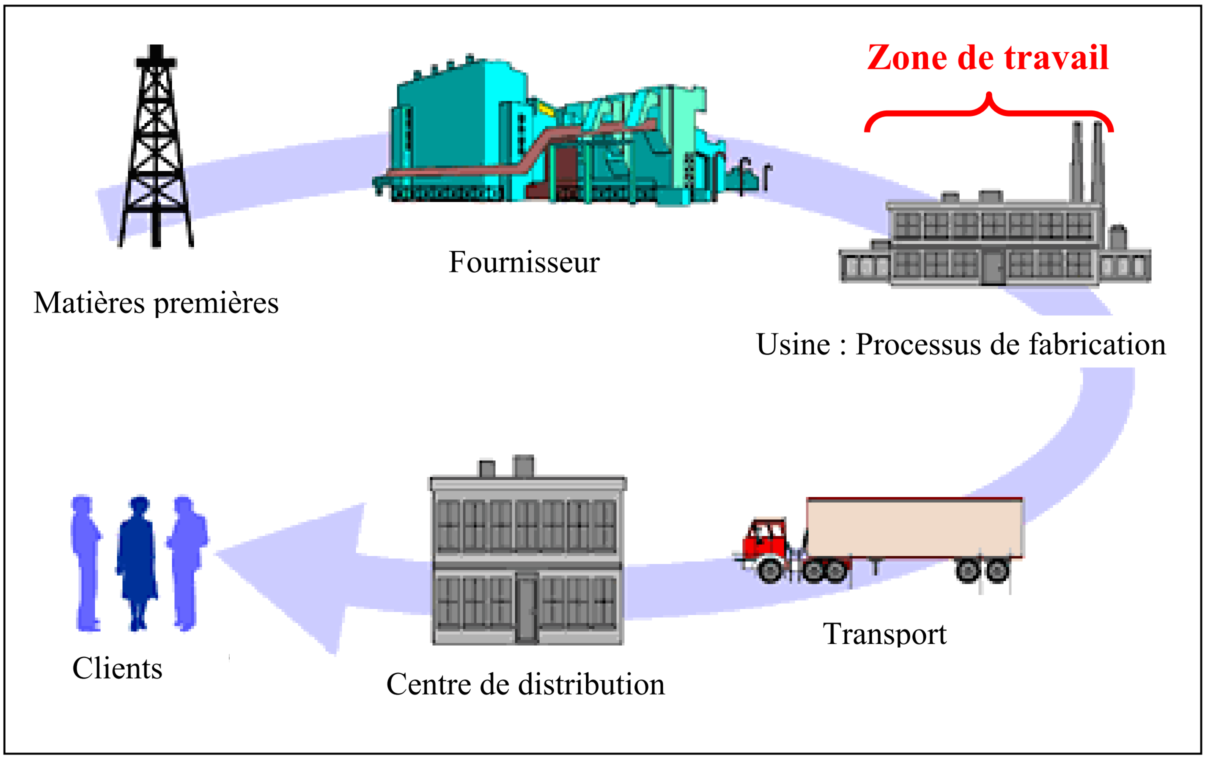

Figure 1.2 Modélisation des chaînes d'approvisionnement.

Une chaîne d'approvisionnement est caractérisée par le flux direct des produits et un flux inverse d'information (Wang *et al.*, 2010). Elle est composée de deux activités principales; la gestion du matériel (acquisition et stockage des matières premières) et le service client. Les entités d'une chaîne d'approvisionnement sont les clients, les entrepôts, les dépôts, les unités de transformation (exemples: usines, sous-traitants) et les fournisseurs. La chaîne comprend aussi les mouvements des produits entre les entités; les flux d'information et financiers.

L'objectif de la gestion de la chaîne d'approvisionnement (*SCM* - *Supply Chain Managemen*t) est d'améliorer l'efficacité opérationnelle, la rentabilité de l'entreprise et la relation entre les différents membres de la chaîne (Mahnam *et al.*, 2009). La prise de décisions dans une chaîne d'approvisionnement est un processus complexe qui doit respecter trois phases (Min et Zhou, 2002). (i) Les décisions stratégiques concernent les localisations et capacités des usines et entrepôts, les produits à fabriquer ou à stocker à divers endroits; les modes de transport et le système d'information; (ii) Pour répondre aux exigences de la question de la satisfaction de la demande, la stratégie de production, la sous-traitante, la campagne de promotion (quand, à quel coût?), on prend les décisions tactiques; (iii) Les décisions opérationnelles permettent d'allouer des commandes à la production, d'allouer une commande à un transporteur, de déterminer les calendriers de livraison, de placer les commandes de réapprovisionnement; ce sont des décisions qui ont moins d'incertitude car elles sont prises sur un horizon à très court terme.

Dans les lignes qui suivent, nous nous intéressons aux unités de transformation d'une chaîne d'approvisionnement. Nous avons noté précédemment que l'un des phénomènes aléatoires en logistique industrielle est la panne des unités de production. Il est donc impératif d'appliquer des stratégies efficaces de maintenance afin d'assurer la disponibilité et la fiabilité des équipements.

1.3.2 Stratégies de maintenance

Le principal objectif de la planification et du pilotage (*MPC* - *Manufacturing Planning* and *Control*) des entreprises manufacturières est de réduire le coût total de la production. Le coût de la maintenance des machines est estimé à 15% du coût total du système pour certaines entreprises et près de 70% pour d'autres (Ling *et al.*, 2007). Dans la littérature, on distingue deux principaux types de maintenance (Li *et al.*, 2007): la maintenance corrective et la maintenance préventive.

La maintenance corrective s'effectue suite à une panne de la machine. Lorsqu'une entreprise n'applique que cette stratégie de maintenance, elle est exposée à de sérieux dommages au niveau des équipements, du personnel et de l'environnement (Ling *et al.*, 2007). La maintenance préventive peut être divisée en deux grands groupes; la maintenance de type âge et la maintenance de type bloc. Dans le premier cas, la maintenance dépend de l'âge de la machine et dans le second cas, les dates de maintenance sont connus à l'avance et ne dépendent ni de l'âge, ni de l'état du système. La maintenance préventive s'exécute avant les pannes; elle permet de maintenir l'équipement sous certaines conditions grâce à des inspections et des préventions systématiques (Wang, 2002).

Selon les techniques de la maintenance préventive, on peut citer la maintenance préventive temporelle, conditionnelle et prédictive (Ling *et al.*, 2007). La maintenance préventive temporelle est planifiée et exécutée sur un horizon périodique afin de réduire les pannes spontanées et d'assurer la fiabilité des équipements. Dans la majeure partie des cas, cette stratégie de maintenance entraîne la détérioration des machines si les activités de maintenance sont imparfaites. La maintenance prédictive permet de prévoir la dégradation de la performance des machines et de prédire les pannes pouvant survenir, par l'analyse des données des paramètres de commande.

Il est à noter que les stratégies de maintenance préventive (temporelle, conditionnelle et prédictive) ne permettent pas d'éviter complètement les pannes et la maintenance corrective à cause de la nature aléatoire des pannes des machines. Il est donc nécessaire de bien choisir les stratégies de maintenance préventive à appliquer. Selon Ling *et al.* (2007), les critères de sélection peuvent être:

- la sécurité (personnel, machines, environnement),
- le coût (matériel, logiciel, formation du personnel),
- la valeur ajoutée (petit stock des pièces de rechange, réduction des pénuries, identification rapide des fautes)
- la faisabilité de la maintenance.

Malheureusement, la plupart des résultats précédents ne prennent pas en compte les contraintes de production et de satisfaction de la demande. Ceci est dû au fait que certains auteurs considèrent que les temps pour effectuer une réparation ou une maintenance préventive sont négligeables. Donc, ces temps n'affectent pas significativement les activités de production.

1.3.3 Optimisation de la production des systèmes manufacturiers

Face à un environnement commercial compétitif, les entreprises sont de plus en plus attirées par la planification efficace de leur production dans le but d’optimiser le stock des produits finis. Un stock est défini comme la différence entre la quantité produite et la demande du client. Lorsqu’il est positif, on parle de stock des produits finis. Sinon, on parle de pénurie. Dans le cas d’un stock positif, le client est satisfait dans les délais (sans retard). S’il y’a pénurie, le temps d’attente du client dépend de la quantité des produits manquants. Il est donc néfaste de faire attendre un client pour un service ou un produit car on peut soit perdre le client, soit avoir une évaluation négative de la qualité du service ou du produit. Gershwin *et al.* (2009) ont fait l’étude d’un système manufacturier en introduisant une fonction de défection qui indique la fraction de clients qui choisissent de retirer leurs commandes lorsque les pénuries ont atteint un certain seuil. Les résultats obtenus ont montré que la politique optimale de production d’un tel problème est de type seuil critique (*Hedging Point Policy* - *HPP*).

Les travaux de Gershwin *et al.* (2009) ne s’appliquent pas aux systèmes plus larges (plusieurs machines et/ou produits). L’étude du problème de contrôle des taux de production d’un système manufacturier constitué de plusieurs machines, plusieurs produits a été faite par Gharbi et Kenne (2003). Leur objectif était de minimiser le coût total des pénuries et du stock des produits finis. Les conditions d’optimum ont été définies par les équations d’Hamilton-Jacobi-Bellman (HJB). Ces équations étant difficiles à résoudre dans le cas de plusieurs produits, les auteurs ont utilisé la combinaison d’une approche analytique et d’une approche de simulation basée sur les plans d’expérience pour trouver une approximation de la politique optimale. Les machines de leur système étant flexibles c'est-à-dire que le temps et le coût de réglage pour passer d'un produit à un autre sont négligeables.

Krasik *et al.* (2008) ont fait l'extension du modèle de Gharbi et Kenne (2003) à un système constitué de plusieurs machines identiques en parallèle et fabricant plusieurs types de produits avec coûts de réglage non nuls car d'après leur politique, pour passer d'un produit à un autre, on arrête le fonctionnement de la machine. Leur objectif étant de minimiser les coûts des stocks, de pénuries et de réglage en utilisant la programmation dynamique.

Aucune de ces approches n'a développé un modèle qui intègre simultanément la gestion optimale de production et de maintenance des ressources du système manufacturier. En effet, chaque fois qu'une machine produit une pièce, les pannes sont plus fréquentes. Il est donc nécessaire de faire le contrôle combiné des opérations de production et de la dynamique des machines pour réduire de manière efficace le coût total du système.

1.3.4 Gestion simultanée de la production et de la maintenance des machines

Boukas et Haurie (1990) ont fait l'étude combinée de la maintenance préventive et de la production dans le but d'augmenter la disponibilité du système de production et de réduire le coût total encouru. La théorie de commande optimale a été utilisée par Kenne et Gharbi (1999) pour montrer que l'âge de la machine dépend du taux de production et de la maintenance préventive. Cette étude montre que le nombre de pièces à mettre en stock augmente avec l'âge de la machine car la probabilité pour qu'il y ait une panne sur une machine augmente avec l'âge.

Kenne *et al.* (2007) ont fait l'extension des travaux suscités. Leur modèle a permis de déterminer à quel moment devons-nous effectuer les opérations de maintenance préventive et d'identifier le niveau du stock de sécurité optimal qui minimisent le coût total du système manufacturier. Ils ont utilisé une méthode numérique pour résoudre leur problème afin de trouver les valeurs optimales des paramètres de commande. Les travaux de Dehayem Nodem

(2009) ont permis de déterminer la politique optimale de production, de réparation versus remplacement et d'entretien préventif d'un système manufacturier dans un environnement incertain.

Dans les travaux précédents, les auteurs ne posent pas la question de savoir ce qui se passe lorsque les unités de production sont utilisées à leur vitesse maximale pendant une longue période.

1.3.5 Dégradation des unités de production

Les systèmes manufacturiers soumis à des pannes et réparations aléatoires ont été largement étudiés dans la littérature. Dans les travaux de Kimemia and Gershwin (1983) et Bielecki and Kumar (1988), il a été prouvé que sur un horizon infini, le coût total d'un système décrit par un processus de Markov homogène, est minimisé par une politique à seuil critique; politique selon laquelle la machine fonctionne à sa vitesse maximale jusqu'à ce que le stock de sécurité soit atteint. Si le niveau du stock est supérieur au stock optimal, on ne produit pas. Mais si le stock est égal à sa valeur optimale, on produit au taux de la demande.

Le concept de politique à seuil critique décrit par des processus markoviens a été étendu de plusieurs façons au fil des ans par Tan et Gershwin (2004); Dong-Ping (2009). Quelques travaux ont examiné les processus semi-Markov (Hu et Xiang, 1995; Dehayem *et al.*, 2011; Kazaz et Sloan, 2013). L'hypothèse fondamentale dans le modèle de Dehayem *et al.* (2011) est que le système se détériore avec l'âge et le nombre de pannes. Le problème devient plus complexe si la dégradation de la machine est fonction de sa vitesse de production.

Dans Rishel (1991), il a été prouvé que la politique à seuil critique reste optimale, si et seulement si, la dépendance des taux de pannes du taux de production est une fonction quadratique. De même, l'une des réalisations les plus importantes des travaux de Hu *et al.* (1994) a été de trouver les conditions nécessaires et suffisantes pour l'optimalité de la politique à seuil critique dans le cas d'une seule machine produisant un seul type de pièce, lorsque le taux de pannes dépend du taux de production de la machine. Ils ont montré que les politiques à seuil critique sont optimales si le taux de panne est une fonction linéaire du taux de production. Selon leur analyse, les résultats numériques dans le cas général suggèrent de réduire le taux de production de la machine lorsqu'on approche le seuil critique afin de tenir compte de sa fiabilité. Cette conjecture a été confirmée par les résultats numériques présentés dans Martinelli (2007), où l'auteur a considéré un système de production constitué d'une machine, un produit, soumis à deux taux de panne différents dont le second est inférieur au taux de panne correspondant à sa production maximale. Martinelli (2010) généralise le problème de Martinelli (2007) en considérant une machine avec plusieurs taux de pannes différents; plus précisément, le taux de panne est une fonction croissante et constante par morceau du taux de production.

Dans le paragraphe précédent, les auteurs traitant de la dégradation de la machine avec sa vitesse de production, ont fait l'étude des systèmes de production constitués d'une seule machine fabricant un seul type de produit. Leurs modèles doivent être étendus au cas de multiple-machines et/ou multiple-produits. De plus, compte tenu de la rareté et du coût élevé des matières premières, le respect de l'environnement et les législations sur l'environnement, plusieurs pays ont mis sur pied des lois sur le respect de l'environnement et des taxes sur les émissions des gaz à effet de serre. Beaucoup d'entreprises de nos jours prennent en charge les déchets de leurs produits déjà utilisés ou en fin de cycle de vie. Les travaux développés dans les sections précédentes ne prennent pas en compte la chaine des retours dans leurs modèles.

1.3.6 Structure de la ligne inverse

La chaîne de logistique inverse est composée d'une série d'activités; l'important est la récupération des produits ou composants en fin de vie (Bennekrouf *et al.*, 2010). La figure 1.3 est la structure d'une ligne inverse. Les éléments en gras correspondent à ceux de la ligne inverse de la figure 1.1.

D’après la figure 1.3, la mise en œuvre de la ligne inverse se fait en neuf (9) étapes; (i) Les centres de collecte récupèrent les produits utilisés ou en fin de cycle de vie chez les clients; (ii) Les entrepôts servent d’espace de stockage et de consolidation; (iii) Au niveau des centres de retraitement, les modules sont désassemblés, nettoyés et inspectés afin de décider de leur nouvelle direction; (iv) Les pièces de rechange sont destinées aux clients de seconde main; (v) Les usines de *refabrication* font l’assemblage des nouveaux produits à partir des modules *remanufacturés*; (vi) Si ces derniers ne sont pas suffisants pour l’assemblage du nouveau produit, les modules manquants sont commandés chez des fournisseurs; (vii). Les centres de recyclage; (viii) Les sites de destruction; (ix) Les centres de distribution des produits neufs.

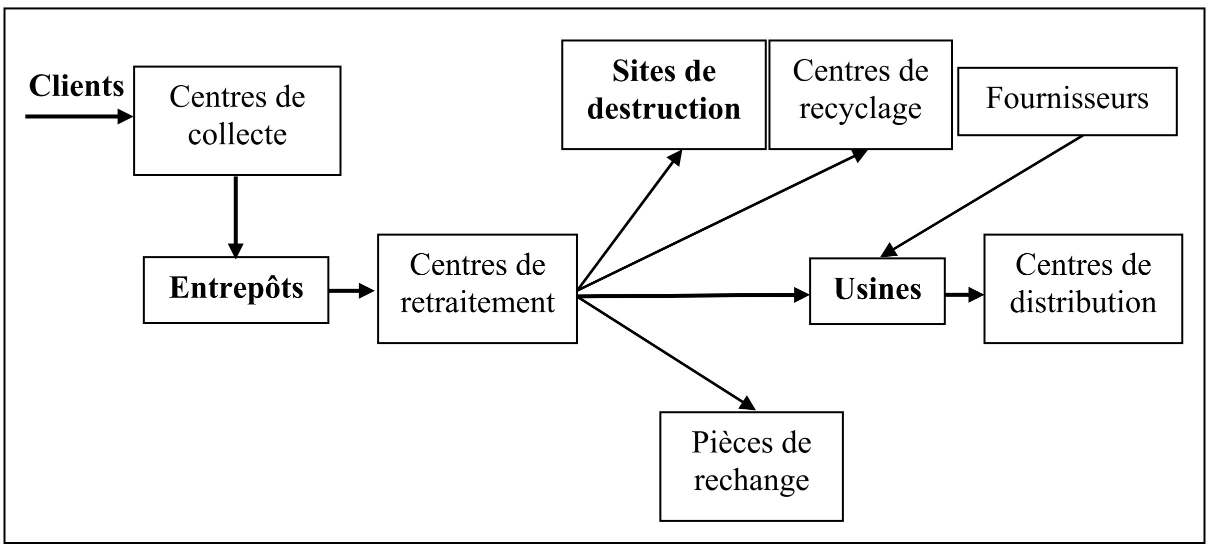

Figure 1.3 Structure de la ligne inverse adaptée de Mutha et Pokharel (2009).

1.3.7 Systèmes hybrides de production/réutilisation des pièces

Bien que l’on ait proposé des modèles depuis les années 1960, les systèmes hybrides de production/réutilisation ont connu une expansion au cours des vingt dernières années avec le respect de l’environnement et les législations sur l’environnement (Mahadevan *et al.,* 2003). La logistique inverse établi la relation entre le marché des produits utilisés et retournés, et le marché des nouveaux produits. Lorsque les deux marchés coïncident, on parle d'un réseau en boucle fermée, sinon d'une boucle ouverte (Salema *et al.*, 2007).

Plusieurs auteurs ont travaillé dans le contexte des systèmes hybrides de production/réutilisation. Van der Laan et Salomon (1997) ont fait l'étude de la stratégie du *Push-disposal* qui consiste à détruire tous les produits collectés lorsque le niveau de stock des produits finis a atteint le seuil fixé. Ils ont aussi étudié le principe du *Pull-disposal* qui consiste à détruire tous les produits collectés lorsque le stock des retours a déjà atteint le seuil fixé. Ils ont démontré que lorsque le flux des produits retournés est inférieur au flux de la demande, le coût total est inférieur au coût total d'un système qui ne tient pas compte des retours détruits. Dans l'approche de Kiesmüller (2003), le processus de *remanufacturing* n'est lancé que lorsqu'on veut satisfaire une demande (principe du *Pull Policy*). Mahadevan *et al.* (2003) ont fait l'étude du principe de P*ush Policy* selon lequel tous les articles retournés sont directement *remanufacturés*; donc pas de stock de retours.

Ces auteurs ne tiennent pas compte de l'état (exemple: fonctionnel, en panne, en réparation) des équipements de production. Pour se rapprocher de la réalité, il est nécessaire de traiter le problème des systèmes hybrides de *manufacturing/remanufacturing* des pièces en tenant compte des aspects stochastiques reliés à la dynamique des machines et à la maintenance des unités de *manufacturing* et de *remanufacturing*.

1.3.8 Systèmes dynamiques de *manufacturing et de remanufacturing*

Le schéma global du système dynamique de *manufacturing*/*remanufacturing* considéré est décrit à la figure 1.4. Le système est constitué de deux centres de production soumis à des phénomènes aléatoires. Les phénomènes aléatoires sont les pannes et les réparations des machines, les activités de maintenance, les retours et les variations de la demande.

Les centres de *manufacturing* et de *remanufacturing* sont constitués des machines, d'unités de stockage des matières premières, du personnel, d'ordinateurs ou tout autre élément mis ensemble pour la fabrication (Gershwin, 1994).

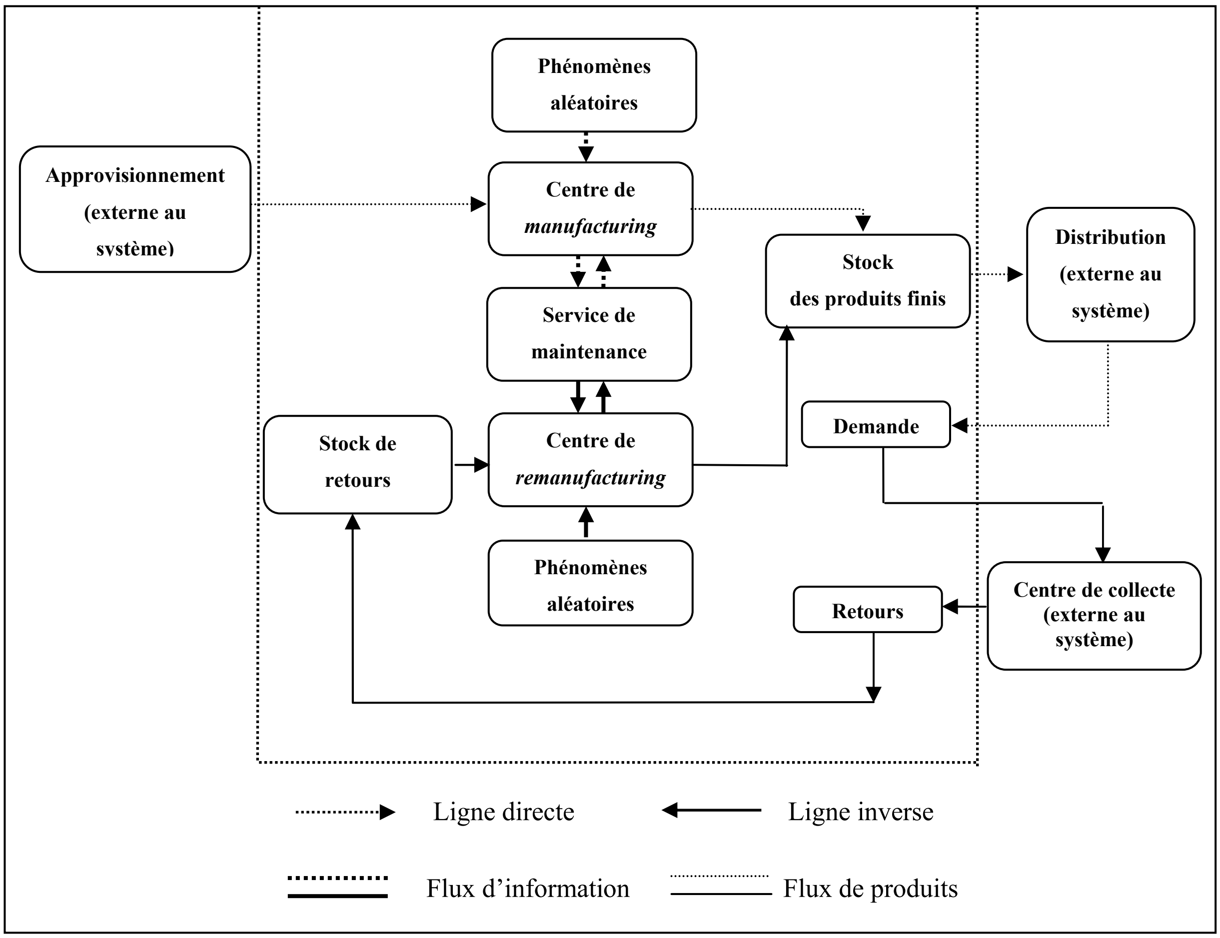

Figure 1.4 Système dynamique de *manufacturing/remanufacturing*

Le stock des produits finis peut être ravitaillé soit par les produits fabriqués à partir de la matière première, soit par *remanufacturing* des pièces retournées. La demande des clients est satisfaite par le stock des produits finis. Le retour des produits est composé des produits en fin de cycle de vie ou déjà utilisés. Les retours ne respectant pas les normes de *remanufacturing* ne sont pas stockés; ils sont détruits (Van der Laan et Salomon, 1997).

De nos jours, peu d'auteurs se sont investis dans l'action combinée du *manufacturing*, du *remanufacturing* et de la maintenance des machines.

Dans les travaux de Gharbi *et al.* (2008), une étude simultanée de planification de la production, des stratégies de remplacement et de réparation de la machine de *remanufacturing* a été proposée. Ils ont utilisé la combinaison de méthodes numériques, la simulation, les plans d'expérience et la méthodologie des surfaces de réponse pour résoudre le problème.

Comme extensions des travaux de Gharbi *et al* (2008), Berthaut *et al.* (2009) et Pellerin *et al.* (2009) ont fait la planification de la production d'un système de *remanufacturing* qui intègre le cas d'indisponibilité des pièces de remplacement. Ils ont supposé que le système de production répond à la demande de chaque pièce de l'équipement en fin de cycle de vie et à la demande imprévue provoquée par une défaillance de l'équipement. Ces auteurs ont formulé leur problème comme une politique de commande à plusieurs niveaux de décision en fonction des seuils critiques du stock. Leur principal objectif de planification de la maintenance et du *remanufacturing* était de maintenir le stock des produits finis au seuil optimal. Cependant, les auteurs ne traitent que du *remanufacturing* sans tenir compte de la ligne directe c'est-à-dire du *manufacturing*.

L'analyse systématique des similitudes et des différences entre le processus de fabrication, de *remanufacturing* et de réparation a été faite par Tongzhu *et al.* (2010). Kenne *et al.* (2012) ont traité la planification de la production d'un système hybride de *manufacturing/remanufacturing* dans un réseau de logistique inverse en boucle fermée. Les machines étant soumises aux pannes et réparations aléatoires. L'objectif était de proposer une politique de *manufacturing* et de *remanufacturing* qui minimise les coûts de mise en stock des produits finis et des pénuries.

Suite à l'analyse de la section précédente, il ressort que l'article de Kenne *et al.* (2012) représente la première tentative dans l'étude des systèmes hybrides de *manufacturing/remanufacturing* avec des machines soumises aux pannes et réparations aléatoires.

La section qui suit est un récapitulatif des modèles mathématiques utilisés par les auteurs suscités.

1.3.9 Modèles mathématiques de la logistique inverse

Les principaux types de modèles mathématiques sont

- les modèles déterministes (la programmation linéaire en nombres entiers et mixte, la programmation non linéaire en temps discret ou continu, état discret ou continu,
- les modèles stochastiques en temps continu ou discret, état discret ou continu ou mixte.

Le tableau 1.1 récapitule les problèmes traités, les types de modèles et les méthodes de résolution présentées dans les revues consultées.

Tableau 1.1 Modèles mathématiques de la logistique inverse

Auteurs	**Problème traité**	**Modèles utilisés**	**Méthodes de résolution**
Van der Laan et Salomon (1997)	Les demandes et les retours suivent une distribution exponentielle	Modèle stochastique continue	Chaîne de Markov
Minner et Kleber (2001)	Coordination de la production lorsque la demande et les retours sont dynamiques	Modèle analytique	Principe du maximum de Pontryagin
Fleischmann (2001)	Gestion des stocks avec retours	Modèle stochastique périodique	Chaîne de Markov
Kiesmüller (2003)	Les demandes suivent une distribution normale et les retours une distribution Gamma	Programmation linéaire	Heuristiques
Mahadevan *et al.* (2003)	Les demandes et les retours sont stochastiques et suivent un processus de Poisson	Modèle stochastique périodique	Heuristiques et Simulation
Min *et al.* (2008)	Quantification des temps d'opérations et des périodes de collecte	Programmation non linéaire en entier mixte à deux échelons	Algorithmes génétiques

Auteurs	Problème traité	Modèles utilisés	Méthodes de résolution
Gharbi *et al.* (2008)	Étude simultanée de la maintenance et du *remanufacturing* avec une capacité limitée	Modèle stochastique continue	Plans d'expérience, Simulation et Surfaces de réponse
Pellerin *et al.* (2009)	Étude d'un système de *remanufacturing* caractérisé par des demandes et des réparations des machines aléatoires.	Modèle stochastique continue	Chaîne de Markov
Kenné *et al.* (2012)	Planification de la production d'un système hybride de *manufacturing/remanufacturing*	Programmation dynamique stochastique	Algorithme basée sur les méthodes numériques

1.3.10 Synthèse de la revue de la littérature

Tableau 1.2 Synthèse de la revue critique de la littérature

<table>
<tr><th rowspan="2">Revues</th><th colspan="4">Domaines étudiés</th></tr>
<tr><th>Production</th><th>Dynamique des machines (opération, panne, maintenance)</th><th>Dégradation des machines et taux de production</th><th>Logistique inverse (recyclage, destruction, remanufacturing, etc.)</th></tr>
<tr><td rowspan="2">Chaîne d'approvisionnement et logistique industrielle</td><td colspan="4">Étude de la chaîne globale allant du fournisseur au client en passant par l'entreprise, les sous-traitants et les distributeurs</td></tr>
<tr><td colspan="4">Pas de modèle universel à cause de la diversité des paramètres</td></tr>
<tr><td>Stratégies de maintenance</td><td></td><td>✓</td><td></td><td></td></tr>
<tr><td>Optimisation de la production</td><td>✓</td><td></td><td></td><td></td></tr>
<tr><td>Gestion de la production et de la maintenance</td><td>✓</td><td>✓</td><td></td><td></td></tr>
<tr><td>Dégradation du système et taux de production</td><td>✓</td><td>✓</td><td>✓</td><td></td></tr>
<tr><td>Chaine de logistique inverse</td><td></td><td></td><td></td><td>✓</td></tr>
<tr><td>Production / réutilisation des pièces</td><td>✓</td><td></td><td></td><td>✓</td></tr>
<tr><td>Manufacturing / remanufacturing et maintenance des machines</td><td>✓</td><td>✓</td><td></td><td>✓</td></tr>
</table>

Le tableau 1.2 donne une synthèse de notre revue de la littérature. Ce tableau montre que la revue de la littérature sur les stratégies de maintenance traite le domaine de la dynamique des machines sans tenir compte conjointement de la production, de la logistique inverse et de la dégradation des machines en fonction de leur taux d'utilisation. De même, les auteurs qui ont travaillé sur la gestion de la production et de la réutilisation des produits usagers n'ont pas étudié la dégradation des unités de production en fonction de leur vitesse d'utilisation. Les contributions de cette thèse permettront d'intégrer tous ces aspects dans des modèles continus d'optimisation conjointe de la production, de la disponibilité des machines, de la réutilisation et de la maintenance en contexte dynamique et stochastique.

Cette revue sans être totalement exhaustive visait à couvrir l'aspect général des approches rencontrées dans la littérature. Elle aura permis de prendre connaissance des forces et des faiblesses des méthodes proposées. En intégrant simultanément tous les aspects du tableau 1.2 dans un même modèle, le problème d'optimisation devient complexe et nous proposons à la section suivante la méthodologie pour le résoudre.

1.4 Méthodologie proposée

Dans cette section, nous détaillons la méthodologie adoptée pour l'atteinte des objectifs incluant les outils nécessaires pour la réalisation de ce projet de recherche.

La méthodologie proposée comprend quatre étapes :

1. Revue critique de la littérature. Nous commençons par une revue critique de la littérature sur les modes de fabrication intégrant la production, la dégradation des systèmes et la logistique inverse. Cette étape permet de situer notre travail de recherche par rapport à l'ensemble des travaux et d'en ressortir l'originalité de la thèse.

2. Théorie de commande optimale stochastique. Nous formulons des modèles mathématiques qui minimisent le coût total du système (coûts de stockage des produits finis et des produits retournés, de pénuries, de maintenance préventive et corrective des machines) dans un contexte de production intégrant la réutilisation et la

dégradation des machines de production en fonction de leur taux d'utilisation. Les processus des demandes et des retours sont déterministes, les pannes et les réparations des machines sont aléatoires.

3. Résolution des conditions d'optimum. Nous développons des approches de résolution des conditions d'optimum des équations d'Hamilton-Jacobi-Bellman obtenues en nous basant sur les méthodes numériques itératives.

4. Simulation, analyse des résultats, extensions et rédaction d'articles scientifiques. Nous appliquons dans cette étape les algorithmes de résolution développés à l'étape 3 sur des exemples tirés de la littérature. Des analyses de sensibilité sont faites pour confirmer les structures des politiques obtenues. Pour valider les modèles développés, nous visons quelques domaines industriels tels que les usines de fabrication des pièces mécaniques et des cartouches d'imprimantes.

Le schéma global de notre méthodologie est présenté à la figure 1.5

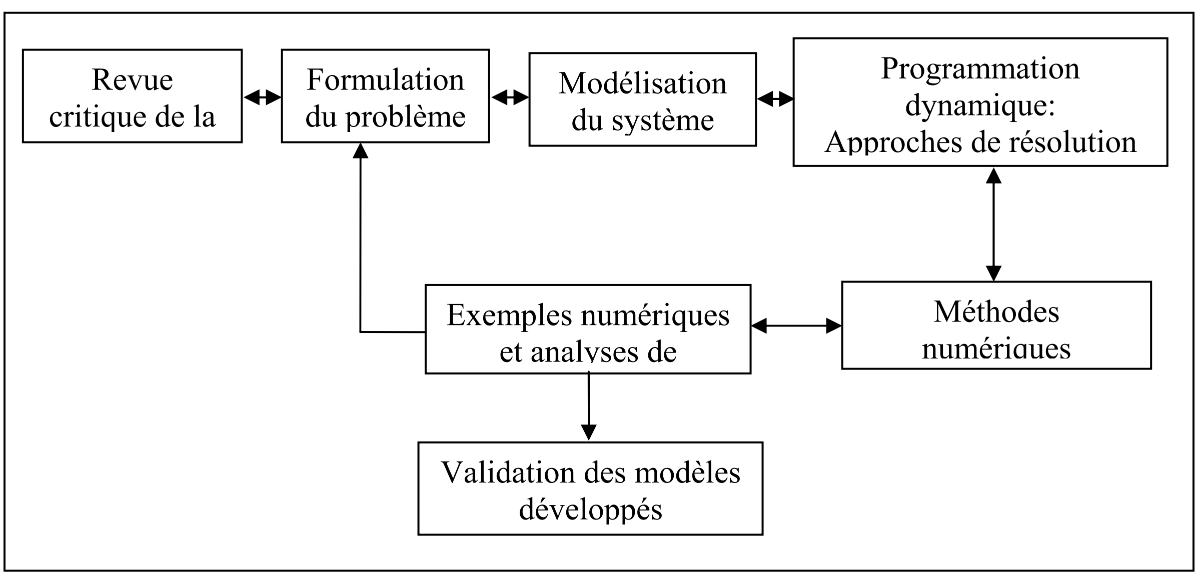

Figure 1.5 Méthodologie proposée

La section qui suit présente les contributions et la structure de la thèse.

1.5 Contributions et structure de la thèse

Cette thèse a fait l'objet de quatre (4) articles de revues et la participation à huit (8) conférences avec comité de lecture. Ces dernières sont récapitulées en annexe

1.5.1 Articles de revues

Les articles de revues sont présentés dans les quatre (4) prochains chapitres. La figure 1.6 récapitule le sujet traité dans chaque article.

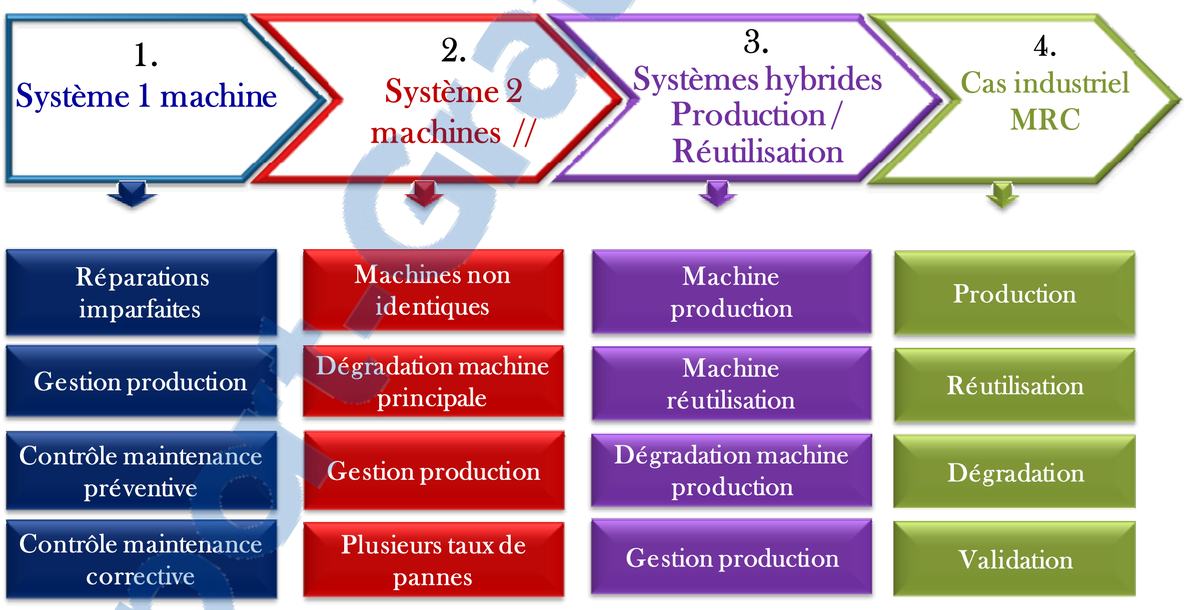

Figure 1.6 Sujets traités

L'article du chapitre deux (2) présente un modèle de l'optimisation conjointe de la production et des stratégies de maintenance (préventive et corrective) d'un système manufacturier soumis à des réparations imparfaites. Le système est constitué d'une machine produisant un seul type de pièce. Les variables de décision sont le taux de production, le taux de maintenance préventive et le taux de maintenance corrective. Cet article a été resoumis dans la revue International Journal of Advanced Manufacturing Technology sous la référence:

Kouedeu, A. F., J. P. Kenne, P. Dejax and Songmene, V. 2014. “Production and maintenance planning for a failure prone deteriorating manufacturing system: a hierarchical control approach”. International Journal of Advanced Manufacturing Technology. Resoumis en Janvier 2014. Confirmation de soumission : JAMT-D-14-00169.

Dans l’article du chapitre trois (3), une deuxième machine est ajoutée en parallèle à la machine du chapitre 2. Nous faisons donc l’étude de la planification de la production d’un système manufacturier constitué de deux machines non-identiques en parallèle produisant un seul type de pièce. Le taux de panne de la machine principale dépend de son taux de production. Une modélisation a été faite en utilisant une chaîne de Markov non-homogène. Les variables de décision sont les taux de production des machines. Cet article est publié dans la revue International Journal of Production Economics sous la référence :

Kouedeu, A. F., J. P. Kenne, P. Dejax, V. Songmene and Polotski, V. 2014. “Stochastic optimal control of manufacturing systems under production-dependent failure rates”. International Journal of Production Economics 150C (2014), pp. 174-187: 10.1016/j.ijpe.2013.12.032.

Dans l’article du chapitre quatre (4), la seconde machine du chapitre 3 est considérée comme une machine de *remanufacturing* (réutilisation des pièces récupérées sur le marché). Ainsi, nous faisons une étude combinée des systèmes hybrides de *manufacturing/remanufacturing* avec une dégradation de la machine de *manufacturing* en fonction de son taux d’utilisation. Les machines sont sujettes aux pannes et des réparations aléatoires. Les variables de décision sont les taux de production des machines. Ces variables influencent les stocks des produits finis et des retours. Cet article a été accepté dans la revue Applied Mathematics sous la référence:

Kouedeu, A. F., J. P. Kenne, P. Dejax, V. Songmene and Polotski, V. 2013. “Production planning of a failure-prone manufacturing/remanufacturing system with production dependent failure rates”. Applied Mathematics (October 2013), AM7401876.

L'article du chapitre cinq (5) est la validation des modèles développés dans le chapitre 4 sur les compagnies fabricant des cartouches compatibles, Laser et Jet d'encre, neuves et *remanufacturées*. Cet article a été soumis au N° Spécial du Journal of Manufacturing Systems on "Reverse supply chains" de la revue Journal of Manufacturing Systems, édité par O. Battaia et S.M. Gupta, sous la référence:
Kouedeu, A. F., J. P. Kenne, P. Dejax, V. Songmene and Polotski, V. 2014. "Stochastic models and numerical solutions for manufacturing/remanufacturing systems with applications to the printer cartridges industry". Journal of Manufacturing Systems. Soumis en Janvier 2014. Confirmation de soumission : SMEJMS-D-14-00024.

Nous terminons ce chapitre par les retombées et l'impact industriel de ce travail de recherche.

1.6 Retombées et impact industriel

Les applications des résultats de notre recherche pouvant être, sous certaines hypothèses et extensions de modèles:

- Les lignes d'assemblage d'automobiles et des aéronefs: proposition des politiques optimales de production et de maintenance des unités de production.
- Les usines de fabrication des pièces mécaniques: optimisation conjointe de la production des pièces mécaniques, de la disponibilité et de la fiabilité des outils de coupe.
- Les usines de fabrication des cartouches d'imprimantes: optimisation conjointe des politiques du *manufacturing* et du *remanufacturing* des produits, de la disponibilité et de la fiabilité des unités de production.
- Les autres systèmes de production et/ou de réutilisation dont la configuration correspond à celle des modèles proposés dans cette thèse.

D'ores et déjà, l'application que nous avons faite de nos travaux dans le contexte de l'industrie des *manufacturing* et r*emanufacturing* de cartouches d'encres d'imprimante avec la coopération d'une entreprise française semble très réaliste et donne des résultats encourageants pour la validation de nos travaux et leur mise en œuvre dans un contexte industriel.

