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Glossary of Terms

ACO	Ant Colony Optimisation method
AHP	Analytic Hierarchy Process method
B&B	Branch and Bound algorithm
BCA	Benefit-Cost Analysis method
BCR	Benefit Cost Ratio
CEA	Cost-Effectiveness Analysis
CER	Cost Effectiveness Ratio
CPU	Central Processing Unit
DA	Dichotomic Approach
DECM	Dynamic Epsilon Constraint Method
dTIMS	Deighton's Total Infrastructure Management System
ECM	Epsilon Constraint Method
ELECTRE	Elimination Et Choice Translating Reality
ELSA	Economic Level of Service Assessment
FHWA	Federal Highway Administration
GA	Genetic Algorithm
GRIP	Generalized Regression with Intensities of Preference
H&J	Hooke & Jeeves Move
HDM	Highway Design and Maintenance Standards Model
IAM	Infrastructure Asset Management
IBC	Incremental Benefit-Cost
IIMM	International Infrastructure Management Manual
IP	Integer Programming problem
LTM	Long-Term Memory
MaineDOT	Maine Department of Transportation
MAUT	Multi-Attribute Utility Theory
MCA	Multi-Criteria Analysis
MOO	Multi-Objective Optimisation
MOOT List	a list of MOO techniques that have potential to deal with long-term and network-level decision making in IAM
MTM	Medium-Term Memory

NAMS	New Zealand Asset Management Support
NN	Neural Networks
NPV	Net Present Value
NSGA II	Nondominated Sorting Genetic Algorithm II
PSO	Particle Swarm Optimisation method
QBL	Quadruple Bottom Line method
RAM	Random Access Memory
RNBI	Revised Normal Boundary Intersection method
SA	Simulated Annealing method
SOO	Single-Objective Optimisation
STM	Short-Term Memory
TBL	Triple Bottom Line method
TS	Tabu Search method
U.S. EPA	United States Environmental Protection Agency
WIN	Water Infrastructure Network
WSM	Weighted Sum Method

List of Variables

AB_t	annual budget in year t;
AB^B	annual budget for bridge maintenance;
AB^R	annual budget for road maintenance;
$APPI_t$	acceptable pavement performance index in year t;
B_i	benefit if strategy i is applied;
B_b^B	benefit of strategy b when maintaining a bridge;
B_r^R	benefit of strategy r when maintaining a road segment;
C_i	cost if strategy i is applied;
$CL_{t,i}^l$	condition level of a segment in year t if strategy i is applied;
$\bar{d}$	average distance of all consecutive solutions;
$d(\boldsymbol{x})$	distance of a solution $\boldsymbol{x}$ to its closest Pareto solution;
$d'(\boldsymbol{x})$	Euclidean distance between solution $\boldsymbol{x}$ and its closest solution in objective space;
$d(\boldsymbol{x_p})$	distance of a Pareto solution $\boldsymbol{x_p}$ to its closest identified solution;
$\overline{d_0}$	average distance of initial solutions;
d_{iter}	distance between initial solutions and solutions at $iter$ iteration;
$\overline{d_{iter}}$	average distance of solutions identified at $iter$ iteration;
$d_{Tchebyshev}$	Tchebyshev distance;
$d_{\boldsymbol{x}}$	distance between a solution $\boldsymbol{x}$ and its consecutive solution;
f_k	function of objective k;
$f_{k,i}$	value of objective k if strategy i is implemented;
f_{main}	function of main objective;
f^l	fitness of solution $\boldsymbol{y}^l$;
F, F_1 and F_2	function of a new objective;
$g_{l,i}^U$	value of upper constraint l if strategy i is implemented;
$g_{l,i}^L$	value of lower constraint l if strategy i is implemented;
g_i^{iter}	values of element i of local optimal solution of particle l and global optimal solution at $iter$ iteration;
G_i^l	indicator of the segments in sub-network l. If the segment of strategy i is in sub-network l;

$GD(\boldsymbol{x}, \boldsymbol{x}')$	GD between solutions $\boldsymbol{x}$ and $\boldsymbol{x}'$;
$HV(\boldsymbol{x})$	hypervolume of a solution $\boldsymbol{x}$;
i	index of a strategy;
$iter$	index of iterations;
j	index of a segment;
k	index of an objective;
K	number of objectives;
l	index of a constraint;
L_i	length of the corresponding segment of strategy i is applied;
$Limit_l^U$	upper bound of constraint l;
$Limit_l^L$	lower bound of constraint l;
LYC	largest yearly cost;
$main$	index of a main objective;
M	number of segments;
M^B	number of bridges;
M^R	number of road segments;
N	number of strategies;
N^B	number of strategies for bridge management;
N^R	number of strategies for road management;
N_k	nadir value of objective k;
$N_{\boldsymbol{X}}$	number of solutions in $\boldsymbol{X}$;
NC	number of criteria;
NS_{iter}	number of identified solutions at $iter$ iteration;
p_i	performance of strategy i;
$p_i^{l,iter}$	values of element i of local optimal solution of particle l at $iter$ iteration;
P	progress;
$PPI_{t,i}$	pavement performance index in year t if strategy i is applied;
r, r_1 and r_2	random numbers between 0 and 1;
r_{UL}	uniformity level ratio;
R_k	range of objective k;
sc	slack variable;
s_i	score of element i;

s^i	strength of solution $\boldsymbol{y}^i$;
$size$	size of an objective area;
S	spacing;
$S(\cdot)$	sigmoid function;
SB_k^U and SB_k^L	upper and lower bound of the sub-boundary on objective k;
t	current temperature;
T	number of years;
TB	total budget;
U_k	utopian value of objective k;
$v_i^{l,iter}$	velocity of element i of particle l at $iter$ iteration;
w_i	weight of element i;
w_k	weight of objective k;
w_{sc}	penalty variable of soft constraints;
w_k^{1*} and w_k^{2*}	extreme weight of a shared edge;
x_i	decision variable indicating whether strategy i is selected;
x_b^B	decision variable of strategy b of bridge maintenance;
x_r^R	decision variable of strategy r of road maintenance;
$\boldsymbol{x}$ and $\boldsymbol{x}'$	feasible solutions with binary variables;
$\boldsymbol{x}_{ideal}$	an ideal solution;
$\boldsymbol{x_r}$	reference point;
$\boldsymbol{x_p}$	a Pareto solution;
$\boldsymbol{x}^s$	feasible solution with the index of s;
$\boldsymbol{X}$	set of all identified solutions;
$\boldsymbol{X}_0$	set of all initial solutions;
$\boldsymbol{X}_{iter}$	set of solutions identified at iteration $iter$;
$\boldsymbol{X_p}$	set of all Pareto solutions;
$\boldsymbol{y}$ and $\boldsymbol{y}_{new}$	feasible solutions with integer variables;
$y_i^{l,iter}$	value of element i of particle l at $iter$ iteration;
$\boldsymbol{Y}_{ND}$	set of non-dominated solutions in the solution pool;
$YC_{t,i}$	yearly cost in year t if strategy i is applied;
$YC_{b,t}^B$	maintenance cost of strategy b in year t when maintaining a bridge;

$YC_{r,t}^{R}$	maintenance cost of strategy r in year t when maintaining a road segment;
$\mathcal{S}_j$	set of alternative strategies for segment j
$\mathcal{S}_j^B$	set of available strategies for bridge j;
$\mathcal{S}_j^R$	set of available strategies for road segment j;
δ	uniformity level;
ε_k	value of epsilon on objective k;
ϵ	coverage error;
η_i	heuristic information of strategy i;
$\tau_{k,i}$	pheromone of strategy i with respect to objective k;
$\mathbf{\Omega}$	set of all feasible solutions; and
$c_1, c_2, \alpha, \beta, \rho, \omega, \Delta\tau, \gamma$	parameters.

CHAPTER 1 INTRODUCTION

1.1 Background

Infrastructure assets are described as a “stationary system forming a network and serving whole communities, such as roads, bridges, pipelines, etc.” (NAMS, 1998). They are the socio-economic backbone that supports everyone’s daily life. To keep its functionality, the infrastructure asset network as a whole needs to be managed with continuing maintenance and rehabilitation of its components.

Infrastructure Asset Management (IAM) is proposed to systematically manage an infrastructure asset network so that it is able to provide its users with the required level of service in the most efficient manner (NAMS, 2011). Hence, IAM is imperative for a better utilisation of infrastructure assets and raising the return on management investment. More specifically, for a specific infrastructure asset network, IAM attempts to achieve its management goals by generating, determining and implementing an appropriate management decision (Hassanain *et al.*, 2003). Figure 1.1 demonstrates the structure of IAM.

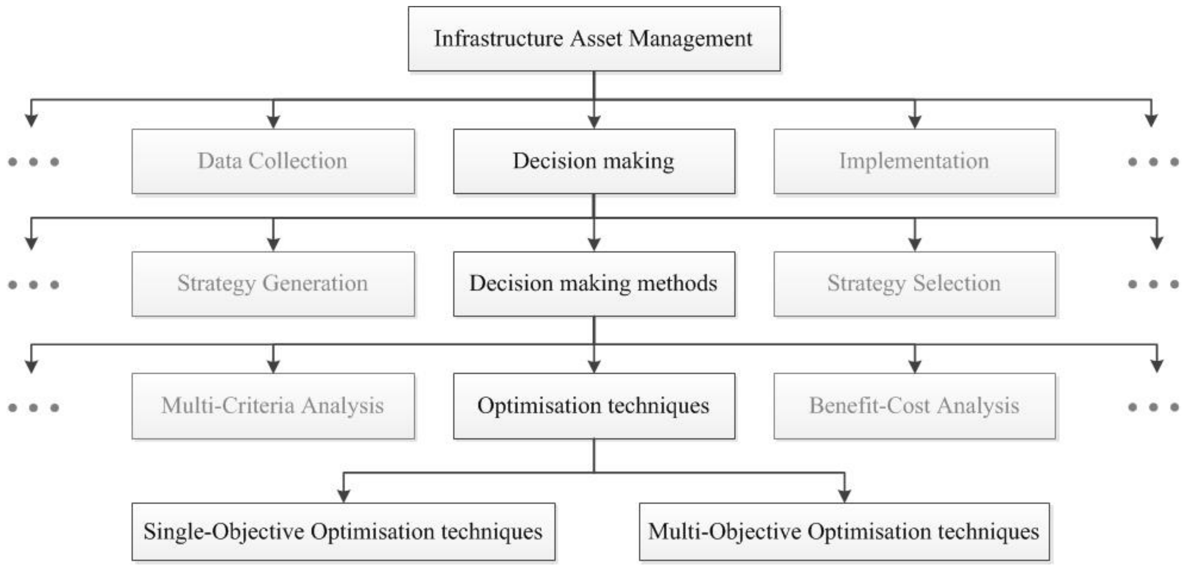


Figure 1.1 Structure of IAM (NAMS, 2011)

Decision making as shown in Figure 1.1 is a part of IAM, which determines a management decision for an infrastructure asset network. An infrastructure asset network is divided into small segments, and decision making generates alternative management strategies and selects an appropriate one to each segment of an infrastructure asset network so that the goals of IAM can be achieved with the selected strategies. However, complex in its nature, decision making balances a number of trade-offs, faces many challenges and deals with various requirements from asset owners, agencies, and users. Therefore, **decision making methods** are introduced to aid decision making. More specifically, these methods evaluate alternative strategies and suggest the selections of strategies. With the help of decision making methods, decision makers are able to make their management decisions easier and more accurate.

Therefore, different types of decision making methods are developed to help with decision making in IAM and meet different requirements. One type of decision making methods is **optimisation techniques** which can be used to analyse decision making problems and enhance the understanding of these problems. To begin with, optimisation techniques require establishing a mathematical model called optimisation problem to describe a decision making problem, which clarifies the inherent features of the decision making problem during the modelling process (Pohekar and Ramachandran, 2004). More importantly, optimisation techniques solve the established optimisation problem and generate high-quality solutions. These solutions are available selections of strategies, which can be directly selected as management decisions or adjusted based on specific considerations. Last but not least, optimisation techniques are based on mathematical computation, which is able to analyse a large number of segments and strategies. In summary, optimisation techniques can assist decision making in IAM, and therefore, they are increasingly applied.

There are two types of optimisation techniques: Single-Objective Optimisation techniques, and Multi-Objective Optimisation techniques. Many studies apply **Single-Objective Optimisation (SOO) techniques** in decision making in IAM (Fwa *et al.*, 2000) because their applications are easy. However, SOO techniques can only optimise one objective. When multiple objectives are pursued, which is common in modern decision making in IAM, SOO cannot be directly applied. Some researchers such as Fwa and Chan (1993) integrate all objectives as an overall objective, therefore enabling SOO techniques. This integration requires that the relationship of objectives is clear and can be correctly expressed by mathematical formulae. However, in practice, the objectives of decision making are often conflicting or incommensurable; hence, they cannot be rationally integrated and have to be analysed individually. Therefore, **Multi-Objective Optimisation (MOO) techniques** are needed.

MOO techniques are able to handle conflicting and incommensurable objectives, and therefore describe decision making problems more directly and practically (Ge, 2010). Furthermore, they not only suggest the selections of strategies but also present the relationship between outcomes and reduce possible subjectivities in decision making process. Thus, they are more useful than SOO techniques in decision making in IAM.

However, when multiple objectives are simultaneously optimised, the solving process can be complex. Much research (Fang *et al.*, 2005; Abiri-Jahromi *et al.*, 2009; Sharma, 2010) describes the decision making problems as multi-objective optimisation problems and then deliberately simplifies their problems into SOO problems. The simplification may cause controversial issues and reduce the accuracy of optimisation results. Hence, a robust technique that directly and effectively solves MOO problems is necessary. Also the optimisation results of MOO could be too complicated to be understood. MOO techniques identify a set of solutions, each with much information on the outcomes. Thus, it is important to report the optimisation results in a meaningful way so that the results can be easily understood and employed. In general, there is a need to find a robust MOO technique that properly handles decision making problems of IAM and a tool to properly interpret optimisation results.

1.2 Problem Statement

Long-term and network-level decision making is an important but complicated type of decision making in IAM. It focuses on life-cycle of infrastructure assets and discusses overall return on management investment. It is holistic decision making that improves the sustainability and integration of IAM and therefore generates large benefits (Sherwin, 2000; Marlow et al., 2010). Hence, long-term and network-level decision making is required by advanced IAM (NAMS, 1998) and sustainable IAM (Haarmeyer, 2011). However, this decision making is also complicated because it requires considering a wide range of factors, analysing a large amount of decision making data, facing many challenges, etc. (Sinha and Eslambolchi, 2006). Thus, compared to other types of decision making such as short-term or project-level decision making, assistance is more needed for long-term and network-level decision making.

MOO has the potential to assist long-term and network-level decision making in IAM. However, it is realised that a comprehensive study of MOO in the context of decision making in IAM is lacking. Current research mainly focuses on the applications and improvement of individual techniques for research purposes; while a completed education of MOO techniques from a practical decision making point of view is hardly provided. Therefore, when dealing with a specific decision making problem in IAM, the applications of MOO may still be confusing.

This research attempts to improve the practical decision making in IAM by enhancing the understanding of MOO in this context. More specifically, this research introduces MOO and its philosophy from a decision making point of view and develops a robust MOO technique for practical long-term and network-level decision making in IAM.

Introducing MOO and its philosophy:

The understanding of MOO and its philosophy is critical when applying it in decision making in IAM. MOO has the great sophistication and can effectively assist decision making in IAM. It is important to clarify MOO, its techniques and its applications in the context of decision making in IAM. In this regard, some important questions have to be answered, including:

(1) What is the value of adopting MOO in decision making in IAM over and above the use of SOO?
(2) What are the applicable MOO techniques and what is the difference in their performance?
(3) How to apply MOO techniques to practical decision making problems?
(4) How to measure and assess MOO techniques in the context of decision making in IAM?
(5) How to correctly understand and use optimisation results?

Even though many researchers apply MOO techniques to their decision making process; they mainly discuss individual MOO techniques, which cannot provide an overall status and comprehensive knowledge of MOO for decision making in IAM. Many studies discuss small decision making problems or use created data. Their findings may not be applicable to large or practical decision making problems such as practical long-term and network-level decision making. Also, there is little discussion on the benchmark of the performance of MOO techniques when dealing with IAM decision making problems, so the measurement and comparison of different MOO techniques could be difficult and confusing. Finally, optimisation results are the mathematical expression of the possible decisions of IAM, which contain a wide range of information. Hence, properly and comprehensively understanding the optimisation results is very important so that MOO can be fully functioning.

Through this research, a comprehensive review of MOO in decision making in IAM is provided which helps to understand MOO, select MOO techniques and read optimisation results. It is the basis of the applications of MOO techniques and its findings are applicable to other decision making problems in IAM.

Developing a robust MOO technique for practical long-term and network-level decision making in IAM.

Practical long-term and network-level decision making in IAM is important but complicated. It involves a wide range of considerations and affects a variety of factors. An appropriate decision of long-term and network-level decision making can largely improve management efficiency and generates huge return on management investment. Thus, a suitable decision making method is necessary when dealing with long-term and network-level decision making.

MOO techniques have the potential to assist practical long-term and network-level decision making in IAM, but existing research is not sufficient in this regard. It is important to study this decision making and develop a robust MOO technique to assist it. The following questions have to be answered:

(1) What are necessary characteristics of a robust MOO technique for practical long-term and network-level decision making in IAM?
(2) What are the applicable MOO techniques for long-term and network-level decision making?
(3) What is the performance of existing MOO techniques?
(4) Which MOO technique is a robust technique for long-term and network-level decision making in IAM?

In this research, a robust MOO technique will be provided to assist practical long-term and network-level decision making in IAM. It is based on investigating existing MOO techniques. If an existing MOO technique can obtain satisfying optimisation results for long-term and network-level decision making in IAM, this technique will be recommended; otherwise, a new technique will be proposed based on the discussion of existing techniques. Typical tests based on practical long-term and network-level decision making are conducted to assess MOO techniques.

1.3 Research Objectives

Given the problem statement above, the aim of this research is to improve decision making in IAM by introducing MOO in the context of practical decision making in IAM and developing a robust MOO technique to assist practical long-term and network-level decision making in IAM. The introduced technique will analyse the MOO problems in decision making process and provide satisfying optimisation results in order to aid long-term and network-level maintenance and rehabilitations planning on the basis of multiple objectives.

Based on this aim, the objectives of this research are to:

(1) Enhance the knowledge of MOO in the context of decision making in IAM;
(2) Examine the existing MOO techniques that have been applied to help with decision making in IAM and recognise the research status quo of their applications;
(3) Investigate other MOO techniques that have not been applied to decision making in IAM but have application potential, therefore, extend the knowledge of MOO and provide more choices for decision making in IAM;
(4) Measure, compare and assess the existing MOO techniques in the context of decision making in IAM based on the typical tests of practical decision making problems and a prototype measurement framework;
(5) Introduce a robust MOO technique to handle multiple objectives, solve optimisation problems and provide satisfying optimisation results for practical long-term and network-level decision making in IAM; and
(6) Develop a communication tool to interpret optimisation results in an understandable and meaningful way, and enable decision makers to explore their management preferences and tailor the results.

1.4 Scope

This research attempts to enhance the understanding of MOO in the context of practical decision making in IAM and introduces a robust MOO technique for long-term and network-level decision making in IAM. The scope of this research is as follows:

- Long-term and network-level decision making is analysed and life-cycle strategies are set. As stated in Section 1.2, long-term and network-level decision making considers infrastructure assets in a holistic manner and creates large benefits. Therefore, this research mainly focuses on this type of decision making while its research results may also be employable to others.

- This research focuses on linear infrastructure assets such as roads and pipelines. These infrastructure assets are capital intensive and cover the most asset values, Hence they play a greatly important role and have a large potential to be improved (Vojnovic, 2000).
- This research involves various interventions including the treatments of maintenance, renewal and rehabilitation. This research focuses on analysing the interventions and does not review any mechanisms or treatment logic that generates the interventions.
- Different types of MOO techniques are covered. This research investigates and examines different types of MOO techniques to obtain a comprehensive study of existing MOO techniques. The traditional decision making methods and SOO techniques are also introduced in this research.

1.5 Structure of the Thesis

Figure 1.2 illustrates the structure of this thesis, where eight chapters are constructed to cover the research scope and achieve the research objectives. The following chapters are introduced below.

Chapter 2 introduces the background knowledge of decision making in IAM and the application status of optimisation through a comprehensive literature review. It explains IAM and its decision making, summarises main decision making methods, reviews the applied optimisation techniques and outlines the commonly analysed outcomes.

Chapter 3 presents the methodology undertaken to conduct this research. A framework is provided. The generic nature of this chapter facilitates the objectives of this research being achieved.

Chapter 4 provides a detailed knowledge of MOO and its techniques in the context of decision making in IAM. It introduces the concepts of optimisation from the viewpoint of decision making in IAM, outlines the significance of the applications of MOO in decision making in IAM and specifies the application process.

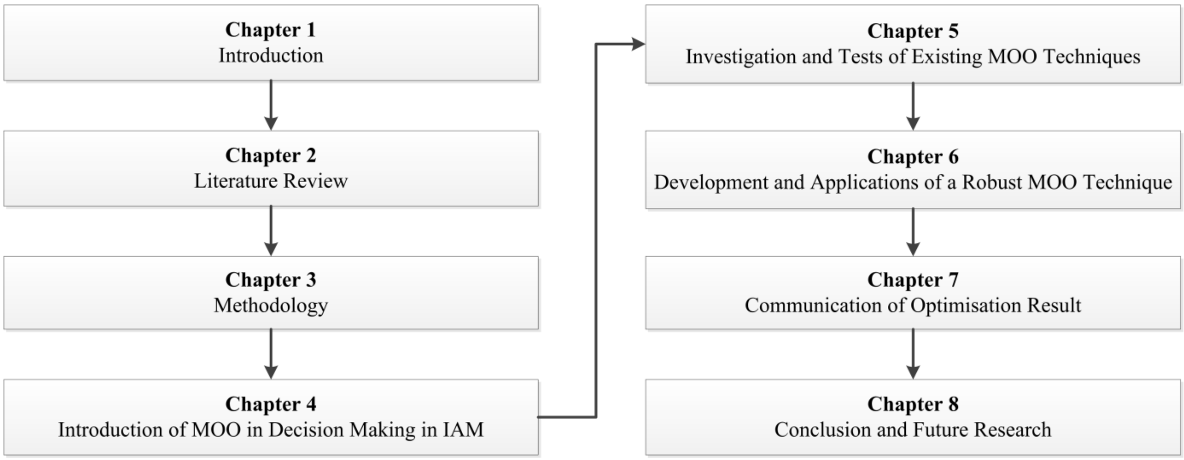


Figure 1.2 Research structure

In **Chapter 5**, existing MOO techniques are studied in the context of decision making in IAM. A list of MOO techniques including the applied ones and new potential ones are investigated and examined. Tests are conducted based on practical decision making problems, and a measurement framework is established to measure and compare MOO techniques when dealing with the tests. This chapter ends with an assessment of the existing MOO techniques and the necessity of a new MOO technique for long-term and network-level decision making in IAM.

Then according to the completed analysis, in **Chapter 6** a robust MOO technique is developed based on existing MOO techniques for practical long-term and network-level decision making in IAM. Tests are also conducted to compare the developed technique with other existing MOO techniques. Specific IAM questions are also discussed and answered with the developed MOO technique.

Chapter 7 develops a communication tool to help decision makers to understand the optimisation results. This tool presents the optimisation results in an understandable and meaningful way and enables decision makers to explore their management preferences and refine the result so that the management decisions can be made better and easier.

Chapter 8 concludes the main findings of this research, provides suggestions and discusses future work.

CHAPTER 2 LITERATURE REVIEW

This chapter provides comprehensive background knowledge of decision making in IAM and reveals the application status of optimisation techniques in decision making in IAM with the intention of guiding this research and future studies in this regard. This chapter reviews and discusses related literature in order to:

- Summarise the concepts of IAM and its decision making process;
- Outline the commonly used decision making methods;
- Review existing optimisation techniques and their applications in decision making in IAM; and
- Survey the main decision making outcomes analysed with optimisation in terms of objectives and constraints.

2.1 Concepts of Infrastructure Asset Management and Decision Making

2.1.1 Infrastructure Asset Management

As a terminology, IAM may sound new; but, according to Sharma (2010), it has a history of around three hundred years and has been employed by U.S. utilities for a long time. It does not have a unified definition. Organisations describe IAM in different words. For example, New Zealand Asset Management Support (NAMS) defines IAM as "the systematic and coordinated activities and practices of an organisation to optimally and sustainably deliver on its objectives through the cost-effective life-cycle management of assets" (NAMS, 2011). The Federal Highway Administration (FHWA), according to Sinha and Eslambolchi (2006), defines IAM as a business process that "incorporates the economic assessment of trade-offs among alternative investment options and uses this information to help make cost-effective investment decisions".

Even IAM is differently defined; it is widely admitted that IAM works on long-lived capital assets that are operated as a network and maintained at a particular level of service in order to deliver essential service to whole communities such as the road network of a city (NAMS, 1998; Sinha and Eslambolchi, 2006). The level of service is an important criterion in IAM, which measures the service quality for a particular activity (i.e. transporting) or service area (i.e. water delivery) against which service performance may be measured (NAMS, 1998). It is a set of standards used to describe the status of an infrastructure asset network in consultation with agencies and users (van Hofwegen, 1999). IAM is supposed to efficiently manage infrastructure assets so that the infrastructure assets are able to provide an acceptable level of service (Sinha and Eslambolchi, 2006; Uddin *et al.*, 2013).