Pour clôturer, nous dresserons en guise de conclusion, le récapitulatif des principales contributions de ce travail et nous présenterons nos travaux futurs.

CHAPITRE 2

ARTICLE 1: PRODUCTION AND MAINTENANCE PLANNING FOR A FAILURE PRONE DETERIORATING MANUFACTURING SYSTEM: A HIERARCHICAL CONTROL APPROACH

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Résumé

Le travail présenté dans cet article est l'étude de l'optimisation conjointe de la production et des stratégies de maintenance (préventive et corrective) d'un système manufacturier soumis à des réparations imparfaites. Le système est constitué d'une machine produisant un seul type de pièce. Suite aux défaillances du système, des réparations imparfaites sont effectuées. Ainsi, le taux de défaillance croit avec le nombre de pannes. Une approche hiérarchique de prise de décision permettant au premier niveau de déterminer le taux de panne de la machine et au second niveau les politiques de production, de maintenance préventive et corrective est utilisée. Le problème d'optimisation est résolu par des méthodes numériques. Et pour illustrer l'utilité de nos résultats, une analyse de sensibilité a été faite.

Mots-clés: Réparations imparfaites, Systèmes manufacturiers flexibles, Production, Politiques de maintenance, Méthodes numériques.

Abstract

The work presented in this paper examines the joint analysis of the optimal production and maintenance planning policies for a manufacturing system subject to random failures and repairs. When a machine fails, an imperfect corrective maintenance is undertaken. The objective of this study is to minimize a discounted overall cost consisting of preventive and corrective maintenance costs, inventory holding cost and backlog cost. A two-level hierarchical decision making approach is proposed, based on the determination of the mean time to failure (first level) and the statement of a joint optimization of production, preventive and corrective maintenance policies (second level). Hence the production, preventive and corrective maintenance rates are determined in the second level, given the failure rates obtained from the first level. In the proposed model, the machine's failure rate depends on the number of failures, and as a result, the control policies of the considered planning problems therefore depend on the number of failures. The structure of the optimal control policies and the usefulness of the proposed approach are illustrated through a numerical example and a sensitivity analysis.

Keywords: Imperfect repairs, Flexible manufacturing systems, Production rate, Maintenance policies, Numerical methods.

2.1 Introduction

The quality of a manufacturing system's design and the maintenance actions undertaken during its operation (production activities) are crucial factors determining its reliability. This paper models and illustrates the control problem of a stochastic manufacturing system. The stochastic nature of the system is due to the fact that the machine is subject to random breakdowns and repairs. The machine produces one part type; when one of the machine's components fails, an imperfect corrective maintenance action is undertaken. Here, the machine dynamics is assumed to be described by a finite-state semi-Markov chain. The decision variables are the production rate, the preventive maintenance rate, and the corrective maintenance rate, which influence the system's availability and the stock level. Many authors have contributed to the production planning and maintenance policies of manufacturing systems without considering the failure rates, depending on the number of failures, and the simultaneous control of production, preventive and corrective maintenance rates in the same model.

Based on the work of Rishel (1975) on production planning for a system affected by jump disturbances, Boukas and Haurie (1990) combined production and preventive maintenance planning in cases where the machine's failure probability increases with its age, using the hedging point policy concept introduced by Kimemia and Gershwin (1983). For more details on this concept, we refer the reader to the age-dependent hedging point concept presented by Boukas (1998); Gharbi and Kenne (2000). Boukas and Haurie (1990) determined production rate and maintenance rules which minimize the total expected cost of a two-machine system over infinite horizon. However, with the numerical scheme adopted in their work, it remains computationally difficult to realize optimal control of a large scale manufacturing system. To cope with this difficulty, Kenne and Boukas (2003) formulated a hierarchical control problem based on production and preventive maintenance planning in manufacturing systems, and obtained a limiting problem that was numerically more tractable. Gharbi and Kenne (2005) extended this approach to cover a large case of non-identical machine manufacturing systems. The paper of Zied et al. (2011) investigated the result of a general class of stochastic production planning and maintenance scheduling problems via optimal procedure. The objective was to satisfy economically a random demand under some constraints like random failure rate and a subcontracting constraint. The manufacturing system considered was prone to random failures. Minimal repairs were adapted at every failure. So as to reduce the failure frequency, preventive maintenance actions were programmed according to the production rate.

Many systems deteriorate with age, and are subject to stochastic random failures. This degradation may result in higher operating costs and less competitive products, thus making maintenance action highly essential (Yan et al., 2004). Conventional maintenance policies assume that the system is restored after repair or preventive maintenance activities, making it as good as new (see Boukas and Haurie, 1990; Kenne and Boukas, 2003 and Kenne et al., 2007) or as bad as an old machine (Nakagawa and Kowada, 1983). The main limitation of these models is that they take into account only extreme maintenance actions (perfect or minimal), and do not consider the real efficiency of repairs, which can significantly improve the state of the system without returning it to an *as good as new* condition. Such a repair is called an *imperfect repair*. The level of such repair is known as the *intensity of repair*, as in Dehayem et al. (2009). The repair intensity could reflect the impact of the k repairs and be a function of the k^{th} repair, as described in Love et al. (2000), or be stochastic, as in Kijima (1989). This would depend on the quality of intervention performed and the skill level of the maintenance team as well as the number and nature of the components repaired (see Shin et al., 1996). However, little has been done in terms of developing a model taking into account the case where this factor is stochastic (Mohafid and Castanier, 2006). The repair intensity used to model the effectiveness of maintenance action undertaken is assumed to be known and constant in deterministic cases.

In Kijima (1989), the author proposed that upon a failure, the repair undertaken could serve to reset the age of the machine only as far back as its age at the start of the last failure, called the virtual age. In the literature, this repair model is called Kijima's Type I imperfect repair model, and it has largely been used in cumulative damage models. The virtual age is equal to or less than the real age, as in Dehayem et al. (2009). Dehayem et al. (2009) extended Kijima’s Type I imperfect repair model; they determined the production rate and the repair/replacement policy that minimizes the total expected cost when the system deteriorates with age, and is subject to damage failures. Jiwen and Lifeng (2011) modeled and analyzed various maintenance policies by incorporating the economic effects of maintenance actions, product deviation-related quality loss and tool obsolescence cost. They provided a comparative analysis of various maintenance policies using the long-term average cost criterion and employed a quadratic loss function to characterize the cost resulting from the deviation of part dimension from its target value.

The main contribution of this work consists in its joint analysis of the optimal production and maintenance (preventive and corrective) planning problems for a manufacturing system under uncertainties and imperfect repairs, when the failure rate increases with the number of failures. Following a preventive maintenance activity, the machine is as good as new. The proposed hierarchical approach involves developing a model in which, at the first level, the parameters of the stochastic machine failure process are derived for each number of failures; at the second level, the optimal production, preventive and corrective maintenance policies are determined for a system that deteriorates with the number of failures. The production and maintenance rates are obtained for the system by minimizing inventory, backlog, preventive and corrective maintenance costs over an infinite planning horizon. The formulation, the approaches, and the numerical procedures used in this paper could possibly be applied to production planning in many industries, where resources can be subject to random failures and their production rates can also be controlled. The phenomenon has been experienced in machinery and mechanical assemblies, including at automobile, aircraft engine and machine tools, and paper manufacturing plants. Yin et al. (2003) obtained the optimal production policies of the paper manufacturing machine, for different machine capacity and demand processes.

The rest of this paper is organized as follows: Notations and assumptions are presented in Section 2.2. Section 2.3 presents the model of the problem under consideration. The optimality conditions described by the Hamilton-Jacobi-Bellman (HJB) equations and the numerical approach to solve the (HJB) equations obtained are presented in Section 2.4. In Section 2.5, a numerical example and results are presented; sensitivity analyses are presented to illustrate the usefulness of the proposed approach in Section 2.6. Section 2.7 presents and discusses some extensions. Finally, the paper is concluded in Section 2.8.

2.2 Notations and assumptions

This section presents the notations and assumptions used throughout this article.

2.2.1 Notations

k	number of failures
$u(\cdot)$	production rate (products/time unit)
$u_{\max}$	maximal production rate (products/time unit)
x^{+}	inventory (products)
x^{-}	backlog (missing products)
d	demand rate (products/time unit)
c^{+}	inventory cost (\$/product/time unit)
c^{-}	backlog cost (\$/missing product/time unit)
c_r	corrective maintenance cost (\$)
c_m	preventive maintenance cost (\$)
$\omega_r^{\min}$	minimal corrective maintenance rate
$\omega_r^{\max}$	maximal corrective maintenance rate
$\omega_m^{\min}$	minimal preventive maintenance rate
$\omega_m^{\max}$	maximal preventive maintenance rate
$\lambda_{\alpha\beta}$	transition rate from state α to β
Q	transition rate matrix
π	vector of limiting probabilities
$g(\cdot)$	instantaneous cost function
$J(\cdot)$	total cost (\$/time unit)
$v(\cdot)$	value function
ρ	discount rate

2.2.2 Assumptions

The following assumptions are made in this paper.

1. The failure rate increases with the number of failures of the machine.
2. The lifetime of the machine decreases after each breakdown.
3. Corrective maintenance activities are imperfect.
4. Preventive maintenance activities are perfect.
5. Corrective and preventive maintenance activities are controlled (minimal and maximal rates).

Assumptions 1, 2, 3 and 5 are the major motivations of our approach. Other works often consider that the failure rate is constant and the corrective maintenance activities restore the machine as good as new state.

6. The customer demand is known and subject to a constant rate over time.

This assumption is common to deterministic demand models.

7. The maximal production rate of the machine is known.

This assumption is common in production planning.

8. The backlog cost depends on the shortage quantity and time (average value ($/product/unit of time)).
9. The holding cost depends on the mean inventory level (average value ($/product/unit of time)).

Assumptions 8 and 9 are common in inventory models.

2.3 Problem statement

The manufacturing system considered consists of a single machine which produces one part type. This machine is subject to random breakdowns and repairs. Its state can be classified as operational, denoted by 1, under repair, denoted by 2, and under preventive maintenance, denoted by 3. Let $\xi(t)$ denote the state of the machine with value in $B=\{1,2,3\}$. The dynamics of the machine is described by a continuous time semi-Markov process, with a transition rate from state α to state β denoted by $\lambda_{\alpha\beta}$ with $\alpha,\beta \in B$. The transition diagram, describing the dynamics of the machine considered is presented in Figure 2.1.

The repairable systems concerned in this paper are complex and consisting of several components or subsystems. The failure of a component causes system failure and replacing or repairing the faulty component lead the system in operating state. Failure rate is deterministic and results from all interactions between the unities constituting the system.

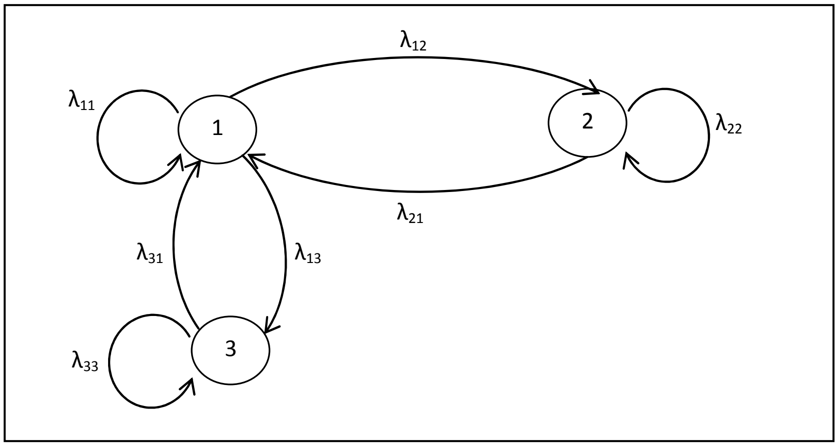

Figure 2.1 States transition diagram of the considered system

Let T, a non-negative random variable, be the first failure time of a system with continuous density function $f(t)$

We assume that before the first failure, the failure rate λ_0 (also called the initial intensity) is a continuous function of time strictly increasing and deterministic.

The failure rate of the global system at time t, over a finite planning horizon, is described by a Weibull distribution with two parameters μ and η. The density function and the failure rate function before the first failure are given by:

$$f(t) = \mu\eta^{-1}\left(\frac{t}{\eta}\right)^{\mu-1}\exp\left(-\left(\frac{t}{\eta}\right)^{\mu}\right) \text{ and } \lambda_0(t) = \frac{\mu}{\eta}\left(\frac{t}{\eta}\right)^{\mu-1} \tag{2.1}$$

The plot of $f(t)$ at time t is presented in figure 2.2 for $\mu = 3$ and $\eta = 500$.

The Weibull law is often used in maintenance due to its flexibility to model survival times of systems and its ability to characterize their wear level through its shape parameter μ.

After system failures, imperfect repairs (between “as good as after the previous overhaul” and “as good as before the overhaul or repair”) are performed to repair or replace the faulty component.

The Kijima virtual age of the failed system is adjusted by a factor that reflects the degree of repair so as to bring it to a desired state somewhere between as good as new and as bad as old. The repair efficiency (improvement factor) θ is a value between 0 and 1. θ equal to 1 indicates that the component is repaired to a condition that is as good as new, while an θ equal to 0 indicates that no rejuvenation takes place after the maintenance action (minimal repair).

In our study, repair efficiency used to model the effectiveness of maintenance actions, is assumed known and deterministic, it can be estimated by the maximum likelihood method based on operation data as in Shin et al. (1996) and Doyen and Gaudoin (2004).

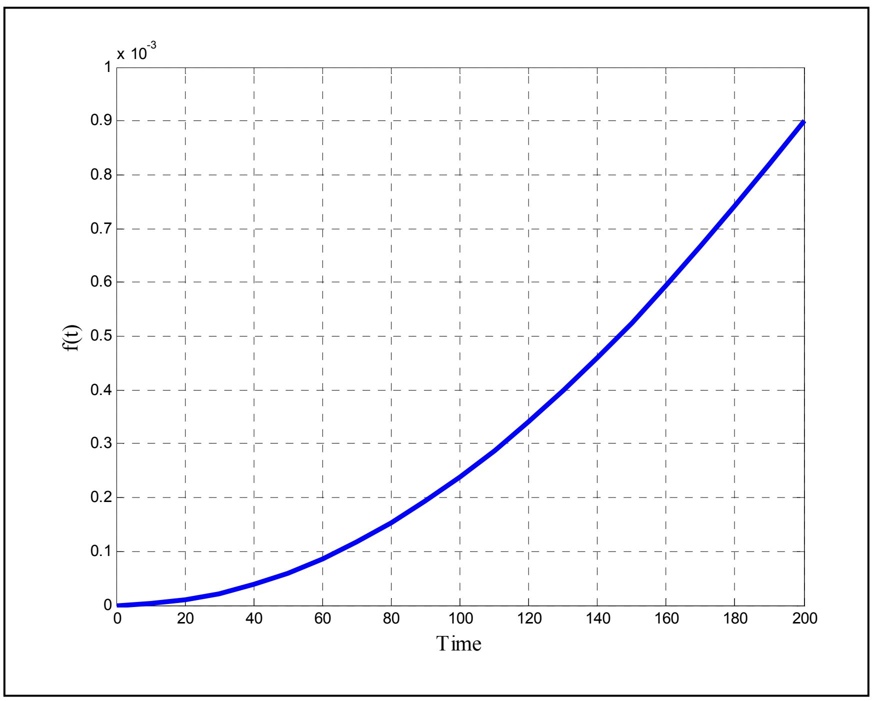

Figure 2.2 The density function $f(t)$

The behavior of the failure rate of the global system at time t according to the random failures, over a finite planning horizon, is presented in Figure 2.3 for $\mu = 3$, three values of θ (i.e. $0, 0.4, 0.7$) and for $\eta = 500$.

Let $\{T_k\}$ $(k \geq 1)$ be the successive failure times of a repairable system, starting from $T_0 = 0$
The failure rate between the k^{th} and $(k+1)^{th}$ repair is given by :

$$\lambda_k(t) = \lambda_{k-1}\left(t - \theta.(t_k - t_{k-1})\right)$$

When all the t_k times are known and considering the conditional distributions of successive inter-failure times, this failure rate becomes:

$$\lambda_k(t) = \lambda_0(t - \theta.t_k) \tag{2.2}$$

where λ_0 is the initial intensity.

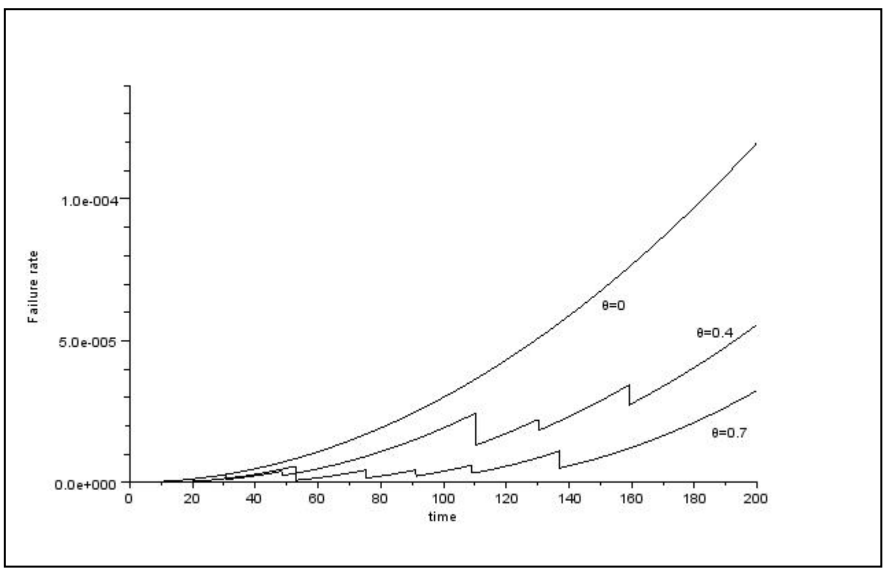

Figure 2.3 Behavior of the failure rate for imperfect repairs

Let $F(t) = \int_0^t f(s)ds$ be a distribution function of the failure time T – defined above random variable with the density function $f(t)$. Let us also define a survival function (or Reliability) $R(t) = 1 - F(t)$ verifying that $\lim_{t \to \infty} t.R(t) = 0$, and $R(t) > 0$.

Consider a reparable system that is put into operation at time $t=0$ and is still functioning after the time of maintenance repair t_k. The probability that this item of age t_k survives an additional interval of length t is:

$$R(t\,|\,t_k)=P(T>t+t_k\,|\,T>t_k)=\frac{R_k(t+t_k)}{R_k(t_k)} \text{ and } \lambda_0(t)=\frac{\mu}{\eta}\left(\frac{t}{\eta}\right)^{\mu-1} \tag{2.3}$$

The expected remaining lifetime (mean time to failure : $MTTF=E_k$) after time t_k, given that the system has survived after t_k is:

$$\mathrm{E}_k(T)=\mathrm{E}(t-t_k|\,t>t_k)=\int_0^\infty P(T>t+t_k\,|\,T>t_k)\,dt \tag{2.4}$$

$$E_k(T)=\int_0^\infty \frac{R_k(t+t_k)}{R_k(t_k)}dt=\frac{1}{R_k(t_k)}\int_0^\infty R_k(t+t_k)dt=\frac{1}{R_k(t_k)}\int_{t_k}^\infty R_k(t)dt$$

When lifetimes are distributed according to Weibull model

$$\mathrm{E}_k(T)=e^{(\psi_k)\mu}\left(\eta\int_0^\infty e^{-t}t^{\frac{1}{\mu}}dt-\psi_k\right) \text{ where } \psi_k=\tfrac{(1-\theta)t_k}{\eta}$$

$$\mathrm{E}_k(T)=e^{(\psi_k)\mu}\left(\eta\Gamma\left(1+\frac{1}{\mu}\right)-\psi_k\right) \text{ where } \Gamma(y)=\int_0^\infty e^{-t}t^{y-1}dt \tag{2.5}$$

Table 2.1 Values of t_k

k	1	2	3	4	5	6	7	8	9	10
t_k	0	642	689	874	957	1020	1256	1270	1307	1385
k	11	12	13	14	15	16	17	18	19	20
t_k	1388	1394	1454	1559	1562	1577	1582	1623	1648	1662

The mean time to failure of the machine for each value of incurred number of failures is described in Figure 2.4, with values of $\mu=3$, $\eta=1000$, $\theta=0.4$. The values of t_k used to generate figure 2.4 are given in table 2.1.

The hierarchical approach proposed in this paper consists of a second level determination of the production and maintenance rates of the system, given the failure rate obtained at the first level. Figure 2.5 presents the different levels:

- Level 1: Determination of the mean time to failure according to equation (2.5).
- Level 2: Joint determination of optimal production, preventive and corrective maintenance policies.

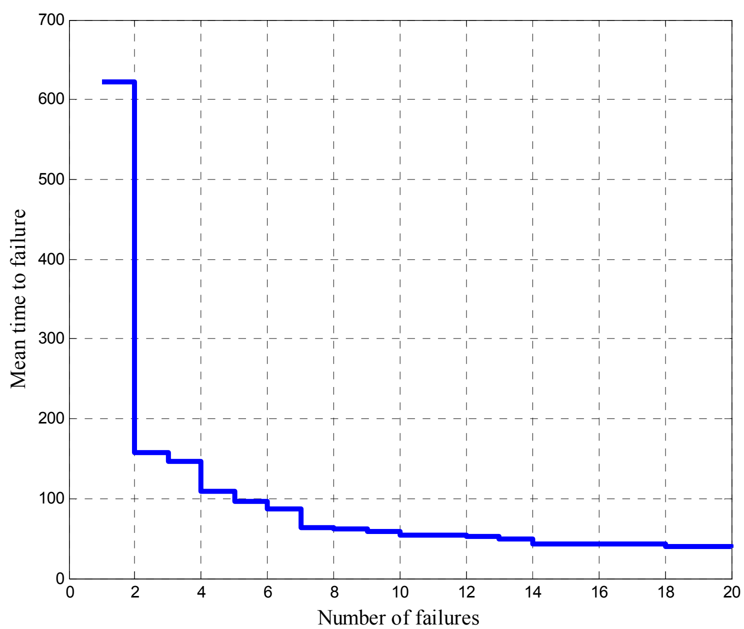

Figure 2.4 Mean time to failure of the machine

The system capacity is increased by controlling the transition rate from node 1 to 3 (preventive maintenance) and from node 2 to 1 (corrective maintenance). Hence, the transition rates matrix Q depends on ω_m and ω_r, defined as preventive and corrective maintenance rates, respectively. For the considered system, the corresponding 3×3 transition matrix Q is one of an ergodic process. Hence, $\xi(t)$ is described by the matrix $Q=\left[\lambda_{\alpha\beta}\right]$, where $\lambda_{\alpha\beta}$ verifies the following conditions:

$$\lambda_{\alpha\beta}(k,\omega_m,\omega_r)\geq 0 \quad (\alpha\neq\beta) \tag{2.6}$$

$$\lambda_{\alpha\alpha}(k,\omega_m,\omega_r) = -\sum_{\beta\neq\alpha}\lambda_{\alpha\beta} \tag{2.7}$$

The transition probabilities are given by:

$$\mathrm{P}\left[\xi(t+\delta t)=\beta \middle| \xi(t)=\alpha\right] = \begin{cases} \lambda_{\alpha\beta}(.)\,\delta t + o(\delta t) & if\,\alpha\neq\beta \\ 1+\lambda_{\alpha\beta}(.)\,\delta t + o(\delta t) & if\,\alpha=\beta \end{cases} \tag{2.8}$$

with $\lim\limits_{\delta t\to 0}\dfrac{o(\delta t)}{\delta t}=0$ for all $\alpha,\beta\in B$.

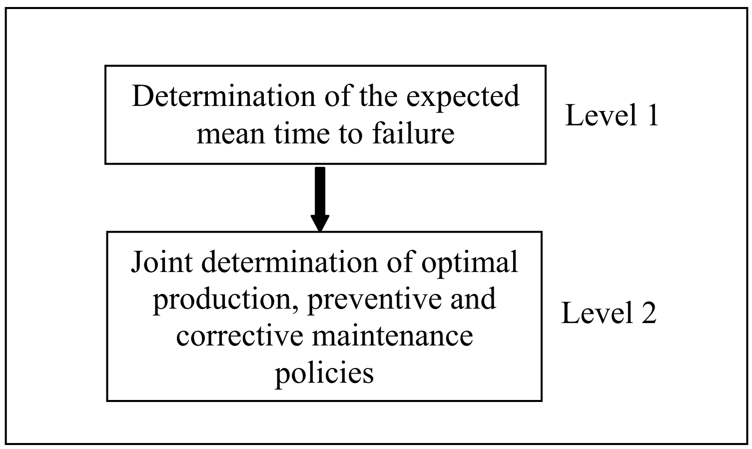

Figure 2.5 Hierarchical control approach

A hybrid state comprising both a discrete and a continuous component, describes the system behavior. The discrete component consists of the discrete stochastic process $\xi(t)$ and the continuous component is the stock level defined later in this section. Let $u(x,k,\alpha,t)$ denote the production rate of the machine in mode α and at time t for a given stock level x and a given number of failures k. The set of the feasible control policies $\mathrm{A}(\alpha)$, including $u(\cdot)$, $\omega_m(\cdot)$ and $\omega_r(\cdot)$ is given by:

$$\mathrm{A}(\alpha) = \begin{Bmatrix} \left(u(\cdot),\omega_m(\cdot),\omega_r(\cdot)\right)\in\Re^3, 0\le u(\cdot)\le u_{\max}, \\ \omega_r^{\min}\le\omega_r(\cdot)\le\omega_r^{\max}, \omega_m^{\min}\le\omega_m(\cdot)\le\omega_m^{\max} \end{Bmatrix} \tag{2.9}$$

where $u(\cdot)$, $\omega_m(\cdot)$ and $\omega_r(\cdot)$ are known as control variables, and constitute the control policies of the problem under study, $u_{\max}$ is the maximal production rate, $\omega_m^{\min}$ and $\omega_m^{\max}$ are the minimal and maximal preventive maintenance rates, and $\omega_r^{\min}$ and $\omega_r^{\max}$ are the minimal and maximal corrective maintenance rates, respectively ($\xi(t)=\alpha$ in equation (2.9)). The transition rates $\lambda_{\alpha\beta}(k,\omega_m,\omega_r)$ of the machine after the k^{th} failure from mode $\xi(t)=\alpha;\quad \alpha\in B$ to mode $\xi(t)=\beta;\quad \beta\in B$ at instant t are defined by:

$$\lambda_{12}(k,\omega_m,\omega_r)=\frac{1}{E_k(T)}=\lim_{\delta t\to 0}\left[\frac{1}{\delta t}\left(P\left[\xi(t+\delta t)=2\,|\,\xi(t)=1)\right]\right)\right] \tag{2.10}$$

$$\lambda_{13}(k,\omega_m,\omega_r)=\omega_m(\cdot)=\lim_{\delta t\to 0}\left[\frac{1}{\delta t}\left(P\left[\xi(t+\delta t)=3\,|\,\xi(t)=1)\right]\right)\right] \tag{2.11}$$

$$\lambda_{21}(k,\omega_m,\omega_r)=\omega_r(\cdot)=\lim_{\delta t\to 0}\left[\frac{1}{\delta t}\left(P\left[\xi(t+\delta t)=1\,|\,\xi(t)=2)\right]\right)\right] \tag{2.12}$$

The behavior of the failure rate $\lambda_{12}(k,\cdot)$ is shown in Figure 2.6.

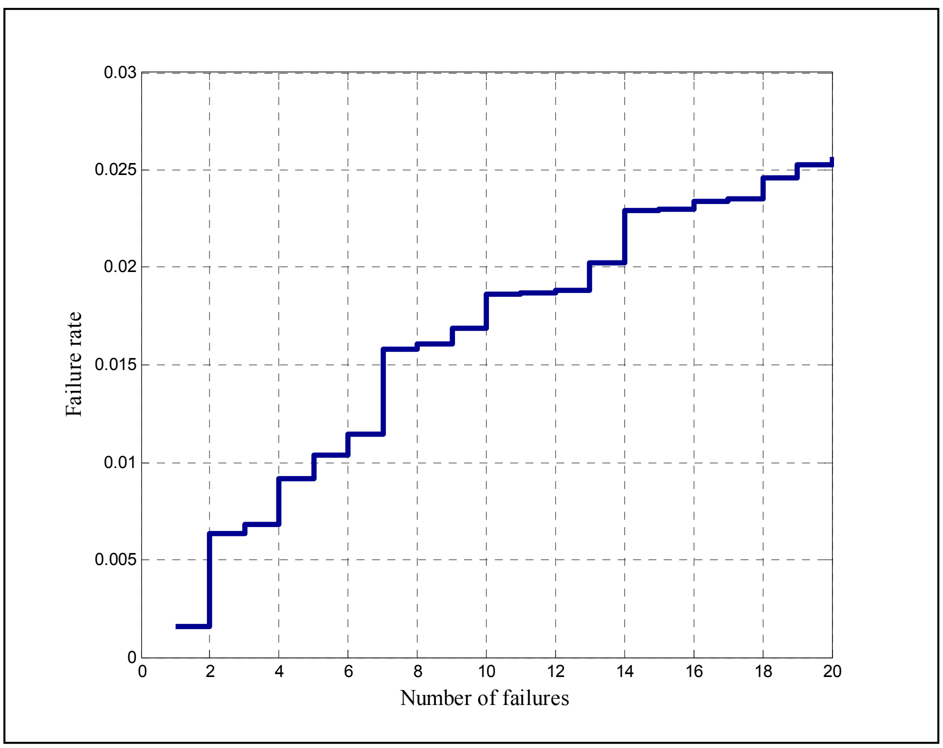

Figure 2.6 Failure rate of the machine

While the machine is submitted to preventive or corrective maintenance, the production has to be stopped. Then, the surplus (stock level) could be positive (inventory), or negative (backlog).

The stock level is given by the state equation:

$$\frac{dx(t)}{dt} = u(t) - d, \qquad x(0) = x_0 \tag{2.13}$$

where x_0 and d are given initial surplus and demand rate, respectively.

Let $g(\cdot)$ be the cost rate defined as follows:

$$g(\alpha, x, \cdot) = c^+ x^+ + c^- x^- + c_m \omega_m Ind\{\alpha = 1\} + c_r \omega_r Ind\{\alpha = 2\} \tag{2.14}$$

with $Ind\{\Theta(\cdot)\} = \begin{cases} 1 & \text{if } \Theta(\cdot) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$

for a given proposition $\Theta(\cdot)$. The constants c^+, c^-, c_m and c_r are used to penalize inventory, backlog, preventive and corrective maintenance, respectively, $x^+ = \max(0, x)$, $x^- = \max(-x, 0)$.

The objective here is to control the production rate $u(\cdot)$, the preventive and the corrective maintenance rates $\omega_m(\cdot)$, and $\omega_r(\cdot)$, respectively, in order to minimize the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha, x, k, u, \omega_m, \omega_r) = E\left\{\int_0^\infty e^{-\rho t} g(\alpha, x, \cdot) dt \,\middle|\, x(0) = x, \xi(0) = \alpha, k(t) = k\right\} \tag{2.15}$$

where ρ is the discounted rate. The value function of such a problem is defined as follows:

$$v(\alpha, x, k, \cdot) = \inf_{(u(\cdot), \omega_m(\cdot), \omega_r(\cdot)) \in \mathrm{A}(\alpha)} J(\alpha, x, k, u, \omega_m, \omega_r) \;\; \forall \alpha \in B \tag{2.16}$$

Section 2.4 presents the properties of the value function $v(\cdot)$ given by equation (2.16) and the numerical methods used to solve the proposed optimality conditions.

2.4 Optimality conditions and numerical methods

This section presents the optimality conditions satisfied by the value function presented in equation (2.16). The properties of the value function and the manner in which such equations are obtained can be found in Kenne and Nkeungoue (2008). They describe the optimal control policies (optimality conditions) for production, preventive and corrective maintenance planning problems. Regarding the optimality principle, we can write the optimality conditions, given by Hamilton-Jacobi-Bellman (HJB) equations as follows:

$$\rho v(\alpha,k,x,\cdot) = \min_{(u,\omega_m,\omega_r)\in A(\alpha)} \left[g(\alpha,x,u,\omega_m,\omega_r) + \sum_{\beta\in B} \lambda_{\alpha\beta} v(\beta,k,x,\cdot) + (u-d)\frac{\partial v(\alpha,k,x,\cdot)}{\partial x} \right] \tag{2.17}$$

The optimal control policies over $A(\alpha)$ of the right hand side of equation (2.17) are $(u^*(\cdot),\omega_m^*(\cdot),\omega_r^*(\cdot))$. When the value function described by equation (2.16) is available, optimal control policies can be obtained as in equations (2.17). However, obtaining an analytical solution for equations (2.17) is almost impossible. The numerical solution of the HJB equations (2.17) used to be considered an insurmountable challenge, but Boukas and Haurie (1990) showed that implementing Kushner's method can solve such a problem in the context of production planning.

The numerical methods for solving the optimality conditions given by equations (2.17) are based on the Kushner approach, as in Kenne et al. (2003), Hajji et al. (2009), and references therein. We should recall that the primary premise of this approach consists in using an approximation scheme for the gradient of the value function $v(\alpha,k,x)$. Let h denote the length of the finite difference interval of the variable x. The value function $v(\alpha,k,x)$ is approximated by $v^h(\alpha,k,x)$, and $\dfrac{\partial v(\alpha,k,x)}{\partial x}$ is approximated using the following equation:

$$\frac{\partial v(x,\alpha)}{\partial x}\times(u-d)=\begin{cases}\frac{1}{h}\left(v^h(\alpha,x+h,k)-v^h(\alpha,x,k)\right)\times(u\text{ - }d)\ if\\(u\text{ - }d)>0,\\\frac{1}{h}\left(v^h(\alpha,x,k)-v^h(\alpha,x-h,k)\right)\times(u\text{ - }d)\\otherwise.\end{cases}\tag{2.18}$$

With approximations given by equation (2.18), and after a couple of straightforward manipulations, the HJB equations can be rewritten as follows:

$$v^h(\alpha,x,k)=\min_{(u,\omega_m,\omega_r)\in A^h(\alpha)}\left\{\frac{g(\alpha,x,u,\omega_m,\omega_r)}{\Omega_h^\alpha(1+\rho/\Omega_h^\alpha)}+\frac{1}{(1+\rho/\Omega_h^\alpha)}\begin{pmatrix}p_x^\pm(\alpha)v^h(\alpha,x\pm h,k)\\+\sum_{\beta\neq\alpha}p^\beta(\alpha)v^h(\alpha,x,k)\end{pmatrix}\right\}\tag{2.19}$$

where $A^h(\alpha)$ is the numerical control grid. The other terms used in equation (2.19) are given as follows:

$$\Omega_h^\alpha=|\lambda_{\alpha\alpha}|+\frac{|u-d|}{h},\qquad p_x^+(\alpha)=\begin{cases}\frac{u-d}{h\Omega_h^\alpha} & if\ u-d>0\\0 & otherwise,\end{cases}$$

$$p^\beta(\alpha)=\frac{\lambda_{\alpha\beta}}{\Omega_h^\alpha},\qquad p_x^-(\alpha)=\begin{cases}\frac{d-u}{h\Omega_h^\alpha} & if\ u-d\leq 0\\0 & otherwise.\end{cases}$$

The system of equations (2.19) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process, as in Kenne and Nkeungoue (2008). The discrete event dynamic programming equations obtained can be solved using either policy improvement or successive approximation methods. In this paper, we use the value iteration procedure to approximate the value function given by equation (2.19). Kenne et al. (2003) and references therein provide details on such methods.

2.5 Numerical example and results

This section presents a numerical example for the manufacturing system presented in Section 2.3. A three-state semi-Markov process with the modes in $B=\{1,2,3\}$ describes the system capacity. The instantaneous cost is described by equation (2.14). The values of c^+, c^-, c_m and c_r will be given later in this section. The transition rate matrix $Q(\cdot)$ is explicitly defined as follows:

$$Q(k,\omega_m,\omega_r)=\begin{pmatrix} -(\lambda_{12}(k)+\omega_m) & \lambda_{12}(k) & \omega_m \\ \omega_r & -\omega_r & 0 \\ \lambda_{31} & 0 & -\lambda_{31} \end{pmatrix}$$

where $\lambda_{12}(k)=1/E_k(T),$ with $E_k(T)$ defined in equation (2.5).

The following three equations are the discrete dynamic programming equations obtained from equation (2.19) for $\alpha=1,2,3$:

$$v^h(1,x,k)=\min_{(u,\omega_m,\omega_r)\in A^h(1)}\left\{\frac{c^+x^+ + c^-x^- + c_m\omega_m}{\Omega_h^1(1+\rho/\Omega_h^1)}+\frac{1}{(1+\rho/\Omega_h^1)}\left(p_x^{\pm}(1,x\pm h,k)\right)+\frac{1}{(1+\rho/\Omega_h^{\alpha})}\begin{pmatrix} p^2(1)v^h(2,x,k)+ \\ p^3(1)v^h(3,x,k)\end{pmatrix}\right\} \quad (2.20)$$

$$v^h(2,x,k)=\min_{\omega_r\in A^h(2)}\left\{\frac{c^+x^+ + c^-x^- + c_r\omega_r}{\Omega_h^2(1+\rho/\Omega_h^2)}+\frac{1}{(1+\rho/\Omega_h^2)}\left(p_x^-(2)v^h(2,x-h,k)+p^1(2)v^h(1,x,k)\right)\right\} \quad (2.21)$$

$$v^h(3,x,k)=\min\left\{\frac{c^+x^+ + c^-x^-}{\Omega_h^3(1+\rho/\Omega_h^3)}+\frac{1}{(1+\rho/\Omega_h^3)}\left(p_x^-(3)v^h(3,x-h,k)+p^1(3)v^h(1,x,k)\right)\right\} \quad (2.22)$$

We use the computational domain D given by:

$$D=\{(x,k):\ -10\le x\le 50;\quad 1\le k\le 20\} \quad (2.23)$$

The condition to meet the customer demands, over an infinite horizon and reach a steady state is given by:

$$\pi_1 * u_{\max} > d \quad (2.24)$$

where π_1 is the limiting probability at the operational mode of the machine. Note that the limiting probabilities of modes 1, 2 and 3 (i.e., π_1, π_2 and π_3), are computed as follows:

$$\pi\cdot Q(\cdot)=0 \quad \text{and} \quad \sum_{i=1}^{3}\pi_i=1 \quad (2.25)$$

where $\pi=(\pi_1,\pi_2,\pi_3)$ and $Q(\cdot)$ is the corresponding 3×3 transition rate matrix. Table 2.2 summarizes the parameters of the numerical example for which the feasibility condition given by equation (2.24) is satisfied. The policy improvement technique is used to solve the system of equations (2.20)-(2.22). The results obtained for the values in Table 2.2 are presented in Figures 2.7 to 2.10.

Table 2.2 Parameters of numerical example

c^+	c^-	c_r	c_m	$u_{\max}$	d	λ_{31}	$\omega_m^{\min}$	$\omega_m^{\max}$	$\omega_r^{\min}$	$\omega_r^{\max}$	ρ
1	100	5,000	10	0.27	0.25	0.5	10^{-4}	0.5	0.02	0.1	0.01

For illustrative purposes, the production rate for five failures of the machine, in its operational mode (i.e., mode 1), is presented in Figure 2.7. This figure shows that the production rate is set to zero for comfortable stock levels. Then, there is no need to produce parts for comfortable stock levels.

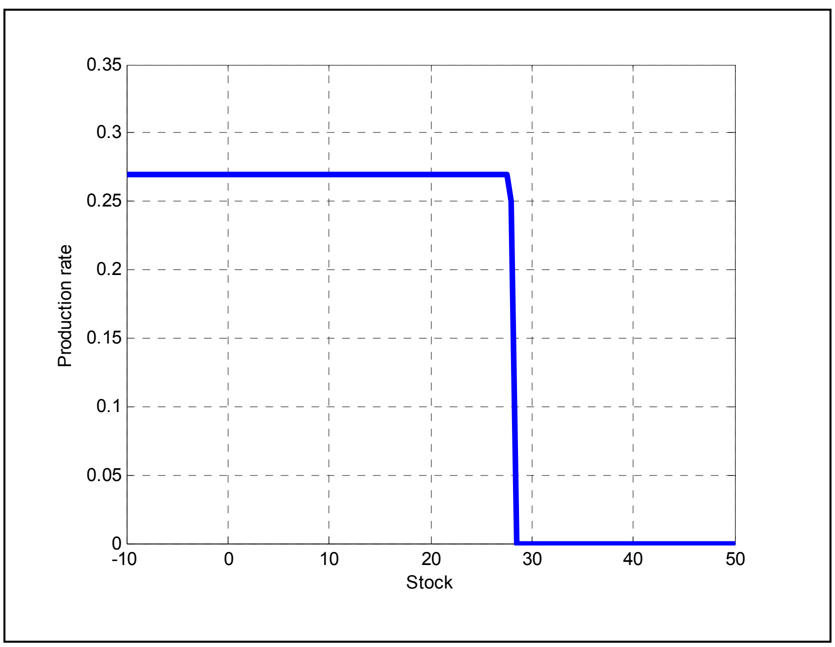

Figure 2.7 Production rate of the machine at mode 1

The production rate is thus set to zero when there are more than 28 products in inventory. Figure 2.8 illustrates the trend of threshold value versus number of failures. This figure shows that the effect of large failure probabilities at large machine number of failures values is minimized by assigning large values to the stock threshold. From the results obtained, the computational domain is divided into three regions where the optimal production control policy consists of one of the following rules:

1. Set the production rate of the machine to its maximal value when the current stock level is under the threshold value;
2. Set the production rate of the machine to the demand rate when the current stock level is equal to the threshold value;
3. Set the production rate of the machine to zero when the current stock level is larger than the threshold value.

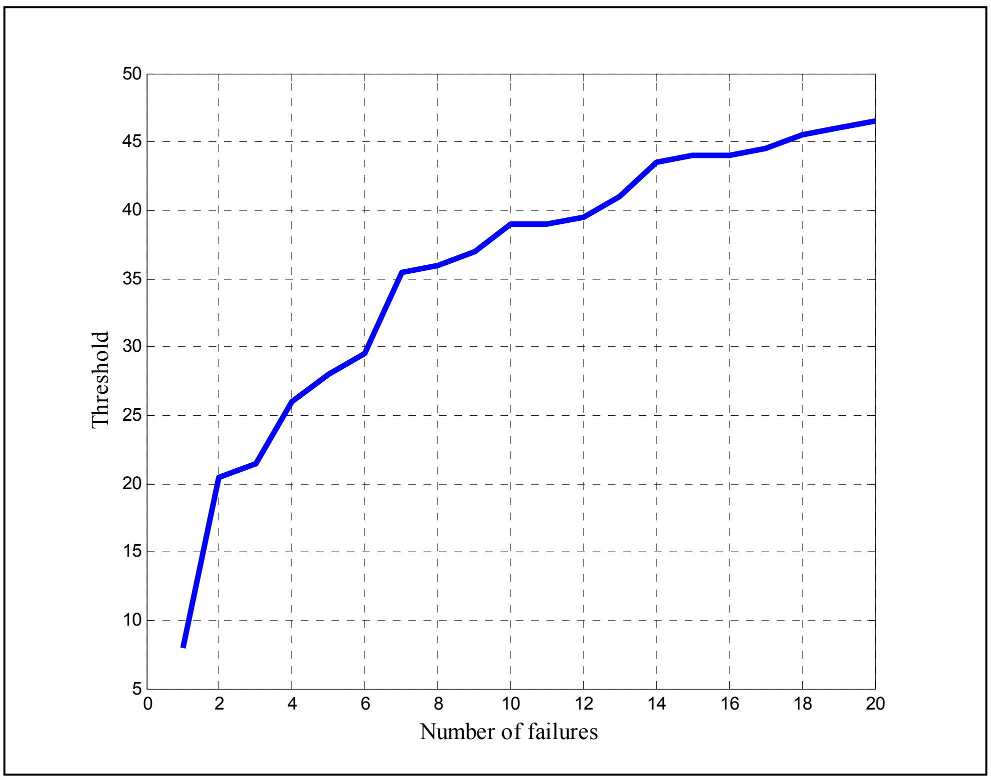

Figure 2.8 Trend of threshold value versus number of failures

The control policy obtained is an extension of the hedging point policy, given that the previous three rules respect the structure presented in Akella and Kumar (1986) for production planning without the control of preventive and corrective maintenance activities.

As shown within the numerical results and in Figure 2.7, the optimal production rate can be expressed as follows:

$$u(x,k,1) = \begin{cases} u_{\max} & \text{if } x(\cdot) < X^*(k) \\ d & \text{if } x(\cdot) = X^*(k) \\ 0 & \text{otherwise,} \end{cases} \tag{2.26}$$

where $X^*(k)$ is the optimal threshold value for each value of the k number of machine failures.

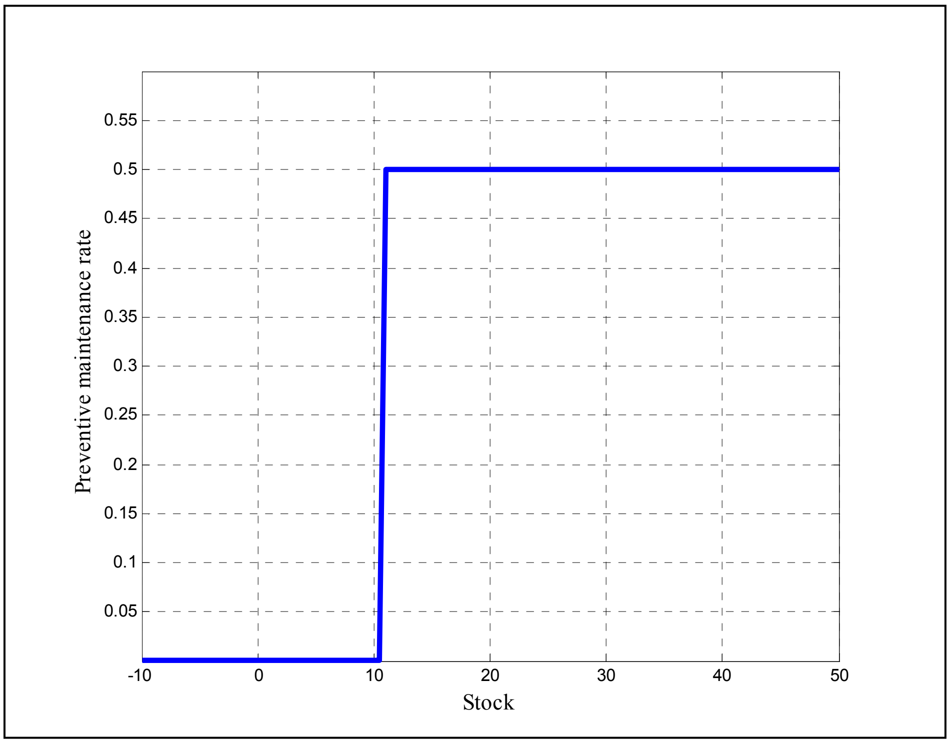

Figure 2.9 Preventive maintenance rate of the machine at mode 1

The preventive maintenance policy, plotted in Figure 2.9, divides the computational domain into two regions where the preventive maintenance rate is set to its minimal and maximal values for uncomfortable stock levels (or for backlog situations) and for large stock levels, respectively. The optimal preventive maintenance policy, like the production policy, has a bang-bang structure, and is described as follows:

$$\omega_m(x,k,1)=\begin{cases}\omega_m^{\min} & \text{if } x(\cdot)<Y^*(k)\\ \omega_m^{\max} & \text{otherwise,}\end{cases} \tag{2.27}$$

where $Y^*(k)$ is the optimal stock level at which the preventive maintenance rate must be switched from $\omega_m^{\min}$ to $\omega_m^{\max}$.

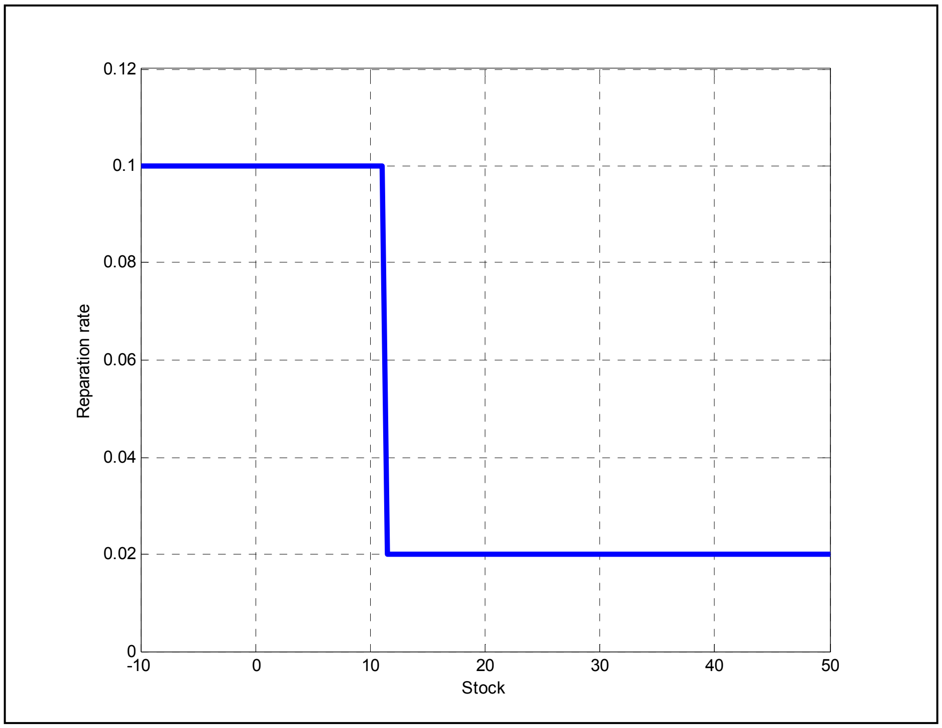

Figure 2.10 Corrective maintenance rate of the machine at mode 2

Figure 2.10 presents the corrective maintenance policy. The computational domain is divided into two regions where the corrective maintenance rate is set to its maximal and minimal values for backlog situations and for large stock levels, respectively. The optimal corrective maintenance policy, like the production and the preventive maintenance policies, has a bang-bang structure, and is defined as follows:

$$\omega_r(x,k,2)=\begin{cases}\omega_r^{\max} & \text{if } x(\cdot)<Z^*(k)\\ \omega_r^{\min} & \text{otherwise,}\end{cases} \tag{2.28}$$

where $Z^*(k)$ is the optimal stock level at which the corrective maintenance rate must be switched from $\omega_r^{\max}$ to $\omega_r^{\min}$.

Using the control policies given by equations (2.26), (2.27) and (2.28), the company will be able to minimize the total cost due to production, allowing it to eventually maximize its total profit.

Table 2.3 Variation of the optimal threshold with the number of failures

c^+	c^-	c_m	c_r	k	X^*	Y^*	Z^*	$Cost$
1	**100**	**10**	**5,000**	**2**	**20.50**	**9.09**	**10.00**	**16,926**
1	100	10	5,000	5	28.00	10.31	10.62	19,179
1	100	10	5,000	10	39.00	13.12	13.12	23,436
1	100	10	5,000	20	46.50	14.69	14.69	26,882

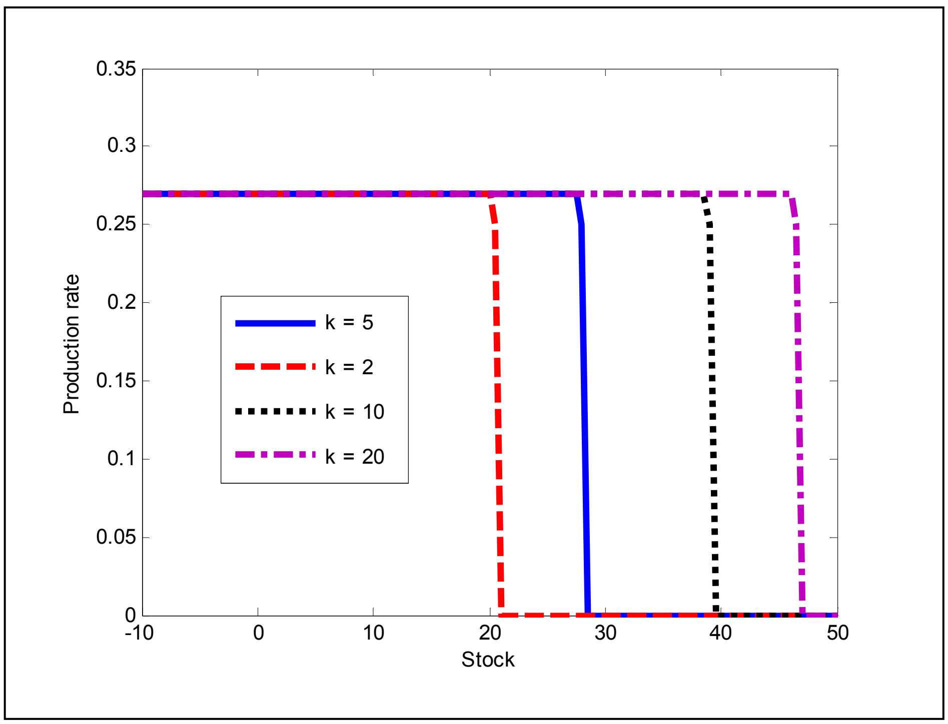

Figure 2.11 Production rate versus number of failures

By combining a k-dependent failure rate, preventive and corrective maintenance actions with production activities, we obtained that the optimal threshold, and the other parameters of the control policy (X^*, Y^* and Z^*) increase as the number of failures increases (see Table 2.3 and figures 2.11 to 2.13). We can then avoid backlogs when the machine is at modes 2 and 3. These results illustrate the contribution of the proposed model compared to one in which one value of the optimal threshold is used for production planning, without considering the fact that the failure rate depends on the number of failures combined with control of the corrective and the preventive maintenance. In Section 2.6, we confirm such an observation through a sensitivity analysis, which can also validate and illustrate the usefulness of the model developed in this research.

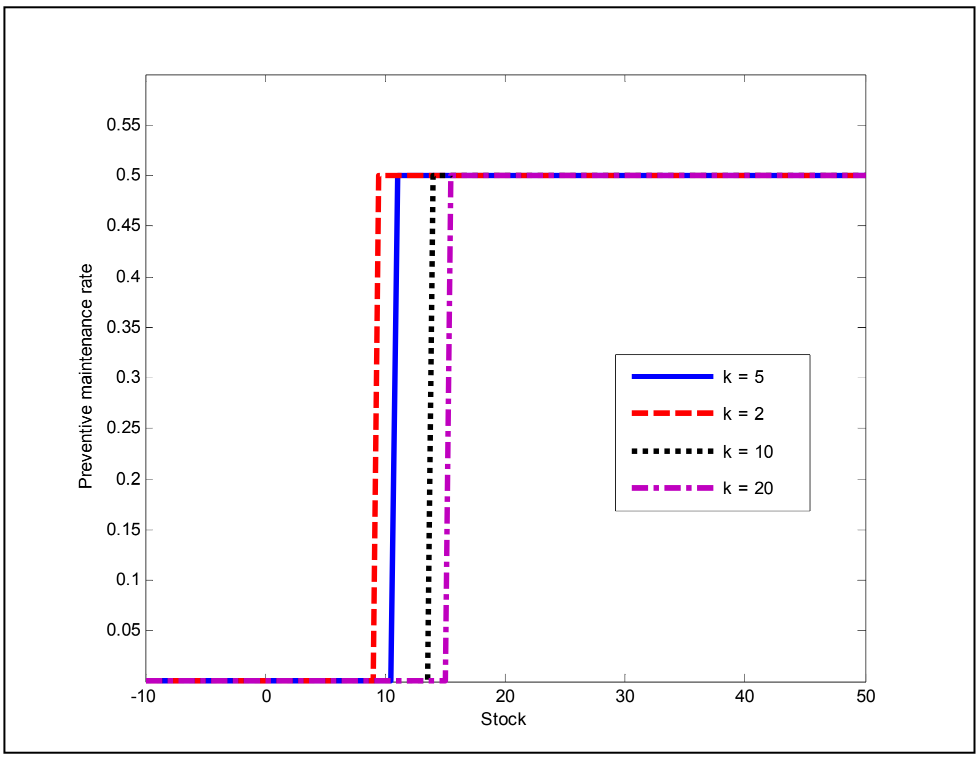

Figure 2.12 Preventive maintenance rate versus number of failures

2.6 Sensitivity analyses and extensions

Backlog, inventory, preventive and corrective maintenance cost parameters are considered in the sensitivity analyses in order to gain insight into the proposed stochastic model. The numerical example presented previously was used to perform a couple of experiments, and the results shown in Table 2.4 illustrate four scenarios. The following variations are explored and compared to the basic case (highlighted lines).

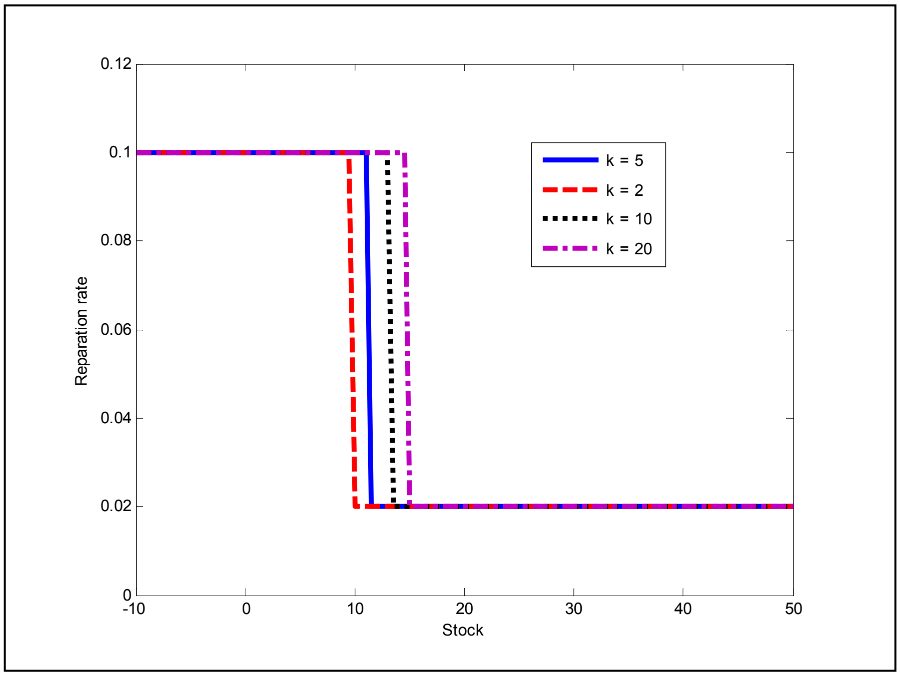

Figure 2.13 Corrective maintenance rate versus number of failures

2.6.1 Variation of the backlog cost

- Increasing c^- : X^*, Y^* and Z^* increase. This must result in a tendency to increase the threshold value and the other parameters of the control policy in order to avoid further backlog costs. The overall cost increases as well.
- Decreasing c^- : The stock level decreases in order to avoid further inventory costs (second line of Table 2.4 : $c^- = 50$).

2.6.2 Variation of the inventory cost

- Increasing c^+ : The threshold value decreases and other parameters of the control policy move as predicted, from a practical view point, in order to avoid further inventory costs (second block of Table 2.4).

Table 2.4 Sensitivity analysis and policy parameters

c^+	c^-	c_m	c_r	k	X^*	Y^*	Z^*	$Cost$
1	**100**	**10**	**5,000**	**5**	**28.00**	**10.31**	**10.62**	**19,179**
1	50	10	5,000	5	25.50	5.00	5.00	18,829
1	200	10	5,000	5	30.50	13.13	13.75	19,571
1	300	10	5,000	5	32.50	14.38	15.00	19,821
1	**100**	**10**	**5,000**	**5**	**28.00**	**10.31**	**10.62**	**19,179**
5	100	10	5,000	5	12.50	6.88	10.00	55,213
10	100	10	5,000	5	8.50	5.00	8.75	99,221
20	100	10	5,000	5	5.50	3.13	8.13	186,278
1	**100**	**10**	**5,000**	**5**	**28.00**	**10.31**	**10.62**	**19,179**
1	100	30	5,000	5	27.00	12.50	10.63	20,202
1	100	50	5,000	5	25.00	16.88	10.63	21,181
1	100	70	5,000	5	22.50	$\omega_m = \omega_m^{\min}$	10.63	21,376
1	**100**	**10**	**5,000**	**5**	**28.00**	**10.31**	**10.62**	**19,179**
1	100	10	10,000	5	34.50	8.75	8.75	27,772
1	100	10	15,000	5	39.00	7.50	7.50	36,145
1	100	10	20,000	5	42.00	6.25	6.25	44,398

2.6.3 Variation of the preventive maintenance cost

- Increasing c_m: The threshold value decreases in order to avoid further inventory costs. The overall cost increases. For high values of preventive maintenance costs compared to the basic case, no preventive maintenance is required (the preventive maintenance rate is set to its minimal value). In these cases, the corrective maintenance parameter remains constant (third block of Table 2.4).

2.6.4 Variation of the corrective maintenance cost

- Increasing c_r: The corrective maintenance policy parameter decreases in order to avoid further repair cost. The threshold level increases in order to avoid further backlog costs with high levels of repair costs. The preventive maintenance parameter decreases and the overall cost increases (last block of Table 2.4).

The above sensitivity analyses validate the proposed approach and show that the control policy and parameters obtained from the results analyses are consistent.

2.7 Extensions

For given k-dependent failure rate parameters $X(k), Y(k)$ and $Z(k)$, the control policy described by equations (2.26) to (2.28) is completely known for the system proposed in this paper (one-machine and one-product). For a manufacturing system consisting of m machines producing n different part types, the production, preventive and corrective maintenance policies could be defined by 3^{n+m} parameters or input factors because the control policy would depend on $k_1,...,k_m$, $X_1^\alpha,...,X_n^\alpha$, $Y_1^\alpha,...,Y_n^\alpha$ and $Z_1^\alpha,...,Z_n^\alpha$ with $\alpha \in \{1,2,3\}$. In that case, the HJB equations (such as equation (2.17)) are impossible to solve for large values of m and n since the dimension of the numerical scheme to be implemented increases exponentially with the complexity of the system. The analytical models combined to simulation can be used to determine the effects of the factors considered on the incurred cost and to obtain a near-optimal control policy (see Gharbi and Kenne, 2000 and Boulet et al., 2009).

For purposes of extension, the structure of a new approach to defining a near-optimal control policy in the context of a multiple-machine, multiple-product manufacturing system could consist of the following six sequential steps.

1. The control problem statement of the manufacturing system. Here, the objective is to find the production, preventive and corrective maintenance control variables.
2. The structure of the HJB equations, the numerical methods, the policy improvement techniques and the optimal control policies are obtained.

3. The control production, preventive and corrective maintenance factors for small size manufacturing systems (as in this paper) are determined.

4. The structure of the parameterized control policies is described and defined in simple cases. Then, the extension to more complex manufacturing systems is obtained.
5. The incurred cost is obtained from the simulation modeling according to the values of the control factors. The variations of the control factors, the effects of the main factors and their interactions on the cost are defined using the experimental design approach and an analysis of variance. Then, the response surface methodology is used to obtain the relationship between the cost and the significant main factors and interactions given in step 1. The optimization of the regression model obtained allows the determination of the best values of factors.
6. The near-optimal policies describing the production, preventive and corrective maintenance parameters are approximated. Then, the robustness of the proposed approach is validated through a sensitivity analysis.

2.8 Conclusion

This paper studied the impact of imperfect repairs, preventive and corrective maintenance scenarios for a single machine, and single product manufacturing system under uncertainties. We developed a stochastic optimization model of the problem considered, with three decision variables (production rate, preventive maintenance rate and corrective maintenance rate) and one state variable (stock level). By controlling both production and maintenance rates, we obtained a near-optimal control policy for the system through the implementation of the policy improvement algorithm (numerical methods). We have shown that the number of parts to hold in inventory, and preventive and corrective maintenance parameters, increase when the number of breakdowns increases.

We believe that this work represents a significant contribution to the literature on the production control of flexible two-level manufacturing systems, where, at the higher level, the parameters of the machine failure stochastic process are derived for each number of failures. At the lower level, the optimal production, preventive and corrective maintenance policies are determined for a system that deteriorates with the number of failures. We illustrated and validated the proposed approach using a numerical example and sensitivity analyses yielding logical conclusions.

The proposed model is developed in the case of a constant demand rate, one-machine and one-product manufacturing system. To cope with a real industrial environment case, we discussed the extensions of the proposed model to the case of manufacturing systems involving multiple products and multiple machines.

CHAPITRE 3

ARTICLE 2: STOCHASTIC OPTIMAL CONTROL OF MANUFACTURING SYSTEMS UNDER PRODUCTION-DEPENDENT FAILURE RATES

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Résumé

Dans cet article, nous faisons la planification de la production d'un système manufacturier constitué de deux machines non-identiques en parallèle produisant un seul type de pièce. Le taux de panne de la machine principale (machine dont le taux de production est le plus élevé) dépend de son taux de production. Une modélisation a été faite par une chaîne de Markov non-homogène, et une résolution numérique à travers des équations différentielles d'Hamilton-Jacobi-Bellman (HJB) a conduit à la solution du problème étudié. Les résultats ont montré que pour tenir compte de la fiabilité de la machine et réduire le coût total encouru, il est nécessaire de réduire le taux de production lorsqu'on approche le stock optimal des produits finis. Cette approche est très importante dans le cas des systèmes manufacturiers où la vitesse de la production influence considérablement l'usure de l'outil de coupe (exemple des machines d'usinage). Nos résultats ont été validés par le biais d'une analyse de sensibilité.

Mots-clés: Planification de la production, Programmation dynamique stochastique, Méthodes numériques.

Abstract

A production system consisting of two parallel machines with production-dependent failure rates is investigated in this paper. The machines produce one type of final product and unmet demand is backlogged. The objective of the system is to find a productivity policy for both machines that will minimize the inventory and shortage costs over an infinite horizon. The failure rate of the main machine depends on its productivity, while the failure rate of the second machine is constant. In the proposed model, the main machine is characterized by a higher productivity. This paper proposes a stochastic dynamic programming formulation of the problem and derives the optimal policies numerically. A numerical example is included and sensitivity analyses with respect to the system parameters are examined to illustrate the importance and effectiveness of the proposed methodology.

Keywords: Production planning; Stochastic dynamic programming; Numerical methods.

3.1 Introduction

The number of scientific publications covering failure-prone manufacturing systems has been growing steadily as a result of the intensive search for increased productivity and better customer service. A complete analytical solution was given in Akella and Kumar (1986), for a manufacturing system characterized by a homogeneous Markov process. The authors showed that the hedging point policy is the optimal control policy for minimizing discounted cost. In such a policy, the machine operates at a maximal rate until the inventory hits a safety stock level. If the current inventory level exceeds this level, no production should be carried out, but if it is equal to this level, then production should be just enough to meet demand. For a single machine, single part-type system, the expression of the optimal safety stock level was derived by Akella and Kumar (1986). This basic result has been extended in several ways over the years, with most such extensions relating to the Markovian case (see Tan and Gershwin, 2004, Dong-Ping, 2009, etc.). Only a few papers have examined semi-Markov processes (see Hu and Xiang, 1995; Dehayem et al., 2011); Kazaz and Sloan, 2013; etc.). In the Markovian case, however, a frequent assumption is that the underlying Markov process is homogeneous. The assumption in the semi-Markov processes is that the system deteriorates with age and number of failures. While these are reasonable assumptions, which in some cases provide simple and appealing mathematical solutions, the authors did not address the question of what happens if the machine is used to its maximum production capacity for a long period. The problem becomes much more pertinent if the failure rate depends on the productivity. In Rishel (1991), it was proven that the hedging point policy remains optimal if and only if the dependence of the failure rate on productivity is quadratic.

Similarly, one of the most important achievements of the research of Hu et al. (1994) was the investigation of the necessary and sufficient conditions for the optimality of the hedging point policy for a single machine, single part-type problem, when the failure rate of the machine is a function of productivity. They showed that hedging point policies are only optimal under linear failure rate functions. As per their discussion, numerical results in the general case suggest that as the inventory level approaches a hedging level, it may be beneficial to decrease productivity in order to realize gains in reliability. This conjecture was confirmed by the numerical results reported in Martinelli (2007), where the author considered a long average cost function and a machine characterized by two failure rates: one for low and one for high productivities. Martinelli (2010) generalizes the problem of Martinelli (2007) by considering one machine with different failure rates: more specifically, the failure rate is assumed to depend on productivity, through an increasing, piecewise constant function. Dahane et al (2012) studied the problem of dependence between production and failure rates in the context of a multi-product manufacturing system and the analysis was performed in discrete time. The results provided an answer about how to produce and what to produce over a finite horizon. The authors considered a manufacturing system consisting of a single randomly failing and repairable machine producing two products. A method for integrating load distribution decisions and production planning in the context of multi-state systems was presented by Nourelfath and Yalaoui (2012). The authors considered the load versus failure rate relationship while optimizing planning of production systems. Their integrated objective was to minimize the sum of capacity change costs, unused capacity costs, setup costs, holding costs, backorder costs, and production costs over a finite horizon.

A stochastic deteriorating production system consisting of two parallel machines with the productivity-dependent failure rates of the main machine is investigated in this paper. The stochastic nature of the system is due to machines that are subject to a non-homogeneous Markov process resulting from the dependence of failure rates on the production rate (productivity). The machines produce a single part type. Whenever a breakdown occurs, a corrective maintenance is performed. A repair action renews the machines. Our objective is to find the productivities of both machines such as to minimize the inventory and the shortage costs over an infinite horizon. To solve the optimization problem, we propose a stochastic dynamic programming formulation and derive the optimal production policies numerically. Numerical examples are included and sensitivity analyses with respect to the system parameters are also examined to illustrate the significance and effectiveness of the proposed methodology. As an extension, we apply this methodology to discuss the optimal productivities of manufacturing systems consisting of two machines with five failure rates depending on the productivity of the main machine.

This work distinguishes itself from the literature in three ways. First, the paper extends the work of Liberopoulos and Caramanis (1994), Martinelli (2010) and Dahane et al. (2012) to manufacturing systems consisting of more than one machine subject to a non-homogeneous Markov failure/repair process with productivity-dependent failure rates. We also extend the work of Dahane et al. (2012) and, Nourelfath and Yalaoui (2012) to the production planning over an infinite horizon. Secondly, the case of manufacturing systems consisting of two machines with multiple failure rates is discussed. Lastly, we study the possible industrial applications of the formulation and the approaches used.

The rest of this paper is broken down as follows. In Section 3.2, we present the industrial context of the problem under study. Section 3.3 covers notations and assumptions used in this research, and presents the problem statement. In Section 3.4, numerical results and sensitivity analyses are presented. Section 3.5 examines an extension to the case of multiple failure rates. Discussions and policies implementation are presented in Section 3.6, and the paper is finally concluded in Section 3.7.

3.2 Industrial context

The formulation, the approaches, and the numerical procedures used in this paper could be applied to many industries in which machines can be subjected to random failures and their production rates can also be controlled. The phenomenon has been experienced in machinery and mechanical assemblies, including automobile, aircraft engine and machine tools, and paper manufacturing plants. For example, in the metallic parts machining industries, where basic turning lathes and computer numerically controlled (CNC) lathes are used, the reliability of the machine-tools will depend on how they are used – the type of workpieces, cutting tools, process parameters selected.

The most basic turning lathe is the engine lathe, which is used for single, prototype, and low-quantity parts. The major lathe used in production today is the CNC lathe. Such lathes can produce a variety of parts requiring surfacing, turning, boring, grooving, drilling, threading, and chamfering in single or combined motions.

Any motion that can be expressed mathematically can be programmed into the lathe's computer control. CNC lathes machining provide parts characterized by great precision and low variability. It allows the machining of mechanical parts at high cutting speeds, which improves the productivity and the part surface finish. However, high speed machining (HSM) has some disadvantages: For instance higher acceleration and deceleration rates require precise forecasting and highly capable controllers. As well, constant spindle starting and stopping results in faster wear of guide ways, ball screws and spindle bearings, leading to higher maintenance costs. HSM also requires specific process knowledge, programming equipment and interfaces for the fast data transfer needed. Finding suitably trained staff can be difficult, and HSM can involve a considerable "trial and error" period. Good work and process planning is necessary, along with significant safety precautions and safety enclosing (bullet-proof covers). Tools, adapters and screws need to be checked regularly for fatigue cracks. Only tools with posted maximum spindle speeds can be used.

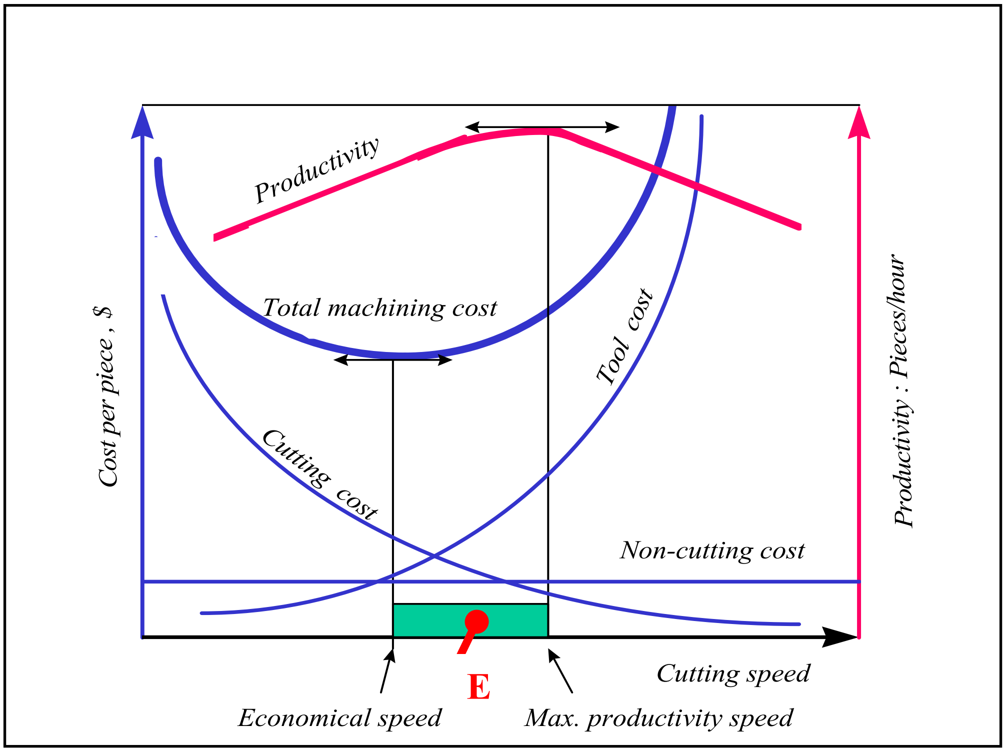


Figure 3.1 Schematic representation of machining costs and productivity as a function of the cutting speed (adapted from Groover, 2007)

The dependency of the machining cost and productivity (parts per hour) as a function of the cutting speed are presented in Figure 3.1. Examining Figure 3.1, we see that to minimize production costs and take into account the reliability of the CNC lathe, it would be advantageous to reduce the machining speed from its maximal productivity value to its economical value (see zone E).The machining costs are broken down into:

- Non-cutting costs (loading, unloading, assembly, rapid movements of approach, return to the table). These costs are independent of the cutting parameters.
- Tool costs (purchasing price, tool holders, tool changing costs, resharpening costs). When machining at higher speeds, tools wear out quickly, leading to short tool life and frequent tool change.

- Cutting costs (real metal removal cost). These decrease when the speed increases.

The total machining costs, which are at their lowest at the speed called the *economical speed*, represent the sum of all tool costs, cutting costs and non-cutting costs. Similarly, the productivity varies with the cutting speed and is at a maximum at the speed called the *maximum productivity speed*. Two important observations should be made here. First, the cost increase is due mainly to tool costs related to machine maintenance and repairs resulting from tool deterioration and operator mistakes induced by insufficient training. Secondly, in the shaded area, the total machining cost is a growing function of productivity. At the level of production optimization, details such as the machine speed cannot be taken into account, but the described phenomena can be addressed by considering the machine failure rate as dependent on the machine's productivity. Below, we formulate an optimization model and develop appropriate techniques for its solution.

3.3 Problem statement and optimality conditions

Before delving into the problem statement, we first present the notations and assumptions used throughout this article.

3.3.1 Notations

The model under consideration is based on the following notations:

u_1: productivity of the main machine M_1

u_2: productivity of the second machine M_2

u_{1max}: maximal productivity of M_1

U: economical productivity (in terms of machine's reliability) of M_1

u_{2max}: maximal productivity of M_2

x: stock level

d : customer demand rate

ξ : stochastic process (manufacturing system)

c^+ : inventory cost

c^- : backlog cost

$\lambda_{\alpha\beta}$: transition rate from mode α to mode β

Q : transition rate matrix

π : vector of limiting probabilities

$g(\cdot)$: instantaneous cost function

$J(\cdot)$: total cost

$v(\cdot)$: value function

ρ : discount rate

n : number of failure rates

3.3.2 Assumptions

This section presents the assumptions used throughout this paper.

(1) For the considered two-machine single-product environment, the machines are subject to random breakdowns and repairs. The failure rate of one machine depends on its productivity. This assumption represents the original characteristic of our approach. Other works consider one machine with a productivity-dependent failure rate or two machines that deteriorate with age and number of failures.

(2) The shortage cost depends on parts produced for backlog (average value ($/unit)).

(3) The inventory cost depends on parts produced for positive inventory (average value ($/unit)).

Assumptions 2 and 3 are common in inventory management.

(4) The productivity of the main machine is higher than that of the second machine.

(5) The second machine alone cannot satisfy customer demand.

This machine is a supporting machine. The main machine is unable to satisfy customer demand with its economical productivity, which is why another machine (second machine) is called upon.

3.3.3 Problem formulation

As illustrated in Figure 3.2, the manufacturing system studied consists of two parallel machines denoted as M_1 and M_2, which produce a single part type. When the main machine works at a faster rate, it is more likely to fail. The mode of the machine M_i can be described by a stochastic process $\xi_i(t),\ i=1,2$ with value in $B_i=\{1,2\}$. Such a machine is available when it is operational ($\xi_i(t)=1$) and unavailable when it is under repair ($\xi_i(t)=2$).

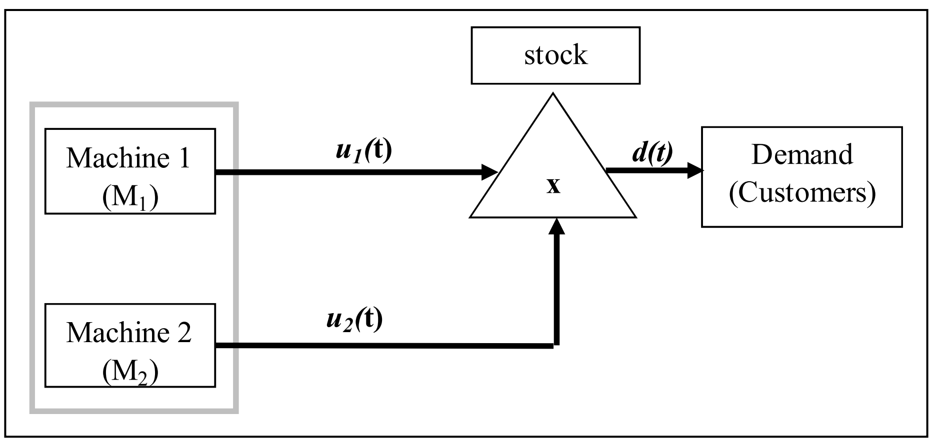
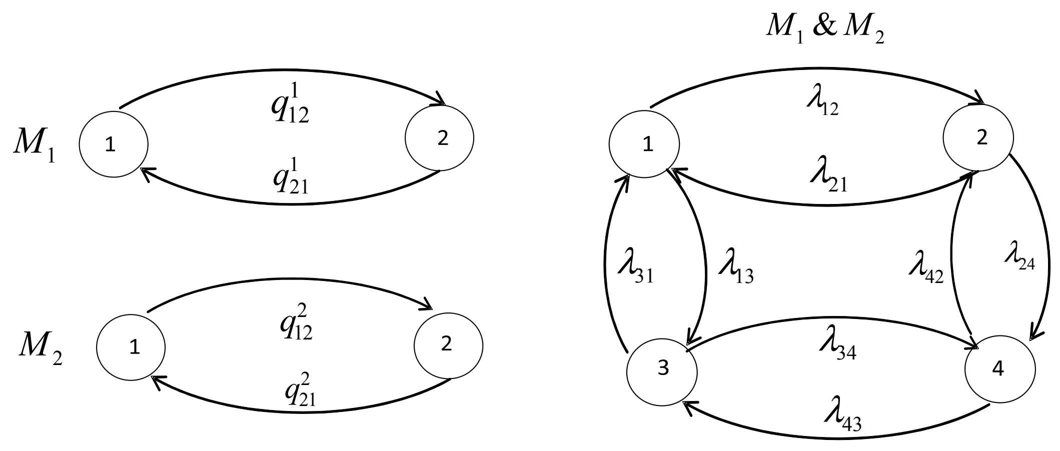


Figure 3.2 Structure of the production system

The transition diagram, which describes the dynamics of the considered manufacturing system, is presented in Figure 3.3. We then have $\xi(t)\in B=\{1,2,3,4\}$. With $\lambda_{\alpha\beta}$ denoting a jump rate of the system from state α to state β, we can describe $\xi(t)$ statistically by the following state probabilities:

$$P[\xi(t+\delta t)=\beta|\xi(t)=\alpha]=\begin{cases}\lambda_{\alpha\beta}(.)\,\delta t+o(\delta t) & if\,\alpha\neq\beta\\ 1+\lambda_{\alpha\beta}(.)\,\delta t+o(\delta t) & if\,\alpha=\beta\end{cases} \tag{3.1}$$

where $\lambda_{\alpha\beta} \geq 0 \quad (\alpha \neq \beta)$, $\lambda_{\alpha\alpha} = -\sum_{\beta \neq \alpha} \lambda_{\alpha\beta}$ and $\lim_{\delta t \to 0} \frac{o(\delta t)}{\delta t} = 0$ for all $\alpha, \beta \in B$.



a) States transition of each machine

b) States transition of the system

$q_{12}^1 = \lambda_{13} = \lambda_{24}$ (failure rate of M_1)

$q_{12}^2 = \lambda_{12} = \lambda_{34}$ (failure rate of M_2)

$q_{21}^1 = \lambda_{31} = \lambda_{42}$ (corrective maintenance rate of M_1)

$q_{21}^2 = \lambda_{21} = \lambda_{43}$ (corrective maintenance rate of M_2)

Figure 3.3 States transition diagram of the considered system

The operational mode of the manufacturing system can be described by the random vector $\xi(t) = (\xi_1(t), \xi_2(t))$. Given that the dynamics of each machine is described by a 2-state stochastic process, the set of possible values of the process $\xi(t)$ can be determined from the values of $\xi_1(t)$ and $\xi_2(t)$ as illustrated in Table 3.1, with:

- Mode 1: M_1 and M_2 are operational
- Mode 2: M_1 is operational and M_2 is under repair
- Mode 3: M_1 is under repair and M_2 is operational
- Mode 4: M_1 and M_2 are under repair

The dynamics of the system is described by a discrete element, namely $\xi(t)$, and a continuous element $x(t)$. The discrete element represents the status of the machines and the continuous one represents that of the stock level. It can be positive for an inventory or negative for a backlog.