It is also widely acknowledged that IAM is very important and described as a “contributor to profits” rather than “a necessary evil” by Sherwin (2000). Many reasons contribute to its importance (Mostert, 2008; NAMS, 2011; Uddin *et al.*, 2013), including:

- Infrastructure assets delivering essential services to the public are important for users, agencies and owners (Haarmeyer, 2011). Users require the services delivered by the assets; owners want to create profit from the assets, and agencies need to increase the return on the asset investment. IAM deals with all the parties and makes sure their goals and requirements are achieved.
- IAM enhances the understanding of infrastructure assets. During the IAM process, the infrastructure assets are examined and their future behaviour is estimated, which clarifies the performance of the infrastructure assets.
- IAM manages the risk of failure. Many infrastructure assets, such as water pipelines, play a critical role in daily lives. Once they fail, heavy losses are incurred on the economy, society, environment and culture (Mostert, 2008). IAM keeps the risk of failures at a low level, thus improves the safety and reliability of the assets (Hall *et al.*, 2006).
- IAM improves the sustainability of infrastructure assets. It attempts to better utilise the available resources (i.e. budget), improve the delivered services and reduce the impact of infrastructure assets on other aspects such as the environment, so that infrastructure assets are more sustainable.

- IAM increases the financial efficiency of infrastructure assets. It produces benefits by reducing the expenses of resources, extending the servicing life of infrastructure assets and delivering improved service (Berardi *et al.*, 2008). With appropriate management decisions, large benefits could be generated and the infrastructure assets can be more cost-efficient.
- IAM improves users' satisfaction. It is able to improve the level of service with limited resources so that the delivered service has better quality, stability and reliability. This also improves the community environment.

Despite its great importance, the situation of IAM will remain one of the challenges for future generations (Marlow *et al.*, 2010). The main reasons include:

Lack of investment: This is one of the biggest challenges of IAM. Infrastructure assets are highly capital intensive, from installation to maintenance and finally to replacement. However, adequate investment is not always made (Marlow *et al.*, 2010). Moreover, owing to economic growth, infrastructure assets are getting more expensive, which enlarges the gap between required and available investment.

Complex business environment: Because of the economic recession, investments become conservative and government budgets become more constrained (Haarmeyer, 2011). To increase the financial support, private investors are allowed in some cities (Too and Too, 2010). Yet private and local investors have different goals and requirements of IAM, which makes IAM more complex.

Growing demand: Owing to the growing population, more services are required to satisfy all users. Moreover, because of the improvement in living standards, users want better services. Hence, IAM needs to improve the infrastructure assets so that their service is enhanced in both quantity and quality. A survey (Haarmeyer, 2011) estimates that hundreds of billions of dollars are needed for managing and constructing public infrastructure assets to meet the growing and stricter demands in the next twenty years.

Ageing problem: This is a widespread issue in IAM (Marlow *et al.*, 2010). Fenner and Ainger (2014) point out in many cities infrastructure assets were built a long time ago and are reaching or even exceeding their designed servicing life. For example, in New Zealand a vast amount of roads were constructed during the 1940s to 1970s and are reaching the end of their service life (NAMS, 2011). In the next few years, a significant amount of roads may fail to provide required level of service without proper management.

Many considerations: Infrastructure assets have a wide range of interventions and affect a variety of factors and aspects (Lim *et al.*, 2010; Jaffe, 2011). IAM is required to consider all possible interventions and related outcomes in order to make appropriate management decisions. This process can be difficult, especially when intangible outcomes are analysed.

Lack of skilled managers and tools: A study reveals many organisations still manage their infrastructure assets based on asset managers' experience, yet the most asset managers do not have sufficient knowledge of IAM (Vanier and Rahman, 2004). Hence, the quality of IAM is limited. The lack of skilled managers and management tools increases the subjectivity and reduces the reliability of IAM.

2.1.2 Decision Making in Infrastructure Asset Management

Decision making is an essential part of IAM. It clarifies the goals and requirements of IAM, generates alternative strategies and makes management decisions for an infrastructure asset network. A management strategy indicates the types of interventions that are designed to be implemented to a segment of an infrastructure asset network during an analysis period. A management decision is a set of strategies selected for all segments of a network. When selecting different strategies, the outcomes of the management decisions are different. Decision making is required to select appropriate strategies so that the outcomes of the selected strategies satisfy the goals and requirements of IAM.

2.1.2.1 Decision Making in Organisations

IAM decision making is described in different ways. The International Infrastructure Management Manual (IIMM) suggests decision making is based on understanding and defining the goals and requirements of IAM and covers accountability, sustainability, risk management, service management and financial efficiency (NAMS, 2011). The United States Environmental Protection Agency (U.S. EPA) states decision making is to maximise the benefits based on the estimation of infrastructure asset deterioration, available funds and risk management (Sinha and Eslambolchi, 2006). The Maine Department of Transportation (MaineDOT) in the U.S. points out decision making should be a customer-focused process and considers safety, infrastructure condition and level of service (MaineDOT, 2012). The Water Infrastructure Network (WIN) in the U.S. recommends long-term, sustainable and reliable decision making that deals with the partnerships between governments and private sectors at different levels and considers a wide range of factors (Water Infrastructure Network, 2011).

2.1.2.2 Importance and Challenges of Decision Making

Even decision making has different descriptions; its importance in IAM is widely admitted. The main reasons are below (Sinha and Eslambolchi, 2006; Marlow *et al.*, 2009; NAMS, 2011):

- Decision making decides the outcomes of IAM. Decision making decides the implemented interventions and their implementation time by selecting strategies. After decision making, all the interventions are decided. Accordingly the outcomes, such as benefits and costs, which are calculated based on the interventions are also determined.
- Decision making helps in achieving the goals and requirements of IAM. Decision making attempts to select the appropriate strategies from alternative ones so that the outcomes of the selected strategies meet the requirements and achieve the goals of IAM.
- The risk is an important consideration in decision making. IAM is based on the predicted information; hence, the risk of failures exists and may cause heavy losses (Berardi *et al.*, 2008). Decision making can control the risk by reducing the exposure of the risk events and/or consequence, therefore improving the asset reliability (Lindhe *et al.*, 2011; Sitzenfrei *et al.*, 2011).
- Decision making helps to tackle the challenges of IAM. A variety of challenges of IAM is mentioned in Section 2.1.1. Decision making needs to generate and select appropriate strategies under these challenges and therefore supports IAM.

In summary, decision making is an essential part of IAM. Complex in its nature, decision making not only handles the challenges of IAM but also has its own difficulties:

- Data are scarce (McDonald and Zhao, 2001; Vanier, 2004; Elliott *et al.*, 2006). Decision making is a result of analysing data. Accurate data are critical and can be difficult to obtain. Firstly, the utilisation of the infrastructure assets may disturb the data collection and increase data noise. Secondly, some infrastructure assets are concealed, such as underground pipelines; hence their data collection is difficult. Finally, some cities began to collect data only decades ago; it is not sufficient for analysis.
- Decision making deals with a wide range of outcomes. Infrastructure assets are the basis of everyday lives, of which management impacts economic, environmental, social and cultural aspects (Maunsell Limited, 2004). The outcomes originating from these aspects

are often related, yet their relationships may be unclear or filled with uncertainties. Decision making needs to clarify these outcomes and their relationships.

- Decision making may analyse a large number of segments and strategies. IAM focuses on an infrastructure asset network that may be divided into thousands of segments. Accordingly a large number of strategies are likely to be generated for all the segments. Decision making is required to analyse alternative strategies and select strategies in a reasonable time.
- Decision making plans the future. Strategies are to guide future interventions and their implementation is filled with uncertainties. Decision making should take the uncertainties into account and make practical decisions.
- Various decision making methods are developed to assist decision making but none of them offers a panacea (Haarmeyer, 2011). Each method has its own characteristics. The selection of a proper method can be difficult without sufficient knowledge of the applicable methods.

2.1.2.3 Decision Making Process

The process of decision making in IAM means to make an appropriate management decision for an IAM project. Generally, it has four steps: clarifying a decision making problem, generating alternative strategies, measuring strategy outcomes and selecting strategies (Maunsell Limited, 2004; NAMS, 2011).

Clarifying a decision making problem: Decision making attempts to help to achieve the goals and requirements of IAM. Hence in the first step, these goals and requirements should be clearly defined and presented with the proper decision making criteria.

Generating alternative strategies: Strategies are generated by implementing different interventions at different points of time. Decision making needs to generate the alternative strategies that have the potential to result in good outcomes and achieve the goals and requirements of IAM. Normally a large number of strategies are generated to ensure all the potential strategies are covered.

Measuring strategy outcomes: The outcomes determine the priority of strategies. They should be carefully measured. Because IAM is related to many factors; a wide range of outcomes should be measured, including yearly outcomes (i.e. yearly maintenance cost) and overall outcomes (i.e. overall cost). Often these outcomes have different terms and cannot be simply combined, especially intangible and indirect outcomes (Robert, 2010).

Selecting strategies: After measuring the outcomes of each strategy, some strategies are selected to achieve the goals and requirements of IAM. However, the selection of strategies is not always easy because of the difficulties of decision making (see Section 2.1.2.2). Thus, decision making methods are developed to evaluate strategies and suggest the selections of strategies, therefore improving decision making process.

2.1.3 Summary

This section introduces IAM and its decision making process. IAM attempts to efficiently manage an infrastructure asset network, which is important and challenging. Decision making, as an essential and necessary part of IAM, determines management decisions by generating and selecting appropriate management strategies. It determines management outcomes and helps in achieving the goals and requirements of IAM. However, decision making is a complex process and has many difficulties.

Because of the importance and difficulties of decision making in IAM, methods are developed to assist decision making in IAM. Proper decision making methods can efficiently and effectively analyse a decision making problem and suggest reasonable selections of strategies. The next section outlines and discusses the common decision making methods.

2.2 Outline of Common Decision Making Methods

Because of the importance and difficulties of decision making, many decision making methods are developed to assist decision making in making management plans. This section provides an overview of common decision making methods and discusses their applications.

Decision making methods can evaluate the outcomes of alternative strategies, discuss their achievements, handle the challenges of decision making and suggest the selection of strategies for a decision making problem so that this decision making problem becomes easier and simpler. Then a decision maker can directly use a suggested selection of strategies or adjust the selections based on other considerations. Hence, decision making methods are very helpful for decision making in IAM.

Nowadays a variety of decision making methods are developed and each has different performance and characteristics. This section investigates the commonly used decision making methods and provides the application status of these methods.

2.2.1 Benefit-Cost Analysis

Benefit-Cost Analysis (BCA) "has been traditionally used as an accepted method of optimising a decision" (Maunsell Limited, 2004). It selects strategies by analysing their benefits and costs. Generally, benefits and costs are defined as positive and negative outcomes of strategies measured in monetary value. Then all strategies are measured by a benefit-cost criterion based on their benefits and costs. Table 2.1 shows some benefit-cost criteria of BCA (Shahin *et al.*, 1985; NAMS, 2011). One benefit-cost criterion is selected and computed, and the priority of alternative strategies is determined by their values on the selected criterion. Strategies with higher priority are selected for decision making.

Table 2.1 Summary of criteria in BCA

Criteria	Definition[1,2]	Measurement
Net Present Value (NPV)	$\mathrm{Benefit} - \mathrm{Cost}$	Financial profit of strategy
Benefit Cost Ratio (BCR)	$\frac{\mathrm{Benefit}}{\mathrm{Cost}}$	Financial return on investment Cost-efficiency
Incremental Benefit-Cost (IBC)	$\frac{\Delta\mathrm{Benefit}}{\Delta\mathrm{Cost}}$	Cost-efficiency comparing with the benchmark strategy Time value of cash flow
Internal Rate of Return (IRR)	Discount rate at which discounted benefit equals discounted cost	Desirability of investment strategy

[1] $\mathrm{Benefit}$ and Cost are net benefit and cost in present value

[2] $\Delta\mathrm{Benefit}$ and $\Delta\mathrm{Cost}$ are the increased benefit and cost comparing with a benchmark strategy in present value.

BCA is an easy and efficient method when evaluating the financial performance of strategies. After calculating the benefits and costs and selecting a benefit-cost criterion, strategies can be directly measured and easily compared according to this single measure. This method is straightforward and objective.

BCA is the most popular decision making method and has a long history (Prest and Turvey, 1965). According to Maass (1966), local governments in the U.S. officially applied BCA in the management of urban infrastructure assets such as highways in the 1960s. Mishan and Quah (1976) and Smith (1986) discuss the characteristics and applications of BCA. However, the classic BCA focuses on the financial benefits and costs. Some studies measure the benefits and costs of other aspects including the environment (Smith, 1984; Hanley and Spash, 1993; Pearce *et al.*, 2006), society (Feldstein, 1964; Baram, 1979) and others (Walton *et al.*, 2004; Damart and Roy, 2009) when applying BCA.

Other decision making methods are also proposed based on BCA. Cost effectiveness analysis (CEA) uses the cost effectiveness ratio (CER) to find the most cost efficient way of achieving the goals of IAM (Dodgson *et al.*, 2009; Lindhe *et al.*, 2011). It is able to analyse the intangible outcomes and risk. Economic Level of Service Assessment (ELSA) is another decision making method based on BCA (Smith, 2005), which focuses on the service provided by infrastructure assets. As shown in Figure 2.1, it measures user demand and infrastructure asset capability by analysing the gap between the curves of service provider cost and customer demand benefit. Strategies with a larger gap have a higher priority to be selected.