Table 3.1 Modes of a two-machine manufacturing system

$\xi_1(t)$	1	1	2	2	Machine 1	Stochastic process
$\xi_2(t)$	1	2	1	2	Machine 2	Stochastic process
$\xi(t)$	1	2	3	4	Manufacturing system	Stochastic process

We assume that the failure rate of M_1 depends on its productivity, and is defined by:

$$q_{12}^{1} = \begin{cases} \theta_1 & \text{if } u_1 \in (U, u_{1\max}] \\ \theta_2 & \text{if } u_1 \in [0, U] \end{cases} \qquad \text{with } \theta_1 \geq \theta_2 \geq 0 \text{ and } 0 \leq U \leq u_{1\max}$$

Hence, $\xi(t)$ is described by the following matrix:

$$Q = \begin{cases} \Theta_1 & \text{if } u_1 \in (U, u_{1\max}] \\ \Theta_2 & \text{if } u_1 \in [0, U] \end{cases} \quad \text{with} \tag{3.2}$$

$$\Theta_1 = \begin{pmatrix} -(q_{12}^2+\theta_1) & q_{12}^2 & \theta_1 & 0 \\ q_{21}^2 & -(q_{21}^2+\theta_1) & 0 & \theta_1 \\ q_{21}^1 & 0 & -(q_{21}^1+q_{12}^2) & q_{12}^2 \\ 0 & q_{21}^1 & q_{21}^2 & -(q_{21}^1+q_{21}^2) \end{pmatrix}; \text{ with } \theta_1 = \lambda_{13} = \lambda_{24}$$

$$\Theta_2 = \begin{pmatrix} -(q_{12}^2+\theta_2) & q_{12}^2 & \theta_2 & 0 \\ q_{21}^2 & -(q_{21}^2+\theta_2) & 0 & \theta_2 \\ q_{21}^1 & 0 & -(q_{21}^1+q_{12}^2) & q_{12}^2 \\ 0 & q_{21}^1 & q_{21}^2 & -(q_{21}^1+q_{21}^2) \end{pmatrix}; \text{ with } \theta_2 = \lambda_{13} = \lambda_{24}$$

The continuous part of the system dynamics is described by the following differential equation:

$$\frac{dx(t)}{dt} = u_1(t) + u_2(t) - d,\ x(0) = x_0 \tag{3.3}$$

where x_0 and d are the given initial stock level and demand rate, respectively.

The set of the feasible control policies A, including $u_1(\cdot)$ and $u_2(\cdot)$, is given by:

$$\mathrm{A} = \left\{ \left(u_1(\cdot), u_2(\cdot)\right) \in \Re^2,\ 0 \le u_1(\cdot) \le u_{1\max},\ 0 \le u_2(\cdot) \le u_{2\max} \right\} \tag{3.4}$$

where $u_1(\cdot)$ and $u_2(\cdot)$ are known as control variables, and constitute the control policies of the problem under study. The maximal productivities of the main machine and the second machine are denoted by $u_{1\max}$ and $u_{2\max}$, respectively.

Let $g(\cdot)$ be the cost rate defined as follows:

$$g(\alpha, x) = c^+ x^+ + c^- x^- \tag{3.5}$$

where constants c^+ and c^- ($ per part per unit of time) are used to penalize inventory and backlog respectively, $x^+ = \max(0, x), x^- = \max(-x, 0)$.

The production planning problem considered in this paper involves the determination of the optimal control policies ($u_1^*(t)$ and $u_2^*(t)$) minimizing the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha, x, u_1, u_2) = E\left\{ \int_0^\infty e^{-\rho t} g(\alpha, x) dt \,\middle|\, x(0) = x_0, \xi(0) = \alpha \right\} \tag{3.6}$$

where ρ is the discount rate. The value function of such a problem is defined as follows:

$$v(\alpha, x) = \inf_{(u_1(\cdot), u_2(\cdot)) \in \mathrm{A}(\alpha)} J(\alpha, x, u_1, u_2) \ \ \forall \alpha \in B \tag{3.7}$$

The properties of the value function and the manner in which the Hamilton-Jacobi-Bellman (HJB) equations are obtained can be found in Kenné et al. (2003), with a constant failure rate.

3.3.4 Optimality conditions

Regarding the optimality principle, we can write the HJB equations as follows:

$$\rho v(\alpha,x) = \min_{(u_1,u_2)\in \mathrm{A}(\alpha)} \left[g(\alpha,x) + \sum_{\beta\in B} \lambda_{\alpha\beta} v(\beta,x) + (u_1+u_2-d)\frac{\partial v(\alpha,x)}{\partial x} \right] \tag{3.8}$$

where $\frac{\partial v(\alpha,x)}{\partial x}$ is the partial derivative of the value function $v(\alpha,x)$

The optimal control policy $\left(u_1^*(\cdot),u_2^*(\cdot)\right)$ denotes a minimizer over A of the right hand of equation (3.8). This policy corresponds to the value function described by equation (3.7). When the value function is available, an optimal control policy can then be obtained by solving equation (3.8). However, an analytical solution of equations (3.8) is almost impossible to obtain. The numerical resolution of the HJB equations (3.8) represents a challenge which was considered insurmountable in the past. Boukas and Haurie (1990) showed that implementing Kushner's method can solve such a problem in the context of production planning. In the Appendix 3.A, we present the numerical methods used to solve the proposed optimality conditions. In this research, the development contribution of Hamilton-Jacobi-Bellman (HJB) equations lies in the fact that at modes 1 and 2, where M_1 is operational, we have four equations instead of two as in the case of a manufacturing system without a productivity-dependent failure rate (see equations (3.A.2) and (3.A.3)). The next section provides a numerical example to illustrate the structure of the control policies.

3.4 Simulation and numerical example

Here, we illustrate the resolution of the model above with a numerical example.
Sensitivity analyses with respect to the system parameters are also presented to illustrate the importance and effectiveness of the proposed methodology.

3.4.1 Numerical results

In this section, we present a numerical example for the manufacturing system presented in Section 3.3. A four-state Markov process with the modes in $B=\{1,2,3,4\}$ describes the system capacity. The instantaneous cost is described by equation (3.5).

The considered computation domain D is given by:

$$D=\{x:-20\leq x\leq 40\} \tag{3.9}$$

The limiting probabilities of modes 1, 2, 3 and 4 (i.e., π_1,π_2,π_3 and π_4) are computed as follows:

$$\pi\cdot Q(\cdot)=0 \quad \text{and} \quad \sum_{i=1}^{4}\pi_i=1 \tag{3.10}$$

where $\pi=(\pi_1,\pi_2,\pi_3,\pi_4)$ and $Q(\cdot)$ is the corresponding 4×4 transition rate matrix given by equation (3.2).

Table 3.2 Parameters of numerical example

c^+	c^-	h	U	$u_{1\max}$	$u_{2\max}$	d	θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	ρ
1	50	3	0.75	1.2	0.65	1	0.03	0.02	0.04	0.1	0.2	0.03

The condition for meeting customer demands, over an infinite horizon is given by:

$$\pi_1\cdot(U+u_{2\max})+\pi_2\cdot U+\pi_3\cdot u_{2\max}>d \tag{3.11}$$

where (π_1,π_2 and π_3) constitute the limiting probability at the operational modes of the machines. Equation (3.11) is also satisfied with $u_{1\max}$ because $U<u_{1\max}$. Table 3.2 summarizes the parameters of the numerical example for which the feasibility conditions given by equation (3.11) are satisfied.

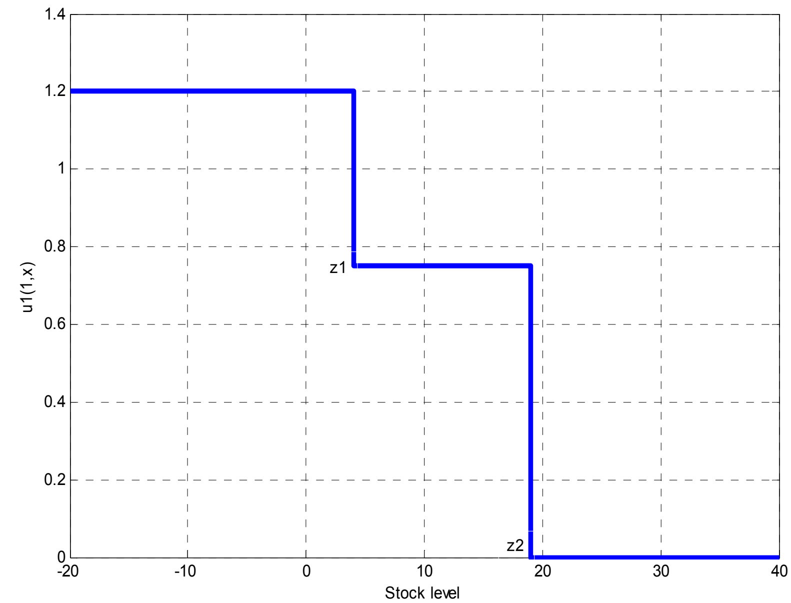


Figure 3.4 Productivity of M_1 at mode 1

The productivities at mode 1 of machines M_1 and M_2 are presented in Figures 3.4 and 3.5, respectively. Examining these figures, we can see that the threshold z_1 is low because both machines are operational. The results show that the productivities are set to zero for comfortable stock levels. At this point, there is no need to produce parts to ensure comfortable stock levels. According to the classical results as in Kenne et al. (2012) and references therein, the computational domain is expected to be divided into two stages, such as in Figure 3.5. Our results show however that the computational domain of Figure 3.4 is divided into three stages, which represents a specific finding of this paper. The optimal production control policy consists of one of the following rules for M_1:

1. Set the productivity of M_1 to its maximal value when the current stock level is under the first threshold value ($z_1 = 4.0$);
2. Reduce the productivity of M_1 to its economical value when the current stock level approaches the second threshold value ($z_2 = 19.0$);

3. Set the productivity of M_1 to zero when the current stock level is greater than the second threshold value.

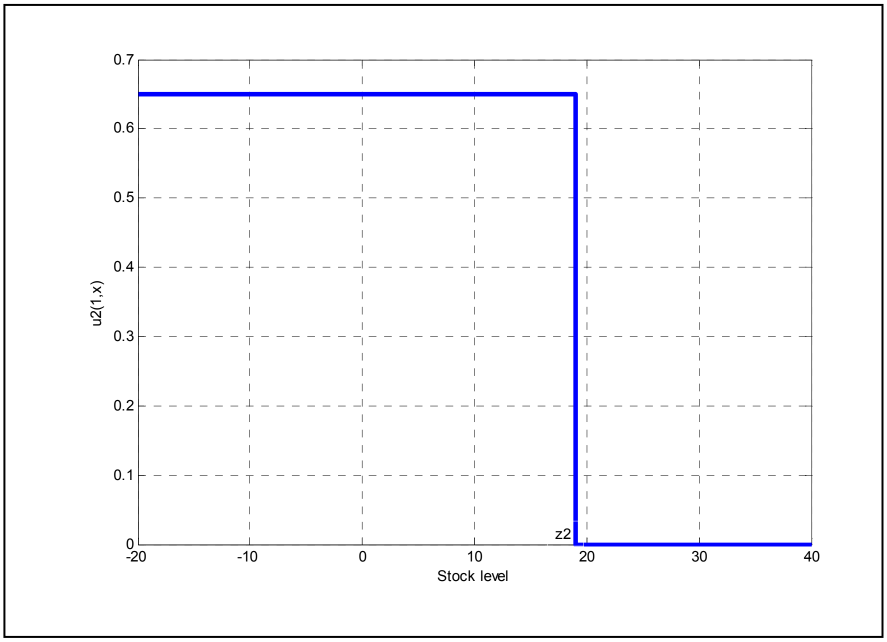


Figure 3.5 Productivity of M_2 at mode 1

The control policies obtained in Figure 3.4 are of multi-hedging point policy form. According to these results, the optimal productivities for the two machines can be expressed as follows:

$$u_1(x,1) = \begin{cases} u_{1\max} & \text{if } x < z_1 \\ U & \text{if } z_1 \le x < z_2 \\ 0 & \text{if } x > z_2 \end{cases} \tag{3.12}$$

where z_1 and z_2 are the first and the second threshold values of M_1, respectively.

$$u_2(x,1) = \begin{cases} u_{2\max} & \text{if } x < z_2 \\ 0 & \text{if } x > z_2 \end{cases} \tag{3.13}$$

where z_2 is the optimal threshold value at mode 1.

The productivity of M_1 at mode 2 of the system is presented in Figure 3.6. Unlike the case illustrated in Figure 3.4, where the tendency was to use the maximal productivity of the main machine less, at mode 2, the first threshold ($z_3 = 13.0$) is higher than z_1 in Figure 4 because the machine works alone. However, the control policy is still a multi-hedging point policy, and is defined by:

$$u_1(x,2) = \begin{cases} u_{1\max} & \text{if } x < z_3 \\ U & \text{if } z_3 \le x < z_4 \\ 0 & \text{if } x > z_4 \end{cases} \tag{3.14}$$

where z_3 and z_4 are the first and second threshold values of M_1 at mode 2, respectively.

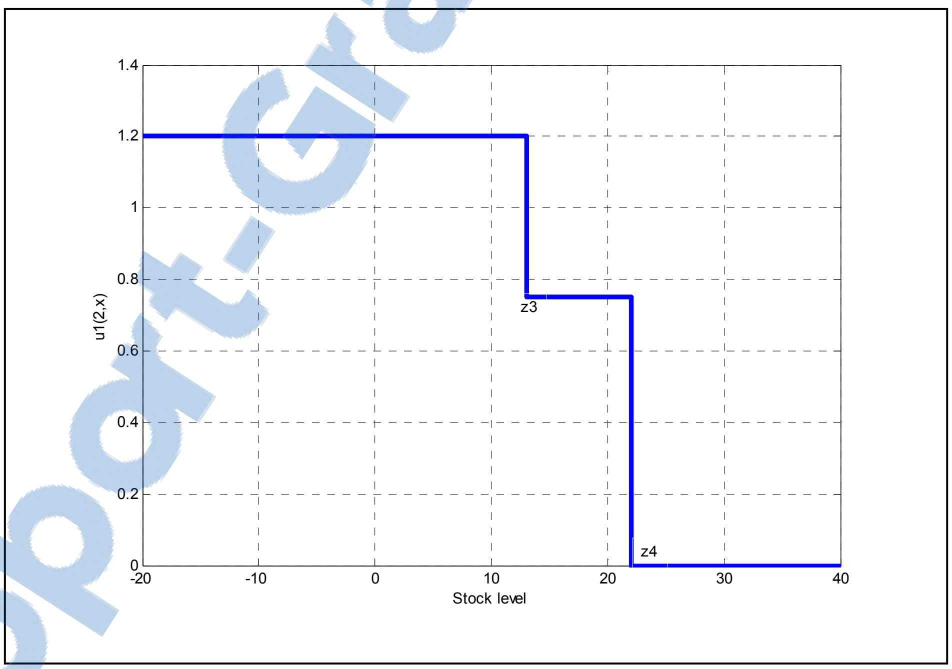


Figure 3.6 Productivity of M_1 at mode 2

The productivity of M_2 at mode 3 is plotted in Figure 3.7. The results of this figure show that the threshold value ($z_5 = 25$) is higher than the thresholds z_2 and z_4 because at mode 3, M_1 is under repair. The second machine must use its maximum productivity over a long period to avoid over-shortages. With numerical methods, the results show $z_5 = 25$.

Physically, however, the system cannot exceed the value of $z_4 = 22$. Hence, the threshold value z_5 will be ignored.

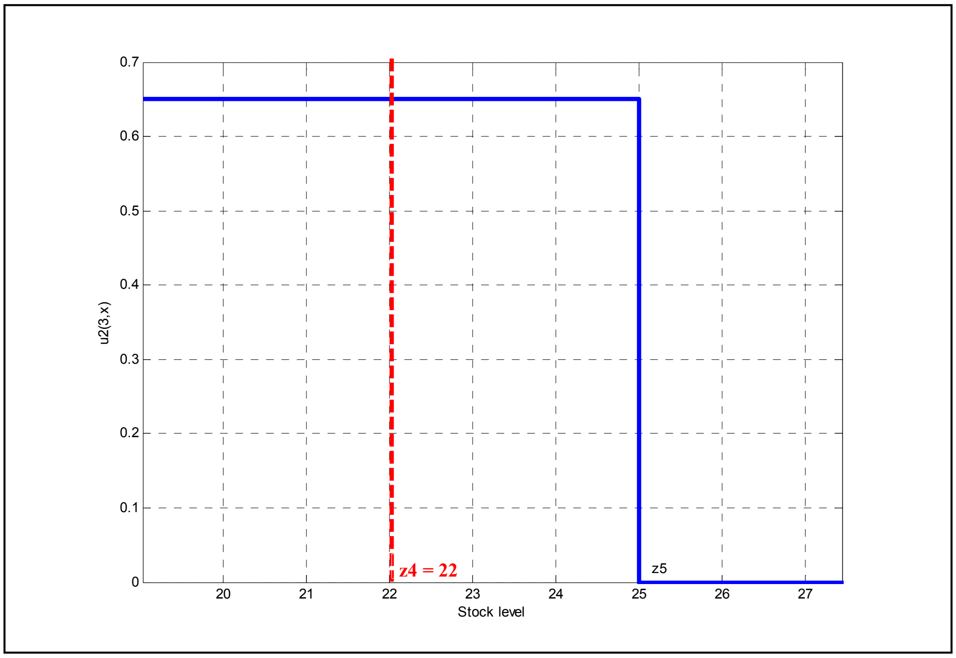


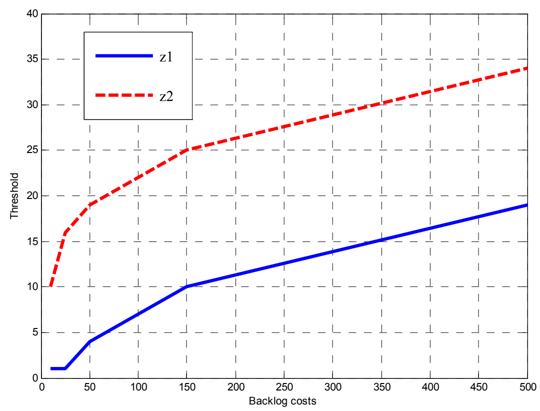
Figure 3.7 Productivity of M_2 at mode 3

In the manufacturing system consisting of two machines and one type of product, with a constant failure rate, the optimal control policy is characterized by two threshold values (Ouaret et al., 2013). The results obtained in this paper show that the optimal control policy is characterized by four different threshold parameters (z_1,z_2,z_3 and z_4) because the main machine degrades according to its productivity speed. This is a main finding of this paper.

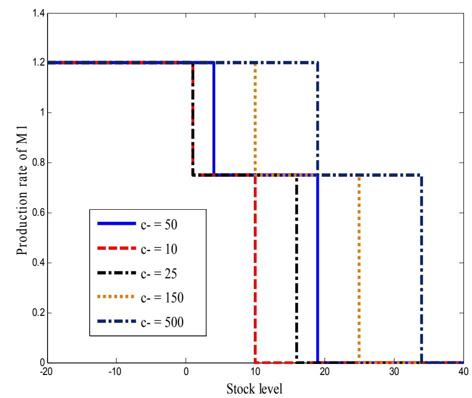
The next section analyzes the sensitivity of the policies obtained with respect to the various parameters of the model. Several experiments were conducted to ensure that the structure of the obtained policies is maintained under parameter variation, and therefore, can be used in practice.

3.4.2 Sensitivity analyses

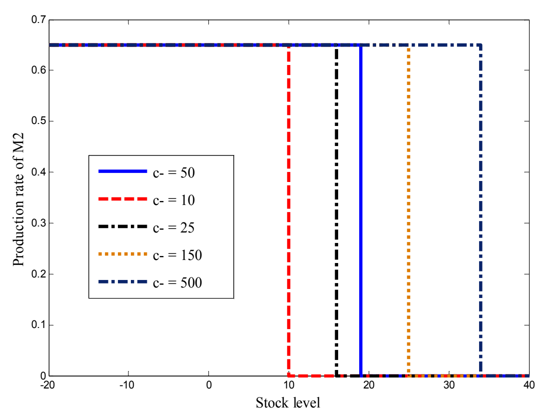
A set of numerical examples were considered to measure the sensitivity of the control policies obtained and to illustrate the contribution of this paper. The sensitivity of the control policies is analyzed according to the variation of the backlog costs and the machine parameters.



a) Effect on the threshold values



b) Effect on the production rate of M_1

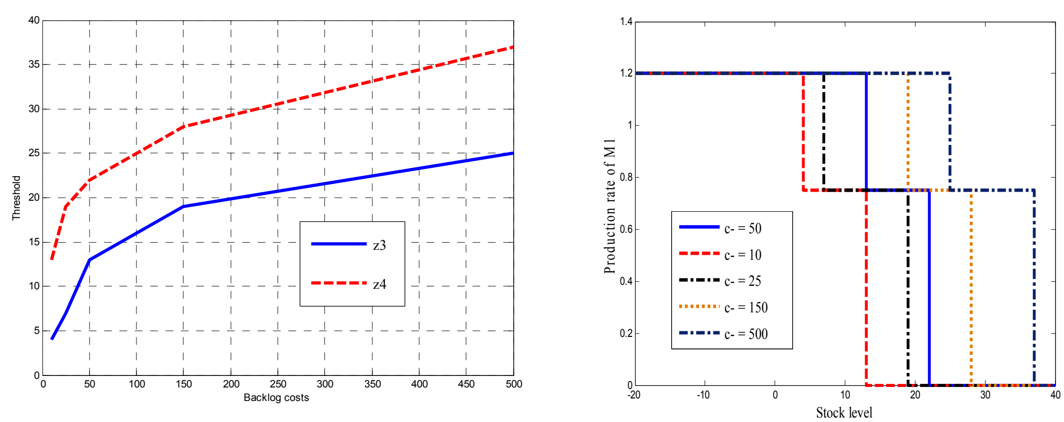


c) Effect on the production rate of M_2

Figure 3.8 Sensitivity to the variation of backlog costs at mode 1

3.4.2.1 Sensitivity analysis with respect to backlog costs

The results presented in Figures 3.8 and 3.9 show the behavior of the productivities of machines according to variations of backlog costs. Based on these results, we can see that low backlog cost values ($0 \leq c^{-} \leq 25$) do not affect the threshold z_1. This is logical because at mode 1, when both machines are operational, the system does not use the first machine enough to its maximal productivity in order to take account of its reliability. The thresholds z_1, z_2, z_3 and z_4 increase as the backlog costs increase in order to avoid further backlog costs. Figure 3.9 shows that the threshold values of M_1 at mode 2 (z_3 and z_4) are higher than the thresholds at mode 1 (z_1 and z_2) because the second machine is under repair. We therefore need a lot of parts in stock to avoid further backlog costs.



b) Effect on the threshold values

a) Effect on the production rate of M_1

Figure 3.9 Sensitivity to the variation of backlog costs at mode 2

3.4.2.2 Sensitivity analysis with respect to machine parameters

This section analyzes the sensitivity of the threshold values with the respect to the parameters of the two machines, as shown in Figures 3.10 to 3.17. The results show that the variation of the parameter q_{12}^2 does not affect the thresholds z_1 and z_3 . This adequately reflects the phenomenon of degradation of our system. The productivity of M_1 should be reduced to its economical value when closing to a comfortable stock level in order to ensure its reliability. We recall that z_1 and z_3 are the first hedging point policies of M_1 at mode 1 and mode 2, respectively. Let us now analyze the sensitivity of the thresholds according to each machine parameter.

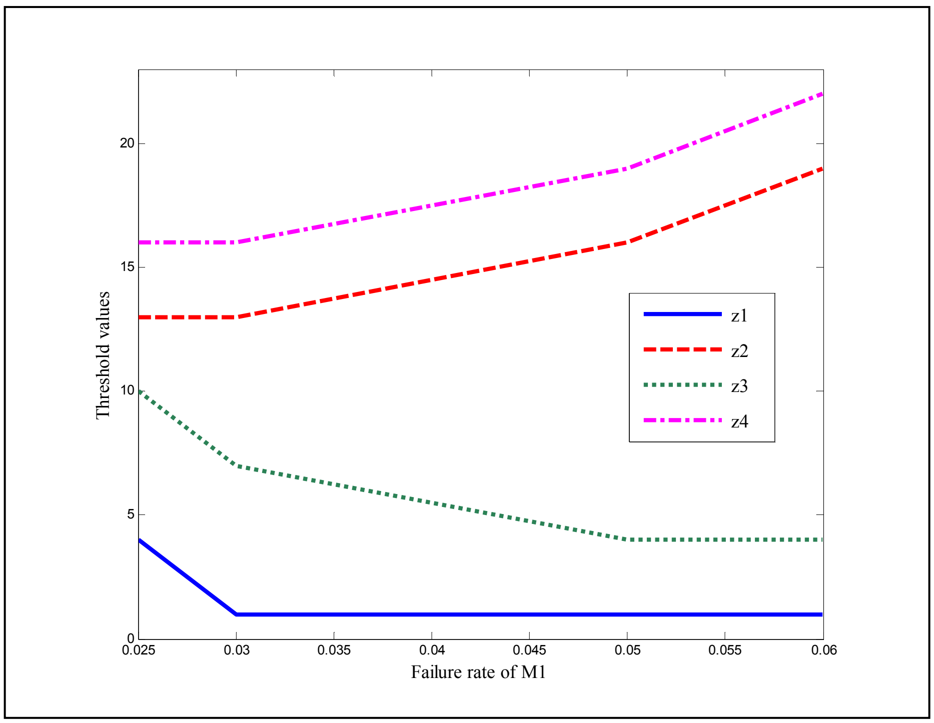


Figure 3.10 Sensitivity analysis with respect to failure rate of M_1 for $u_1 \in (U, u_{1\max}]$

a. Varying θ_1 (failure rate of M_1 for $u_1 \in (U, u_{1\max}]$)

When θ_1 increases, z_1 remains constant, z_3 decreases, and z_2 and z_4 increase. M_2 will necessarily tend to be more commonly used at mode 1 (both machines are producing) and M_1 will be at its economical productivity level ($u_1 = U$) at mode 2 (M_1 runs alone) in order to account for the reliability (the probability of failure at the maximum value is high). When

θ_1 decreases, z_1 and z_3 increase because the probability of failure, for $u_1 \in (U, u_{\max}]$, is low. The other parameters of the control policy move as predicted, from a practical perspective (see Figure 3.10).

b. Varying θ_2 (failure rate of M_1 for $u_1 \in [0, U]$)

When θ_2 increases, the thresholds z_1 and z_3 increase, while z_2 and z_4 remain constant. This means that we must limit the use of M_1 at its economical productivity level because doing so increases the second failure rate; the threshold z_1 remains constant, as do the other parameters of the control policy, when θ_2 decreases (Figure 3.11).

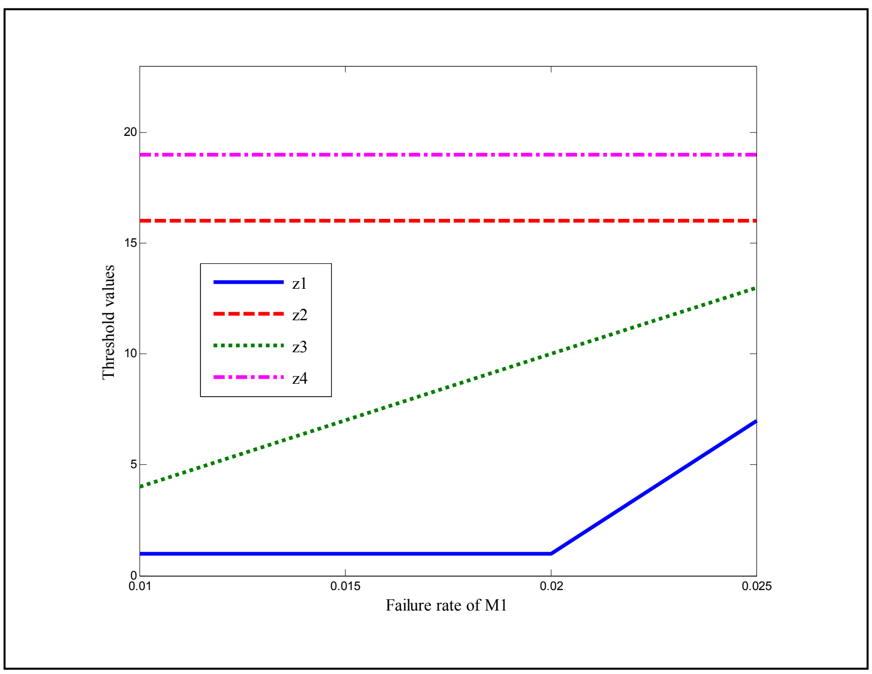


Figure 3.11 Sensitivity analysis with respect to failure rate of M_1 for $u_1 \in [0, U]$

c. Varying q_{12}^2 (failure rate of M_2)

When q_{12}^2 decreases, the thresholds z_2 and z_4 decrease. This means that the system will stay at mode 1 for a long time before transitioning to mode 2 because the probability of failure of the second machine decreases. As for z_1 and z_3, their values remain constant. As a result, M_1 will tend to be used to its maximal productivity in order to avoid backlogs. The thresholds z_1, z_2, z_3 and z_4 remain constant when q_{12}^2 increases (see Figure 3.12).

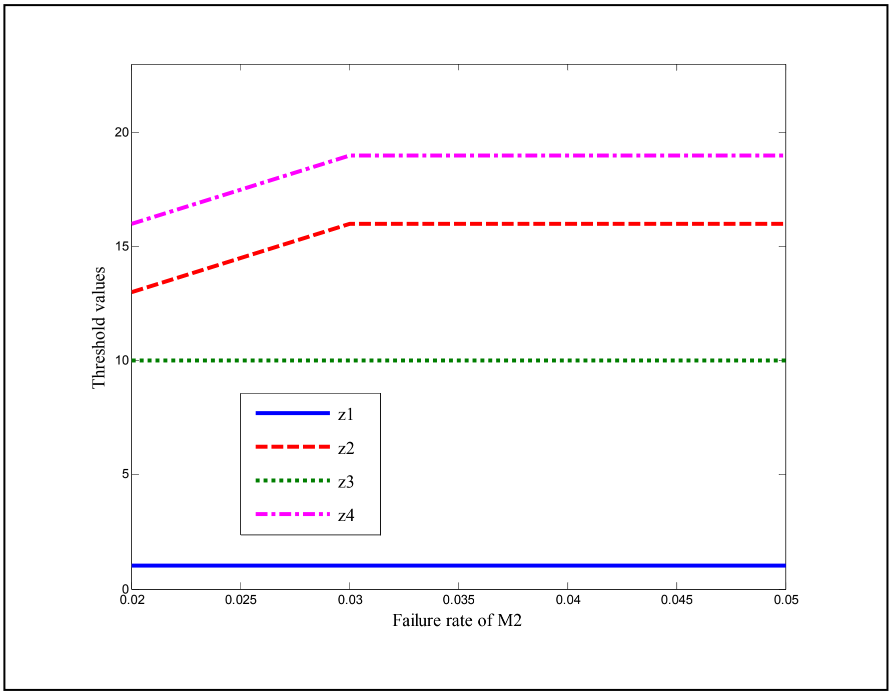


Figure 3.12 Sensitivity analysis with respect to failure rate of M_2

d. Varying q_{21}^1 (repair rate of M_1)

When q_{21}^1 increases, the thresholds z_1, z_2, z_3 and z_4 decrease in order to avoid over-stocking because the probability of repairing M_1 is high. There is a tendency to use M_1 and M_2 less when the repair rate of the main machine increases. If M_1 breaks down, it soon returns to the operational state. The parameters of the control policy move as predicted, from a practical perspective when q_{21}^1 decreases (Figure 3.13).

e. Varying q_{21}^{2} (repair rate of M_2)

The parameters z_1 and z_3 remain constant when q_{21}^{2} increases; when q_{21}^{2} decreases, z_1, z_2, z_3 and z_4 increase in order to avoid backlogs because the repair time of M_2 is long. If this machine fails, it will later return to its operational mode (Figure 3.14).

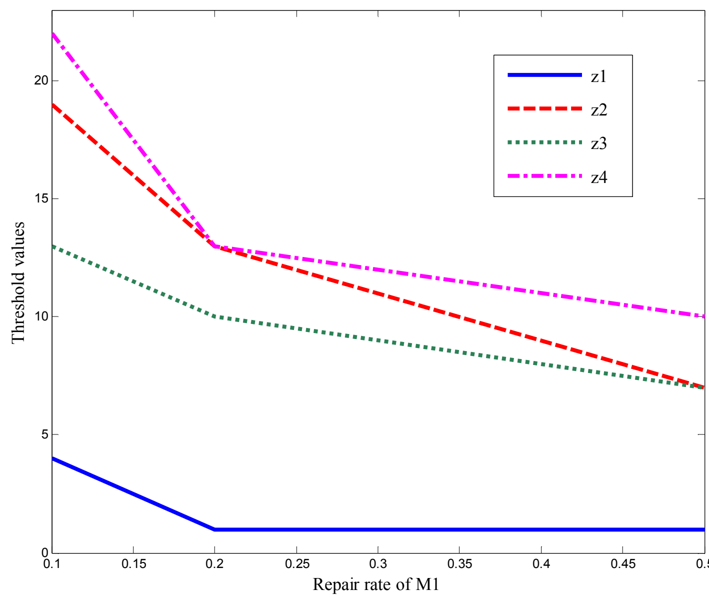


Figure 3.13 Sensitivity analysis with respect to repair rate of M_1

f. Varying $u_{1\max}$ (maximal productivity of M_1)

The values of z_1 increases when $u_{1\max}$ increases. This inevitably increases the chances of M_1 being used to its maximal productivity at mode 1. At mode 2, where M_2 is under repair, z_3 decreases and z_4 remains constant. The productivity of M_1 must be reduced to its economical value to take account of its reliability. When $u_{1\max}$ decreases, the threshold z_1 decreases and the other parameters remain constant. This means M_1 must be used less to its maximum productivity (Figure 3.15).

g. Varying U (economical productivity of M_1)

The values of z_1 and z_3 increase and z_2 and z_4 remain constant when the economical productivity of M_1 decreases. This must increase the likelihood of M_1 being used to its maximum productivity at mode 1 and mode 2. The parameters of the control policy move as predicted, from a practical perspective when U decreases. See Figure 3.16.

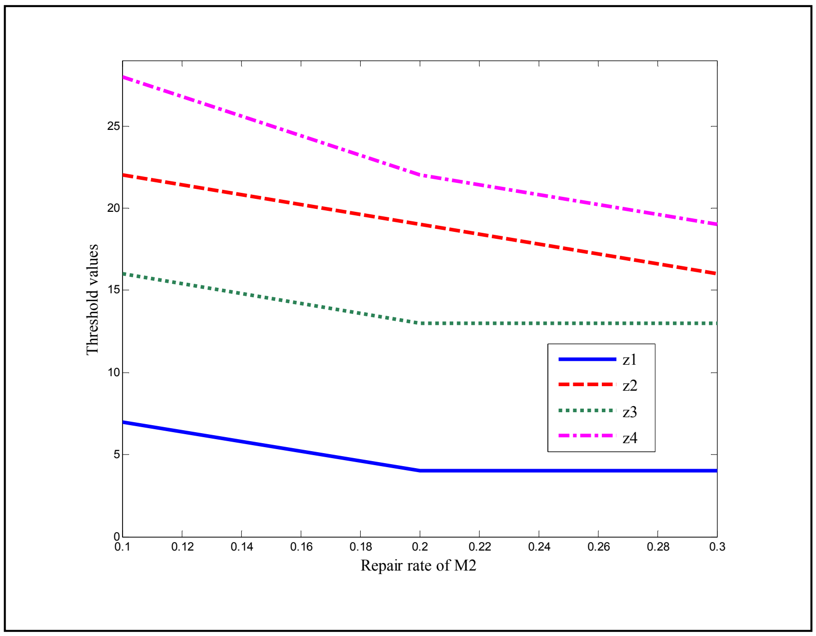


Figure 3.14 Sensitivity analysis with respect to repair rate of M_2

h. Varying $u_{2\max}$ (maximal productivity of M_2)

When $u_{2\max}$ increases, z_1 remains constant, and z_2, z_3 and z_4 decrease in order to avoid over-stocking. The parameters of the control policy move as predicted, from a practical perspective when $u_{2\max}$ decreases, in order to avoid over-shortages (Figure 3.17).

Through the observations drawn made from the sensitivity analysis, it clearly appears that the results obtained make sense, and confirm and validate the proposed approach. They show the usefulness of the proposed model, given that the parameters of the control policies move as predicted, from a practical perspective. The next section studies the case of production rate-dependent multiple failure rates.

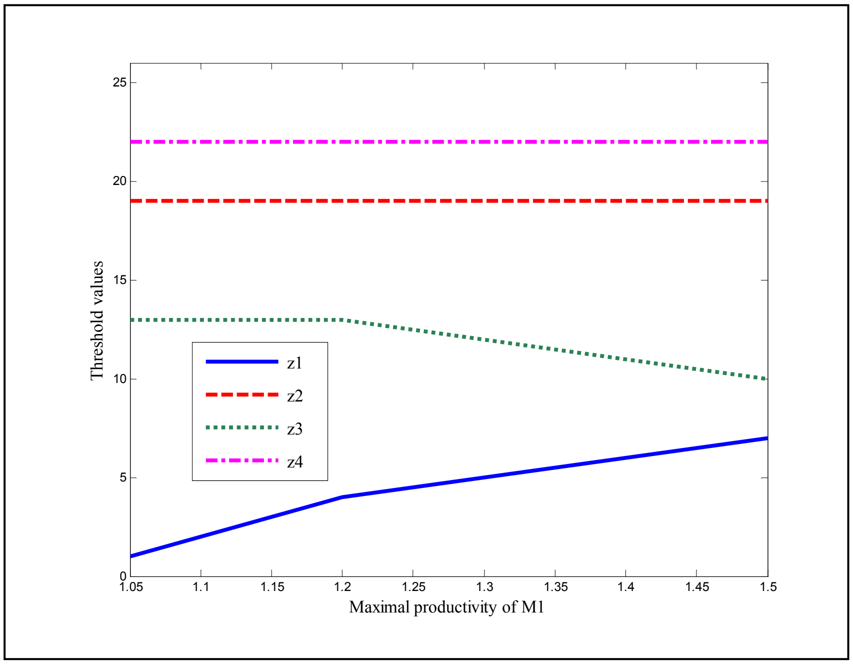


Figure 3.15 Sensitivity analysis with respect to maximal productivity of M_1

3.5 Extensions to the case of multiple failure rates

3.5.1 Numerical example

Section 3.4 showed that the hedging point policies are optimal for two failure rates of the main machine. In this section, we study the case of manufacturing systems consisting of two machines with five failure rates depending on the productivity of the main machine. These failure rates are given as follows:

$$q_{12}^{1}(u_1)=\begin{cases}\theta_1 & \text{if } u_1 \le U_1\\ \theta_2 & \text{if } u_1 \in (U_1,U_2]\\ \ldots & \\ \theta_5 & \text{if } u_1 \in (U_4,u_{1\max}]\end{cases} \tag{3.15}$$

where $0<\theta_1<\theta_2<\ldots<\theta_5$ and $0<U_1<U_2<U_3<U_4<U_5=u_{1\max}$.

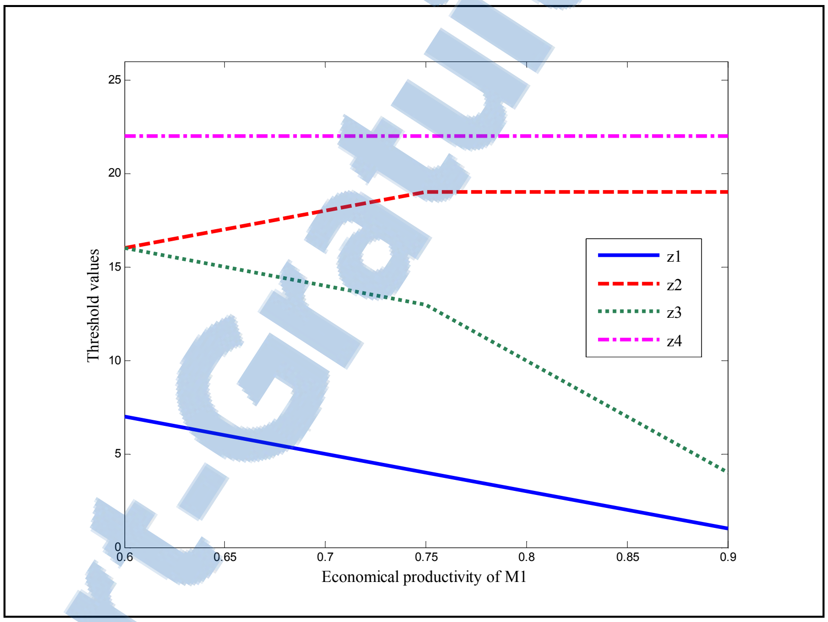


Figure 3.16 Sensitivity analysis with respect to economical productivity of M_1

The failure rate in equation (3.15) has the general form considered in Liberopoulos and Caramanis (1994),

$$q_{12}^{1}(u_1)=a\left(\frac{u_1}{u_{1\max}}\right)^{b} \tag{3.16}$$

where a and b are non-negative constants. The results for $a=0.02$ and different values of b are plotted in Figure 3.18.

The curves plotted in Figure 3.18 illustrate the impact of the machine's productivity on its dynamics. The solid sections represent the feasible productivity values (values for which the condition to meet customer demand is satisfied) when both machines are operational. The dashed sections represent the unfeasible values. The concave curve is represented by $b < 1$ and the convex curve by $b > 1$.