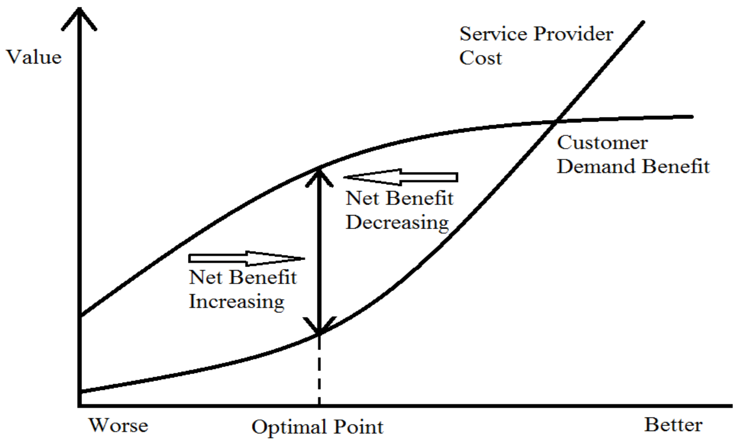


Figure 2.1 Illustration of ELSA (Smith, 2005)

BCA is not always effective. It requires measuring all outcomes as benefits and costs. It may be difficult to precisely quantify intangible outcomes against a predetermined monetary scale (Espeland and Stevens, 1998). Secondly, the classic BCA evaluates strategies using one benefit-cost criterion while in practice decision making is related to many factors. However, one benefit-cost criterion may not be enough to describe the priority of strategies. Thirdly, BCA requires all outcomes to be commensurable and their relationships to be clear, which is not true in practice.

2.2.2 Multi-Criteria Decision Making Methods

Multi-Criteria Analysis (MCA) is a classic decision making method that deals with multiple criteria (Belton and Stevart, 2003; Maunsell Limited, 2004; Mostert, 2008). When using MCA, a set of criteria is defined based on the goals and requirements of IAM to evaluate alternative strategies. It allows considering different outcomes and selecting strategies with respect to a decision maker’s preference.

MCA has four steps (Thompson *et al.*, 2008). Firstly, criteria are established to describe the requirements and goals of IAM. Next, weights are given to present the importance of these criteria. Then, strategies are measured by the criteria and scores are assigned indicating the achievement of network-wide strategies on each criterion. Finally, the scores on all criteria are weighted summed as the overall score of a strategy. Strategies with highest scores are selected.

MCA is able to handle multiple outcomes especially the outcomes that are difficult to be quantified (Gampera and Turcanuc, 2007). Compared with BCA, MCA does not necessary qualify criteria as financial benefits and costs and can analyse different types of outcomes, even the intangible or indirect ones (Espeland and Stevens, 1998).

Because of the strengths of MCA, it is recommended by the local governments in the U.S. (Joubert *et al.*, 1997), Europe (European Union, 2003) and Japan (Morisugi, 2000) and is applied to analyse the outcomes of the environment (Janssen, 2001), risk (Janssen, 2001; Fullera *et al.*, 2003), etc.

Another method based on MCA is Analytic Hierarchy Process (AHP) that is widely applied in decision making in IAM to analyse a series of decision making criteria. AHP compares criteria, explores the criterion structure and then defines a sound weight for the criteria. The priority of strategies is determined by weighting criteria and scoring strategies, and the high-priority strategies are selected. AHP is applied when managing different types of infrastructure assets, including pavements (Sikow *et al.*, 1993; Holguín-Veras, 1995; Santos *et al.*, 2009), bridges (Dabous and Alkass, 2010), pipelines (Morais and Almeida, 2010; Marlow *et al.*, 2012) and railways (Nyström and Söderholm, 2010). Other methods are also developed based on MCA. Triple bottom line (TBL) (Kenway *et al.*, 2007) measures strategies from -life-cycle perspective using the criteria of finance, society and the environment. A similar method named quadruple bottom line (QBL) adds a criterion of culture in decision making in IAM. According to Maunsell Limited (2004), QBL is often used to provide management guidelines and TBL is often used by private investors.

MCA and the methods based on MCA also have drawbacks (Belton and Stevart, 2003; Bäckstrand, 2011). Firstly, weights and scores may cause subjectivities and controversies; as they are determined by a decision maker. Secondly, a rational scoring of criteria is difficult. Criteria vary from case to case. The clear knowledge of the addressed decision making problem and sufficient experience of MCA is necessary when scoring the strategies on the criteria. Finally, the relationship of all criteria should be clear and can be correctly described by weights.

2.2.3 Multi-Attribute Utility Theory

Multi-Attribute Utility Theory (MAUT) is developed by Keeney and Raiffa (1976). It can also analyse multiple criteria in decision making in IAM. Different from MCA, MAUT defines additive utility functions based on decision making criteria and then assess strategies by possessing the subjective expected utility value (Dodgson *et al.*, 2009; Ishizaka and Nemery, 2013; Sarin, 2013).

Firstly, criteria, including descriptive and intangible criteria, are defined based on the goals and requirements of IAM. Then a utility function is defined to mathematically evaluate the performance of strategies on each criterion. Similarly with MCA, it also requires a given weight based on the importance of criteria. Finally, an overall utility is defined by weighted summing the values of all utility functions. Strategies with better overall utility are selected.

MAUT defines utility functions to measure the "degree of well-being" of strategies (Ishizaka and Nemery, 2013). It integrates different outcomes including uncertainties, intangible and indirect outcomes, as well as decision makers' preference. MAUT also breaks down a decision making problem and measures each criterion individually, therefore, simplifying complex decision making problems.

According to Dodgson *et al*. (2009), MAUT is "potentially demanding to apply". Many researchers apply MAUT in their decision making in IAM to analyse a wide range of quantitative and qualitative criteria (Torrance *et al.*, 1995; Zietsman *et al.*, 2006; Brito and Almeida, 2009). Based on MAUT, a method named UTA is proposed to evaluate the stability and sensitivity of strategies in decision making in IAM (Jacquet-Lagreze and Siskos, 1982). Figueira *et al*. (2009) presents a decision making method named Generalized Regression with Intensities of Preference (GRIP) based on MAUT to deal with incomplete preference information in decision making in IAM.

When applying MAUT, the definition of utility is critical and difficult. The utility functions are mathematical formulae that may not be able to describe some criteria and their relationships. Secondly, MAUT requires defining weights, which, similarly with MCA, may cause subjectivities and controversies. Finally, MAUT measures strategies with utility functions. Utility functions simplify decision making problems but some information of the strategies may be lost when defining utility functions.

2.2.4 Outranking methods

Outranking methods are a class of decision making methods that assess strategies using pairwise comparison (Vincke, 1999). According to Dodgson *et al.* (2009), the idea of outranking was proposed by Roy in mid-1960s and then extended by Belgian and Dutch researchers.

Outranking methods select strategies based on pair-wise comparison. For example, if enough evidence shows strategy A is at least as good as strategy B, then strategy A outranks strategy B. Strategies with higher outranking are selected.

To example outranking method, one of the best known ones, named Elimination Et Choice Translating Reality (ELECTRE), is introduced (Figueira *et al.*, 2005). ELECTRE starts at defining the criteria based on the goals and requirements of IAM. Weights are given based on the importance of these criteria. Then ELECTRE introduces a so-called concordance index and discordance index to measure the degree that a strategy outranks all others or is outranked by the others. Strategies with higher outranking status are recommended.

Outranking methods follow an interactive process, which involves decision makers' opinions. These methods leave the final choice to decision makers through fine-tuning in terms of outranking (Dodgson *et al.*, 2009). Tsamboulas *et al*. (1999) compares five decision making methods and points out outranking methods can analyse any number of criteria and deal with incomplete information and uncertainties for decision making in IAM.

Now outranking methods are applied to measure different levels of qualitative criteria (Van Delft and Nijkamp, 1977; Voogd, 1983), deal with uncertainties (D'Avignon and Vincke, 1988; Aouam *et al.*, 2003; Pires *et al.*, 2011) and improve decision makers' judgement (Tille and Dumont, 2003). Other decision making methods are also proposed based on outranking methods. Doumpos *et al*. (2009) hybridises an evolutionary approach and ELECTRE to improve the outranking classification. PROMETHEE is another outranking method that helps to explore decision makers' preferences (Brans and Vincke, 1985; Brans *et al.*, 1986).

Outranking methods can help with decision making in IAM, yet they are not efficient. They are based on the pair-comparisons of all strategies. Hence, decision makers need to have sufficient knowledge to correctly define the pairwise comparison for outranking. When the criteria are complicated, a proper definition of the pairwise comparison may be difficult. Outranking methods also require weighting criteria, which may be controversial.

2.2.5 Optimisation

Optimisation is a discipline of Operations Research that applies scientific methods to analyse decision making problems in IAM (Sherwin, 2000). It concerns "how to conduct and coordinate the operations" for a decision making problem and attempts to find the best solutions (Hillier and Lieberman, 2005). Optimisation has a variety of techniques that mathematically analyse a decision making problem and generate optimisation results so as to assist decision making in IAM.

Taking the advantage of mathematical analysis, optimisation has great strengths when assisting decision making in IAM.

- Optimisation is able to examine different outcomes and achieve the goals and requirements of IAM by formulating objectives and constraints;
- Optimisation can analyse a large number of segments and strategies and identify solutions in reasonable time;
- Optimisation has different types of variables and formulae to correctly describe practical decision making problems; and,
- Optimisation handles a decision making problem from a mathematical point of view; thus decision makers' preference is not necessary, which reduces the subjectivities and enhances the reliability of decision making process.
- Taking the advantage of cheap computing power, optimisation, when well implemented on a computer, is able to analyse large-size and complicated problems and obtain correct solutions in short time.

To date many optimisation techniques are developed and applied in decision making in IAM, especially when other decision making methods such as BCA and MCA cannot obtain satisfying results (Silva and Tatam, 1996). Their applications are discussed in the next section. Optimisation techniques have different principles, characteristics and performance. A suitable optimisation technique can effectively and efficiently solve decision making problems and provide good solutions. However, the selection of a suitable technique can be difficult because of the lack of sufficient knowledge of the existing techniques.

2.2.6 Discussion of Decision Making Methods

This section reviews the main decision making methods for decision making in IAM including BCA, MCA, MAUT, outranking methods and optimisation. These methods have their characteristics and performance while the only optimisation has the application potential to assist long-term and network-level decision making in IAM. The main reasons are:

Firstly, BCA only measures the criteria that are quantitated into benefits and costs. Long-term and network-level decision making has a wide range of criteria that are often conflicting or incommensurable. BCA cannot analyse these criteria, and therefore is not applicable.

Secondly, BCA, MCA, MAUT and outranking methods need large effort from decision makers. These methods require enumerating all the possible decisions of strategy selections and measure every decision. Long-term and network-level decision making problems may have thousands of segments and millions of strategies and therefore have numerous solutions. It may be impossible to enumerate all the decisions; hence, these methods are not applicable.

Thirdly, MCA, MAUT and outranking methods require weighting, which may involve subjectivities. The weight is given by a decision maker. When decision makers do not have sufficient knowledge or they have different opinions on the weighting system, the solving process may lead to a controversial result.

Optimisation, different from the other decision making methods, can analyse a large number of segments and strategies, and generate a good solution or a set of solutions for a decision making problem in IAM. Decision makers can save time and energy by only focusing on the optimised solution(s) rather than enumerating and measuring all possibilities. Optimisation can be implemented on a computer and then speed up the decision making speed. Some optimisation techniques can deal with many criteria even conflicting and incommensurable ones; where weights are not necessary. Hence, the subjectivities when giving weights are avoided. Hence, optimisation is the most suitable decision making method for long-term and network-level decision making in IAM.

Nowadays a variety of optimisation techniques has been developed and is applicable to assist decision making in IAM. To improve and guide their applications, the knowledge of current applications in decision making in IAM is important. Hence, in the next section, the applications of optimisation and its techniques are reviewed and summarised in the context of decision making in IAM.

2.3 Review of the Applications of Optimisation in Decision Making in Infrastructure Asset Management

According to the analysis completed, optimisation can greatly assist decision making in IAM including long-term and network-level decision making. Hence, it gets increasing attention in decision making in IAM. However, its applications still can be difficult without sufficient knowledge of existing optimisation techniques and their applications in decision making in IAM. This section attempts to enhance this knowledge through a comprehensive literature review in this regard.

In this research, a total of 302 publications including journal papers, conference proceedings, theses, book chapters, etc. are reviewed and summarised. These publications are presented in Appendix A. Through an analysis of these publications; this section discusses optimisation techniques applied in decision making in IAM.

2.3.1 Introduction of Types of Optimisation

According to Appendix A, a large body of publications applies optimisation techniques in decision making in IAM. These techniques can be divided into two types: Single-Objective Optimisation (SOO) techniques and Multi-Objective Optimisation (MOO) techniques. Figure 2.2 illustrates these techniques and their targeted solution(s).

Single-Objective Optimisation techniques: SOO techniques analyse optimisation problems with only one objective and attempt to obtain the optimal solution that satisfies all the constraints and achieves the best objective value. In decision making in IAM, the most important goal of a decision making problem is often defined as the objective, and other goals and requirements are defined as constraints. Then a SOO technique is applied to solve this problem and identify the optimal solution. In the context of IAM, an optimal solution

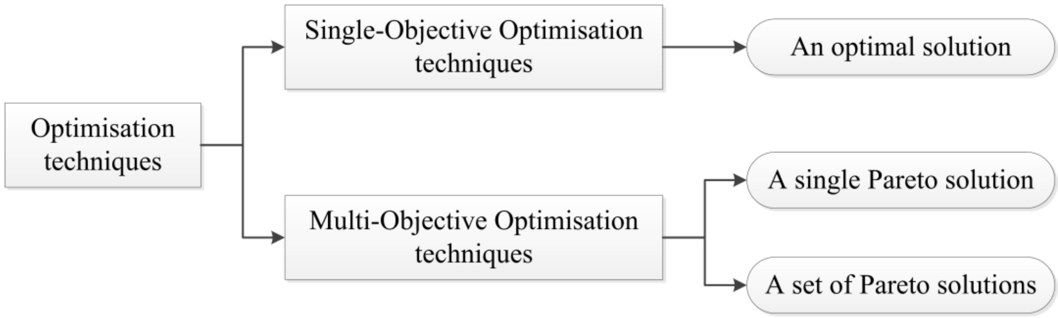


Figure 2.2 Types of optimisation techniques and their targeted solution(s)

corresponds to a selection of strategies that the outcomes of the selected strategies satisfy all constraints and generate the best possible objective value. SOO techniques are often applied when a decision making problem only has one main goal or its goals can be integrated. Compared to MOO techniques, their optimisation process is relatively easy. However, if a decision making problem has conflicting or incommensurable goals, SOO techniques are not applicable and a MOO technique is needed.

Multi-Objective Optimisation techniques: MOO techniques are able to simultaneously optimise multiple objectives and attempt to obtain a single Pareto solution or a set of Pareto solutions. Pareto solutions are the best feasible solutions, of which the constraints are satisfied, and no objective value of a solution can be improved without worsening at least one other objective value. In the context of decision making in IAM, the goals can be expressed as individual objectives and the requirements can be expressed as constraints. MOO techniques are able to optimise all the objectives even when they are conflicting or incommensurable, and then identify the Pareto solution(s), each corresponding to a selection of strategies whose outcomes satisfy all constraints and achieve the objectives in the best possible manner. They are applied when decision making has incommensurable goals or requires trade-offs of conflicting outcomes. According to Ge (2010), MOO describes decision making problems more rationally and practically.

Often a MOO problem has more than one Pareto solution. Some MOO techniques only generate one Pareto solution based on the interaction with decision makers, while others can obtain a set of Pareto solutions or even all existing Pareto solutions.

2.3.2 Application Status of Optimisation in Decision Making in Infrastructure Asset Management

This section attempts to specify the previous applications of optimisation in decision making in IAM and therefore guides further applications and research. More specifically, the previous applications are summarised and discussed based on a list of 302 publications in this regard (see Appendix A) and the main findings are presented in the following paragraphs.

Application history of optimisation techniques: The applications of optimisation techniques in decision making in IAM can be traced back to 1964 (Magee, 1964), and became popular in recent decades. Figure 2.3 shows the number of the publications published by decades. After 1990, the publications on the optimisation applications increase dramatically. During 2000-2009, a number of 182 publications are found to apply optimisation techniques in decision making in IAM.

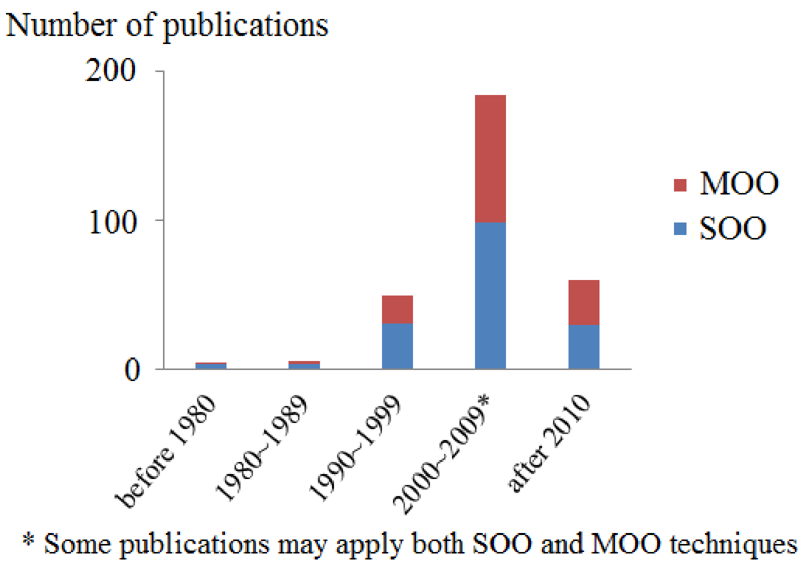


Figure 2.3 Number of publications applying optimisation techniques in decision making in IAM

Figure 2.3 also includes the applications of SOO and MOO. Before 2009, SOO was the dominating optimisation in decision making in IAM, because its optimisation procedure was relatively easier. With the development of decision making, the needs of balancing multiple objectives were increasing. Therefore researchers begin to apply MOO techniques to assist their decision making in IAM. After 2010, of the 60 publications examined, half discuss the applications of MOO in decision making in IAM.

Targeted result when applying MOO techniques: MOO techniques may obtain one single solution or a set of solutions depending on their algorithms. When applying MOO in decision making in IAM, most publications only generate a single solution; while the publications aiming at a set of solutions are increasing. Figure 2.4 shows the numbers of publications that aim at one single Pareto solution or a set of Pareto solutions. Because obtaining one Pareto solution is easier than obtaining a set of Pareto solutions, MOO techniques identifying one solution are the dominating choices in current decision making in IAM. Beginning from 1997, Halhal *et al*. (1997) successfully identifies a set of solutions using Genetic Algorithm (GA) and uses these solutions to clarify the relationship between outcomes. Then increasing numbers of publications try to generate a set of Pareto solutions with MOO techniques especially in the last decade. With the development of MOO, more techniques will be proposed to obtain a set of solutions for decision making in IAM.

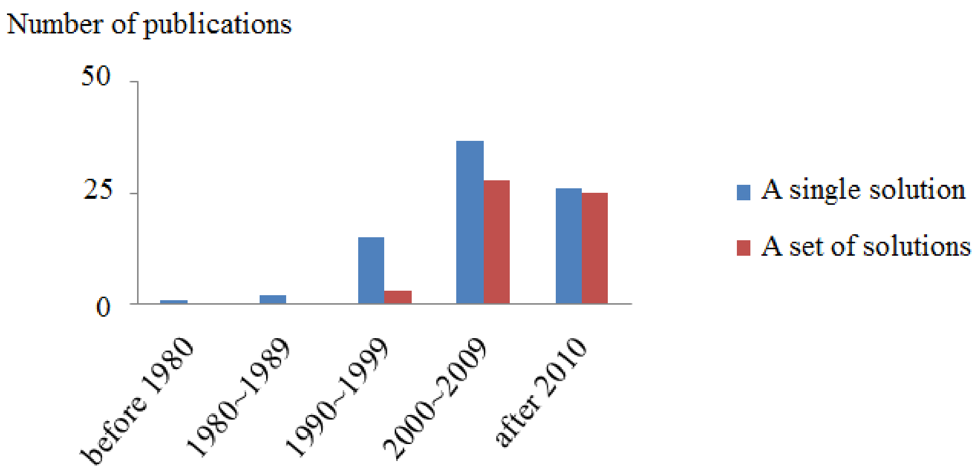


Figure 2.4 Number of publications applying MOO in decision making in IAM

Number of optimised objectives: Figure 2.5 presents the numbers of objectives optimised in the reviewed publications. Over half of them only had one objective, which certifies the current dominance of SOO in decision making in IAM. In terms of MOO applications, most publications define 2-3 objectives for their decision making problems. Furthermore, with the increase of the objective numbers, research tends to generate one single solution rather than a

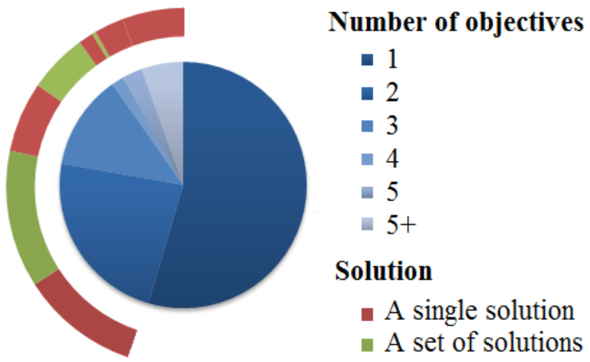


Figure 2.5 Number of the optimised objectives with optimisation in decision making in IAM

set of solutions, especially when four or more objectives are optimised. The main reasons behind this are (1) when more objectives are optimised, the identification of a Pareto solution set becomes more difficult; and (2) even if a set of solutions is obtained, understanding these solutions can be difficult as each solution corresponds to several objectives.

Project- and network- level decision making: Decision making in IAM has two levels: project- and network- level. In project-level decision making, an infrastructure asset network is regarded as a whole and each strategy is a management decision for the whole network. This decision making only needs to select one strategy that satisfies the goals and requirements of IAM. It is relatively simple. However, project-level decision making requires generating strategies for the whole network. This can be rigid and difficult because each network segment may have different management interventions and objectives and constraints may be applicable to the whole network or sometimes for individual segments.

In network-level decision making, segments of an infrastructure asset network are individually treated, each with its own strategies. Then decision making selects one strategy for each segment so that the overall outcomes of the selected strategies can satisfy the goals and requirements of decision making for the whole network. It is more flexible and practical and allows examining individual segments. Network-level decision making needs to decide the best combination of strategies for the segments of a network. It is more complicated than project-level decision making, and cannot be easily handled.

The review suggests that with the help of optimisation techniques, network-level decision making is successfully handled as well as project-level decision making. Figure 2.6 shows the number of publications that analyse project- and network- level decision making using optimisation techniques. With the increase in the applications of optimisation, after 1980 over half of the publications analyse network-level decision making with the optimisation techniques.

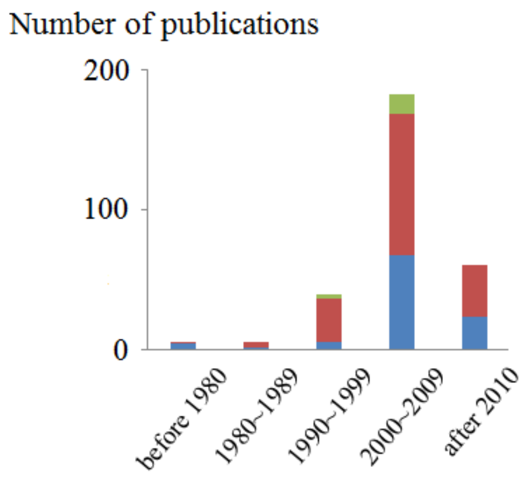


Figure 2.6 Number of publications dealing with project- and network-level decision making

Types of infrastructure assets: IAM may manage different types of infrastructure assets. According to the reviewed publications, optimisation techniques have been applied to help to manage various types of infrastructure assets. Figure 2.7 shows the types of infrastructure assets analysed with optimisation in their decision making problems. Almost half of the publications apply optimisation techniques in decision making in pavement management. 21.2% and 9.9% of publications apply optimisation techniques in decision making in bridge and pipe management. Moreover, 20.2% of the publications focus on the general infrastructure assets and attempt to find a general method for decision making in IAM.

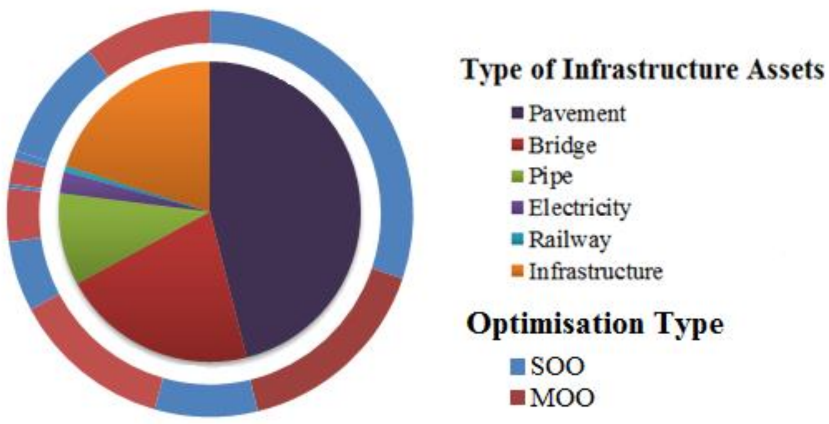


Figure 2.7 Publications managing different types of infrastructure assets using optimisation

Figure 2.7 also presents the types of optimisation applied when managing different types of infrastructure assets. In pavement management, over half of the publications discuss applications of SOO, especially during 1990-2009. On the contrary, the publications in bridge management mainly discuss the applications of MOO. One reason is that pavement management has a longer history than the others and in the early decades SOO was the dominating optimisation approach. Because of the development of IAM and MOO, the management of other types of infrastructure asset were studied and MOO techniques began to be used in these IAM projects.