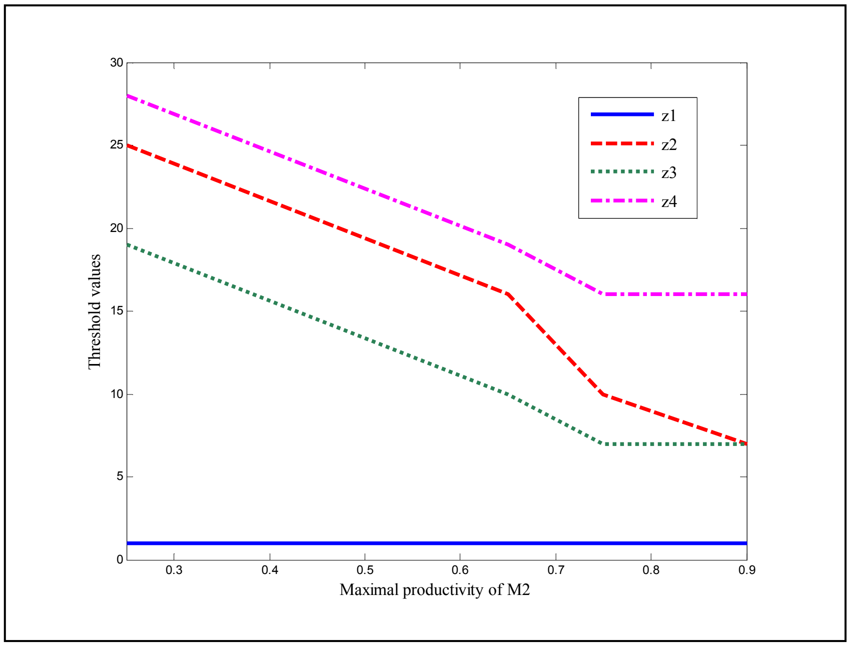


Figure 3.17 Sensitivity analysis with respect to maximal productivity of M_2

For a single machine, single product manufacturing system, where the failure rate depends on the production rate, it was concluded that an optimal feedback policy control does not exist if $U_n < d$, $n = 1,2,3,4,5$ (see Liberopoulos and Caramanis, 1994 and Martinelli, 2010). For a manufacturing system consisting of two machines, with a single product, such as the one studied in this paper, we examine the case of the main machine's productivity lower than the demand rate ($\frac{u_1}{u_{1\max}} < 0.9$). Typical results for productivity with values of $b = 0.4$ and $b = 3$ are shown in Figures 3.19 and 3.20, respectively. The values used for $\frac{u_1}{u_{1\max}}, U_n$ and θ_n are presented in Table 3.3.

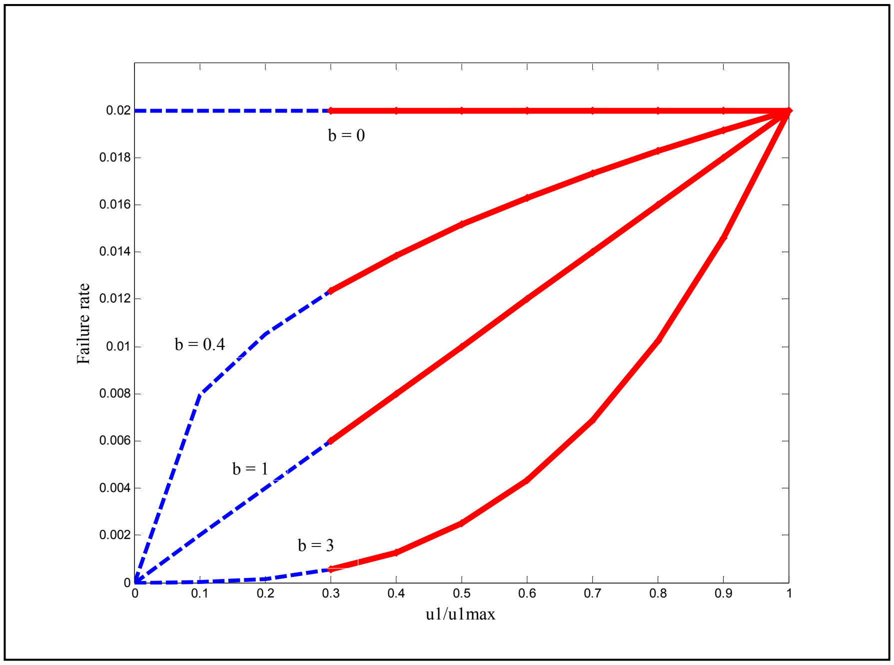


Figure 3.18 Failure rate of the main machine

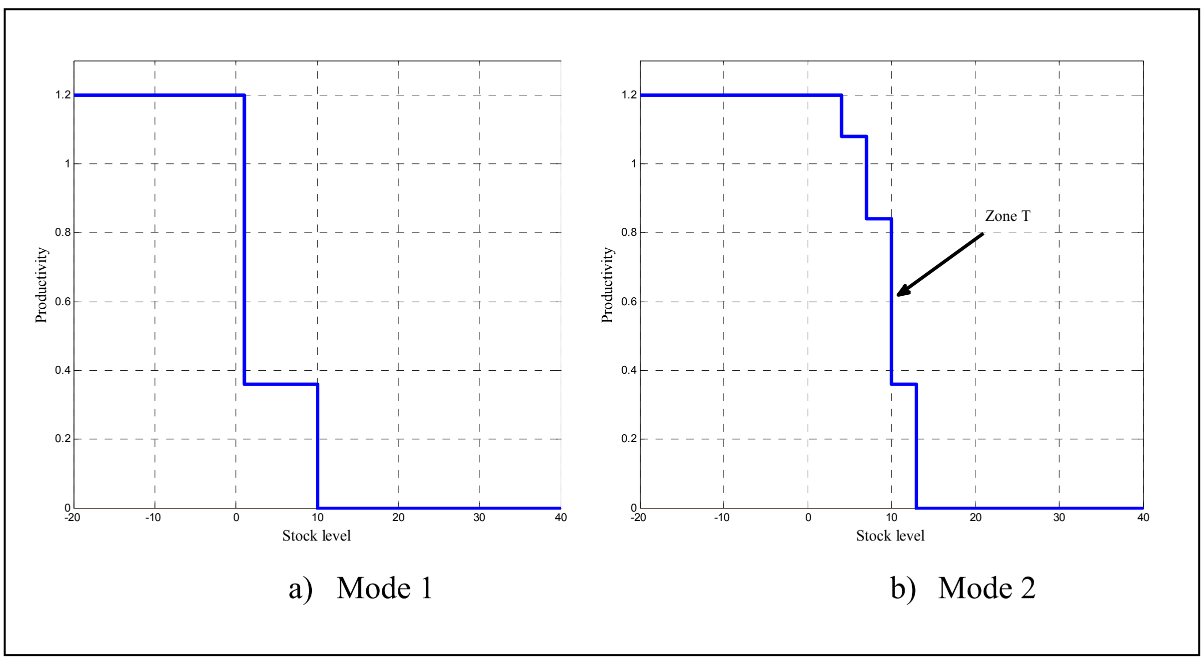
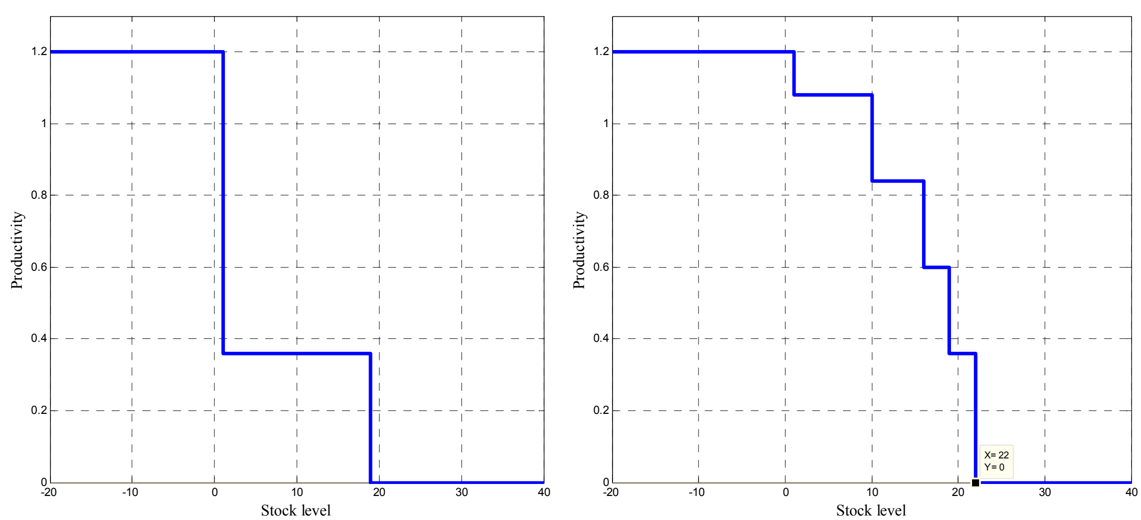


Figure 3.19 Productivity of M_1 at mode 1 and mode 2, $b = 0.4$

Table 3.3. Parameter values

$\frac{u_1}{u_{1\max}}$	0.3	0.5	0.7	0.9	1
U_n	0.36	0.60	0.84	1.08	1.2
$\theta_n(b=0.4)$	0.01236	0.01516	0.01734	0.01917	0.0200
$\theta_n(b=3)$	0.00054	0.0025	0.00686	0.01458	0.0200



a) Mode 1

b) Mode 2

Figure 3.20 Productivity of M_1 at mode 1 and mode 2, $b=3$

Based on the results presented in Figures 3.19 and 3.20, the productivity policy of M_1 defines three control rules at mode 1 (Figures 3.19a and 3.20a) and four (Figure 3.19b) or five (Figure 3.20b) control rules at mode 2. More specifically, these rules state that:

i) When the stock level is higher than the optimal threshold point, M_1 does not produce.

ii) If the stock level is lower than the first threshold point ($y_{11}, y_{12}, z_{11}, z_{12}$), M_1 should be set to its maximal productivity. Note that the requirement $d < u_1$ is not imposed for the machine M_1 because the system has a supporting machine M_2 (both machines can satisfy the customer demand together).

According to Figure 3.19, the corresponding multiple threshold point policy has the structure of equations (3.17) and (3.18) for mode 1 and mode 2, respectively.

$$u_1(x,1) = \begin{cases} u_{1\max} & \text{if } x < y_{11} \\ U_1 & \text{if } \mathrm{y}_{11} \le x < y_{21} \\ 0 & \text{if } x > y_{21} \end{cases} \tag{3.17}$$

where y_{11} and y_{21} are the first and second threshold values of M_1 at mode 1, respectively.

$$u_1(x,2) = \begin{cases} u_{1\max} & \text{if } x < y_{12} \\ U_4 & \text{if } \mathrm{y}_{12} \le x < y_{22} \\ U_3 & \text{if } \mathrm{y}_{22} \le x < y_{32} \\ U_1 & \text{if } \mathrm{y}_{32} \le x < y_{42} \\ 0 & \text{if } x > y_{42} \end{cases} \tag{3.18}$$

where $y_{i2}, i = 1,2,3,4$ is the ith threshold value of M_1 at mode 2.

The optimal policy of Figure 3.20 is defined by equations (3.19) and (3.20) for mode 1 and mode 2, respectively.

$$u_1(x,1) = \begin{cases} u_{1\max} & \text{if } x < z_{11} \\ U_1 & \text{if } \mathrm{z}_{11} \le x < z_{21} \\ 0 & \text{if } x > z_{21} \end{cases} \tag{3.19}$$

where z_{11} and z_{21} are the first and second threshold values of M_1 at mode 1, respectively.

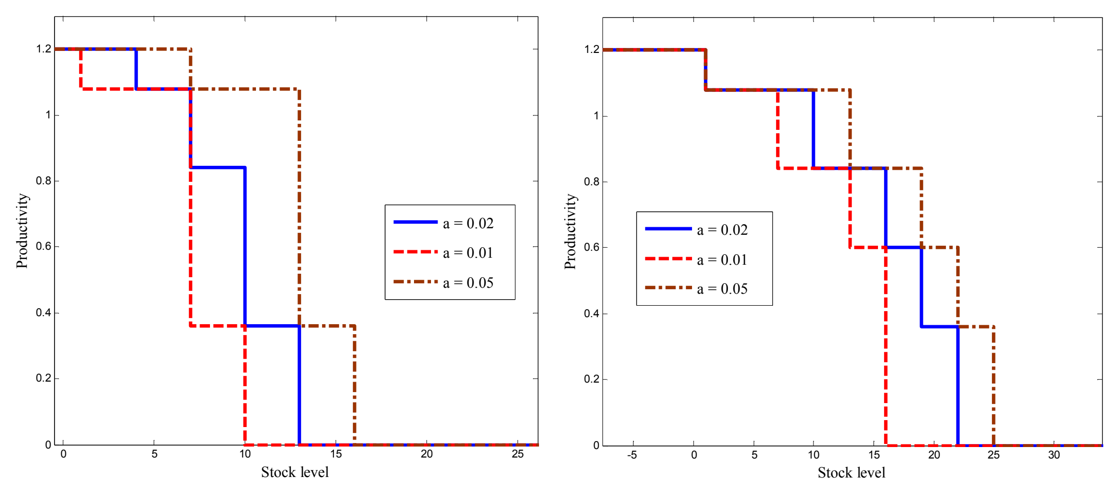
$$u_1(x,2) = \begin{cases} u_{1\max} & \text{if } x < z_{12} \\ U_4 & \text{if } \mathrm{z}_{12} \le x < z_{22} \\ U_3 & \text{if } \mathrm{z}_{22} \le x < z_{32} \\ U_2 & \text{if } \mathrm{z}_{32} \le x < z_{42} \\ U_1 & \text{if } \mathrm{z}_{42} \le x < z_{52} \\ 0 & \text{if } x > z_{52} \end{cases} \tag{3.20}$$

where $z_{i2}, i=1,2,3,4,5$ is the i[th] threshold value of M_1 at mode 2.

It clearly appears that at mode 2 (M_1 produces alone), when $u_1 = U_3 = 0.84$ (meaning that the machine begins to produce at a rate lower than the demand rate), the system switches directly from $U_3 = 0.84$ to its minimal value $U_1 = 0.36$ (see Zone T in Figure 3.19b). This is logical because the system has to avoid shortages and ensure its reliability at the same time.

3.5.2 Sensitivity analyses

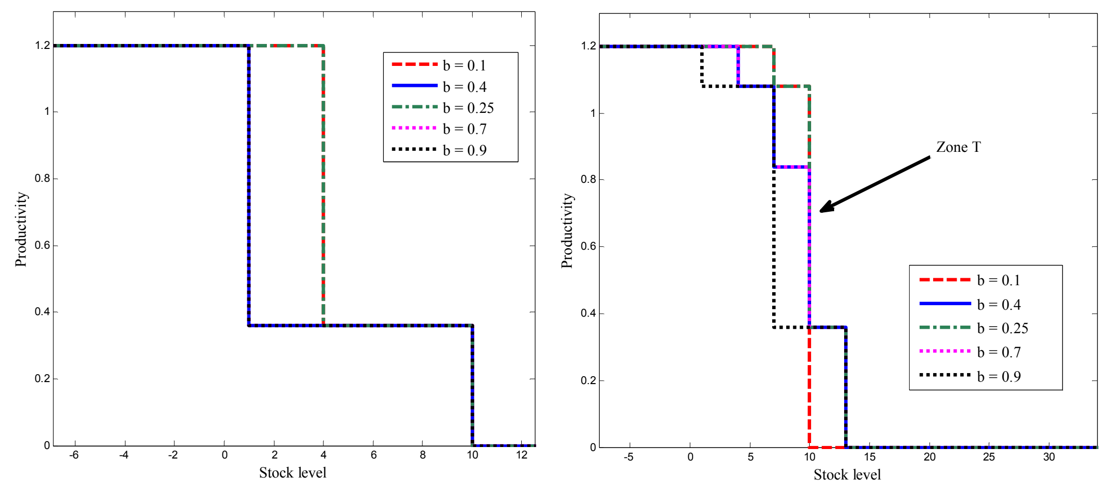
This section explains the usefulness of the obtained control policy. We perform a sensitivity analysis according to the variation of the parameters "a", "b" and "n" to illustrate the contribution of the proposed approach, and also to confirm the structure of the control policy. The productivity is presented in Figures 3.21 to 3.24.



a) Concave case "$b = 0.4$" b) Convex case "$b = 2$"

Figure 3.21 Sensitivity to the variation of "a"

The effect of the variation of the parameter a on the productivity policy is illustrated in Figure 3.21. The parameter takes three values: $a = 0.01,$ 0.02 and 0.05. When the parameter is low, $a = 0.01$, it means the system experiences fewer failures. Thus, the threshold value is low. If the parameter is set to $a = 0.05$, the system needs more protection against failures, leading to an even greater increase in the threshold value. It is worth mentioning that when a increases, the probability of failure of the system increases and the reliability of the machine is reduced. This necessarily leads to a high likelihood of increasing the threshold level in order to avoid shortages.



a) Mode 1 b) Mode 2

Figure 3.22 Sensitivity to the variation of "b"; Concave case

From the results obtained in Figures 3.22 and 3.23, we notice that when parameter b increases, the threshold values increase. If parameter b is increased, then for the same productivity value, the failure rate decreases. This means that the system produces for a long time before failure. The variation of parameter b does not affect the second threshold value at mode 1.

At mode 2, when b decreases ($b = 0.1$ and $b = 0.25$), zone T increases and is the same (see Figure 3.22b). Decreasing b in the concave case means that the failure rate increases. The system must store to avoid shortages.

The results of Figure 3.23 show that in the convex case, decreasing parameter b does not affect the productivity trend at mode 1. However, when b increases ($b=5$ and b=7), Figure 3.23a shows three stages instead of two stages, as in the basic case ($b=3$). It is clear that higher values of b reduce the deterioration of the system. The system can produce to its intermediate speeds before reaching the minimal productivity.

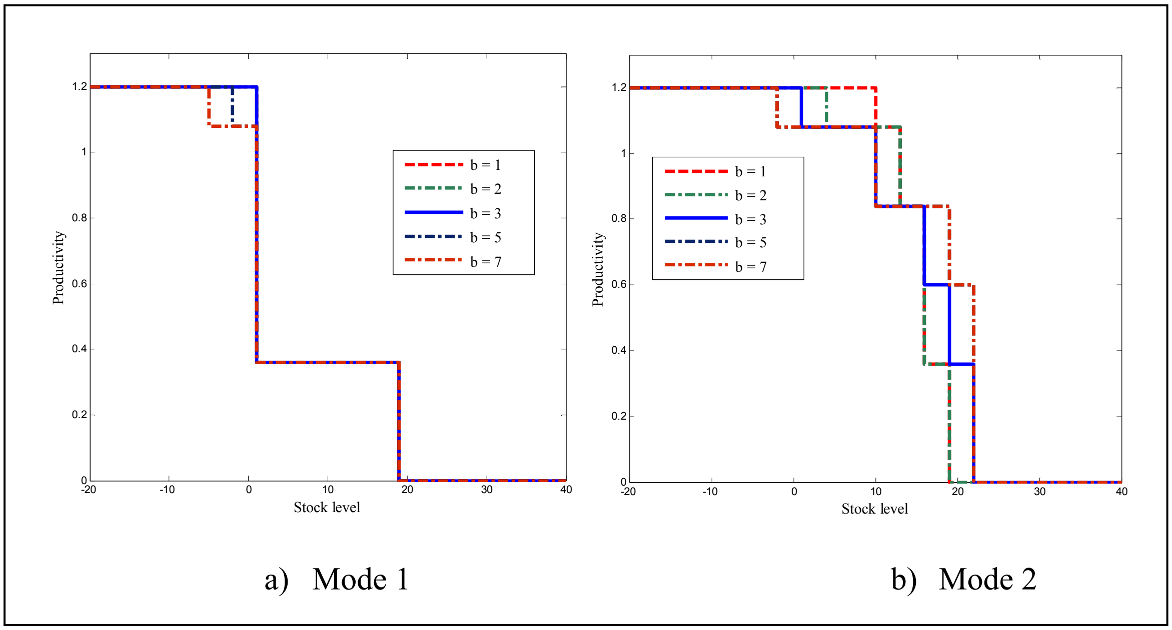


Figure 3.23 Sensitivity to the variation of "b"; Convex case

The results of Figure 3.24a show that the variation of parameter n does not affect the threshold values at mode 1. At mode 2 (Figure 3.24b), the number of stages does not change when n increases. For example, when $n>5$, we have five threshold parameters, such as in the case of five failure rates. However, when the number of failures decreases, the number of stages decreases. The next section presents the discussions and how to implement the obtained control policies.

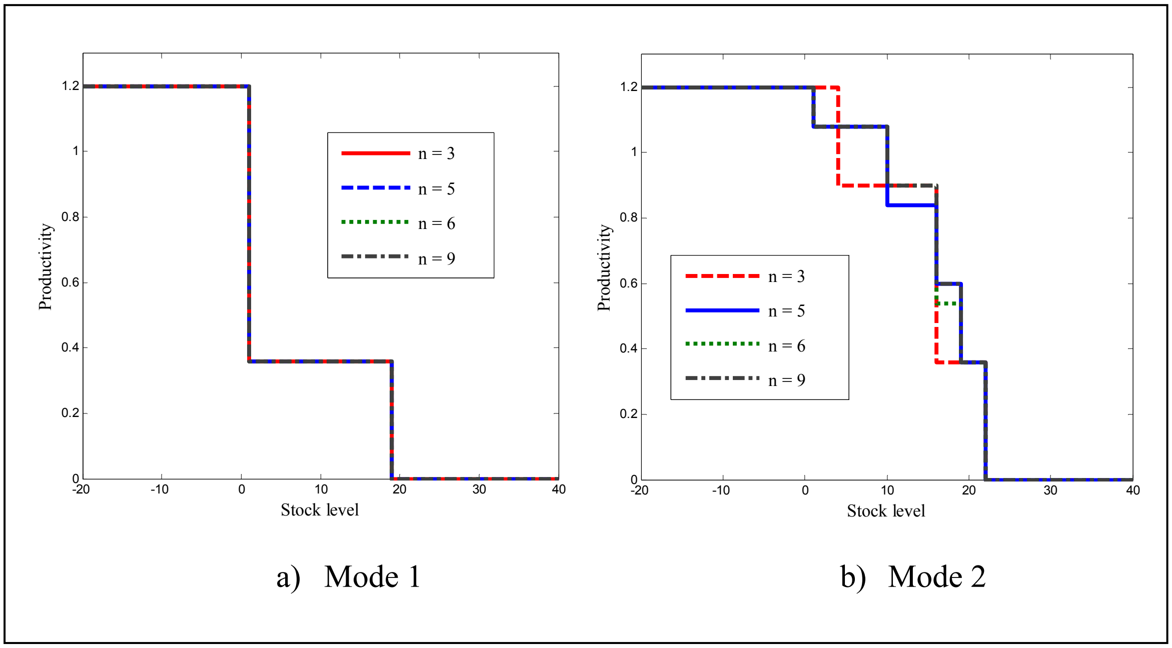


Figure 3.24 Sensitivity to the variation of "n"

3.6 Discussions and policies implementation

The results of Figures 3.4 (M_1 at mode 1) and 3.6 (M_1 at mode 2) confirm the possible practical suggestion based on the analysis of Figure 3.1. The results suggest that to obtain gains in availability of the main machine and to reduce the total machining cost incurred, it may be beneficial to decrease the productivity speed from the maximal value to the economical value when the inventory level approaches the maximal threshold values.

In the sensitivity analysis, we observe that the threshold values increase as the backlog costs increase (Figures 3.8 and 3.9). This seems natural in order to avoid further backlog costs. Figures 3.10 to 3.17 show that the parameters of the control policy move as predicted, from a practical perspective, when the machine parameters change.

For the manufacturing system considered, the optimal control policies are characterized by four different threshold parameters (z_1,z_2,z_3 and z_4) for two failure rates of M_1, which constitute a main finding of this paper. For n different failure rates of the main machine, the control policies will depend on more than n different threshold parameters. In Section 3.5, the case of five failure rates was studied. The most important observation from the results is that the optimal policy exists and is still equivalent to the multiple threshold policy. In the concave case, the structure of the productivity policy is the multiple hedging point policy because the system consists of two machines. The effect of the second machine reduces the concavity of the curve. Therefore, the concave curve is close to linear ($b=1$), or is nearly convex. At mode 2, where M_2 is under repair, we have multiple thresholds in both cases. With two machines, even if one machine fails, the system knows that it exists and will return to the operational state in a relatively short time.

At mode 1, where both machines produce, Figures 3.19a and 3.20a show that the optimal control policy of the main machine is characterized by two threshold parameters, such as in the system described in Section 3.4 with two failure rates. In the case of five failure rates, the system prefers to skip the intermediate productivities of the main machine and to produce directly to the minimal value ($U_1 = 0.36$) over a long period. It can then use the supporting machine to fill demand. This is logical because the probability of failure of the main machine increases with high productivity, while the failure rate of the supporting machine is constant. However, at mode 2, the optimal policy is characterized by four (concave case) or five (convex case) threshold parameters (Figures 3.19b and 3.20b). The number of stages in the concave case is less than the number of stages in the convex case because from $U_3 = 0.84$ to $U_2 = 0.60$, Table 3.3 shows that the differences between the failure rates are lower. In contrast, this difference is higher in the convex case. The system must rapidly reduce the productivity of the machine to account for its reliability. Hence, the system passes through $U_2 = 0.60$ before reaching $U_1 = 0.36$ (see Figure 3.20b). Another remark regarding Figure 3.20b is that $z_{52} = 22 = z_4$. The threshold parameter is the same with five failures. This means that to achieve gains in availability of the main machine and to reduce the total machining costs incurred, the system does several speed jumps before reaching the optimal stock level.

Through the sensitivity analysis conducted, Figure 3.21 shows that the threshold values increase when the parameter a increases. This necessarily leads to a sustained increase in the threshold level in order to avoid shortages. According to Figures 3.22 and 3.23, the results show that the parameters of the control policy move as predicted, from a practical perspective, when parameter b changes. Unlike in the concave case, where Figure 3.22a shows two stages, as in the basic case, Figure 3.23a shows three stages when b increases. This is due to the fact that the failure rate decreases faster in the convex case than it does in the concave case. We should recall that the failure rate decreases when b increases. The system can use its intermediate speeds before achieving minimal productivity. The results of Figure 3.24a show that at mode 1 where both machines are operational, for gaining in availability of the main machine, the system maintains the same threshold values and uses the supporting machine to fill the customer demand. Increasing of parameter n ($n > 5$) does not change the number of stages at mode 2 (see Figure 3.24b) because for several values of productivity comprised between $U_1 = 0.36$ and $u_{1\max} = 1.2$, the differences between values are too low. In this way, the system skips some intermediate values. If the value of $u_{1\max}$ increases, the number of threshold parameters will be increased as well. However, when $n = 3$, there is three threshold parameters. The relevance of the sensitivity analysis is apparent, since it seems that our results are logical and consistent, and this enables us to confirm the structure of the control policies obtained.

Figure 3.25 illustrates the implementation of the control policy when the number of failure rates is $n = 2$. This illustration shows the actions that should be taken by the manager when both machines are producing (mode 1), and when the supporting is under repair (mode 2). Based on the diagram of Figure 3.25, we can see how the production speed of the main machine is set to different values depending on the both machine modes (functioning or failure) and stock level. Thus, the obtained policies have a direct managerial implication, namely the manager can use obtained results to define the parameters of the manufacturing system in order to optimize the production process.

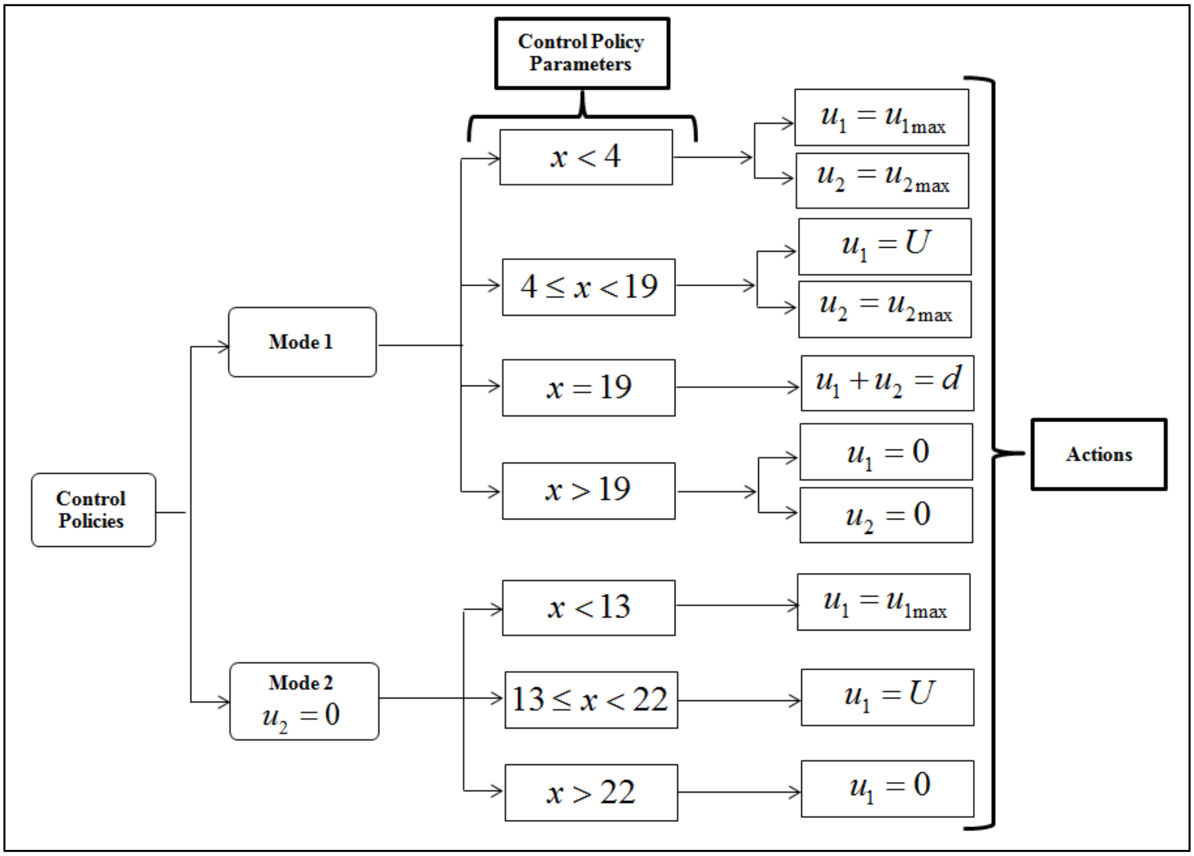


Figure 3.25 Illustration of the production policy

3.7 Conclusion

The number of scientific publications in the field of deteriorated systems is growing steadily, reflecting the increasing importance of this subject. However, the reported works are mostly based on systems which deteriorate with the age and the number of failures. This paper investigates the problem of minimizing a cost function which penalizes both the presence of waiting customers and the inventory surplus. The manufacturing system studied comprises parallel machines subject to a non-homogeneous Markov process, with the failure rate depending on the productivity. The machines produce a single part type. We developed the stochastic optimization model of the considered problem with two decision variables (productivities of the main and the supporting machines) and one state variable (stock level of final products). From the numerical study, it has been found that for two parallel machines systems, when the failure rate of the main machine depends on its productivity, the hedging point policies are optimal within a four-threshold feedback policy, and the reliability of the machines is enhanced. The results also show that to reduce the total machining cost, it may be beneficial to decrease the productivity of the main machine from its maximal value to its economical value when the inventory level approaches the threshold value. We illustrated and validated the proposed approach using a numerical example and a sensitivity analysis. We have studied the case of manufacturing systems involving multiple failure rates, and the results obtained are very satisfactory and may be productive for future research to address the issue of multiple-part-type, random demand rates and multiple-machine (more than two machines) systems.

CHAPITRE 4

ARTICLE 3: PRODUCTION PLANNING OF A FAILURE-PRONE MANUFACTURING/REMANUFACTURING SYSTEM WITH PRODUCTION-DEPENDENT FAILURE RATES

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Résumé

Cet article est l'étude combinée des systèmes hybrides de *manufacturing/remanufacturing* soumis aux pannes et réparations aléatoires. Les machines produisent un seul type de pièce et les pénuries sont permises. A la fin de leur cycle de vie ou après utilisation, les pièces usagées sont récupérées chez les clients pour être remises dans le circuit du système de production. L'objectif ici est de trouver les politiques optimales de *manufacturing* et de *remanufacturing* qui permettent d'avoir un coût total minimal incluant les coûts du stock des produits finis, les coûts de pénurie et les coûts du stock des produits retournés. La machine de *manufacturing* se dégrade en fonction de son taux d'utilisation. Donc son taux de panne dépend de son taux de production. Le taux de panne de la machine du *remanufcaturing* est constant. En réduisant le taux de production de la machine de *manufacturing* pour tenir compte de sa fiabilité, celle-ci ne peut plus satisfaire la demande, d'où le *remanufacturing* des produits retournés pour combler la demande manquante. Le modèle est résolu par des méthodes numériques et les conditions d'optimums sont obtenues par programmation dynamique stochastique. Un exemple numérique et des analyses de sensibilité sont élaborés pour montrer la pertinence de l'approche proposée. Nos résultats montrent clairement que le système est robuste par rapport à la performance dynamique du système et les processus de *manufacturing/remanufacturing* peuvent aider à améliorer les performances du système dynamique. Ainsi, les avantages perçus de *remanufacturing* de produits, tant environnemental qu'économiques, tel que cités dans la littérature se confirment lorsque les taux de pannes de la machine dépendent de son taux de production.

Mots-clés: Processus stochastique, Commande optimal, Logistique inverse, Planification de la production, Méthodes numériques.

Abstract

This paper deals with the production-dependent failure rates for a hybrid manufacturing/remanufacturing system subject to random failures and repairs. The failure rate of the manufacturing machine depends on its production rate, while the failure rate of the remanufacturing machine is constant. In the proposed model, the manufacturing machine is characterized by a higher production rate. The machines produce one type of final product and unmet demand is backlogged. At the expected end of their usage, products are collected from the market and kept in recoverable inventory for future remanufacturing, or disposed of. The objective of the system is to find the production rates of the manufacturing and the remanufacturing machines that would minimize a discounted overall cost consisting of serviceable inventory cost, backlog cost and holding cost for returns. A computational algorithm, based on numerical methods, is used for solving the optimality conditions obtained from the application of the stochastic dynamic programming approach. Finally, a numerical example and sensitivity analyses are presented to illustrate the usefulness of the proposed approach. Our results clearly show that the optimal control policy of the system is obtained when the failure rates of the machine depend on its production rate.

Keywords: Stochastic process, Optimal control, Reverse logistics, Production planning, Numerical methods.

4.1 Introduction

With markets globalization and technological advancement, manufacturing systems are faced with optimization problems in their global supply chain of production. Problems of production planning become more complex when the environmental constraints require optimization of the production and reuse parts returned by customers after use (reverse logistics). Compared to a situation where customer demand is only satisfied by the direct line of production (production from raw materials), the simultaneous control of production and product recovery is very complex (Kiesmüller, 2003). Product recovery management deals with the collection of used and End of Life products in order to remanufacture the products, reuse the components or recycle the materials. Remanufacturing is one of the most desirable options of product recovery (Aksoy and Gupta, 2005). Accordingly, Aksoy and Gupta (2005) point out that remanufacturing is an industrial process involving the conversion of worn-out products into like-new conditions. While for Kumar and Putnam (2008), remanufacturing is restoring a product to like-new condition by reusing, reconditioning and replacing parts. A number of firms have focused on remanufacturing. Rolls-Royce, MTU aero Engines, General Electric, Caterpillar and Cummins Engine are only a few prominent examples (Jian et al., 2010). Savaskan et al. (2004) are prominent examples of successful remanufacturing initiatives. Hybrid manufacturing-remanufacturing systems are often subject to random events such as equipment (production facilities) failure. The past decade, many authors have provided works in the area of reverse logistics systems without considering the stochastic aspects related to the dynamic of the manufacturing and the remanufacturing machines and production-dependent failure rates. This paper deals with a stochastic manufacturing/remanufacturing system consisting of two parallel machines (manufacturing and remanufacturing machines) which produce one part type. The stochastic nature of the system is due to machines that are subject to a non-homogeneous Markov process resulting from the dependence of failure rates on the production rate. Whenever a breakdown occurs, a corrective maintenance is performed to restore the machines to their operational mode. The main contribution of this paper is to joint control of the manufacturing and remanufacturing policies with production-dependent failure rates. Our objective is to find the production rates of both machines so as to minimize a discounted overall cost consisting of serviceable inventory cost, backlog cost and holding cost for returns. A computational algorithm, based on numerical methods, is used for solving the optimality conditions obtained from the application of the stochastic dynamic programming approach. Finally, a numerical example and sensitivity analyses are presented to illustrate the usefulness of the proposed approach.

The remainder of the paper is organized as follows. A literature review is presented in Section 4.2. Section 4.3 consists of notations and assumptions of the model. The problem statement is also described in detail in Section 4.3. Section 4.4 provides numerical results and sensitivity analyses to illustrate the usefulness of the proposed approach. The paper ends with conclusion in Section 4.5.

4.2 Literature review

Several authors have worked on the study of a combined manufacturing and remanufacturing system. However, nowadays few studies have been included aspects related to the stochastic dynamics of machines and its maintenance activities. Stochastic dynamics models allow approaching more real cases characterized by the presence of random phenomena. Until now, few papers have studied a non-homogeneous Markov process (dependence of failure rates on the production rate) for a hybrid manufacturing-remanufacturing system. Literature on the combined manufacturing, remanufacturing and maintenance; the manufacturing system with production-dependent failure rates is discussed below.

In the Esterman et al. (2006) paper, a general framework for reliability prediction in a remanufacturing environment was proposed. A case study of a remanufactured engine cylinder head that has had a fatigue crack repaired by a welding process was presented in order to illustrate the process. Their approach combined the use of Failure Modes and Effects Analysis (FMEA), Experimental Model Building, Monte Carlo Simulation and Linear Elastic Fracture Mechanics (LEFM) to generate a reliability estimate. The FMEA and physical modeling was used to generate a model that relates the welding process control parameters to the fatigue performance of the test specimens. Monte Carlo Simulation techniques and LEFM was built on the above model to relate the process control parameters to the reliability performance. Min and Ko (2008) develop a mathematical model and genetic algorithm (GA) which aim to provide a minimum cost solution for the reverse logistics network design problem involving product returns for repairs. The model considered, explicitly, savings due to the use of existing warehouses as repair facilities and costs associated with location or expansion. Computational experimentation revealed that GA presented a promise in solving practical size problems with multi-commodities, 90 customers, 10 potential sites, and 10-year periods. Also, the model and solution procedure produced multi-echelon reverse logistics configurations that consider the options of both direct product returns from customers to manufacturing plants and indirect returns through either repair facilities or regional warehouses. Their study found that the location/allocation decision of repair facilities or regional warehouses should be re-evaluated and changed over time. Berthaut et al. (2009); Pellerin et al. (2009) extended the work for repair/remanufacturing system, considered by Gharbi et al (2008) to the production control problem for a remanufacturing system executing capital assets repair and remanufacturing in a single system that integrates the replacement unavailability case. They assumed that the production system responds to planned demand at the end of the expected life cycle of each individual piece of equipment and unplanned demand triggered by a major equipment failure. The authors formulated their problem as a multi-level control policy based on inventory thresholds triggering the use of different execution modes and propose a suboptimal policy. Their main objective of the maintenance and remanufacturing organisation was to maintain the number of serviceable items above the operating firms' service levels. Tongzhu et al. (2010) improved the system reliability of remanufactured products. The authors pointed out the reliability requirements of remanufactured systems. Then, they analysed systematically the similarities and differences between the manufacturing, remanufacturing and repairing process. The reliability design of remanufacturing was defined and the reliability prediction and allocation methods were investigated. Kenne et al. (2012) treated the production planning and control involving combined manufacturing and remanufacturing operations within a closed-loop reverse logistics network with machines subject to random failures and repairs. The objective was to propose a manufacturing/remanufacturing policy that would minimize the sum of the holding and the backlog costs for manufacturing and remanufacturing products. Ouaret et al. (2013) extended the model considered by Kenne et al. (2012) to the production control problem for hybrid manufacturing/remanufacturing systems subject to random demand.

In the preceding paragraph, we note that the work of Kenne et al. (2012) represents the first attempt to consider hybrid manufacturing/remanufacturing systems where machines are subject to random failures and repairs. However, the authors did not address the question of what happens if the machines are used to their maximum production capacity for a long period, and they did not consider the stock of returns.

One of the most important achievements of the research of Hu et al. (1994) was the investigation of the necessary and sufficient conditions for the optimality of the hedging point policy for a single machine, single part-type problem, when the failure rate of the machine is a function of productivity. They showed that hedging point policies are only optimal under linear failure rate functions. As per their discussion, numerical results in the general case suggest that as the inventory level approaches a hedging level, it may be beneficial to decrease productivity in order to realize gains in reliability. This conjecture was confirmed by the numerical results reported in Martinelli (2007), where the author considered a long average cost function and a machine characterized by two failure rates: one for low and one for high productivities. Martinelli (2010) generalizes the problem of Martinelli (2007) by considering one machine with different failure rates: more specifically, the failure rate is assumed to depend on productivity, through an increasing, piecewise constant function.

The results of Hu et al. (1994), Martinelli (2007) and Martinelli (2010) are limited to one manufacturing machine. Based on the literature review, we point out that in the context of reverse logistics, it would be of interest to analyze systems consisting of at least two machines, taking into account the gradual deterioration along the production process – this is the main topic addressed in our paper.

4.3 System under study

This section presents the assumptions used throughout this article, as well as the problem statement.

4.3.1 Assumptions

(1) The failure rate of the manufacturing machine depends on its production rate. This assumption is the major motivation of our paper. Other works consider one machine with a production-dependent failure rate or two machines (manufacturing and remanufacturing machines) without deterioration with production speed.
(2) The shortage cost depends on parts produced for backlog ($/unit).
(3) The inventory cost depends on parts produced for positive inventory ($/unit).
(4) The production rate of the manufacturing machine is higher than that of the remanufacturing machine.
(5) The remanufacturing machine cannot satisfy customer demand alone.
The manufacturing machine is unable to satisfy customer demand with its economical productivity, which is why the remanufacturing machine is called upon to fill the demand rate.
(6) Manufacturing processes convert the raw materials to finished items.
(7) Remanufacturing processes convert used products to as good as new parts
(8) New parts (manufactured and remanufactured) satisfy the serviceable inventory.
(9) Backorders of unsatisfied demands are permitted.

4.3.2 Problem statement

The system under study as depicted in Figure 4.1 consists of a hybrid manufacturing and remanufacturing system. The whole system faces one part type demands. Manufacturing and remanufacturing resources (denoted by M_1 and M_2 respectively) in parallel are subject to random breakdowns and repairs. When the manufacturing machine works at a faster rate, it is more likely to fail. Then, the failure rate of M_1 depends on its production rate, while the failure of M_2 is constant. The repair rates of both machines are constant. The maximum production rates of the machines are known and the demand process for finished products is deterministic. At the expected end of their usage, products are collected, cleaned and disassembled by a third party for possible reuse. The return process is deterministic (percentage of the demand rate). During inspection, used products can be segregated into different quality levels. The products can then either be remanufactured or kept in recoverable inventory for future remanufacturing, or disposed of.

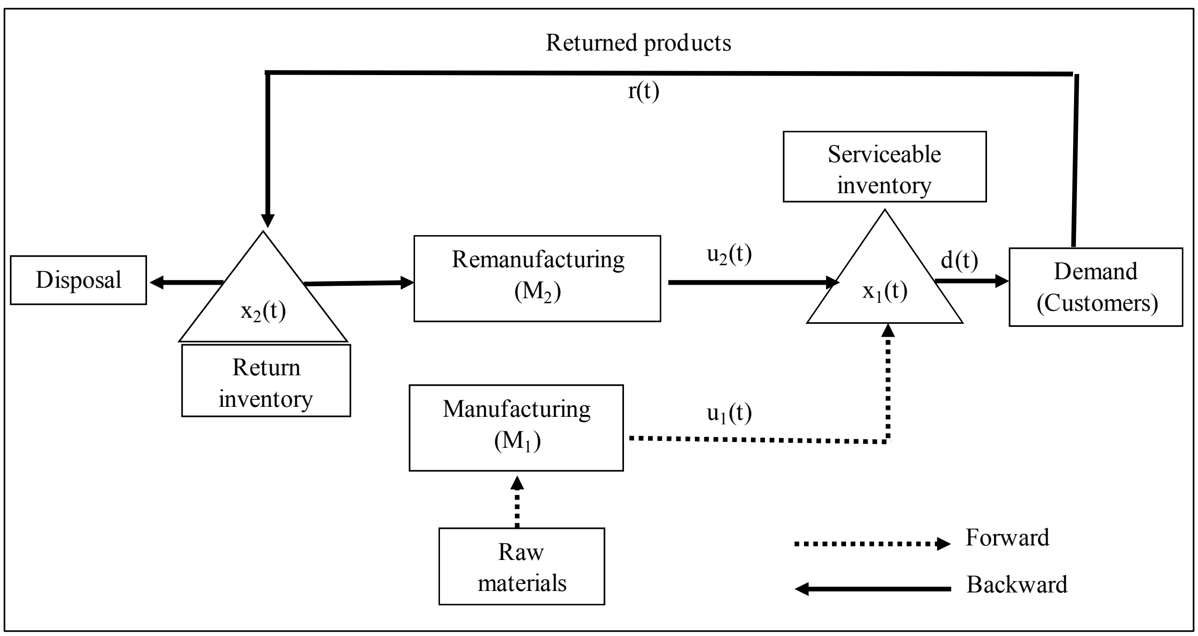

Figure 4.1 Hybrid manufacturing/remanufacturing system

The mode of the machine M_i can be described by a stochastic process $\xi_i(t),\ i=1,2$. Such a machine is available when it is operational ($\xi_i(t)=1$) and unavailable when it is under repair ($\xi_i(t)=2$).

The operational mode of the system can be described by the random vector $\xi(t)=(\xi_1(t),\xi_2(t))$. Given that the dynamics of each machine is described by a 2-state stochastic process, we define a combined stochastic process $\xi(t) \in B = \{1,2,3,4\}$; its possible values are determined from the values of $\xi_1(t)$ and $\xi_2(t)$, as follows:

- Mode 1: M_1 and M_2 are operational
- Mode 2: M_1 is operational and M_2 is under repair
- Mode 3: M_1 is under repair and M_2 is operational
- Mode 4: M_1 and M_2 are under repair

With $\lambda_{\alpha\beta}$ denoting a jump rate of the system from state α to state β, we can describe $\xi(t)$ statistically by the following state probabilities:

$$\mathrm{P}\left[\xi(t+\delta t)=\beta \middle| \xi(t)=\alpha\right]=\begin{cases}\lambda_{\alpha\beta}(.)\,\delta t+o(\delta t) \ \ if\, \alpha\neq\beta \\ 1+\lambda_{\alpha\beta}(.)\,\delta t+o(\delta t) \ \ if\, \alpha=\beta\end{cases} \tag{4.1}$$

where $\lambda_{\alpha\beta} \geq 0 \quad (\alpha \neq \beta)$, $\lambda_{\alpha\alpha} = -\sum_{\beta\neq\alpha} \lambda_{\alpha\beta}$ and $\lim_{\delta t\to 0} \frac{o(\delta t)}{\delta t} = 0$ for all $\alpha, \beta \in B$.

The transition diagram, which describes the dynamics of the considered manufacturing system, is presented in Figure 4.2, with: $q_{12}^1 = \lambda_{13} = \lambda_{24}$ (failure rate of M_1), $q_{12}^2 = \lambda_{12} = \lambda_{34}$ (failure rate of M_2), $q_{21}^1 = \lambda_{31} = \lambda_{42}$ (corrective maintenance rate of M_1) and $q_{21}^2 = \lambda_{21} = \lambda_{43}$ (corrective maintenance rate of M_2).

The dynamics of the system is described by a discrete element, namely $\xi(t)$, and continuous elements $x_1(t)$ and $x_2(t)$. The discrete element represents the status of the machines and the continuous one, the stock level of serviceable inventories and returned items. The stock level $x_1(t)$ can be positive for an inventory or negative for a backlog. We assume that there is no shortage of returned products, then $x_2(t) \geq 0$.

We assume that the failure rate of M_1 depends on its production rate, and is defined by:

$$q_{12}^{1} = \begin{cases} \theta_1 & \text{if } u_1 \in (U, u_{1\max}] \\ \theta_2 & \text{if } u_1 \in [0, U] \end{cases} \qquad \text{with } \theta_1 \geq \theta_2 \geq 0 \text{ and } 0 \leq U \leq u_{1\max}$$

Hence, $\xi(t)$ is described by the following matrix:

$$Q = \Theta_i; \quad \begin{cases} i = 1 \text{ if } u_1 \in (U, u_{1\max}] \\ i = 2 \text{ if } u_1 \in [0, U] \end{cases} \quad \text{with} \tag{4.2}$$

$$\Theta_i = \begin{pmatrix} -(q_{12}^2 + \theta_i) & q_{12}^2 & \theta_i & 0 \\ q_{21}^2 & -(q_{21}^2 + \theta_i) & 0 & \theta_i \\ q_{21}^1 & 0 & -(q_{21}^1 + q_{12}^2) & q_{12}^2 \\ 0 & q_{21}^1 & q_{21}^2 & -(q_{21}^1 + q_{21}^2) \end{pmatrix}; \text{ where } \theta_i = \lambda_{13} = \lambda_{24}$$

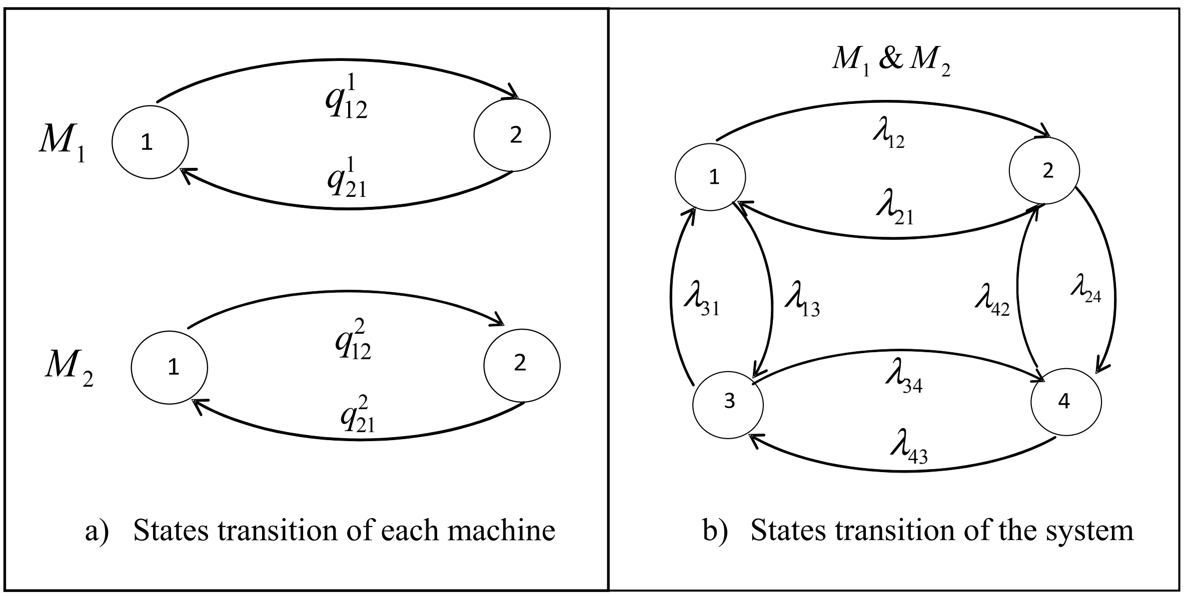

Figure 4.2 States transition diagram of the considered system

Let $u_1(t)$ and $u_2(t)$ denote the production rates of the machines 1 and 2, respectively, in mode α and at time t for a given stock levels x_1 and x_2.

The set of the feasible control policies A, including $u_1(\cdot)$ and $u_2(\cdot)$, is given by:

$$\mathrm{A} = \left\{ (u_1(\cdot), u_2(\cdot)) \in \Re^2, 0 \leq u_1(\cdot) \leq u_{1\max}, 0 \leq u_2(\cdot) \leq u_{2\max} \right\} \tag{4.3}$$

where $u_1(\cdot)$ and $u_2(\cdot)$ are known as control variables, and constitute the control policies of the problem under study. The maximal productivities of the manufacturing machine and the remanufacturing machine are denoted by $u_{1\max}$ and $u_{2\max}$, respectively.

The continuous part of the system dynamics is described by the following differential equations:

$$\frac{dx_1(t)}{dt} = u_1(t) + u_2(t) - d,\ x_1(0) = x_{10} \tag{4.4}$$

$$\frac{dx_2(t)}{dt} = r(t) - u_2(t) - disp,\ x_2(0) = x_{20},\ x_2(t) \geq 0 \tag{4.5}$$

where $x_{10}, x_{20}, r, disp$ and d are the given initial stock level of serviceable inventories and returned items, return rate, disposal rate and demand rate, respectively.

Let $g(\cdot)$ be the cost rate defined as follows:

$$g(\alpha, x_1, x_2, \cdot) = c_1^+ x_1^+ + c_1^- x_1^- + c_2 x_2 \tag{4.6}$$

where constants c_1^+, c_1^- and c_2 are used to penalize serviceable inventory and backlog, and inventory of returns, respectively. These holding and backlog costs are such that $c_1^- > c_1^+ > c_2$. $x_1^+ = \max(0, x_1), x_1^- = \max(-x_1, 0)$.

The production planning problem considered in this paper involves the determination of the optimal control policies ($u_1^*(t)$ and $u_2^*(t)$) minimizing the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha, x_1, x_2, u_1, u_2) = E\left\{\int_0^\infty e^{-\rho t} g(\alpha, x_1, x_2)\,dt \,\middle|\, x_1(0) = x_{10}, x_2(0) = x_{20}, \xi(0) = \alpha\right\} \tag{4.7}$$

where ρ is the discount rate. The value function of such a problem is defined as follows:

$$v(\alpha, x) = \inf_{(u_1(\cdot), u_2(\cdot)) \in A} J(\alpha, x, u_1, u_2) \quad \forall \alpha \in B \tag{4.8}$$

Based on the value function presented in equation (4.8), the optimality conditions and the numerical methods used to solve them (in order to determine the optimal manufacturing and remanufacturing rates) are presented in Appendix 4.A.

The next section provides a numerical example to illustrate the structure of the control policies.

4.4 Analysis of results and sensitivity analysis

Here, we illustrate the resolution of the model above with a numerical example. Sensitivity analyses with respect to the system parameters are also presented to illustrate the importance and effectiveness of the proposed methodology.

4.4.1 Optimal control results of numerical illustration

This section gives a numerical example for a hybrid manufacturing/remanufacturing system presented in Section 4.3. A four-state non-homogeneous Markov process with the modes in $B=\{1,2,3,4\}$ describes the system capacity. The instantaneous cost is described by equation (4.6).

The considered computation domain D is given by:

$$D=\{x_1 : -10 \le x_1 \le 30; \quad x_2 : 0 \le x_2 \le 25\} \tag{4.9}$$

The production line will be able to meet the demand rate over an infinite horizon and reach a steady state if the following condition of limiting probability or availability (π_1, π_2, π_3 and π_4) of the production line at operational modes is fulfilled: $\pi_1 \cdot (U + u_{2\max}) + \pi_2 \cdot U + \pi_3 \cdot u_{2\max} > d$. With $\pi \cdot Q(\cdot) = 0$ and $\sum_{i=1}^{4} \pi_i = 1$, and the data presented in Table 4.1, with $r = 0.5 * d$ and $disp = 0.1 * r$. The condition for meeting customer demands is also satisfied with $u_{1\max}$ because $U < u_{1\max}$.

Table 4.1 Numerical data of the considered system

c_1^+	c_1^-	c_2	h_1	h_2	U	$u_{1\max}$	$u_{2\max}$
2	50	1	0.5	0.5	1.2	1.3	1.15
d	θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	ρ	
1.25	1/80	1/100	1/60	1/15	1/15	0.09	

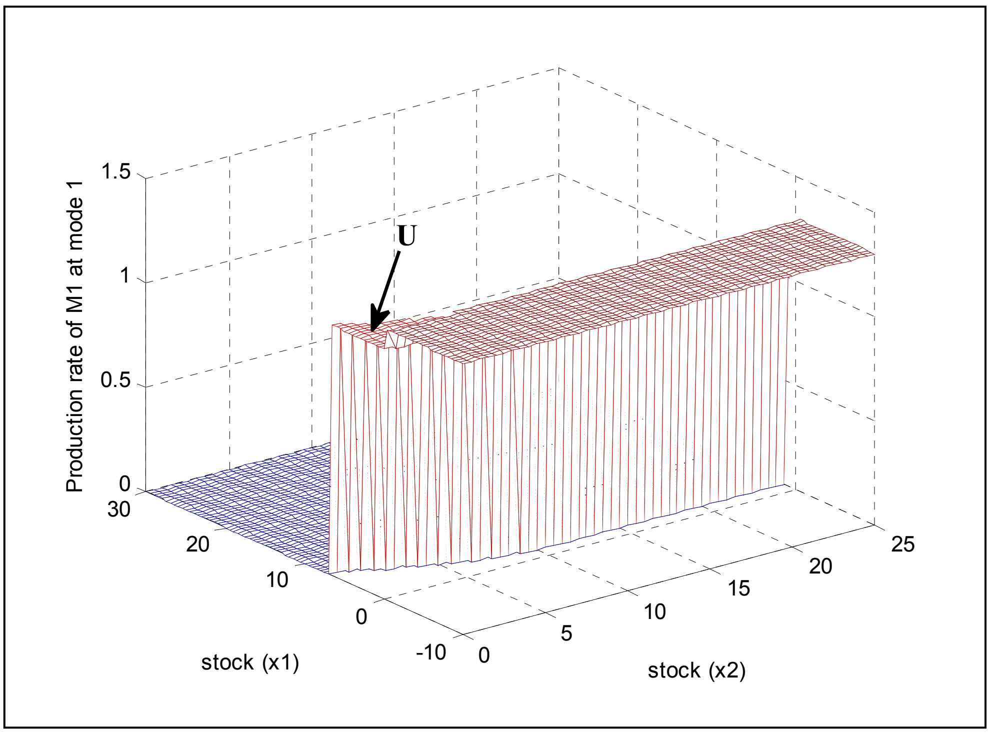

Figure 4.3 Production rate of M_1 at mode 1

The production policies $u_i^*(x_1, x_2, \alpha)$, $(i = 1,2)$, illustrated in Figures 4.3, 4.5, 4.7 and 4.9 indicate the production rates of the manufacturing/remanufacturing system for a given stock of return products $x_2(t)$ and stock level $x_1(t)$. Based on the results, there is no need to produce at a comfortable stock level capable of meeting demand; we do not need to produce if the stock level is greater than 7.5, 10.5, 5.5 and 19 parts at modes 1, 2 and 3, respectively.

Figures 4.4 and 4.6 illustrate the optimal production rate boundary of M_1 at mode 1 and mode 2, which is the optimal stock level, such that even with stock levels below 7.5 and 10.5, we need to produce at the economical and at the maximum production rates. If the stock of the return product increases, the stock level decreases. The traces of M_1 at mode 1 (Figure 4.4) and mode 2 (Figure 4.6) show that for a quantity of returned products greater than 9 (12.5), regardless of the level of serviceable stock, the production rate will need to be set to its maximal rate. Unlike the case illustrated in Figure 4.3, where the tendency was to use the maximal productivity of M_1 less, at mode 1, the first threshold in Figure 4.5 is higher than in Figure 4.3 because the machine works alone.

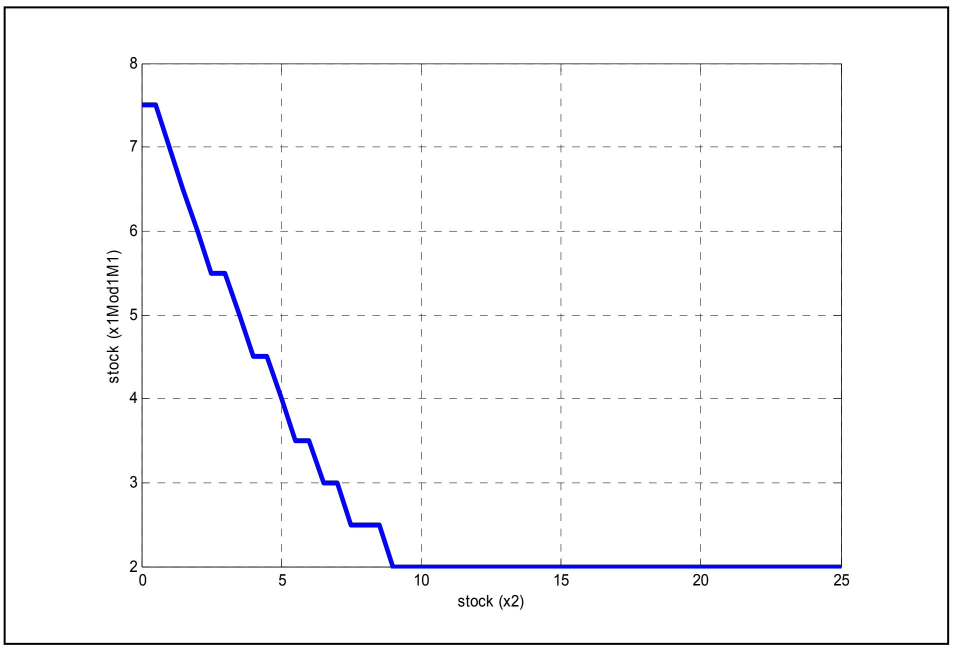

Figure 4.4 Boundary of M_1 at mode 1

According to the classical results as in Kenne et al. (2012) and Ouaret et al. (2013), the computational domain is expected to be divided into two stages. The results of Figure 4.3 and 4.5 show however that the computational domain is divided into three stages, which represents a specific finding of this paper.

Examining Figures 4.3 to 4.6, we see that the optimal stock levels depend directly on the level of returned products. Consequently, the optimal production control policy consists of one of the following rules:

1. Set the productivity of M_1 to its maximal value when the current stock level is under the first threshold value ($z_1(x_2)=2$ and $z_3(x_2)=8.5$, respectively);
2. Reduce the productivity of M_1 to its economical value when the current stock level approaches the second threshold value ($z_2(x_2)=7.5$ and $z_4(x_2)=10.5$, respectively);
3. Set the productivity of M_1 to zero when the current stock level is greater than the second threshold value

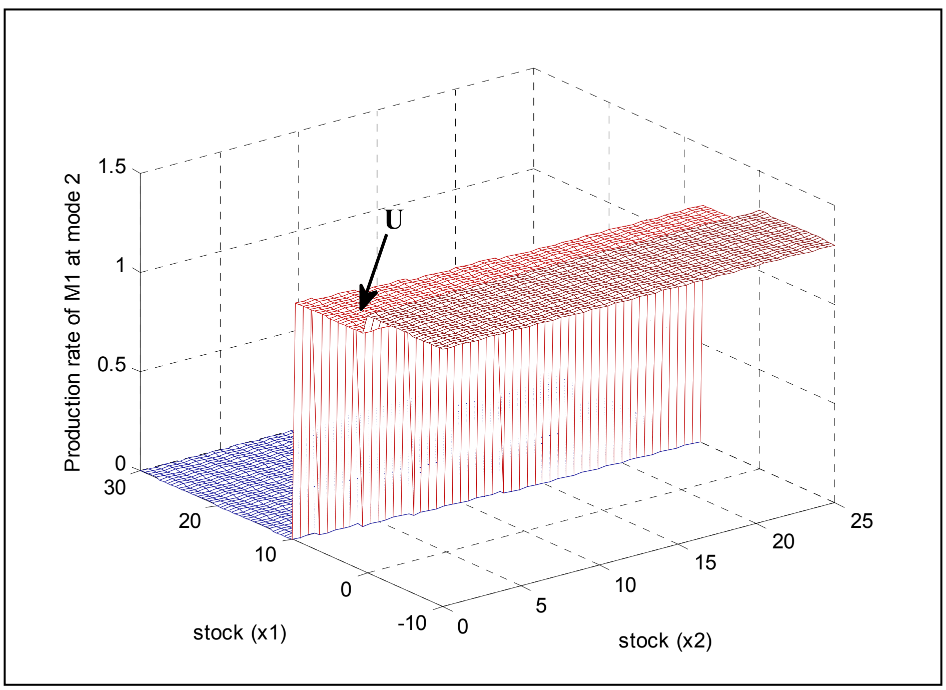

Figure 4.5 Production rate of M_1 at mode 2

In Figures 4.7 to 4.10, the optimal policy of M_2 at mode 1 and mode 3 is presented. At mode 3, the zone where the machine is set to its maximal production rate is larger than that at mode 1. This illustrates the difference between operational modes 1 and 3. The gap between states 1 and 3 is due to the fact that at mode 3, the manufacturing machine is under repair and the remanufacturing machine cannot satisfy the customer demand alone. In Figure 4.8, we can see that for $0 \leq x_2 \leq 1$, the production rate of M_2 is set to $r - disp$ (see zone T)

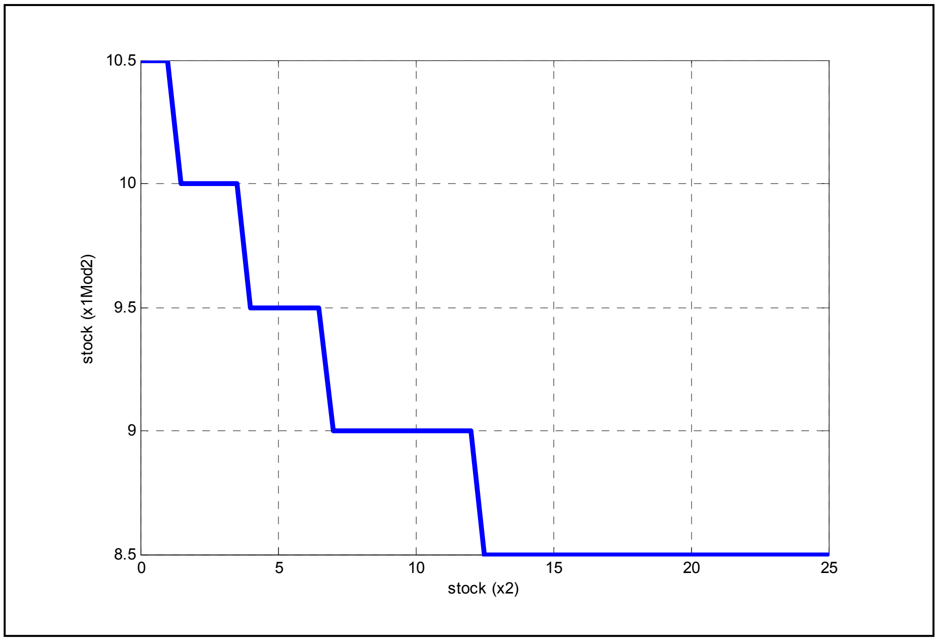

Figure 4.6 Boundary of M_1 at mode 2

The relation between inventory, stock of returned products and production rate of M_2 at operational mode 1 (mode 3) is illustrated in Figures 4.7 and 4.8 (Figures 4.9 and 4.10). The results show that when the stock level is 5.5 (16.5) and the stock of returned products is 21 (16.5), the production rate is set to zero. If the stock level is 2.0 (8.5) and the stock of products is greater than 21 (11.5), the production rate is set to its maximal value. The results of Figure 4.7 show that the zone where the production rate is set to zero is restricted when the stock of returned products increases. The effect of large quantity of x_2 is minimized by assigning large values of the stock threshold at mode 1.

Figure 4.8 (Figure 4.10) illustrates the optimal production rate boundary of M_2 at mode 1 (mode 3), which is the optimal stock level. The results show that the threshold values also depend on the level of returned products. The computational domain of M_2 at mode 1 and 3 is divided into two regions where the optimal production control policy consists of the following two rules:

1. Produce at the maximal rate (or at $r-disp$ if $0 \le x_2 \le 1$) when the current stock level is under a threshold value ($z_5(x_2)=5.5$ and $z_6(x_2)=19$, respectively).
2. Set the production rate to zero when the current stock level is larger than a threshold value.

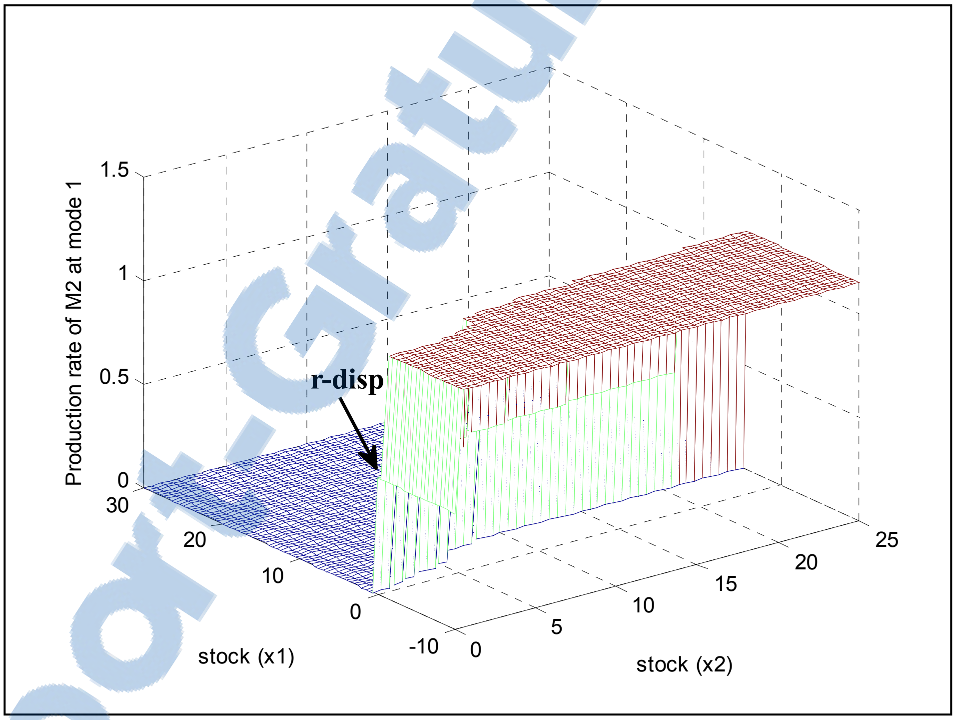

Figure 4.7 Production rate of M_2 at mode 1

The results of Figure 4.10 show that at mode 3 where M_1 is under repair, when x_2 increases, x_1 decreases because M_2 cannot satisfy the customer demand alone. At mode 2, where M_2 is under repair, we still have multiple thresholds because with two machines, even if one machine fails, the system knows that it exists, and will return to the operational state in a relatively short time.

The results of Figure 4.9 show that the threshold value ($z_6(x_2)$) is higher than the thresholds $z_2(x_2), z_4(x_2)$ and $z_5(x_2)$ because at mode 3, M_1 is under repair. The second machine must use its maximum productivity over a long period to avoid over-shortages.

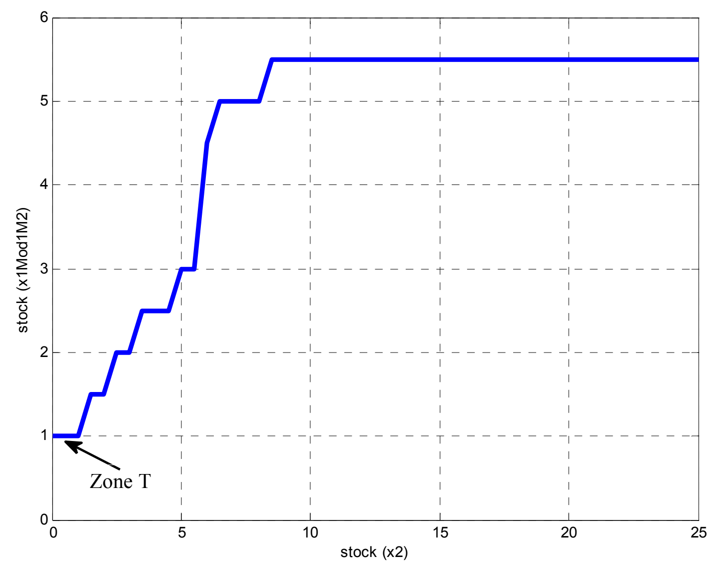

Figure 4.8 Boundary of M_2 at mode 1

Based on the results from Figures 4.3 to 4.10, the production rates of M_1 and M_2 are given by a x_2 dependent hedging point:

$$u_1^*(x_1,x_2,1)=\begin{cases} u_{1\max} & \text{if } x_1 < z_1(x_2) \\ U & \text{if } z_1(x_2) \le x_1 < z_2(x_2) \\ 0 & \text{if } x_1 > z_2(x_2) \end{cases} \tag{4.10}$$

$$u_1^*(x_1,x_2,2)=\begin{cases} u_{1\max} & \text{if } x_1 < z_3(x_2) \\ U & \text{if } z_3(x_2) \le x_1 < z_4(x_2) \\ 0 & \text{if } x_1 > z_4(x_2) \end{cases} \tag{4.11}$$

$$u_2^*(x_1,x_2,1)=\begin{cases} u_{2\max} & \text{if } x_1 < z_5(x_2) \\ 0 & \text{if } x_1 > z_5(x_2) \end{cases} \tag{4.12}$$

$$u_2^*(x_1, x_2, 3) = \begin{cases} u_{2\max} & \text{if } x_1 < z_6(x_2) \\ 0 & \text{if } x_1 > z_6(x_2) \end{cases} \tag{4.13}$$

where $z_1(x_2), z_2(x_2), z_3(x_2), z_4(x_2), z_5(x_2)$ and $z_6(x_2)$ are the first and the second threshold values of M_1 at mode 1, the first and the second threshold values of M_1 at mode 2, the optimal threshold value of M_2 at mode 1 and the optimal threshold value of M_2 at mode 3, respectively.

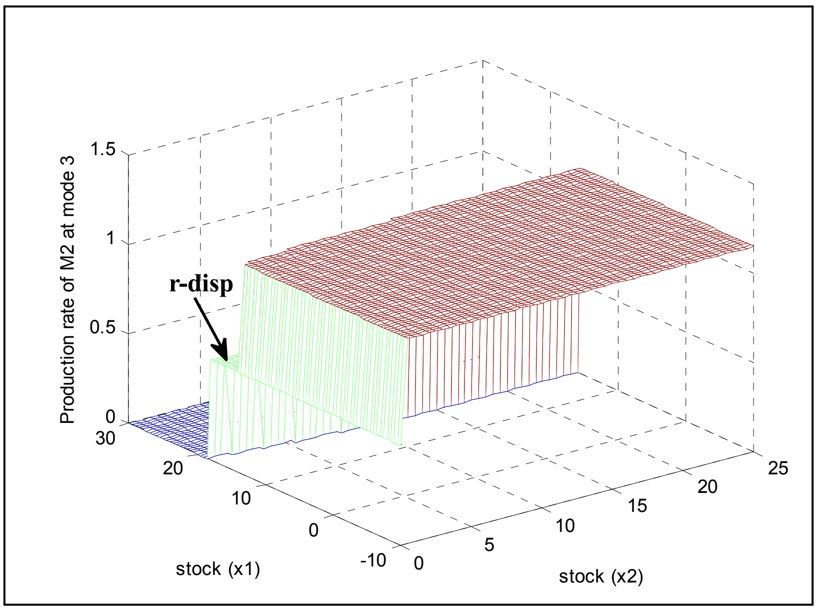

Figure 4.9 Production rate of M_2 at mode 3

With numerical methods, the results show that $z_6(x_2) = 19$. Physically, however, the system cannot exceed the value of $z_4(x_2) = 10.5$ because M_2 cannot satisfy the customer demand alone. Hence, the threshold value $z_6(x_2)$ will be ignored.

In the hybrid manufacturing/remanufacturing system consisting of two machines and one type of product, with a constant failure rate such as the one described in Kenne et al. (2012) and Ouaret et al. (2013), the optimal control policy should be characterized by three threshold values. The results obtained in this paper show that the optimal control policy is characterized by five different threshold parameters: $z_1(x_2), z_2(x_2), z_3(x_2), z_4(x_2)$ and $z_5(x_2)$ because the manufacturing machine degrades according to its productivity speed. This is the main finding of this paper.

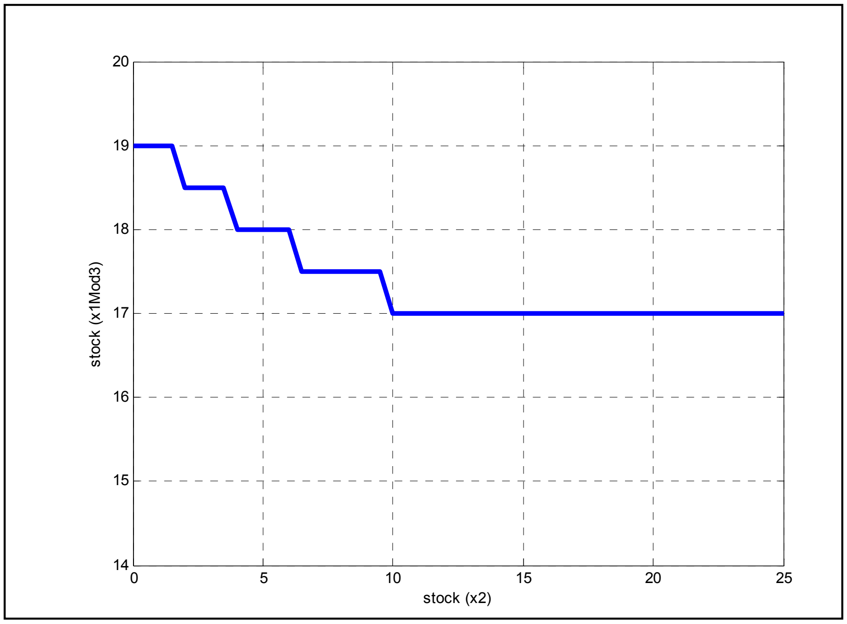

Figure 4.10 Boundary of M_2 at mode 3

The optimal policy of the proposed joint optimization of production and machine reliability is given by equations (4.10)-(4.12). To validate and illustrate the usefulness of the developed model, let us confirm the obtained results through a sensitivity analysis. Several experiments were conducted to ensure that the structure of the policies obtained is maintained under the variation of the model parameters, and can therefore be used in practice.

4.4.2 Sensitivity analysis

A set of numerical examples were considered to measure the sensitivity of the control policies obtained a mode 1 (both machines are producing) and to illustrate the contribution of this paper. We analyze the sensitivity of the control policies according to the backlog costs in the first section. In the second section, we examine the sensitivity of the optimal policies according to different values of the return rate. The sensitivity analysis enables the tracking of variations to the policy boundaries.

4.4.2.1 Sensitivity analysis with respect to backlog costs

In this section, we will perform sensitivity analysis on the backlog cost.

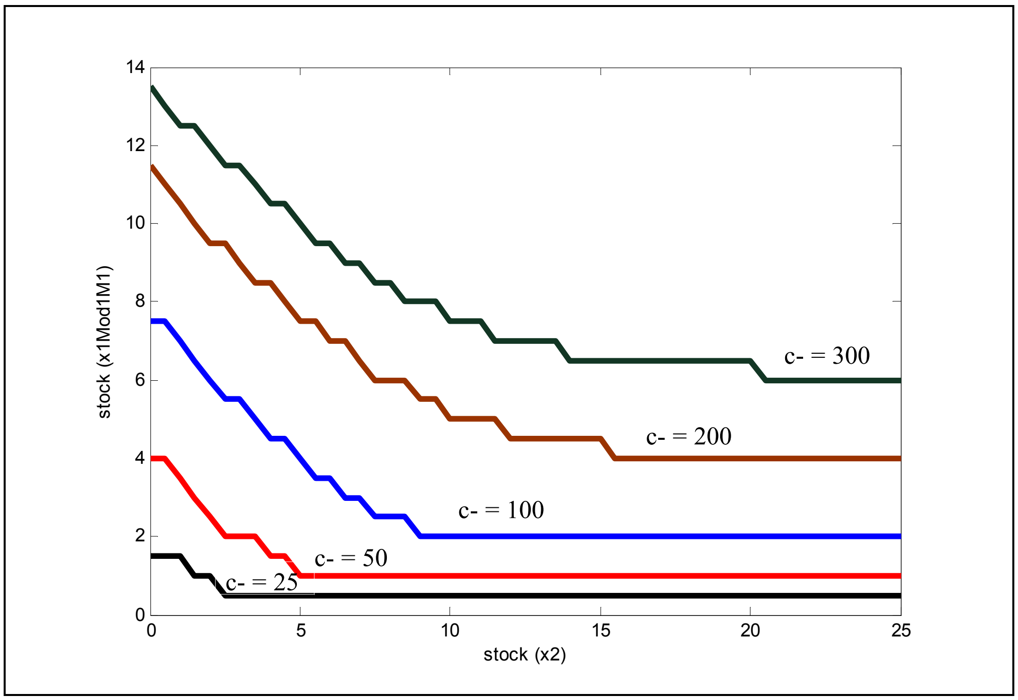

Figure 4.11 Variation of c^- at mode 1: Effect on M_1

Figures 4.11 and 4.12 illustrate the behavior of the optimal threshold values of the machines for five backlog cost values: $c_1^- = 25,\ 50,\ 100,\ 200$ and $c_1^- = 300$. The results show that the thresholds $z_1(x_2), z_2(x_2)$ and $z_5(x_2)$ increase as the backlog costs increase. We therefore need a lot of parts in stock to avoid further backlog costs.

4.4.2.2 Sensitivity analysis with respect to return rate

This section analyzes the sensitivity of the optimal threshold values with respect to the return rates.

When the return rate takes four values: $0.25*d$, $0.50*d$, $0.60*d$ and $0.75*d$ (where d is the demand rate), we obtain the results presented in Figures 13 and 14. The results show that the variation of the parameter r does not affect the threshold $z_1(x_2)$.

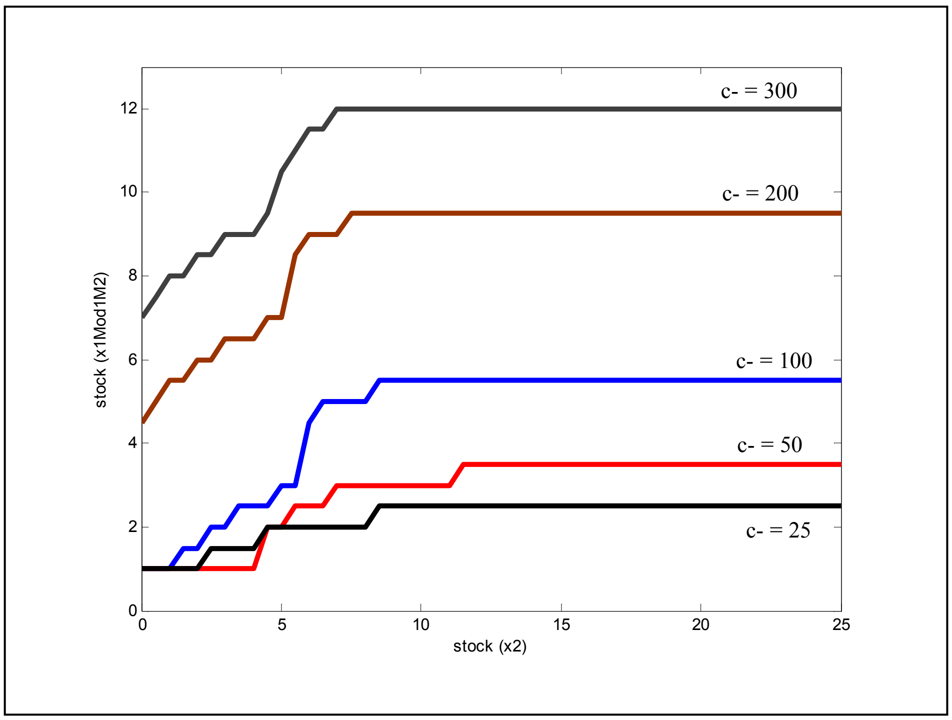

Figure 4.12 Variation of c^- at mode 1: Effect on M_2

When r increases, the thresholds $z_2(x_2)$ decrease in order to avoid over-stocking. The threshold value $z_5(x_2)$ increases as well because x_2 is enough to supply M_2. From a practical perspective, the parameters of the control policy move as predicted when r decreases, in order to avoid over-shortages (see Figure 4.13). Zone T moves in the opposite direction of the return rates. For example, if the return rate increases, zone T will shrink (see

Figure 4.14). Increasing r means that the value of $r-disp$ is close to $u_{2\max}$. Hence, the production rate of the remanufacturing machine is set directly to its maximal value instead of $r-disp$ as in the base case ($r=0.5*d$). Zone T moves as predicted, from a practical perspective when r decreases.