2.3.3 Applied Optimisation Techniques in Decision Making in Infrastructure Asset Management

According to Appendix A, a variety of optimisation techniques have been applied in decision making in IAM. Figure 2.8 summarises the applied optimisation techniques. For both SOO and MOO, there are three classes of optimisation techniques: exact methods, heuristics and other methods and software tools.

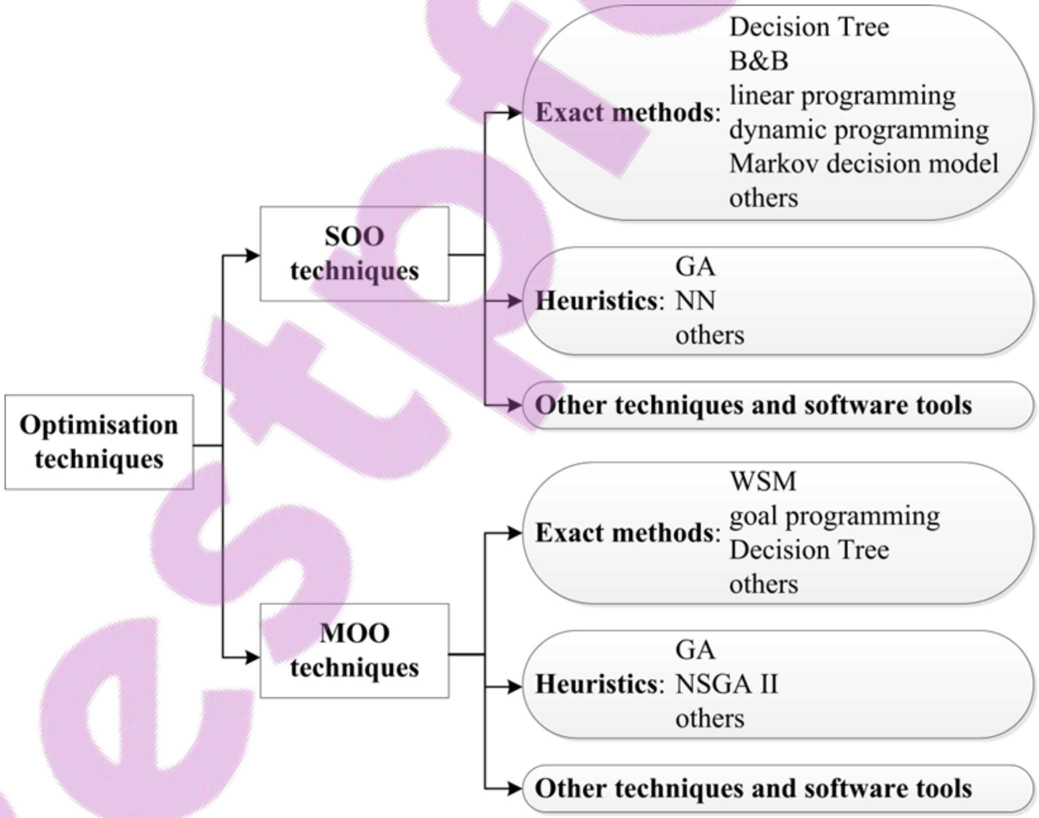


Figure 2.8 Structure of optimisation techniques applied in decision making in IAM

Exact methods are described by Moteleb (2010) as the "empirical or mechanic" algorithms that solve optimisation problems based on the mathematical theorems and corollaries. They follow specific computing algorithms and generate guaranteed solutions for decision making problems in IAM.

Heuristics are described by Silver *et al.* (1980) as "intuitive approaches", where an optimisation problem is "interpreted and exploited intelligently to obtain reasonable solution[s]". Often, heuristics arise from the idea of relatively simple common sense, and iteratively reproduce solutions based on identified ones (Yaseen and Al-Slamy, 2008). Heuristics are normally developed as a general-purpose algorithmic framework, and their algorithms can be adjusted to suit the requirements of different optimisation problems.

Other techniques and software tools are proposed by improving existing techniques, hybridising different techniques methods or others. They are also applied to assist decision making in IAM.

2.3.3.1 Single-Objective Optimisation Techniques in Decision Making in Infrastructure Asset Management

According to the literature review, SOO techniques from all the three technique classes are applied in decision making in IAM and exact methods are the most popular choices. Figure 2.9 specifies the applied SOO techniques. Almost half of the reviewed publications use exact methods while only 24. 1% and 39.7% of the publications use heuristics and other methods and software tools. This is mainly because SOO is relatively easy and many exact methods are applicable and generate good solutions.

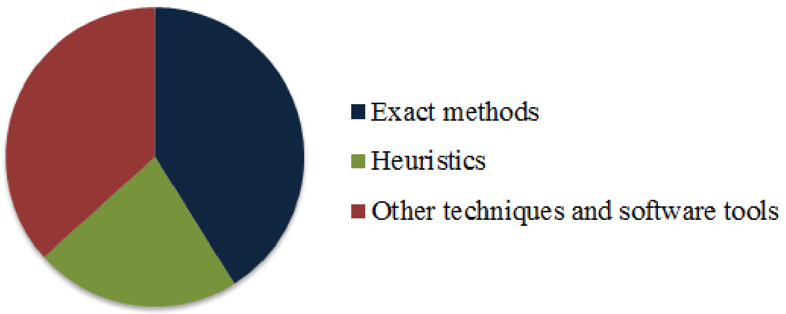


Figure 2.9 SOO techniques applied by the reviewed publications

Exact methods

Decision Tree is the most frequently used exact method in decision making in IAM. It uses a tree-like model to enumerate all the possible solutions and measures their outcomes individually to obtain the best one. Many publications use Decision Tree to find the least-cost management decision (Frangopol *et al.*, 1997; Kong and Frangopol, 2003; Kong and Frangopol, 2005). In addition, each node of the model can grow and split again until all possibilities are explored. In this way, complicated problems can be analysed by extending the model (Bonyuet *et al.*, 2002; Anastasopoulos *et al.*, 2014).

Brach and Bound (B&B) is also a common choice of SOO exact methods. It is an effective method for discrete optimisation problems, which explores possible optimal solutions by defining the bounds of discrete variables (Clausen, 1999). Compared to Decision Tree, B&B is more efficient when solving optimisation problems of decision making as bounding can significantly reduce the space to be explored and improve the efficiency. Hence, it is applied to handle decision making problems with a large number of segments and strategies in IAM (Ferreira *et al.*, 2002; Osorio *et al.*, 2008; Scheinberg and Anastasopoulos, 2010).

Linear programming is an effective method to solve optimisation problems with linear formulae. It optimises a linear function subject to linear equality and inequality constraints (Todd, 2002). It has been applied to assist decision making in IAM since 1975 (Terrell) and was popular in the early years (Way, 1985). It is also incorporated in some decision making tools (Liu and Wang, 1996; Wang and Zaniewski, 1996) or directly applied to practical decision making problems (Babani, 2007).

Dynamic programming decomposes an optimisation problem into a set of simpler sub-problems, then solves the sub-problems and combines their solutions. It can solve the hard optimisation problems. Hence, dynamic programming is applied to help with complex decision making problems, especially when outcome relationships are non-linear or fuzzy (Augusti *et al.*, 1994; Augusti *et al.*, 1998; Kleiner *et al.*, 2001).

Markov decision model is an applicable simulation based method, which simulates the condition of infrastructure assets under uncertainties. More specifically, condition probabilities are explored and expressed using transition probability matrices. Strategies are generated and selected based on the condition probabilities (Guignier and Madanat, 1999; Orcesi and Cremona, 2009).

Other exact methods are also applied in decision making in IAM. They are developed based on different theories and follow different algorithms. For example, Yoo (2004) hybridises dynamic programming and B&B to analyse a series of factors when selecting maintenance strategies for pavement networks. Li and Madanu (2009) develop an uncertainty-based method with Lagrangian relaxation to analyse the life-cycle benefits,costs and risks in highway management. Gao *et al*. (2013) introduces an augmented Lagrangian decomposition approach for infrastructure maintenance and rehabilitation decisions. Other applied methods are shown in Appendix A. These methods are developed to solve a specific decision making problem, and may be not applicable to solve other optimisation problems such as non-linear problems.

Heuristics:

Genetic Algorithm (GA) is a heuristic that generates new solutions based on the identified solutions. It reconstructs well identified solutions to generate new ones. It has been widely applied in decision making in IAM (Itoh *et al.*, 1997; Maji and Jha, 2007; Farran and Zayed, 2012). New algorithms are also proposed based on GA for specific decision making problems such as a stepwise GA for large-scale decision making (Kim *et al.*, 2005), a simulation-based GA to manage the uncertainties in decision making process (Chootinan *et al.*, 2006) and pre-constrained GA for network-level decision making (Tack and Chou, 2002).

Neural Network (NN) is inspired by the structure of human neural networks (McCulloch and Pitts, 1943). It establishes a network model to produce solutions and iteratively improve this model until a satisfying solution is obtained (Yang *et al.*, 2003). To date, NN is applied to many decision making problems in IAM (Flintsch *et al.*, 1996; Razaqpur *et al.*, 1996). New methods are also proposed based on NN for specific problems, such as a Neural Dynamic Model to analyse a series of condition requirements in decision making in IAM (Sirca Jr and Adeli, 2005). However, when applying NN, a large amount of data is needed to improve the model accuracy.

Other heuristics**:** Because of their prosperity, a large body of SOO heuristics is developed and applied to assist decision making in IAM, including Shuffled Complex Evolution Procedure (Nunoo and Mrawira, 2004), Kalman filter algorithm (Durango-Cohen and Tadepalli, 2006), Max-Min Ant System (MMAS) (Lukas and Borrmann, 2011), Simulation Algorithm (Zhang and Wang, 2014), etc. These applications have their pros and cons.

Other methods and software tools:

Other methods and software tools are also applied to solve SOO problems in decision making in IAM. For example, Sirajuddin (1997) develops a tabulated manual procedure to facilitate the allocations of limited funding. Abaza (2007) develops a global network optimisation model to allow modifying and updating of the pavement condition. Software tools, including CA4PRS (Lee *et al.*, 2005), dTIMS (Deighton Associates Limited, 2008), HDM-IV (Archondo-Callao, 2008; Jorge and Ferreira, 2012) and OPTIPAV system (Santos and Ferreira, 2013), are also developed to help with decision making in IAM. More methods are the software tools listed in Appendix A.

2.3.3.2 MOO Techniques in Decision Making in Infrastructure Asset Management

Figure 2.10 shows the applied MOO techniques in the reviewed publications to assist decision making in IAM. Different from SOO, heuristics become the most popular choices, and a number of publications apply exact methods to solve MOO problems in decision making in IAM. One reason for this is that MOO problems are more complicated and heuristics are flexible and can be easily applied to solve MOO problems.

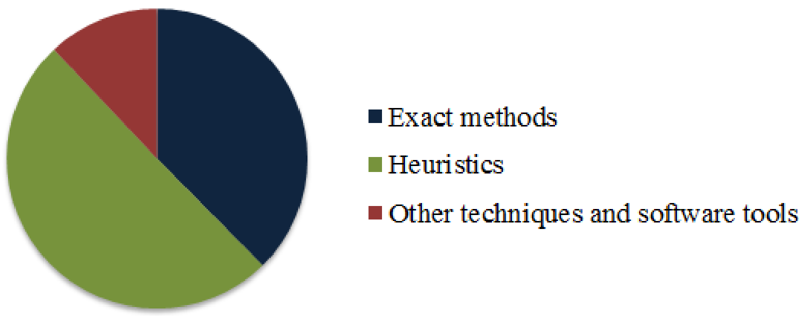


Figure 2.10 MOO techniques applied in the reviewed publications

Exact methods:

Decision Tree is the most frequently applied exact method that searches for all the existing Pareto solutions for a MOO problem. Same as its SOO counterpart, it enumerates all the possibilities using a tree-like model. They are applied to handle uncertainties (Frangopol *et al.*, 1997; Barone *et al.*, 2013) or improve asset condition (Hicks *et al.*, 1999; Amador-Jiménez and Afghari, 2013) in different decision making in IAM.

Weighted Sum Method (WSM) is one of the most popular exact methods for MOO in decision making in IAM. It transforms a MOO problem into a SOO sub-problem or a set of SOO sub-problems. It defines a new objective by weighting summing the original objectives with a given weight and then optimised the new objective. When a preferred weight is given, it identifies one Pareto solution with the given weight (BECA Asset Management Services, 1999; Auckland (N.Z.). Transport Planning, 2000; Transit New Zealand, 2000). When a set of weights are defined, a set of SOO sub-problems is established; therefore a set of Pareto solutions can be obtained to help with the trade-offs of objectives (Gabriel *et al.*, 2006; Wu and Flintsch, 2009; Han *et al.*, 2012).

Goal programming deals with a MOO problem by pursuing a goal point. The goal point contains the best value of every objective and may be not achievable. Goal programming searches for a feasible solution that is the closest to the goal point. Goal programming can easily handle a wide range of objectives and obtain a preferred Pareto solution for decision making in IAM (Stewart, 1992; Ravirala and Grivas, 1995; Ravirala *et al.*, 1996)). It is simple and effective when optimising multiple objectives. However, it only obtains one solution based on the definition of the goal point.

Other exact methods are also applied to achieve specific requirements in decision making in IAM, such as a dominance-based rough set approach to allocate budget in highway management (Augeri *et al.*, 2011) and a parametric method to efficiently use budgets in pavement management (Gao *et al.*, 2010). These methods follow specific algorithms, and may perform poorly when solving other decision making problems.

Heuristics:

GA is the most frequently applied MOO techniques in decision making in IAM. When solving MOO, its algorithm is similar to its SOO counterpart, but it has different fitness definition that enables to handle multiple objectives. Some of them can generate a solution based on a specific preference or requirements (Fwa *et al.*, 1996; Lee and Kim, 2007; Sharma, 2010), while some are able to generate a set of solutions to help with objective trade-offs (Dogaki *et al.*, 2001; Lee and Kim, 2007; Marzouk and Omar, 2013).

Nondominated Sorting Genetic Algorithm II (NSGA II) is based on GA; while it has a superior fitness measurement (Deb *et al.*, 2002). According to Bai *et al*. (2012), NSGA II can effectively solve the optimisation problems and help with the trade-offs of objectives in decision making in IAM. Therefore, it is applied in different decision making in IAM (Sharma *et al.*, 2009; Lee *et al.*, 2011; Orcesi and Frangopol, 2011).

Other heuristics are applicable in decision making in IAM. Because of the effectiveness of GA, many heuristics are proposed based on GA, including a structured messy GA for water pipeline management (Halhal *et al.*, 1997; Halhal *et al.*, 1999), a constraint-based GA for highway management (Herabat and Tangphaisankun, 2005), and a step-wise integrated GA for pipeline management (Halfawy *et al.*, 2008). However, only one of them obtains a set of solutions (Hugo *et al.*, 2005) and others only generate one solution in their decision making in IAM.

Other methods and software tools:

Other methods and software tools are applied to solve MOO problems in decision making in IAM. However, their applications are largely reduced compared to their applications in SOO due to the difficulties of MOO. Only 2 out of 17 publications use hybrid methods to obtain a set of solutions (Pelet *et al.*, 2005; Medury and Madanat, 2012), while others only generate one solution to their decision making problems.

2.3.4 Discussion of Optimisation Techniques

Section 2.3 reviews the applications of optimisation in decision making in IAM. According to Appendix A, a variety of optimisation techniques are applied to assist decision making in IAM, including SOO techniques and MOO techniques.

SOO techniques measure solutions with only one objective and aim at an optimal solution for decision making in IAM. They are relatively simple and can solve different decision making problems. However, SOO techniques only optimise one objective. When multiple objectives exist, especially conflicting or incommensurable objectives, SOO techniques are not applicable and a MOO technique is needed.

MOO techniques can handle multiple objectives and aim at identifying Pareto solutions. Some techniques are able to obtain a single Pareto solution based on the management preference. Some techniques are able to obtain a set of Pareto solutions to help with the analysis of trade-offs of objectives. However, compared to SOO, MOO problems are more difficult and fewer techniques are applicable to produce satisfying optimisation results especially for difficult decision making problems such as long-term and network-level decision making in IAM.

2.4 Survey of Decision Making Outcomes in Terms of Objectives and Constraints When Applying Optimisation

This section enhances the knowledge of the optimisation applications by surveying decision making outcomes analysed using optimisation in decision making in IAM. These outcomes are considered in terms of objectives or constraints.

Objectives and constraints are an important part of optimisation, which describes a decision making problem in a mathematical way (Department of Finance and Personnel, 2012). Objectives are indispensable to optimisation. They focus on functioning and planning of a problem and provide the main evaluation for available solutions. Often they are defined to express the main goals of IAM in decision making. Constraints are an important part of optimisation. They describe the restrictions of a problem based on situations, screening and guidelines. Often they are defined to describe requirements, relationships between outcomes and other considerations in decision making in IAM. Both objectives and constraints can be used to achieve specific outcomes in decision making in IAM, while their main concern is different.

This section investigates the main decision making outcomes and their corresponding objectives and constraints through a survey of the publications in Appendix A. Figure 2.11 summarises these outcomes in terms of objectives and constraints. According to this figure, various outcomes are analysed in decision making in IAM by defining objectives and constraints.

2.4.1 Financial Costs

The financial costs are the most important outcome in decision making in IAM. Around 89% of the reviewed publications consider the financial costs in their decision making in IAM.

There are three main types of financial costs: agency cost, user cost and total cost. Agency cost is incurred by the activities and practices of agencies when managing infrastructure assets. It is heavily affected by the implemented strategies. User cost is the expense incurred when using infrastructure assets. It is affected by the condition of infrastructure assets. Total cost, also named life-cycle cost, is the overall cost during the service life of infrastructure assets.

All of these costs can be analysed by defining objectives or constraints in decision making in IAM. Often when decision making tries to reduce its financial cost and improve its financial

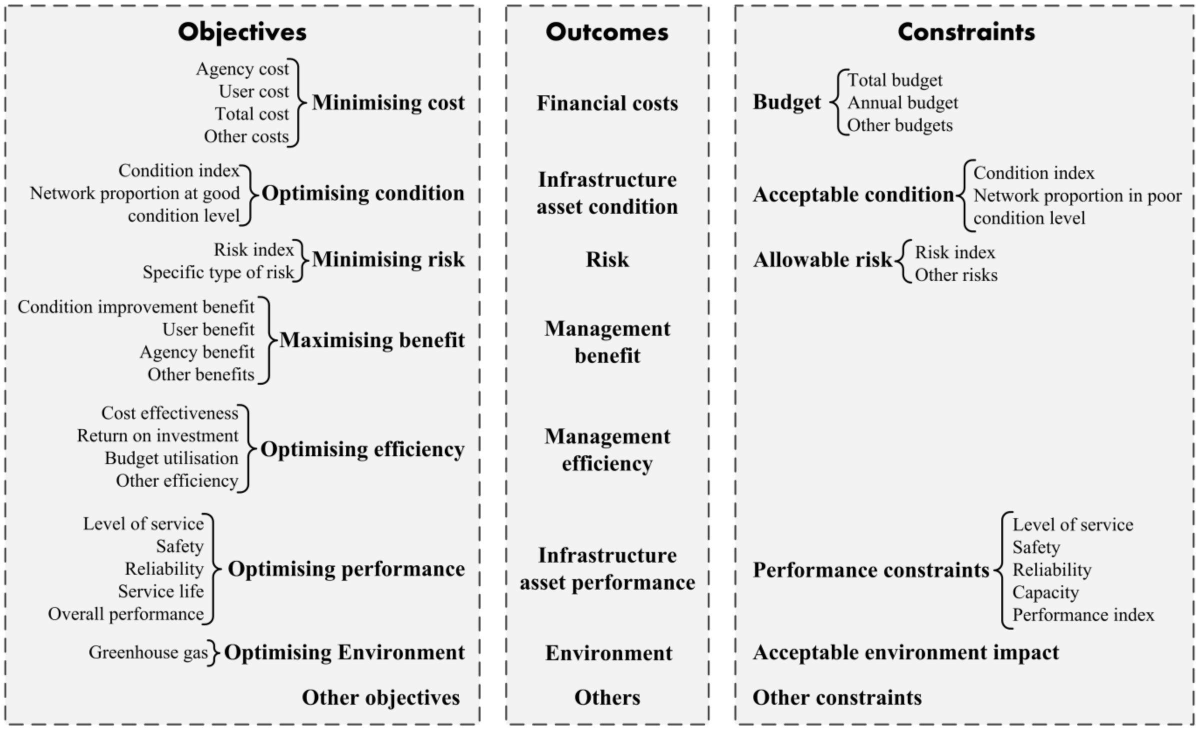


Figure 2.11 Summary of outcomes in terms of objectives and constraints

efficiency, an objective of minimising cost should be defined; while when decision making tries to best use a given funding, a constraint of the budget should be defined.

Objective of minimising cost:

This objective directly measures the financial efficiency of management decisions. All the three types of financial costs can be minimised in decision making in IAM.

The objective of minimising agency cost improves the cost-efficiency of the management of infrastructure assets. It is the most commonly used objective in decision making (Meneses and Ferreira, 2010, 2013; Khan *et al.*, 2014). In particular, some decision making problems may only minimise a part of the agency cost, such as management cost (Jacobs, 1992; Ozbek *et al.*, 2010; Famurewa *et al.*, 2015) and risk cost (Orcesi and Frangopol, 2011; 2011).

The objective of minimising user cost measures the cost-efficiency when using infrastructure assets and improves users' satisfaction. It is an important objective especially for user-oriented decision making in IAM (Bonyuet *et al.*, 2002; Liu and Madanat, 2014).

Different from the others, the objective of minimising total cost measures the overall cost-efficiency of IAM. It is commonly defined in long-term decision making in IAM (Ouyang, 2007; Anastasopoulos, 2009; Osman, 2015).

Other types of costs may be defined and minimised for specific decision making problems, such as intangible costs (Šelih *et al.*, 2008; Rogers and Grigg, 2009) and indirect costs (Kim, 2005).

Constraint of budget:

Budget is the most common constraint in decision making in IAM. Almost half of the reviewed publications have budget constraints. Different from the objectives of minimising different costs, budget constraints are normally based on agency costs. They control the financial expense during the management and help rationally using the available funding. There are two types of budget constraints: total budget and annual budget.

Total budget controls the overall cost of an IAM project. It helps to distribute interventions through the entire analysis period. Agency costs during the analysis period are discounted into present value and are restricted to the available funding (Frangopol and Neves, 2004; Tsunokawa *et al.*, 2006; Yang *et al.*, 2014).

Annual budget controls the agency costs in every year. It helps to reasonably distribute interventions through the analysis period by controlling the funding for annual management of Infrastructure assets (Abaza *et al.*, 2004; Nafi and Kleiner, 2009; Chen *et al.*, 2015). It is more common than the total budget in decision making in IAM

Other budgets are also defined for decision making in IAM such as a budget of user cost (Ferreira *et al.*, 2002) and a budget of intangible costs (Kang *et al.*, 2012). Gao *et al*. (2013) defines an uncertain budget to improve the investment utilisation for their decision making problem.

2.4.2 Infrastructure Asset Condition

The rutting and roughness in pavement management, are measured; and the condition index is defined by integrating these criteria. It is an easy way to numerically describe the condition of both individual segments and the entire network. Condition can also be defined as different levels. The condition of each segment is associated with a condition level and the network condition is described by the proportion of the network in different condition levels.

Condition can be analysed by defining objectives or constraints in decision making in IAM. When decision making tries to obtain the best infrastructure assets, a condition objective should be defined; and when decision making tries to keep the condition of infrastructure assets, a condition constraint should be defined. The author notices that in many decision making problems, asset condition and financial cost are considered together, where one of them is defined as an objective and the other is defined as a constraint.

Objective of optimising condition:

Condition objectives try to improve the infrastructure asset condition with the available resources. They are defined based on the condition measurement. When measuring condition using a condition index, the condition objective is defined as maximising or minimising the condition index based on the quantification system. This condition objective can optimise the condition index of a specific infrastructure such as a road (Shekharan, 2000) or a bridge (Lee *et al.*, 2011), or the average condition index of a network (Lounis, 2005; Abaza and Ashur, 2009; Liu and Madanat, 2014). In some decision making problems, instead of a composite index, only a specific condition criterion is used to measure the asset condition and is directly optimised (Yang *et al.*, 2003; Farhan and Fwa, 2009; Chen *et al.*, 2013).

When measuring condition with condition levels, the condition objective is often defined to keep the largest proportion of an infrastructure asset network in a good condition level (Chan *et al.*, 2003; Gao *et al.*, 2013). This condition objective describes the condition status of an infrastructure asset network.

Some decision making may want to well maintain all the segments of its infrastructure asset network. A condition objective can be defined to improve the worst condition (Frangopol and Neves, 2004; Neves *et al.*, 2006; Lee *et al.*, 2011). This condition objective focuses on individual segments and ensures all segments are in acceptable condition.

Constraint of acceptable condition:

Condition constraints ensure that infrastructure assets are in acceptable condition to deliver a satisfying level of service. When using a condition index, acceptable condition refers to the worst condition index. The condition constraint requires the condition indices of all segments being better than the worst index (Ben-Akiva *et al.*, 1993; Frangopol and Neves, 2004; Marzouk and Omar, 2013) or the average condition index of a network being better than the worst index (Frangopol and Liu, 2007; Zhang, 2009; Ng *et al.*, 2011). This condition constraint manages the average condition of an infrastructure asset network.

When using condition levels, the acceptable condition often refers to the acceptable proportion of a network that is in poor condition level. The condition constraint ensures the proportion in poor condition is smaller than the acceptable proportion (Guignier and Madanat, 1999; 2008; Zhang and Wang, 2014). This condition constraint controls the poor-condition part of a network.

2.4.3 Risk

Risk is a critical outcome in decision making in IAM (Berardi *et al.* 2009). It cannot be avoided; but decision making needs to keep it at a low level therefore improves the reliability of infrastructure assets. Around 40 reviewed publications directly consider risk in decision making in IAM.