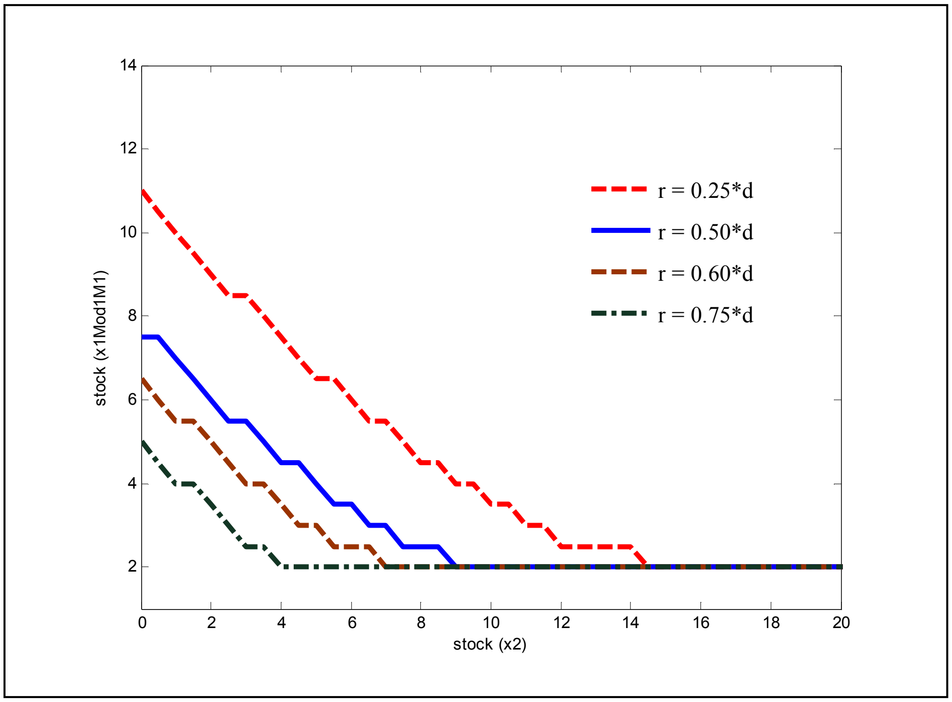

Figure 4.13 Variation of r at mode 1: Effect on M_1

Through the observations drawn from the sensitivity analysis, the results demonstrate conclusively that the resulting policy is optimal and enhances machine reliability. Control policies for our systems consider an extension of the multi-hedging point structure. Without in any way limiting the generality of this proposal, this model is based on certain assumptions relating to a pair of machines (manufacturing and remanufacturing machines) which are not identical and which operate in parallel. Given certain conditions, extended versions of this model might be adopted across a number of industrial sectors.

4.5 Conclusion

Although some of the concepts of reverse logistics, such as the facility location models including return flows, inventory management models, production and transportation planning models, have been put into practice for years, it is only fairly recently that the integration of aspects related to the stochastic dynamics of machines has been a real concern for the management of reverse logistics systems. This paper confirms that it is possible to integrate production-dependent failure rates in a hybrid manufacturing/remanufacturing system subject to random failures and repairs, in order to minimize the overall incurred cost. The machines produce one type of final product.

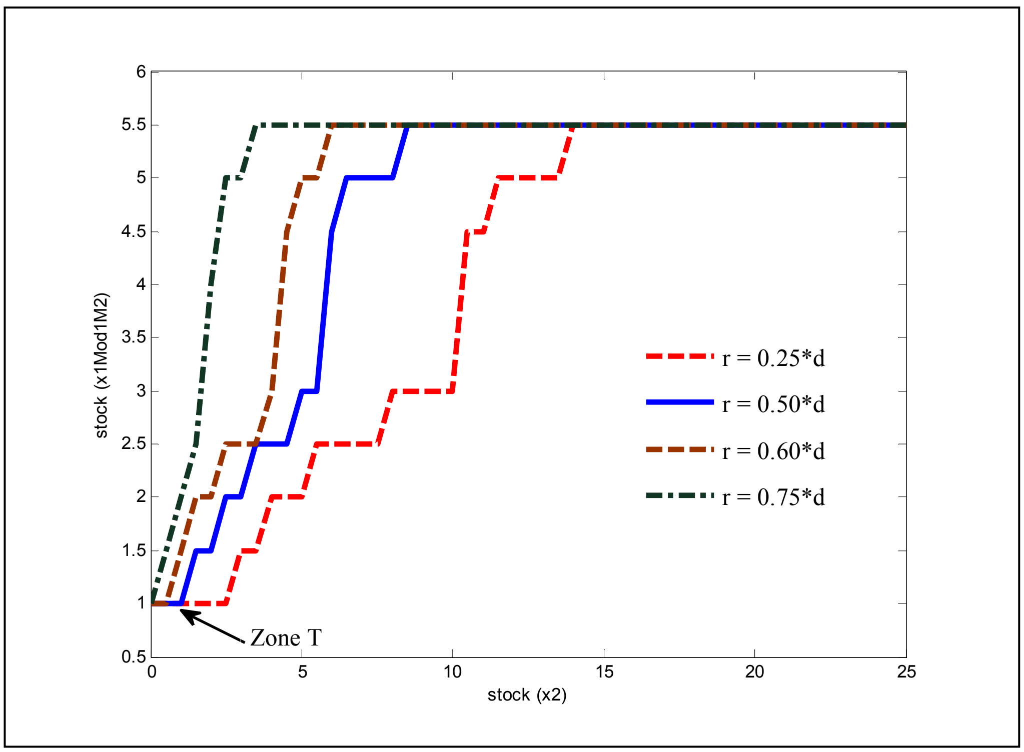

Figure 4.14 Variation of r at mode 1: Effect on M_2

The failure rates of the manufacturing machine depend on its production rate. To take into account its availability, the company will then maximize the recovery of its products used from the market, allowing it to eventually minimize the use of raw materials which become increasingly rare. We developed the stochastic optimization model of the considered problem with two decision variables (production rates of manufacturing and remanufacturing machines). The stock levels of new and returned products were the state variables. From the numerical study, it was found that for two parallel machines, when the failure rates of the machines depend on the production rate, the hedging point policies are optimal within a five-threshold feedback policy, and the reliability of the machines is enhanced. A numerical example is given to illustrate the utility of the proposed approach. The sensitivity analyses show that the structure of the results obtained is maintained. This approach takes into account both the multi-objective aspect and the dynamics of machines. However, the model is far from perfect, and leaves much to be desired, especially in the case involving multiple machines, multiple products, random return rate, the quality of remanufactured products (non-conforming products) and the returns control policy, such as the pricing policy.

CHAPITRE 5

ARTICLE 4: STOCHASTIC MODELS AND NUMERICAL SOLUTIONS FOR MANUFACTURING/REMANUFACTURING SYSTEMS WITH APPLICATIONS TO THE PRINTER CARTRIDGES INDUSTRY

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Résumé

Cet article porte sur l'application des méthodologies développées précédemment et notamment au Chapitre 4 au cas de l'industrie du *manufacturing* et du *remanufacturing* des cartouches d'encre pour les imprimantes à jet d'encre ou impression laser. Le cas étudié est adapté du contexte réel d'une entreprise française très orientée vers le développement durable et qui est l'un des leaders européens dans le domaine. Le système hybride de *manufacturing/remanufacturing* étudié comprend deux usines en parallèle soumis à un processus de Markov non-homogène, avec le taux de panne de l'usine de *manufacturing* qui dépend de son taux production. L'objectif concerne la minimisation de la fonction coût qui pénalise la mise en stock des produits finis, les pénuries et la mise en stock des produits usagés collectés chez les clients. Les usines produisent un seul type de produit. Nous avons développé un modèle d'optimisation avec deux variables de décision (les taux de production des usines de *manufacturing* et de *remanufacturing*). Les stocks des produits finis et des produits retournés étant les deux variables d'état. Nous avons utilisé la méthode de programmation dynamique pour résoudre le problème posé. Sur la base du cas pratique étudié, les politiques optimales de production sont obtenues. Des analyses de sensibilité sont réalisées pour illustrer l'utilité de l'approche proposée et son extension possible à d'autres systèmes du même type.

Mots-clés: Logistique inverse, Remanufacturing, Cartouches d'imprimante, Étude de cas, Planification de la production, Programmation dynamique, Méthodes numériques.

Abstract

This article focuses on the application of the methodologies developed previously to the sector of the printer cartridge industry. The case study is adapted from the real context of a French company, which is very focused on sustainable development and is one of the European leaders in this field. The hybrid manufacturing/remanufacturing system studied comprises parallel factories subject to a non-homogeneous Markov process, with the failure rate depending on the production rate for the manufacturing factory while it is constant for the remanufacturing factory. The overall goal of the problem concerns the minimization of a cost function which penalizes the presence of waiting customers, the inventory surplus and the inventory of returns. The factories produce a single type of product. We developed a stochastic optimization model of the considered problem with two decision variables (production rates of manufacturing and remanufacturing factories). The stock levels of new and returned products are the two state variables. Using a real business case study adapted from a European leading company in the field of printer cartridges, the optimal production policies of the both plants are obtained. Sensitivity analyses are conducted to illustrate the usefulness of the proposed approach and its applicability to other cases.

Keywords: Reverse logistics; Remanufacturing; Printing Cartridges; Case-study; Production planning; Dynamic programming; Numerical methods.

5.1 Introduction

There is an increasing concern in the industrial world as well as in the academic world for reverse logistics. As an example, Ilgin and Gupta (2010) reviewed recently the literature on environmentally conscious manufacturing and product recovery. They discuss the evolution in the last decade and analyse new areas that have emerged. They classify the literature in four categories: environmentally conscious product design, reverse and closed-loop supply chains, remanufacturing, and disassembly; and Ilgin and Gupta (2012) published a comprehensive book on Remanufacturing modeling and analysis.

Traditionally, remanufacturing has been performed within the automotive or aeronautical sectors. Rolls-Royce, MTU (Motoren- und Turbinen-Union) aero Engines, General Electric, Caterpillar and Cummins Engine are only a few prominent examples (Jian et al., 2010). During the past decades, it has spread to other sectors as well (Sundin et al., 2005). Savaskan et al. (2004) are prominent examples of successful remanufacturing initiatives. In this paper, we will focus on the recent works which studied the case of printer cartridges.

Krikke et al. (1999) discussed a business case study carried out at Océ, a copier firm in Venlo (Germany). It concerned the installment of remanufacturing processes. The study was meant to verify whether the strategic decision of Océ to move remanufacturing activities to the Czech Republic is also economically feasible. The authors have optimised the total operational costs over all possibilities and also compared three pregiven managerial solutions (network designs) with a Mixed Integer Linear Programming model. The purpose of Östlin and Ekholm (2007) was to analyze if lean productions principles for material flow could be applied in a remanufacturing environment, and especially at the Swedish remanufacturer Scandi-Toner AB. They concluded that the inherent characteristics of variable processing times and uncertainty in materials recovered have the major negative impact for implementing a lean production process. The project of Rong (2009) focused on remanufactured (refilled) inkjet cartridges for home and small offices. Sixty different remanufactured inkjet cartridges from six aftermarket manufacturers and six original equipment manufacturers (OEM) of cartridges were studied. He investigated the following quality issues: defective rate, optical density, color difference, line raggedness, mottle, streaking, and inter-color bleeding. His results showed that the remanufactured inkjet cartridges rated lower than the OEM cartridges in the areas of evaluation. The most significant differences were in color, mottle, streaking, and inter-color bleeding.

The study of Jung and Hwang (2011) dealt with the interaction between an OEM and a remanufacturer where remanufactured Toner Cartridges cannibalize the OEM's market under the assumption of completion and cooperation of the two parties. Researchers developed the mathematical models with the objective of maximizing the profit of each party. Through numerical experiments, the authors found that competition of the two parties raises the return rate, while the net profits are always larger under cooperation compared to competition. Sundin and al. (2012) explored how manufacturers can develop automatic end-of-life (EoL) processes facilitated by product design methods, e.g. design for disassembly, recycling and remanufacturing. They illustrated this kind of product and EoL process development while maintaining economic and environmental values. The cases of toner cartridges and liquid crystal displays (LCDs) were the focus. Their research methodology consisted of a literature study within design for automatic disassembly, recycling and remanufacturing. In addition, empirical data from industries was added through the case studies of toner cartridge remanufacturing and recycling and component reuse of LCDs.

Kenne et al. (2012) treated the production planning and control involving combined manufacturing and remanufacturing operations within a closed-loop reverse logistics network with machines subject to random failures and repairs. The objective was to propose a manufacturing/remanufacturing policy that would minimize the sum of the holding and the backlog costs for manufacturing and remanufacturing products. Their model is generic and could be applied to the combined manufacturing/remanufacturing of components or parts in different sectors. Ouaret et al. (2013) extended the model considered by Kenne et al. (2012) to the production control problem for hybrid manufacturing/remanufacturing systems subject to random demand. Kouedeu et al. (2013) extended the model of Kenne et al. (2012) to the production planning of a failure-prone closed loop manufacturing/remanufacturing system with production dependent failure rates.

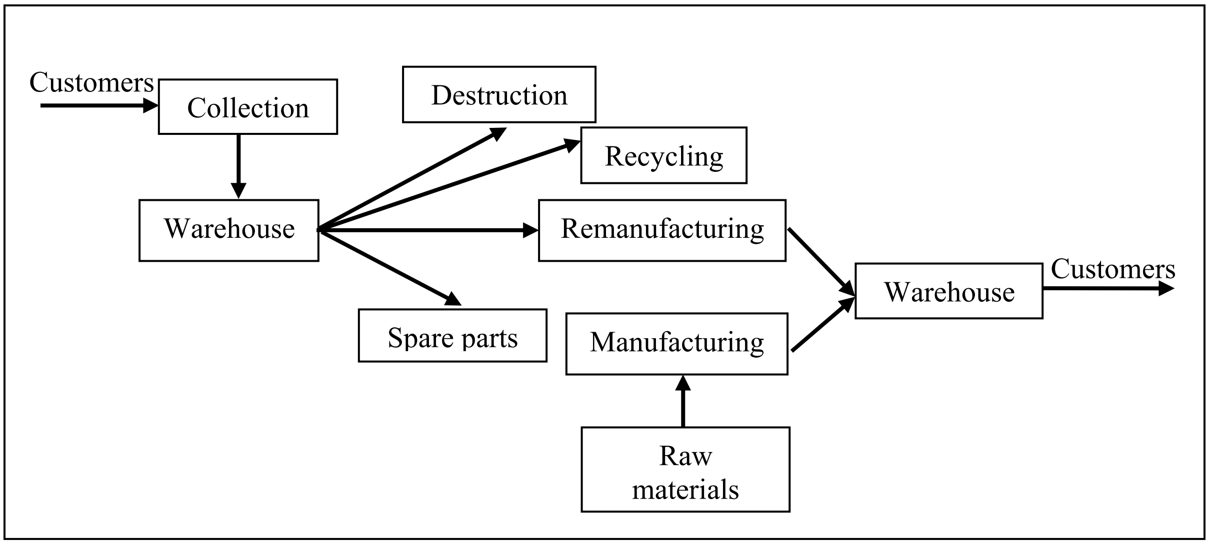

Figure 5.1 The manufacturing/remanufacturing process

In consideration of the previous literature review, we note that production planning that involves making decision is often formulated as an optimal control problem. To solve such a problem usually entails the minimization of the expected value of a cost function. The production system evolves as a dynamic process, and the decision obtained is a feedback control policy. Many dynamic systems consist of discrete-event processes or subject to discrete-event interventions, which lead to jump discontinuities in their evolution. For instance, manufacturing/remanufacturing systems as depicted in Figure 5.1 are often associated with movements involving discontinuities influenced by such random and exogenous discrete events. The random events can be equipment failures and repairs, customers demand and returned products. Due to the possible large number of modes of the system, and the large number of available alternatives of decisions, the problem can be very complex.

It has become clear that there is a great need for methods capable of handling random events in the process and its environment. In this context, we have studied the case of a printer cartridges company in France operating all over Europe and which activity is based upon the combined manufacturing of new cartridges and the remanufacturing of used ones. This company will be referred to as the Manufacturing/Remanufacturing Company (MRC) thereafter. Although Markov processes have been used in many operational management problems, the applications to printer cartridges industry are still scarce. This work intends to contribute in this direction.

Our model describes the process and reflects the dynamic behavior of the underlying system. Contingent upon the mode of the system and its capacity, the decision made is affected by both earlier decisions as well as other random disturbances. We seek optimal long-term planning decisions in the manufacturing/remanufacturing stage for dynamic systems under uncertainties and formulate it as an optimal control problem. We address issues involved in problem formulation and solution procedure; provide the associated dynamic programming equations, and present numerical approximation schemes that lead to an approximation of the optimal policy. The objective function used includes serviceable inventory, backlog and returns holding costs and it can be easily extended to include other costs. We assume a known demand for new products as well as for the supply of used products and consider one type of uncertainty regarding the production system failures, and formulate the problem using finite-state non-homogeneous Markov chains. Such an approach enables us to quantitatively describe the random and jump behavior that is common in many stochastic dynamic systems. The policy obtained allows us to make optimal decisions for each production mode (in operations or not).

This work is motivated by the needs for better production planning in the ink jet and laser printer cartridges companies. We seek mathematical models and numerical procedures applicable to the real processes. The primary tools are non-homogeneous Markov chains and dynamic programming. The rationale for using non-homogeneous Markovian models stems from the fact that the factories capacity, as observed in MRC (the European leader in compatible consumables for inkjet, laser, fax and impact printing, offering remanufactured and new patent-compliant cartridges), often display both uncertainty (randomness) and piecewise constant behavior. In addition such processes also tend to be memoryless. That is, given past information up to the current time is essentially the same as given the current information (the remote past can be ignored). Finally, the process is non-homogeneous because we assume that the failure rate of the manufacturing factory depends on its production rate. Although the remainder of the paper is focused on the practical issues of applying our general model at MRC, the formulation, the approaches, and the numerical procedures can be applied to production planning in other industries as well.

The rest of the paper is organized as follows. Using MRC as an example, we discuss the structure and several important concepts in industrial production planning systems in Section 5.2. The problem formulation and numerical method are also presented in Section 5.2. Applying the dynamic programming principle, the optimal control policies are obtained in Section 5.3. Conclusions are given in Section 5.4.

5.2 Manufacturing/Remanufacturing system

This section presents the industrial context, the production planning, as well as the numerical procedure used in this article. Before delving into the industrial context, we first present the notations used.

5.2.1 Notations

The model under consideration is based on the following notations:

u_1: production rate of the manufacturing factory M_1 ($\times 10^5$ items/week)

u_2: production rate of the remanufacturing factory M_2 ($\times 10^5$ items/week)

$u_{1\max}$: maximal production rate of M_1 ($\times 10^5$ items/week)

U: economical production rate (in terms of factory's reliability) of M_1 ($\times 10^5$ items /week)

$u_{2\max}$: maximal production rate of M_2 ($\times 10^5$ items/week)

x_1 : stock level of finished products ($\times 10^5$ items)

x_2 : stock level of returned products ($\times 10^5$ items)

d : customer demand rate ($\times 10^5$ items/week)

r : return rate ($\times 10^5$ items/week)

$disp$: disposal rate ($\times 10^5$ items/week)

ξ : stochastic process

c_1^+ : inventory cost of finished products (\$/lot of 10^5 items/week)

c_1^- : backlog cost (\$/lot of 10^5 items missing/week)

c_2 : inventory cost of returned products (\$/lot of 10^5 items/week)

$\lambda_{\alpha\beta}$: transition rate from mode α to mode β

Q : transition rate matrix

π : vector of limiting probabilities

$g(\cdot)$: instantaneous cost function (\$/week)

$J(\cdot)$: total cost

$v(\cdot)$: value function

ρ : discount rate

5.2.2 Industrial context

MRC is the European leader in compatible consumables for inkjet, laser, fax and impact printing, offering remanufactured and new patent-compliant cartridges. As an organisation independent of printer manufacturers, MRC offers an optimal alternative solution, regardless of the equipment brand, printer definition, number of pages or characters printed. Its products are marketed under the MRC brand and under distributor own brands. European leader, it invests and innovates to offer new solutions that meet increasingly significant economic but also ecological requirements. MRC is active in about 20 countries, has about 25 industrial and commercial sites, and a yearly turnover of more than €200 million. It holds headquarters in France and employs close to 2000 people world-wide. It operates in several production facilities for new or remanufactured inkjet or laser catridges in Eastern Europe or North Africa We limit ourselves to the production chain, i.e., issues concerning administration, controlling, quality management etc. are not addressed in this study. In this way, the structure of the studied MRC can be depicted by Figure 5.2. Note that this representation is identical to those usually employed for closed loop manufacturing/remanufacturing systems in general.

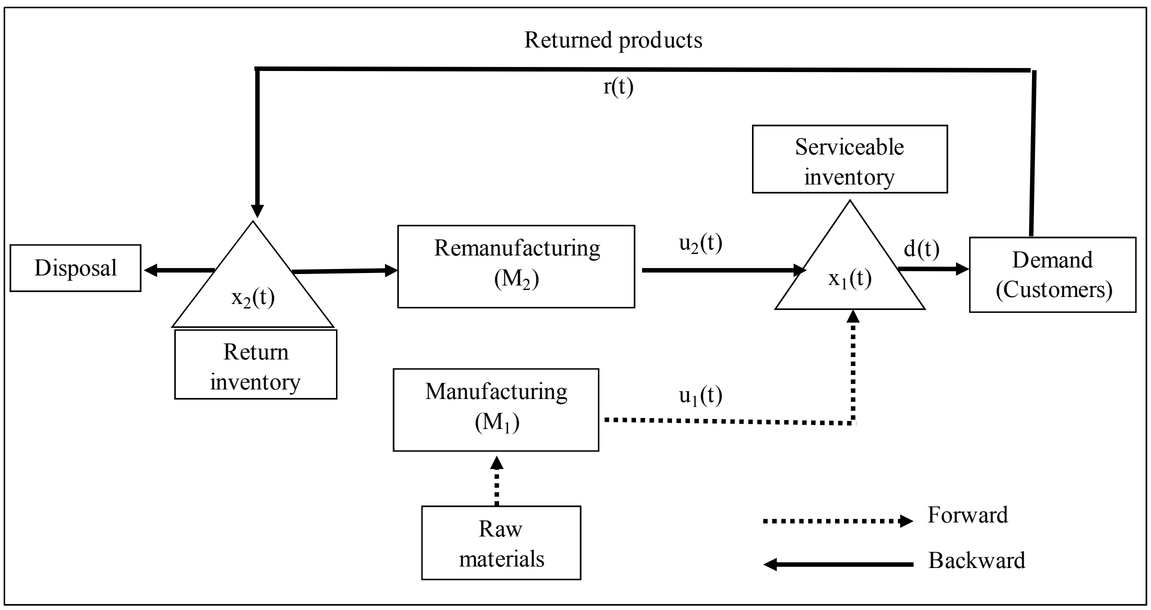

Figure 5.2 Overview of the considered manufacturing/remanufacturing system of MRC

For the considered control problem, the manufacturing and the remanufacturing factories are denoted by M_1 and M_2, respectively. The factories produce one type of product: the Laser printer cartridges given that manufactured and remanufactured products are made at the same industrial location. At the end of their usage, products are collected for possible reuse. Used products are segregated into different quality levels after inspection. They can then either be remanufactured or kept in return inventory x_2 (warehouse on the left of Figure 5.1) for future remanufacturing, or disposed of (recycling, spare parts and destruction blocks in Figure 5.1). However the manufacturing factory makes new products from raw materials, while the remanufacturing factory produces parts “like new” from used products returned from the market. It is interesting to note that the returned products come from the MRC markets as well as the competitor markets such as Armor, HP, Canon, Dell, Epson, Brother, Lexmark, Samsung, Sharp, Toshiba, Xerox, etc. The factories are subject to random non-operational periods considered herein as governed by a failure/repair process. New parts (manufactured and remanufactured parts) are stored in the serviceable inventory x_1 (warehouse on the right of Figure 5.1).

The manufacturing factory is the main factory characterised by a higher production rate. The failure rate of M_1 depends on its production rate. This means that when the manufacturing factory works at a faster rate, it is more likely to be unavailable. In that case, we can not use this factory to its maximum production rate all the time. Thus, we introduce another production rate ($U < u_{1\max}$) called the economical production rate. The failure of M_2 and the repair rates of both factories are assumed constant. The maximum production rates ($u_{1\max}$ and $u_{2\max}$) of the factories and the economical production rate of M_1 are known. The demand process for finished products and returned products process are deterministic. Backorders of unsatisfied demands are permitted.

Involving non-homogenous Markov chains that model production rates, the underlying system is often referred to as a hybrid system since it contains continuous dynamics intertwined with discrete event interventions. The discrete element represents the status of the factories, described by $\xi(t)$ and the continuous one, the stock levels described by $x_1(t)$ and $x_2(t)$. Two decisions have to be taken corresponding to the production rates of M_1 and M_2. To summarize, the objective is to find the optimal production policies for dynamic systems having non-homogeneous Markovian jump processes to minimize the total cost of inventory, backlog and stock of returned products.

5.2.3 Production planning

The modes of the factory M_i can be described by a stochastic process $\xi_i(t)$, $i=1,2$ with value in $B_i=\{1,2\}$. Such a factory is available when it is operational ($\xi_i(t)=1$) and unavailable when it is under repair ($\xi_i(t)=2$). The transition rates, which describe the dynamics of the considered manufacturing system, are presented in Table 5.1. We then have $\xi(t)\in B=\{1,2,3,4\}$.

Table 5.1 Manufacturing/remanufacturing transition rates

$\xi_1(t)$	1	1	2	2	Factory 1	Stochastic process
$\xi_2(t)$	1	2	1	2	Factory 2	Stochastic process
$\xi(t)$	1	2	3	4	Manufacturing/Remanufacturing system	Stochastic process

The operational mode of the system can be described by the random vector $\xi(t)=(\xi_1(t),\xi_2(t))$. The set of possible values of the process $\xi(t)$ can be determined from the values of $\xi_1(t)$ and $\xi_2(t)$ with:

- Mode 1: M_1 and M_2 are operational;
- Mode 2: M_1 is operational and M_2 is non-operational;
- Mode 3: M_1 is non-operational and M_2 is operational,
- Mode 4: M_1 and M_2 are non-operational.

Let $\lambda_{\alpha\beta}$ denotes a jump rate of the system from mode α to mode β, we have the following notations: $q_{12}^1=\lambda_{13}=\lambda_{24}$ (failure rate of M_1), $q_{12}^2=\lambda_{12}=\lambda_{34}$ (failure rate of M_2), $q_{21}^1=\lambda_{31}=\lambda_{42}$ (corrective maintenance rate of M_1) and $q_{21}^2=\lambda_{21}=\lambda_{43}$ (corrective maintenance rate of M_2). We assume that the failure rate of M_1 depends on its production rate, and is defined by:

$$q_{12}^1 = \begin{cases} \theta_1 & \text{if } u_1 \in (U, u_{1\max}] \\ \theta_2 & \text{if } u_1 \in [0, U] \end{cases} \qquad \text{with } \theta_1 \geq \theta_2 \geq 0 \text{ and } 0 \leq U \leq u_{1\max}$$

Hence, $\xi(t)$ is described by the following matrix:

$$Q = \Theta_i; \quad \begin{cases} i = 1 \text{ if } u_1 \in (U, u_{1\max}] \\ i = 2 \text{ if } u_1 \in [0, U] \end{cases} \quad \text{with} \tag{5.1}$$

$$\Theta_i = \begin{pmatrix} -(q_{12}^2 + \theta_i) & q_{12}^2 & \theta_i & 0 \\ q_{21}^2 & -(q_{21}^2 + \theta_i) & 0 & \theta_i \\ q_{21}^1 & 0 & -(q_{21}^1 + q_{12}^2) & q_{12}^2 \\ 0 & q_{21}^1 & q_{21}^2 & -(q_{21}^1 + q_{21}^2) \end{pmatrix}; \text{ where } \theta_i = \lambda_{13} = \lambda_{24}$$

Let $u_1(t)$ and $u_2(t)$ denote the production rates of M_1 and M_2, respectively that may vary with time and the modes of the machines. The production rates are nonnegative. With given stock levels $x_1(t)$ and $x_2(t)$, the system dynamics is given by:

$$\frac{dx_1(t)}{dt} = u_1(t) + u_2(t) - d, \; x_1(0) = x_{10} \tag{5.2}$$

$$\frac{dx_2(t)}{dt} = r(t) - u_2(t) - disp, \; x_2(0) = x_{20} \tag{5.3}$$

where x_{10} and x_{20} are the given initial stock level of serviceable inventories and returned items, respectively. Note that $x_1(t)$ is positive when it represents inventory and negative when it represents shortage. There is no shortage of returned products, then $x_2(t) \geq 0$.

The cost function production J consisting of serviceable inventory and backlog costs c_1^+ and c_1^-, respectively; and c_2 which penalizes the inventory of returns, is defined by:

$$J(\alpha, x_1, x_2, u_1, u_2) = E\begin{Bmatrix} \int_0^\infty e^{-\rho t}(c_1^+ x_1^+ + c_1^- x_1^- + c_2 x_2)dt \,\middle|\, \xi(0) = \alpha \\ x_1(0) = x_{10}, \; x_2(0) = x_{20} \end{Bmatrix} \tag{5.4}$$

where $\rho > 0$ is the discount rate. The holding and backlog costs are such that $c_1^- > c_1^+ > c_2$. $x_1^+ = \max(0, x_1)$ and $x_1^- = \max(-x_1, 0)$ The expectation E is taken over random factories capacity $\xi(t)$. Our goal is to find the optimal policies (the optimal production rates $u_1(t)$ and $u_2(t)$), to minimize the objective function (5.4), subject to dynamics described by equations (5.2) and (5.3), the capacity $\xi(t)$, and certain production constraints for the given initial conditions. It should be noted that the production rates are a function of both factories capacity and the stock levels.

The production system will be able to meet the demand rate over an infinite horizon and reach a steady state if the following condition is satisfied: $\pi_1 \cdot (U + u_{2\max}) + \pi_2 \cdot U + \pi_3 \cdot u_{2\max} > d$ with $\pi \cdot Q(\cdot) = 0$ and $\sum_{i=1}^{4} \pi_i = 1$. The condition for meeting customer demands is also satisfied with $u_{1\max}$ because $U < u_{1\max}$. Let A denote the set of all admissible controls defined by:

$$\mathrm{A} = \left\{ (u_1(\cdot), u_2(\cdot)) \in \Re^2, 0 \le u_1(\cdot) \le u_{1\max}, 0 \le u_2(\cdot) \le u_{2\max} \right\} \tag{5.5}$$

Our objective is to find the admissible feedback control policies $u_1(\cdot)$ and $u_2(\cdot)$ that minimize the cost function $J(\alpha, x_1, x_2, u_1, u_2)$.

Let us define the value function $v(\cdot)$ as the minimum of the cost over $(u_1(\cdot), u_2(\cdot)) \in \mathrm{A}$, i.e.:

$$v(\alpha, x_1, x_2) = \inf_{(u_1(\cdot), u_2(\cdot)) \in \mathrm{A}} J(\alpha, x, u_1, u_2) \quad \forall \alpha \in B \tag{5.6}$$

The next section presents the numerical methods used to solve the optimality conditions for the value function $v(\cdot)$ given by equation (5.6).

5.2.4 Optimality conditions

Using a dynamic programming approach, it can be shown that the $v(\cdot,\alpha)$ are convex and $v(\alpha,x_1,x_2)$ satisfy the Hamilton-Jacobi-Bellman (HJB) equation (Martinelli, 2010):

$$\rho v(\alpha,x_1,x_2)=\min_{(u_1,u_2)\in A(\alpha)}\left[\begin{array}{l} c_1^+x_1^+ + c_1^-x_1^- + c_2x_2 + (u_1+u_2-d)\dfrac{\partial v(\alpha,x_1)}{\partial x_1} \\ +(r-u_2-disp)\dfrac{\partial v(\alpha,x_2)}{\partial x_2}+\sum\limits_{\beta\in B}\lambda_{\alpha\beta}v(\beta,x) \end{array}\right] \tag{5.7}$$

where $\dfrac{\partial v(\alpha,x_1)}{\partial x_1}$ and $\dfrac{\partial v(\alpha,x_2)}{\partial x_2}$ are the partial derivatives of the value functions $v(\alpha,x_1)$ and $v(\alpha,x_2)$, respectively.

Seeking the optimal production policies is related to evaluating the values of the derivative of the value function $\dfrac{\partial v(\alpha,x_1)}{\partial x_1}$ and $\dfrac{\partial v(\alpha,x_2)}{\partial x_2}$ which requires solving (5.7). Similar to many other controlled Markovian systems, however, the closed-form solution of the corresponding HJB equation is difficult or even impossible to obtain. Therefore, using numerical algorithms to approximate the value function becomes a viable alternative.

5.2.5 Numerical procedure for the optimal policy

To design a suitable numerical method, we refer the readers to the technique developed in Kouedeu et al. (2013b). Let h_1 and h_2 denote the length of the finite difference interval of the variables x_1 and x_2, respectively. By approximating $v(\alpha,x_1)$ and $v(\alpha,x_2)$ by functions $v^h(\alpha,x_1)$ and $v^h(\alpha,x_2)$, and the first-order partial derivative of the value functions $\dfrac{\partial v(\alpha,x_1)}{\partial x_1}$ and $\dfrac{\partial v(\alpha,x_2)}{\partial x_2}$ by:

$$\frac{\partial v(x_1,\alpha)}{\partial x_1}=\begin{cases}\frac{1}{h_1}\left(v^h(\alpha,x_1+h_1,x_2)-v^h(\alpha,x_1,x_2)\right) & if\ (u_1+u_2-d)>0\\ \frac{1}{h_1}\left(v^h(\alpha,x_1,x_2)-v^h(\alpha,x_1-h_1,x_2)\right) & otherwise\end{cases}$$

$$\frac{\partial v(x_2,\alpha)}{\partial x_2}=\begin{cases}\frac{1}{h_2}\left(v^h(\alpha,x_1,x_2+h_2)-v^h(\alpha,x_1,x_2)\right) & if\ (r-u_2-disp)>0\\ \frac{1}{h_2}\left(v^h(\alpha,x_1,x_2)-v^h(\alpha,x_1,x_2-h_2)\right) & otherwise\end{cases}$$

and after a couple of straightforward manipulations, the HJB equations can be rewritten as follows:

$$v^h(\alpha,x_1,x_2)=\min_{(u_1,u_2)\in \mathrm{A}^h(\alpha)}\frac{\begin{array}{l}c_1^+x_1^++c_1^-x_1^-+c_2x_2+\frac{(u_1+u_2-d)}{h_1}\begin{bmatrix}v^h(x_1+h_1,x_2,\alpha)Ind\{u_1+u_2-d\geq 0\}\\+v^h(x_1-h_1,x_2,\alpha)Ind\{u_1+u_2-d<0\}\end{bmatrix}\\+\frac{(r-u_2-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,\alpha)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,\alpha)Ind\{r-u_2-disp<0\}\end{bmatrix}+\sum_{\beta\neq\alpha}\lambda_{\alpha\beta}v^h(x_1,x_2,\beta)\end{array}}{\rho+\frac{|u_1+u_2-d_c|}{h_1}+\frac{|r-u_2-disp|}{h_2}+|\lambda_{\alpha\alpha}|} \tag{5.8}$$

with $\lambda_{\alpha\alpha}=-\sum_{\beta\neq\alpha}\lambda_{\alpha\beta}$, $\mathrm{A}^h(\alpha)$ is the numerical control grid and $Ind\{\Phi\}=\begin{cases}1 & if\ \ \Phi\ is\ true\\ 0 & otherwise\end{cases}$.

Section 5.3 provides the application examples to illustrate the structure of the control policies.

5.3 Application examples

The business report of MRC mentions that in 2010, the production capacity of Laser cartridges was over 1.5 million and in 2011, the collection in France was 1,300,000 cartridges with 1,100,000 reusable cartridges. Then, the customers demand was 1.5 million items, returns products was 1.3 million items and disposal products was 200,000 (1,300,000 −1,100,000) items. This means that: $d = 28,846$ items/week (i.e. 1,500,000 items/52 weeks /year), $r = 25,000$ items/week and $disp = 3,846$ items/week.

In Montreal, Canada, to pack away 1,000 cartons which have the same dimensions (600x300x480 mm) as the MRC carton, it costs about \$200/month (contact: http://www.satstorage.ca/, http://www.a1-mini-entrepot.ca/, http://www.stor-wel.com/). This means that 10^5 cartons cost about \$20,000/month (i.e. $\$200\times10^5$cartons/ 1,000 cartons) or about \$4,000/week (i.e. \$20,000/month$\times$12 months/52 weeks). Thus, $c_1^+ = \$4\times10^3$/lot of 10^5items/week. We assume that the inventory cost of returned products is the half of the inventory cost of new products, $c_2 = \$2\times10^3$/lot of 10^5 items/week. For the costs of shortage, we use $c_1^- = \$100\times10^5$/lot of 10^5 items missing/week. This is the average selling price of a laser printer cartridge (see http://www.bestbuy.ca/, http://www.bureauengros.com/, http://www.futureshop.ca/).

Table 5.2 Numerical data of the considered system

c_1^+	c_1^-	c_2	h_1	h_2	U	$u_{1\max}$	$u_{2\max}$
4	100	2	0.025	0.025	0.30	0.27	0.26
d	r	θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	ρ
0.28	0.25	1/80	1/100	1/60	1/1	1/1	0.09

Table 5.2 summarizes the parameters used (the values of $d, disp, r, u_{1\max}, U, u_{2\max}$ and c_1^- are $\times10^5$, c_1^+ and c_2 are $\times10^3$). The values of $c_1^+, c_1^-, c_2, \theta_1, \theta_2, q_{12}^2, q_{21}^1$ and q_{21}^2 do not come from MRC's data. Sensitivity analysis with respect of these values will be made to cover a range of values that may include MRC and thus validate the proposed model, despite a lack of precise information.

The considered computation domain D is given by:

$$D = \{x_1 : -0.5 \le x_1 \le 1;\quad x_2 : 0 \le x_2 \le 2\} \tag{5.9}$$

5.3.1 Results analysis

The production policies $u_i^*(x_1, x_2, \alpha),\ (i=1,2)$, illustrated in Figures 5.3, 5.4 and 5.5 indicate the production rates of the manufacturing/remanufacturing system for a given stock of return products $x_2(t)$ and stock level $x_1(t)$. Based on the results, there is no need to produce at a comfortable stock level capable of meeting long term demand; we do not need to produce if the stock level of finished products is greater than 0.3×10^5, 0.5×10^5 and 0.45×10^5, respectively.

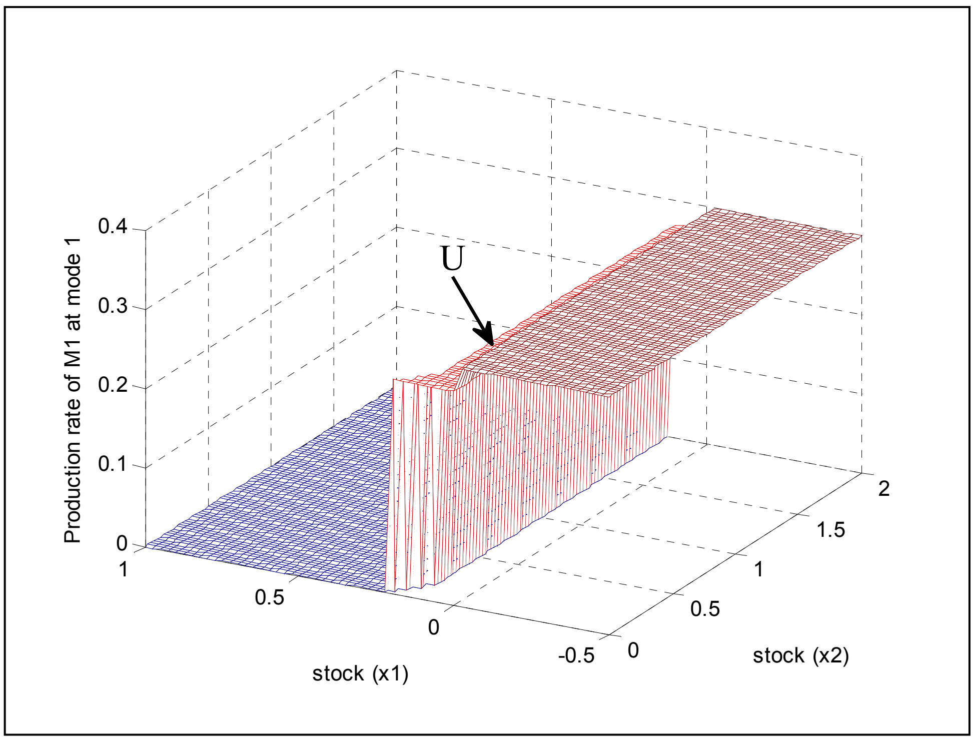

Figure 5.3 Production rate of M_1 at mode 1

Figures 5.3 and 5.4 illustrate the optimal production rate of M_1 at mode 1 and mode 2. These Figures show that when the stock levels of finished products are below 0.15×10^5 and 0.4×10^5, we need to produce at the maximum production and at the economical production rates. If the stock of the return product increases, the stock level of finished products decreases. The results of Figure 5.3 (Figure 5.4) show that for a quantity of returned products greater than 0.15×10^5 (0.525×10^5), regardless of the level of serviceable stock, there is need to set the production rate to its maximal rate. Unlike the case illustrated in Figure 5.3, where the tendency was to use the maximal productivity (production rate) of M_1 less, at mode 2, the first threshold ($z_3 = 0.4\times10^5$ products) in Figure 5.4 is higher than in Figure 5.3 ($z_1 = 0.15\times10^5$ products) because the manufacturing factory works alone.

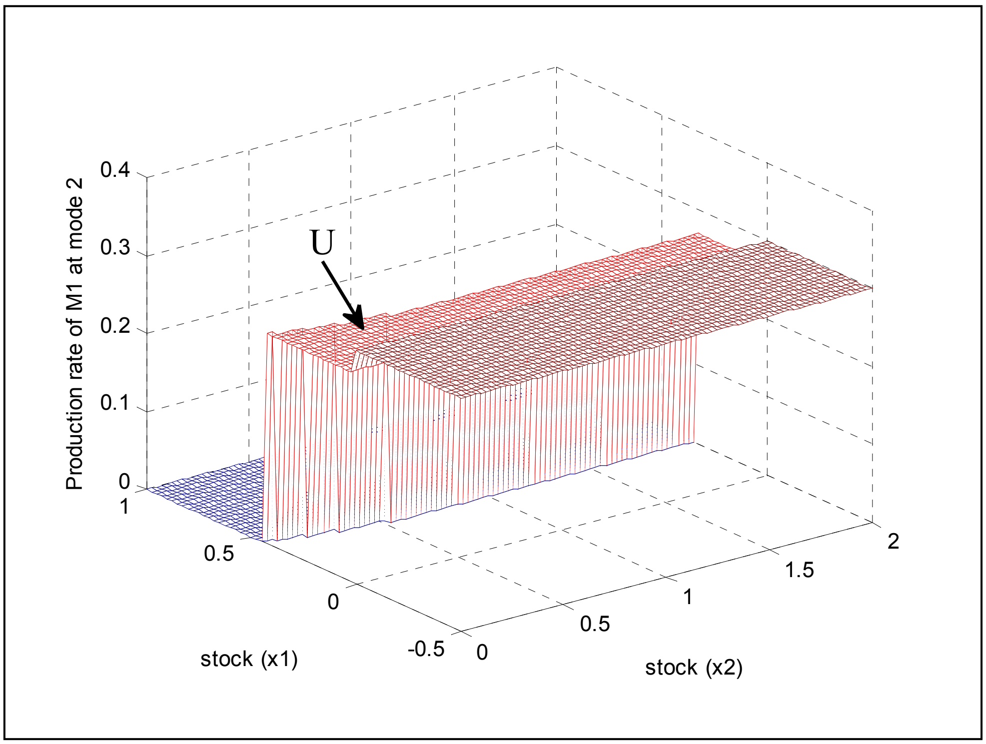

Figure 5.4 Production rate of M_1 at mode 2

Examining Figures 5.3 and 5.4, we see that the optimal stock levels of new products depend directly on the level of returned products. Consequently, the optimal production control policy consists of one of the following rules:

1. Set the productivity of M_1 to its maximal value when the current stock level is under the first threshold value ($z_1(x_2) = 0.15\times10^5$ at mode 1 and $z_3(x_2) = 0.4\times10^5$ at mode 2);

2. Reduce the productivity of M_1 to its economical value U when the current stock level approaches the second threshold value ($z_2(x_2) = 0.3\times10^5$ at mode 1 and $z_4(x_2) = 0.50\times10^5$ at mode 2);
3. Set the productivity of M_1 to zero when the current stock level is greater than the second threshold value.

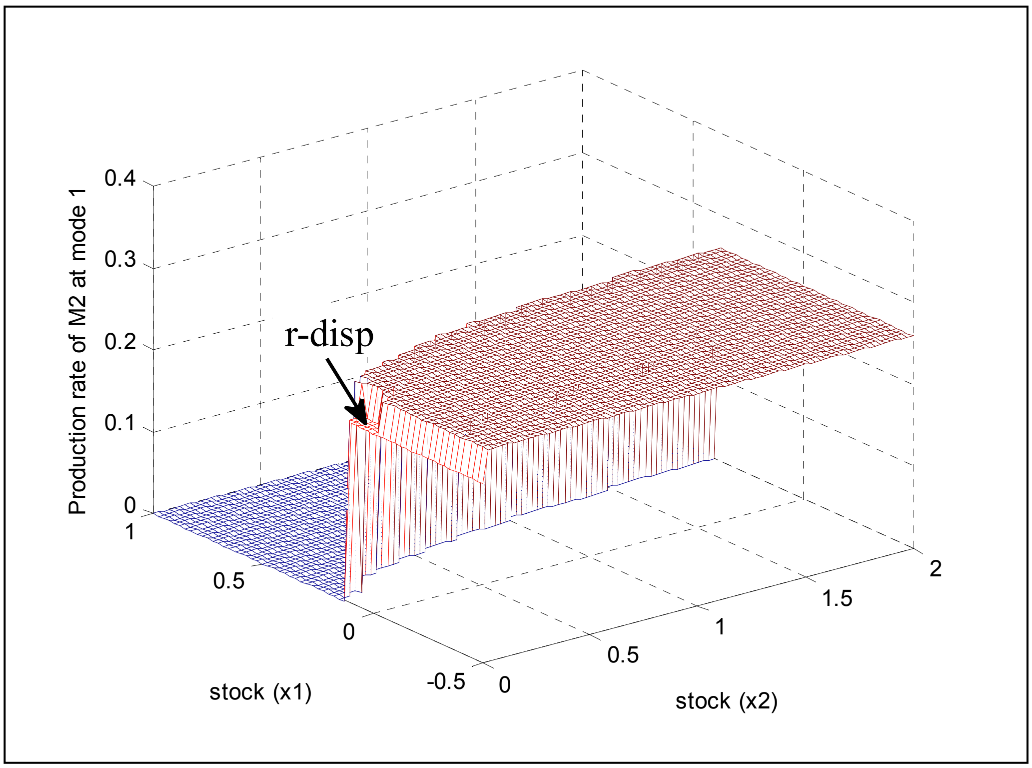

Figure 5.5 Production rate of M_2 at mode 1

In Figure 5.5, the optimal policy of M_2 at mode 1 is presented. We didn't plot the production rate of M_2 at mode 3 because M_1 is under repair and M_2 can not satisfy the customer demand alone. The results of figure 5.5 show that for $0 \le x_2 \le 0.15\times10^5$, the production rate of M_2 is set to $r - disp$. When the stock of new products is 0.45×10^5 products and the stock of returned products is greater than 0.15×10^5 products, the production rate is set to zero.

The zone where the production rate is set to zero is restricted when the stock of returned products increases. The effect of large quantity of x_2 is minimized by assigning large values of the stock threshold at mode 1.

The results of Figure 5.5 show that the threshold values also depend on the level of returned products.

The computational domain of M_2 at mode 1 is divided into two regions where the optimal production control policy consists of the following two rules:

1. Produce at the maximal rate (or at $r-disp$ if $0 \le x_2 \le 0.15\times10^5$) when the current stock level is under a threshold value ($z_5(x_2)=0.45\times10^5$).
2. Set the production rate to zero when the current stock level is larger than a threshold value.

Based on the results from Figures 5.3 to 5.5, the production rates of M_1 and M_2 are given by x_2 dependent hedging point:

$$u_1^*(x_1,x_2,1)=\begin{cases} u_{1\max} & \text{if } x_1 < z_1(x_2) \\ U & \text{if } z_1(x_2) \le x_1 < z_2(x_2) \\ 0 & \text{if } x_1 > z_2(x_2) \end{cases} \tag{5.10}$$

where $z_1(x_2)$ and $z_2(x_2)$ are the first and the second threshold values of M_1 at mode 1, respectively.

$$u_1^*(x_1,x_2,2)=\begin{cases} u_{1\max} & \text{if } x_1 < z_3(x_2) \\ U & \text{if } z_3(x_2) \le x_1 < z_4(x_2) \\ 0 & \text{if } x_1 > z_4(x_2) \end{cases} \tag{5.11}$$

where $z_3(x_2)$ and $z_4(x_2)$ are the first and the second threshold values of M_1 at mode 2, respectively.

$$u_2^*(x_1,x_2,1)=\begin{cases}u_{2\max} & \text{if } x_1<z_5(x_2)\\ 0 & \text{if } x_1>z_5(x_2)\end{cases} \tag{5.12}$$

where $z_5(x_2)$ is the optimal threshold value of M_2 at mode 1.

The optimal policy of the proposed joint optimization of production and factories reliability is given by equations (5.10)-(5.12). To validate and illustrate the usefulness of the model developed, let us confirm the observation through a sensitive analysis.

5.3.2 Sensitivity analysis

A set of numerical examples was considered to measure the sensitivity of the control policies obtained. We analyze the sensitivity of the control policies according to the costs parameters in the first section. In the second section, we examine the sensitivity of the optimal policies according to the factories' parameters. For simplify, z_1, z_2, z_3, z_4 and z_5 will be used in the rest of the paper instead of $z_1(x_2), z_2(x_2), z_3(x_2), z_4(x_2)$ and $z_5(x_2)$, respectively. The following variations are explored and compared to the basic case (highlighted lines).

5.3.2.1 Sensitivity analysis with respect to costs parameters

In this section, backlog, serviceable inventory and inventory of returns cost parameters are considered. The numerical example presented previously was used to perform a couple of experiments, and the results shown in Table 5.3 illustrate three scenarios.

The first block of Table 5.3 shows that the thresholds z_1, z_2, z_3, z_4 and z_5 increase as the backlog costs increase. We therefore need a lot of finished products in stock to avoid further backlog costs. The parameters of the control policy move as predicted, from a practical perspective when backlog costs decrease.

When the serviceable costs increase, the threshold values decrease in order to avoid further inventory costs. The values of z_1, z_2, z_3, z_4 and z_5 increase when c_1^+ decreases. This is logical because the inventory costs are low. See second block of Table 5.3.

Table 5.3 Sensitivity analysis with respect to costs parameters

c_1^-	c_1^+	c_2	z_1	z_2	z_3	z_4	z_5
25	4	2	0.05	0.15	0.20	0.30	0.30
50	4	2	0.10	0.20	0.30	0.40	0.35
75	4	2	0.10	0.25	0.35	0.45	0.40
100	**4**	**2**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
125	4	2	0.15	0.30	0.40	0.55	0.45
150	4	2	0.15	0.30	0.45	0.60	0.50
300	4	2	0.25	0.45	0.55	0.65	0.60
100	3	2	0.15	0.30	0.45	0.50	0.60
100	**4**	**2**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
100	7	2	0.10	0.25	0.35	0.45	0.25
100	4	1	0.15	0.30	0.45	0.50	0.03
100	**4**	**2**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
100	4	3	0.15	0.30	0.40	0.45	0.65

Based on the results of third block of Table 5.3, we can see that the variations of c_2 do not affect the thresholds z_1 and z_2. This is logical because at mode 1, when both factories are operational, the system does not use the manufacturing factory enough in order to take account of its reliability. When the value of c_2 decreases, the values of z_3 and z_4 increase in order to avoid over-shortages, and the value of z_5 decreases. Decreasing the parameter c_2 means that we can over-stock the returned products because the storage costs are low. In this way, the remanufacturing factory is less used. The parameters of control policy move as predicted, when c_2 increases. For example the value of z_5 increases because M_2 is used a lot in order to reduce the stock of returned products.

5.3.2.2 Sensitivity analysis with respect to factories' parameters

This section analyzes the sensitivity of the threshold values with the respect to the parameters of the two factories, as shown in Tables 5.4 to 5.11.

a. Varying θ_1 (failure rate of M_1 for $u_1 \in (U, u_{1\max}]$)

The results of Table 5.4 show that the variation of the parameter θ_1 does not affect the thresholds z_1, z_3, z_4 and z_5. When θ_1 increases, z_1 remains constant in order to avoid further shortages because the probability of failure at the maximum production rate is high, and the value of z_2 decreases. This adequately reflects the phenomenon of degradation of our system. The production rate of M_1 should be reduced to its economical value when closing to a comfortable stock level in order to ensure its reliability. We recall that z_1 is the hedging point policy of M_1 at state 1 when $u_1 \in (U, u_{\max}]$.

b. Varying θ_2 (failure rate of M_1 for $u_1 \in [0, U]$)

When θ_2 decreases, the thresholds z_2, z_3, z_4 and z_5 decrease in order to avoid over-stocking because the probability of failure, for $u_1 \in [0, U]$, is low. The value of z_1 remains constant, as do the other parameters of the control policy, when θ_2 increases (see Table 5.5).

c. Varying q_{12}^2 (failure rate of M_2)

According to Table 5.6, when q_{12}^2 decreases, the thresholds z_2 and z_5 decrease because the probability of failure of the remanufacturing factory decreases. As for z_1, z_3 and z_4, their values remain constant. The thresholds z_1 and z_5 increase in order to avoid backlogs, when q_{12}^2 increases, and the values of z_2, z_3 and z_4 remain constant.

Table 5.4 Sensitivity analysis with respect to θ_1

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/85	1/100	1/60	1/1	1/1	0.30	0.27	0.26	0.15	0.30	0.40	0.50	0.45
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/75	1/100	1/60	1/1	1/1	0.30	0.27	0.26	0.15	0.25	0.40	0.50	0.45
1/70	1/100	1/60	1/1	1/1	0.30	0.27	0.26	0.15	0.25	0.40	0.50	0.45

Table 5.5 Sensitivity analysis with respect to θ_2

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	**1/135**	1/60	1/1	1/1	0.30	0.27	0.26	0.15	0.25	0.35	0.45	0.40
1/80	**1/120**	1/60	1/1	1/1	0.30	0.27	0.26	0.15	0.25	0.35	0.45	0.40
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	**1/95**	1/60	1/1	1/1	0.30	0.27	0.26	0.10	0.30	0.40	0.50	0.45

Table 5.6 Sensitivity analysis with respect to q_{12}^2

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	1/100	**1/70**	1/1	1/1	0.30	0.27	0.26	0.15	0.25	0.40	0.50	0.40
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	**1/50**	1/1	1/1	0.30	0.27	0.26	0.15	0.30	0.40	0.50	0.45
1/80	1/100	**1/30**	1/1	1/1	0.30	0.27	0.26	0.20	0.30	0.40	0.50	0.55

d. Varying q_{21}^1 (repair rate of M_1)

When q_{21}^1 increases, the thresholds z_1, z_2, z_3, z_4 and z_5 decrease in order to avoid over-stocking because the probability of repairing M_1 is high. There is a tendency to use M_1 and M_2 less when the repair rate of the main factory increases. If M_1 breaks down, it will return soon to the operational mode. The parameters of the control policy increase in order to avoid backlogs when q_{21}^1 decreases (see Table 5.7).

e. Varying q_{21}^2 (repair rate of M_2)

When q_{21}^2 decreases, the thresholds z_1 and z_2 remain constant.

The parameters z_3, z_4 and z_5 increase in order to avoid backlogs because the repair time of M_2 is long. If this factory fails, it will later return to its operational mode. All thresholds decrease in order to avoid over-stocking when q_{21}^2 increases (see Table 5.8).

Table 5.7 Sensitivity analysis with respect to q_{21}^1

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	1/100	1/60	**1/4**	1/1	0.30	0.27	0.26	0.50	0.90	0.80	1.00	0.90
1/80	1/100	1/60	**1/2**	1/1	0.30	0.27	0.26	0.30	0.50	0.55	0.75	0.65
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	1/60	**1/0.5**	1/1	0.30	0.27	0.26	0.05	0.15	0.25	0.35	0.35

Table 5.8 Sensitivity analysis with respect to q_{21}^2

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	1/100	1/60	1/1	**1/1.5**	0.30	0.27	0.26	0.15	0.30	0.50	0.60	0.50
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	1/60	1/1	**1/0.5**	0.30	0.27	0.26	0.10	0.25	0.25	0.35	0.40
1/80	1/100	1/60	1/1	**1/0.25**	0.30	0.27	0.26	0.10	0.25	0.20	0.30	0.35

f. Varying $u_{1\max}$ (maximal production rate of M_1)

When $u_{1\max}$ increases, the value of z_2 decreases. This inevitably increases the chances of M_1 being used to its maximal production rate at mode 1. The thresholds z_1, z_3, z_4 and z_5 remain constant (see Table 5.9).

g. Varying U (economical production rate of M_1)

When U increases, the results of Table 5.10 show that at mode 1 where both factories are operational, for gaining in availability of the manufacturing factory, the system maintains the same value of z_1 and the value of z_2 decreases. In this way, the remanufacturing factory is used to fill the customer demand. At mode 2, where M_2 is non-operational, the parameters of the control policy move as predicted, from a practical perspective when the value of U

increases. For example, the value of z_3 decreases and the threshold z_4 remains constant. All thresholds remain constant when U decreases in order to avoid shortages.

Table 5.9 Sensitivity analysis with respect to $u_{1\max}$

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	1/60	1/1	1/1	**0.305**	0.27	0.26	0.15	0.25	0.40	0.50	0.45
1/80	1/100	1/60	1/1	1/1	**0.307**	0.27	0.26	0.15	0.25	0.40	0.50	0.45

Table 5.10 Sensitivity analysis with respect to U

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	1/100	1/60	1/1	1/1	0.30	**0.25**	0.26	0.15	0.30	0.40	0.50	0.45
1/80	1/100	1/60	1/1	1/1	0.30	**0.26**	**0.26**	0.15	0.30	0.40	0.50	0.45
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	1/60	1/1	1/1	0.30	**0.29**	0.26	0.15	0.25	0.35	0.50	0.45

h. Varying $u_{2\max}$ (maximal production rate of M_2)

When $u_{2\max}$ increases, z_1, z_2, z_3 and z_5 decrease in order to avoid over-stocking; the threshold z_4 remains constant. The parameters of the control policy move as predicted, from a practical perspective in order to avoid over-shortages, when $u_{2\max}$ decreases (see Table 5.11).

Table 5.11 Sensitivity analysis with respect to $u_{2\max}$

θ_1	θ_2	q_{12}^2	q_{21}^1	q_{21}^2	$u_{1\max}$	U	$u_{2\max}$	z_1	z_2	z_3	z_4	z_5
1/80	1/100	1/60	1/1	1/1	0.30	0.27	**0.23**	0.25	0.30	0.50	0.50	0.85
1/80	**1/100**	**1/60**	**1/1**	**1/1**	**0.30**	**0.27**	**0.26**	**0.15**	**0.30**	**0.40**	**0.50**	**0.45**
1/80	1/100	1/60	1/1	1/1	0.30	0.27	**0.265**	0.10	0.25	0.35	0.50	0.35
1/80	1/100	1/60	1/1	1/1	0.30	0.27	**0.267**	0.10	0.25	0.35	0.50	0.35

Through the observations drawn from the sensitivity analysis, it clearly appears that the results obtained are robust and thus validate the proposed approach. They show the usefulness of the proposed model, given that the parameters of the control policies move as expected, from a practical perspective.

5.4 Conclusions

This paper discussed the application of a hybrid manufacturing/remanufacturing system to the case of a printer cartridges company, when the manufacturing factory is degraded according to its production rate. The system studied comprised two parallel factories (the manufacturing and the remanufacturing factories) subject to a non-homogeneous Markov process. The manufacturing factory was the main factory characterised by a higher production rate. Its failure rates depend on its production rate. The factories produce the laser printer cartridges. From the numerical study, it has been found that the hedging point policies are optimal within a five-threshold feedback policy, and the reliability of the factories is enhanced. The results also show that to reduce the total cost, it may be beneficial to decrease the production rate of the manufacturing factory from its maximal value to its economical value when the stock level of finished products approaches the threshold value. Numerical examples and certain data of MRC, a manufacturing/remanufacturing firm based in France, which produces manufactured and remanufactured laser printer cartridges, are used to illustrate the utility of the proposed approach. The sensitivity analyses with respect of values which didn't come from MRC have shown that the parameters of the control policy move as expected, from a practical perspective. Thus, the model developed in this paper has been successfully applied to the real case MRC although further investigations and more precise data collections could improve this application. The sensitivity analysis that we have conducted also shows that this work could be applied to other cases in a similar environment.

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CONCLUSION GÉNÉRALE

Les modèles actuelles d'optimisation des systèmes manufacturiers qui intègrent le retour des produits usagés dans leur système de production présentent certaines lacunes. L'utilisation régulière des unités de production à leur pleine capacité présente un impact sur la disponibilité et la fiabilité du système manufacturier. Afin de remédier à ces lacunes, cette thèse a eu pour objectif de proposer des modèles pragmatiques permettant de résoudre les problèmes d'optimisation des systèmes de production / réutilisation dans un contexte dynamique stochastique. Notre travail a été élaboré en cinq (5) chapitres.

Au Chapitre 1, la problématique de notre recherche a été décrite. Nous avons également consulté et critiqué une série de revues scientifiques récentes et pertinentes sur la chaîne d'approvisionnement, les stratégies de maintenance, l'optimisation de la production des systèmes manufacturiers, la gestion simultanée de la production et de la maintenance des machines, la dégradation des unités de production en fonction de leur taux d'utilisation, les systèmes hybrides de production/réutilisation des pièces et les modèles mathématiques utilisés en logistique inverse. Compte tenu du volume des revues consultées, un tableau de synthèse a été donné à la fin du chapitre. Cette revue de la littérature a permis de situer notre travail par rapport à l'ensemble des travaux déjà réalisés et d'en évaluer l'originalité.

Le Chapitre 2 a traité le problème de planification de la production d'un système manufacturier soumis à des réparations imparfaites. Ainsi, le taux de panne dépendait du nombre de panne. Le système était constitué d'une machine produisant un seul type de pièce. En plus du taux de production, la loi de commande incluait les stratégies de maintenance préventive et corrective. Nous avons développé un modèle d'optimisation stochastique du problème considéré, avec trois variables de décision; le taux de production, les taux de maintenance préventive et corrective; et une variable d'état; le niveau du stock des produits finis. A l'aide des méthodes numériques, nous avons obtenu une loi de commande optimale. Nous avons illustré l'approche proposée et validé le concept de commande simultanée de la production, de la maintenance préventive et corrective sur un exemple numérique. Les résultats obtenus ont montré que les politiques optimales recherchées sont de type à seuil critique et que le nombre de pièces à mettre en stock, et les paramètres de maintenance préventive et corrective augmentent lorsque le nombre de pannes augmente. Le critère de performance étant le coût total du système manufacturier. Pour illustrer l'utilité de nos résultats, une analyse de sensibilité a été faite.

Dans le Chapitre 3, nous avons montré que pour tenir compte de la fiabilité du système et réduire le coût total du système manufacturier, il est nécessaire de réduire le taux de production lorsqu'on approche le stock optimal des produits finis. Le système manufacturier était constitué de deux machines non-identiques en parallèle, produisant un seul type de pièce. Par rapport au chapitre 2 où le taux de panne dépendait du nombre de pannes, dans cette étude, le taux de panne de l'une des machines dépend de son taux de production. Donc, une modélisation par un processus de chaîne de Markov non-homogène a été faite. Une résolution numérique des équations d'HJB a conduit à la solution du problème étudié. Nous avons également étudié le cas des systèmes manufacturiers ayant plusieurs taux de panne c'est-à-dire plus de deux taux de panne, et les résultats obtenus ont été très satisfaisants. Les analyses de sensibilité élaborées ont donné des conclusions logiques. L'approche proposée dans ce chapitre est très importante dans les entreprises d'usinage où la vitesse de la production influence considérablement l'usure de l'outil de coupe.

Au Chapitre 4, nous avons établi qu'il est possible d'intégrer la dégradation des machines en fonction de leur vitesse de production dans un système hybride de production / réutilisation soumis aux pannes et réparations aléatoires. En effet, dans cette partie, la deuxième machine du chapitre 3 est remplacée par une machine de réutilisation. Les machines produisaient un seul type de produit. L'objectif du système a été de minimiser les coûts de pénuries, de mise en stock des produits finis et des retours, et d'assurer la disponibilité et la fiabilité des machines de production / réutilisation. Nous avons développé un modèle dynamique stochastique avec deux variables de décision; les taux de production de la machine de production et de la machine de réutilisation, et deux variables d'état; les niveaux du stock des produits finis et du stock des retours. Le problème a été résolu par des méthodes numériques.

Les résultats obtenus ont montré que les politiques optimales sont caractérisées par cinq paramètres au lieu de trois paramètres, comme dans le cas des systèmes manufacturiers avec un taux de panne constant. Un exemple numérique a été donné pour illustrer l'utilité de l'approche proposée, et diverses analyses de sensibilité effectuées ont confirmé la structure des politiques obtenues.

Du Chapitre 5, il ressort que les politiques de gestion proposées dans cette thèse peuvent être validées sur les compagnies fabricant des cartouches compatibles, Laser et Jet d'encre, neuves et refabriquées. Les résultats obtenus permettront à ces compagnies de minimiser les coûts de pénuries, de mise en stock des produits finis et des retours, tout en assurant la disponibilité et la fiabilité de leurs usines de production/réutilisation. Cette application constitue une validation de nos travaux dans un contexte industriel réaliste.

Dans cette thèse, notre travail a apporté une contribution scientifique significative en reformulant les modèles mathématiques existant pour intégrer le taux de panne dépendant du taux de production en contexte de systèmes hybrides de production / réutilisation soumis aux pannes et réparations aléatoires. Les résultats de nos travaux ont été confirmés à travers des études par modélisation, résolution numérique et analyse de sensibilité sur des cas de systèmes manufacturiers flexibles. Ce travail a confirmé qu'en intégrant la dégradation en fonction du nombre de pannes ou de la productivité dans un système manufacturier; et en contrôlant les opérations de maintenance, en plus de la fiabilité du système qui est assurée, le système devient moins vulnérable aux variations des coûts de pénurie, d'inventaire et de maintenance en satisfaisant la demande en permanence. Nos travaux ont été conclus par la validation des politiques proposées aux manufactures des cartouches d'encre pour imprimante. Ces contributions constituent une base solide pour des travaux futurs.

Les systèmes manufacturiers étudiés étaient constitués d'au plus deux (2) machines produisant un seul de produit; les produits refabriqués étaient identiques en termes de qualité aux produits manufacturés; les processus de demande des clients et du retour des produits étaient déterministes. D'où les résolutions proposées dans cette thèse peuvent être étendus à des systèmes plus complexes du point de vue structure et taille afin d'ouvrir une nouvelle piste de recherche.

1. Utiliser la structure des lois de commande obtenue dans cette thèse pour étendre les problèmes résolus à des cas de systèmes manufacturiers plus larges, impliquant plusieurs machines de production et plusieurs machines de réutilisation, voire plusieurs produits. Dans ce cas, une approche combinée intégrant la théorie de commande, la simulation et la méthodologie des surfaces de réponse pourra être utilisée.

2. Intégrer la notion des taux de rejets pour les systèmes hybrides de production / réutilisation. Cette notion de taux de rejet permettra d'introduire les stratégies de contrôle de la qualité des produits refabriqués.

3. Intégrer des aspects reliés aux processus de demandes et retours aléatoires.

4. Faire le contrôle de la maintenance préventive et corrective des machines des systèmes hybrides de production / réutilisation.

5. Perfectionner l'application industrielle de nos travaux et l'étendre à d'autres cas.

ANNEXE I

APPENDIX 3.A. NUMERICAL APPROACH

To solve the HJB equations, we used a numerical method based on the Kushner (1992) approach, such as in Gharbi et al. (2011). By approximating $v(\alpha, x)$ by a function $v^h(\alpha, x)$ and the first-order partial derivative of the value function $\frac{\partial v(\alpha, x)}{\partial x}$ by:

$$\frac{\partial v(x,\alpha)}{\partial x} = \begin{cases} \frac{1}{h}\left(v^h(\alpha, x+h) - v^h(\alpha, x)\right) & \text{if } (u_1 + u_2 - d) > 0 \\ \frac{1}{h}\left(v^h(\alpha, x) - v^h(\alpha, x-h)\right) & \text{otherwise} \end{cases}$$

The HJB equation becomes:

$$v^h(x,\alpha) = \min_{(u_1,u_2)\in \mathrm{A}(\alpha)} \left[\frac{g(x,\alpha) + \sum_{\beta\neq\alpha} \lambda_{\alpha\beta} v^h(x,\beta) + \frac{(u_1+u_2-d)}{h} \left[\begin{array}{l} v^h(x+h,\alpha) Ind\{u_1+u_2-d \geq 0\} \\ + v^h(x-h,\alpha) Ind\{u_1+u_2-d < 0\} \end{array} \right]}{\left(\rho + \frac{|u_1+u_2-d|}{h} + |\lambda_{\alpha\alpha}| \right)} \right] \tag{3.A.1}$$

with $\lambda_{\alpha\alpha} = -\sum_{\beta\neq\alpha} \lambda_{\alpha\beta}$, $\mathrm{A}^h(\alpha)$ is the numerical control grid and $Ind\{\Phi\} = \begin{cases} 1 & \textit{if } \Phi \textit{ is true} \\ 0 & \textit{otherwise} \end{cases}$

The system of equations (3.A.1) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process, as in Boukas and Haurie (1990). In this paper, we use the value iteration procedure to approximate the value function given by equation (3.A.1). Dehayem et al. (2011) and references therein provide details on such methods.

The discrete dynamic programming equation (3.A.1) gives the following six equations:

- **mode 1**

$$v^h(x,1) = \begin{cases} V_1^h(x,1) & \text{if } u_1 \in \left(U, u_{1\max}\right] \\ V_2^h(x,1) & \text{if } u_1 \in \left[0, U\right] \end{cases} \qquad \text{with} \tag{3.A.2}$$

$$V_1^h(x,1) = \min_{\substack{u_1 \in \left(U, u_{1\max}\right] \\ u_2 \in \left[0, u_{2\max}\right]}} \left[\frac{g(x,\alpha) + \dfrac{(u_1+u_2-d)}{h} \begin{bmatrix} v^h(x+h,1) Ind\{u_1+u_2-d \geq 0\} + \\ v^h(x-h,1) Ind\{u_1+u_2-d < 0\} \end{bmatrix} + q_{12}^2 v^h(x,2) + \theta_1 v^h(x,3)}{\left(\rho + \dfrac{|u_1+u_2-d|}{h} + q_{12}^2 + \theta_1\right)} \right]$$

$$V_2^h(x,1) = \min_{\substack{u_1 \in \left[0, U\right] \\ u_2 \in \left[0, u_{2\max}\right]}} \left[\frac{g(x,\alpha) + \dfrac{(u_1+u_2-d)}{h} \begin{bmatrix} v^h(x+h,1) Ind\{u_1+u_2-d \geq 0\} + \\ v^h(x-h,1) Ind\{u_1+u_2-d < 0\} \end{bmatrix} + q_{12}^2 v^h(x,2) + \theta_2 v^h(x,3)}{\left(\rho + \dfrac{|u_1+u_2-d|}{h} + q_{12}^2 + \theta_2\right)} \right]$$

- **mode 2**

$$v^h(x,2) = \begin{cases} V_1^h(x,2) & \text{if } u_1 \in \left(U, u_{1\max}\right] \\ V_2^h(x,2) & \text{if } u_1 \in \left[0, U\right] \end{cases} \qquad \text{with} \tag{3.A.3}$$

$$V_1^h(x,2) = \min_{u_1 \in \left(U, u_{1\max}\right]} \left[\frac{g(x,\alpha) + \dfrac{(u_1-d)}{h} \left[v^h(x+h,2) Ind\{u_1-d \geq 0\} + v^h(x-h,2) Ind\{u_1-d < 0\} \right] + q_{21}^2 v^h(x,1) + \theta_1 v^h(x,4)}{\left(\rho + \dfrac{|u_1-d|}{h} + q_{21}^2 + \theta_1\right)} \right]$$

$$V_2^h(x,2) = \min_{u_1 \in \left[0, U\right]} \left[\frac{g(x,\alpha) + \dfrac{(u_1-d)}{h} \left[v^h(x+h,2) Ind\{u_1-d \geq 0\} + v^h(x-h,2) Ind\{u_1-d < 0\} \right] + q_{21}^2 v^h(x,1) + \theta_2 v^h(x,4)}{\left(\rho + \dfrac{|u_1-d|}{h} + q_{21}^2 + \theta_2\right)} \right]$$

- mode 3

$$v^h(x,3) = \min_{u_2 \in \left[0, u_{2\max}\right]} \left[\frac{\begin{array}{l} g(x,\alpha) + \dfrac{(u_2-d)}{h} \left[v^h(x+h,3) Ind\{u_2-d \geq 0\} + v^h(x-h,3) Ind\{u_2-d < 0\} \right] + \\ q_{21}^1 v^h(x,1) + q_{12}^2 v^h(x,4) \end{array}}{\left(\rho + \dfrac{|u_2-d|}{h} + q_{21}^1 + q_{12}^2\right)} \right] \tag{3.A.4}$$

- mode 4

$$v^h(x,4) = \min\left[\frac{g(x,\alpha)+q_{21}^1 v^h(x,2) + q_{21}^2 v^h(x,3) + \frac{d}{h} v^h(x-h,4)}{\left(\rho + \frac{d}{h} + q_{21}^1 + q_{21}^2\right)}\right] \quad (3.A.5)$$

ANNEXE II

APPENDIX 4.A. OPTIMALITY CONDITIONS AND NUMERICAL APPROACH

This section presents the optimality conditions satisfied by the value function presented in equation (4.8). The properties of the value function and the manner in which the Hamilton-Jacobi-Bellman (HJB) equations are obtained can be found in Martinelli (2010). Regarding the optimality principle, we can write the HJB equations as follows:

$$\rho v(\alpha, x_1, x_2) = \min_{(u_1,u_2)\in \mathrm{A}(\alpha)} \left[\begin{array}{l} g(\alpha,x_1,x_2) + \sum_{\beta\in B} \lambda_{\alpha\beta} v(\beta,x) + (u_1+u_2-d)\dfrac{\partial v(\alpha,x_1)}{\partial x_1} \\ +(r-u_2-disp)\dfrac{\partial v(\alpha,x_2)}{\partial x_2} \end{array} \right] \quad (4.A.1)$$

where $\frac{\partial v(\alpha,x_1)}{\partial x_1}$ and $\frac{\partial v(\alpha,x_2)}{\partial x_2}$ are the partial derivatives of the value functions $v(\alpha,x_1)$ and $v(\alpha,x_2)$, respectively.

The optimal control policy $\left(u_1^*(\cdot),u_2^*(\cdot)\right)$ denotes a minimizer over $\mathrm{A}(\alpha)$ of the right hand of equation (4.A.1). This policy corresponds to the value function described by equation (4.8). When the value function is available, an optimal control policy can then be obtained by solving equation (4.A.1). The proof of optimality conditions to approximate HJB equation follows the same scheme adopted in Martinelli. (2007) for production planning of a manufacturing system with production-dependent failure rates.

To solve the HJB equations, the numerical method based on the Kushner approach (Kushner, 1992) as in Gharbi et al., 2011) and references therein is used. Let h_1 and h_2 denote the length of the finite difference interval of the variables x_1 and x_2, respectively. By approximating $v(\alpha,x_1)$ and $v(\alpha,x_2)$ by functions $v^h(\alpha,x_1)$ and $v^h(\alpha,x_2)$, and the first-order partial derivative of the value functions $\frac{\partial v(\alpha,x_1)}{\partial x_1}$ and $\frac{\partial v(\alpha,x_2)}{\partial x_2}$ by:

$$\frac{\partial v(x_1,\alpha)}{\partial x_1}=\begin{cases}\frac{1}{h_1}\left(v^h(\alpha,x_1+h_1,x_2)-v^h(\alpha,x_1,x_2)\right) & if\ (u_1+u_2-d)>0\\ \frac{1}{h_1}\left(v^h(\alpha,x_1,x_2)-v^h(\alpha,x_1-h_1,x_2)\right) & otherwise\end{cases}$$

$$\frac{\partial v(x_2,\alpha)}{\partial x_2}=\begin{cases}\frac{1}{h_2}\left(v^h(\alpha,x_1,x_2+h_2)-v^h(\alpha,x_1,x_2)\right) & if\ (r-u_2-disp)>0\\ \frac{1}{h_2}\left(v^h(\alpha,x_1,x_2)-v^h(\alpha,x_1,x_2-h_2)\right) & otherwise\end{cases}$$

The HJB equation becomes:

$$v^h(\alpha,x_1,x_2)=\min_{(u_1,u_2)\in A^h(\alpha)}\left[\frac{\begin{array}{l}g(\alpha,x_1,x_2)+\sum_{\beta\neq\alpha}\lambda_{\alpha\beta}v^h(x_1,x_2,\beta)+\frac{(u_1+u_2-d)}{h_1}\begin{bmatrix}v^h(x_1+h_1,x_2,\alpha)Ind\{u_1+u_2-d\geq 0\}\\+v^h(x_1-h_1,x_2,\alpha)Ind\{u_1+u_2-d<0\}\end{bmatrix}\\+\frac{(r-u_2-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,\alpha)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,\alpha)Ind\{r-u_2-disp<0\}\end{bmatrix}\end{array}}{\rho+\frac{|u_1+u_2-d_c|}{h_1}+\frac{|r-u_2-disp|}{h_2}+|\lambda_{\alpha\alpha}|}\right]$$

(4.A.2)

with $\lambda_{\alpha\alpha}=-\sum_{\beta\neq\alpha}\lambda_{\alpha\beta}$, $A^h(\alpha)$ is the numerical control grid and $Ind\{\Phi\}=\begin{cases}1 & if\ \ \Phi\ is\ true\\ 0 & otherwise\end{cases}$

In this paper, we use the value iteration procedure to approximate the value function given by equation (4.A.2). Dehayem et al. (2011) and references therein provide details on such methods.

The discrete dynamic programming equation (4.A.2) gives the following six equations:

- **mode 1**

$$v^h(x_1,x_2,1)=V_i^h(x_1,x_2,1);\quad \text{with}\quad \begin{cases}i=1 & \text{if } u_1\in\left(U,u_{1\max}\right]\\ i=2 & \text{if } u_1\in[0,U]\end{cases}\qquad (4.A.3)$$

$$V_i^h(x_1,x_2,1)=\min_{u_2\in[0,u_{2\max}]}\frac{\begin{array}{l}g(\alpha,x_1,x_2)+\dfrac{(u_1+u_2-d)}{h_1}\begin{bmatrix}v^h(x_1+h_1,x_2,1)Ind\{u_1+u_2-d\geq 0\}\\+v^h(x_1-h_1,x_2,1)Ind\{u_1+u_2-d<0\}\end{bmatrix}\\+\dfrac{(r-u_2-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,1)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,1)Ind\{r-u_2-disp<0\}\end{bmatrix}+q_{12}^2v^h(x_1,x_2,2)+\theta_i v^h(x_1,x_2,3)\end{array}}{\rho+\dfrac{|u_1+u_2-d|}{h_1}+\dfrac{|r-u_2-disp|}{h_2}+q_{12}^2+\theta_i}$$

- **mode 2**

$$v^h(x_1,x_2,2)=V_i^h(x_1,x_2,2);\quad \begin{cases}i=1 & \text{if } u_1\in(U,u_{1\max}]\\ i=2 & \text{if } u_1\in[0,U]\end{cases} \qquad \text{with} \tag{4.A.4}$$

$$V_i^h(x_1,x_2,2)=\min_{u_2\in[0,u_{2\max}]}\frac{\begin{array}{l}g(\alpha,x_1,x_2)+\dfrac{(u_1+u_2-d)}{h_1}\begin{bmatrix}v^h(x_1+h_1,x_2,1)Ind\{u_1+u_2-d\geq 0\}\\+v^h(x_1-h_1,x_2,1)Ind\{u_1+u_2-d<0\}\end{bmatrix}\\+\dfrac{(r-u_2-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,1)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,1)Ind\{r-u_2-disp<0\}\end{bmatrix}+q_{12}^2v^h(x_1,x_2,4)+\theta_i v^h(x_1,x_2,1)\end{array}}{\rho+\dfrac{|u_1+u_2-d|}{h_1}+\dfrac{|r-u_2-disp|}{h_2}+q_{21}^2+\theta_i}$$

- mode 3

$$v^h(x_1,x_2,3)=\min_{u_2\in[0,u_{2\max}]}\frac{\begin{array}{l}g(\alpha,x_1,x_2)+\dfrac{(u_2-d)}{h_1}\begin{bmatrix}v^h(x_1+h_1,x_2,3)Ind\{u_2-d\geq 0\}\\+v^h(x_1-h_1,x_2,3)Ind\{u_2-d<0\}\end{bmatrix}\\+\dfrac{(r-u_2-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,3)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,3)Ind\{r_1-u_2-disp<0\}\end{bmatrix}+q_{21}^1v^h(x_1,x_2,1)+q_{12}^2v^h(x_1,x_2,4)\end{array}}{\rho+\dfrac{|u_2-d|}{h_1}+\dfrac{|r_1-u_2-disp|}{h_2}+q_{21}^1+q_{12}^2} \tag{4.A.5}$$

- mode 4

$$v^h(x_1,x_2,4)=\min\frac{\begin{array}{l}g(\alpha,x_1,x_2)+\dfrac{d}{h_1}v^h(x_1-h_1,x_2,4)+q_{21}^1v^h(x_1,x_2,2)+q_{21}^2v^h(x_1,x_2,4)\\+\dfrac{(r-disp)}{h_2}\begin{bmatrix}v^h(x_1,x_2+h_2,3)Ind\{r-u_2-disp\geq 0\}\\+v^h(x_1,x_2-h_2,3)Ind\{r_1-u_2-disp<0\}\end{bmatrix}\end{array}}{\rho+\dfrac{d}{h_1}+\dfrac{|r-disp|}{h_2}+q_{21}^1+q_{21}^2} \tag{4.A.6}$$

The contribution of this research to the Hamilton-Jacobi-Bellman (HJB) equations lies in the fact that at modes 1 and 2, where M_1 is operational, we have four equations (see equations (4.A.3) and (4.A.4)) instead of two, in the case of a hybrid manufacturing/remanufacturing system without a production-dependent failure rates.

ANNEXE III

ARTICLES DE CONFÉRENCES

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