Theoretically, risk measures the effect of uncertainty. It is determined by the activity risk exposure, more specifically, the level of exposure to the risk and the actions necessary to minimise that risk (NAMS, 2011). In most cases, risk is measured by an index based on the probability and consequence of failure; while its definitions may vary depending on the decision making at hand (Benati and Rizzi, 2007; Orcesi and Frangopol, 2011). Also risk can be measured in monetary terms based on the costs incurred when failures happen (Han *et al.*, 2012).

Risk can be analysed with objectives and constraints. Most of the publications define a risk objective in decision making to reduce the risk of failure. Some decision making problems define risk constraints to keep the risk at a low level.

Objective of minimising risk:

Risk objectives directly manage the risk of failure in decision making in IAM and largely reduce the possible risk with the available resources. Often the average risk index is minimised to reduce the overall risk (Halfawy *et al.*, 2008; Orcesi and Frangopol, 2011). In some decision making, a specific section of the risk is minimised, such as the probability of failure (Stewart, 2011; Essahli and Madanat, 2012) and the failure costs (Yang *et al.*, 2006).

Some publications minimise the specific type of risk, such as society risk (Rosmuller and Beroggi, 2004) or economic risk (Barker and Haimes, 2009). On the contrary, Ibrahim *et al*. (2012) and Lethanh *et al*. (2014) simultaneously minimise different types of risks in their decision making in IAM.

Constraint of allowable risk:

Risk constraints control the possible risk under its tolerance and ensure the reliability of infrastructure assets. Many publications try to keep the average risk index of an infrastructure asset network smaller than an allowable value (Lounis, 2005; Benati and Rizzi, 2007; Almeida *et al.*, 2013). Some publications control the costs of failure (Bucher and Frangopol, 2006), or probability of failure (Frangopol and Okasha, 2009; Furuta *et al.*, 2011) under allowable values in decision making in IAM.

2.4.4 Management Benefit

Benefit is another important outcome of decision making. It directly measures the return on investment and improves the efficiency of IAM. Benefits can be generated in different ways. The most management benefit is generated by improving asset condition and extending asset serving life. Figure 2.12 shows an example of benefit. When a treatment is applied, the asset condition is improved and its serving life is extended. The area between the curves of original condition and new condition is defined as the benefit.

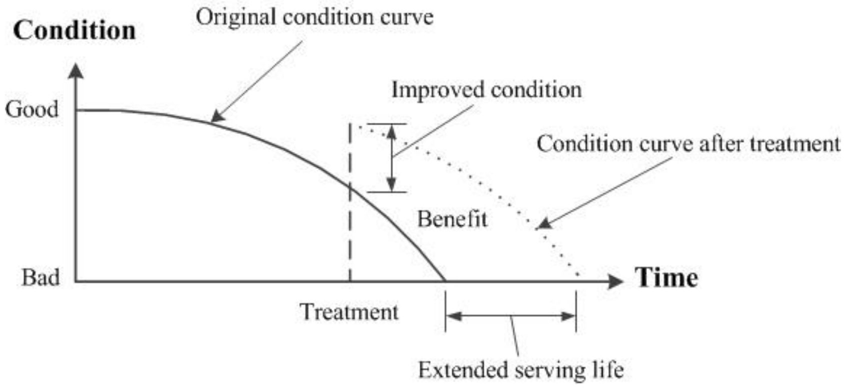


Figure 2.12 An example of benefit of condition improvement

Benefit can also be generated by reducing the user cost. When infrastructure is well maintained, its level of service is improved and user cost is reduced. The reduction of user cost is also a benefit.

In addition, when infrastructure assets are well maintained, agencies may obtain extra revenue by providing better service or reducing management costs (Labi and Sinha, 2003; Anastasopoulos, 2009). The increase in revenue and the saving on management costs are also benefits.

All of these benefits can be analysed using optimisation in decision making in IAM; while they are often pursued through an objective of maximising benefit. Constraints on benefits are hardly defined.

Objective of maximising benefit:

This objective directly measures the return on investment and efficiency of IAM. It is suitable for problems with a large investment and problems with a given funding (Labi and Sinha, 2003). According to the definition, different types of benefits are maximised in decision making in IAM.

If the condition is an important concern in decision making, the benefit measured by condition improvement is maximised, so that strategies that largely improve the asset condition are likely to be selected (Smadi, 2001; Hugo *et al.*, 2005; Scheinberg and Anastasopoulos, 2010). If user satisfaction is an important concern, the benefit measured by the reduction of user cost should be maximised in decision making (Harper, 1996; Petersen, 2002; Szeto *et al.*, 2010). If agency profit is concerned, agency benefit should be maximised (Rosmuller and Beroggi, 2004; Han *et al.*, 2012). Intangible benefits are also maximised in decision making to enhance the positive impact of IAM on the economy (Kerali and Mannisto, 1999; Koo and Ariaratnam, 2008) and the environment (Weber and Allen, 2010). Instead of a specific type of benefit, some publications try to maximise an integrated benefit in their decision making in IAM (Li and Madanu, 2009; Szeto *et al.*, 2010; Adey *et al.*, 2012).

2.4.5 Management Efficiency

Efficiency measures the investment utilisation of IAM. Compared with costs and benefits, efficiency directly evaluates the financial productivity of solutions. It can be defined as cost-effectiveness, return on investment, etc. Cost-effectiveness evaluates the productivity of investments. It is often defined as an index integrating the financial considerations and infrastructure performance. The return on investment evaluates the benefit generated by a given investment. It is often defined in monetary term.

Similarly with management benefit, management efficiency is often analysed as an efficiency objective that directly improves the efficiency of IAM. The efficiency constraint is hardly ever used.

Objective of optimising efficiency:

Efficiency objectives can effectively improve the efficiency of IAM. They are commonly defined in decision making in IAM. According to the definition of efficiency, these objectives are defined in a different way.

When measuring the efficiency with cost-effectiveness, publications often optimise the overall cost-efficiency index of an infrastructure asset network (Li *et al.*, 1997; Wang *et al.*, 2003; Gharaibeh *et al.*, 2006). When measuring efficiency with the return on investment, publications directly maximise the overall return on investment (Hsieh and Liu, 1997; Chen *et al.*, 2006; Dashti *et al.*, 2007). Efficiency objectives are also defined in other ways such as maximising budget utilisation (Gao *et al.*, 2010; Sharma, 2010) and maximising the benefit-cost ratio (Krueger and Garza, 2009; Elhakeem and Hegazy, 2012)

2.4.6 Infrastructure Asset Performance

Performance is defined by Uddin *et al.* (2013) as the degree of which infrastructure assets serve users and fulfil their functions. It not only measures the asset condition but also evaluates the delivered serve. Hence, performance is a composite outcome in decision making in IAM.

Performance can be measured by individual criteria or a composite index. The level of service is an important criterion of performance, which evaluates the service provided by infrastructure assets. It is normally defined based on asset condition, user satisfaction and legislative requirement. Safety is another performance criterion that measures the failure risk from users' point of view. It involves the infrastructure condition, the exposure of the users to possible failure and the consequence of failure. Reliability measures the performance by evaluating asset ability when dealing with risks, hazards and accidents. It is related to a series of factors including infrastructure condition, capacity, possible hazards and accidents. Serving life is a simple way to quantitatively describe asset performance. As shown in Figure 2.12, it can be extended to good management. The assets with longer remaining servicing life have better performance. Finally, performance can also be defined as a composite index by integrating a series of performance criteria.

Performance is a frequently analysed outcome in decision making in IAM. When decision making attempts to improve the network performance, a performance objective should be defined. When decision making attempts to keep the network performance, a performance constraint should be defined.

Objective of optimising performance:

Performance objectives largely improve the overall performance of an infrastructure asset network with the available resources. According to the measurement of performance, the performance objectives are defined in different ways. When the service is concerned, an objective is defined to maximise the average level of service (Sikow *et al.*, 1993; Wu and Flintsch, 2009; Osman, 2015). When the utilisation of assets is concerned, an objective is defined to improve the asset safety (Stewart, 1992; Kulkarni *et al.*, 2004; Durán, 2011) or asset reliability (Yare *et al.*, 2008; Golroo *et al.*, 2010). When the assets are concerned, an objective is defined to maximise the average remaining serving life (Dicdican, 2004; Furuta *et al.*, 2011). When the overall performance is concerned, an objective is defined to optimise the overall or average performance index (Sebaaly *et al.*, 1996; Almeida *et al.*, 2013; Khan *et al.*, 2014).

Constraint of acceptable performance:

Performance constraints ensure that an infrastructure asset network has the acceptable performance to function and deliver required level of service. Normally, an acceptable worst performance is defined and the performance constraint requires the average performance of an infrastructure asset network being better than the acceptable one. The acceptable performance can be defined as the worst level of service (Orcesi *et al.*, 2010; Farran and Zayed, 2012), the lowest safety index (Orcesi *et al.*, 2010; Lukas and Borrmann, 2011), the lowest reliability index (Stewart *et al.*, 2004; Abiri-Jahromi *et al.*, 2009) or the poorest performance index (Smilowitz and Madanat, 2000; Zhang, 2006; Gao and Zhang, 2013).

2.4.7 Environment

This outcome gets increasing attention in decision making in IAM. Decision making wants to reduce the impact of IAM on the environment. As an intangible outcome, the environment is commonly measured using the emission of greenhouse gas. It can be defined as objectives of providing the maximum protection of the environment by minimising the emission of greenhouse gas (Németh *et al.*, 2012; Deb, 2014; Yepes *et al.*, 2015), or as constraints of keeping the pollution to an acceptable level (Hugo *et al.*, 2005; Han *et al.*, 2012).

2.4.8 Discussion of Objectives and Constraints

This section surveys the main decision making outcomes analysed with optimisation in terms of objectives and constraints with the intention of improving the applications of optimisation in decision making in IAM. According to the completed discussion, a range of outcomes are successfully analysed with optimisation techniques, and expected results are produced for decision making in IAM. Other outcomes can also be analysed by defining objectives and constraints. Similarly, their objectives try to achieve these outcomes in the best possible manner; and their constraints try to keep these outcomes to an acceptable or allowable level.

The International Infrastructure Management Manual (IIMM) (NAMS, 2011) recommends considering economic, cultural, environmental and social outcomes in decision making in IAM. With the help of optimisation, this recommendation can be achieved by properly defining objectives and constraints.

2.5 Summary

This chapter provides the background knowledge of decision making in IAM and commonly used decision making methods. It firstly introduces IAM and its decision making. IAM is important but faces many challenges. Decision making, as an essential part of IAM, helps to handle the challenges and to achieve the goals and requirements of IAM. However, decision making, attempting to generate and select appropriate management strategies, also has many difficulties. Therefore, decision making methods are proposed to help with decision making.

In the second section of this chapter, commonly used decision making methods are reviewed. BCA evaluates strategies by analysing their benefits and costs. It requires converting all criteria into monetary terms, which is difficult in practical decision making. MCA, MAUT and outranking methods are able to analyse multiple criteria and therefore achieve different goals at same time. They are affected by subjectivities. In addition, all of these methods require enumerating all possible solutions and measuring each of them. Hence they are not suitable for large decision making including long-term and network-level decision making. Optimisation can handle various criteria and analyse a large number of segments and strategies. It has the application potential to assist long-term and network-level decision making in IAM.

Because of the strengths of optimisation, this chapter specifies the applications of optimisation in decision making in IAM through a comprehensive literature review in this regard. The application status of optimisation is discussed in the context of decision making in IAM, followed by a brief introduction of applied optimisation techniques.

Finally, this chapter surveys the main decision making outcomes that are analysed with optimisation in decision making in IAM. These outcomes are summarised based on the reviewed publications and outlined in terms of objectives and constraints.

CHAPTER 3 METHODOLOGY

The goal of this research was to improve decision making in IAM by enhancing the understanding of MOO in the context of decision making in IAM and developing a robust MOO technique for practical long-term and network-level decision making in IAM. Five research objectives were defined based on this goal (see Section 1.3). To achieve the goal and objectives, a research methodology was proposed to guide this research. Figure 3.1 presents the framework of this methodology. Three stages were established to gradually clarify the applications of MOO in decision making process and thereafter introduce a robust MOO technique and illustrate the optimisation result. The three stages are:

- Stage 1: Study of optimisation techniques;
- Stage 2: Development of a robust MOO technique; and
- Stage 3: Communication of optimisation result.

3.1 Stage 1: Study of Optimisation Techniques

This stage aims at a comprehensive knowledge of MOO in the context of decision making in IAM. It was the basis of the applications of MOO. This stage introduced MOO from a viewpoint of practical decision making in IAM and clarified the previous research on this subject. More importantly, it investigated both frequently applied techniques and new techniques, examined their abilities and assessed their performance in order to strengthen and simplify decision making in IAM. Four questions were answered at this stage, including:

(1) What is MOO?

(2) Why should MOO be used for decision making in IAM?

(3) What are the values of MOO to assist decision making in IAM?

(4) What are the applicable MOO techniques and how do they perform when dealing with problems of decision making in IAM?

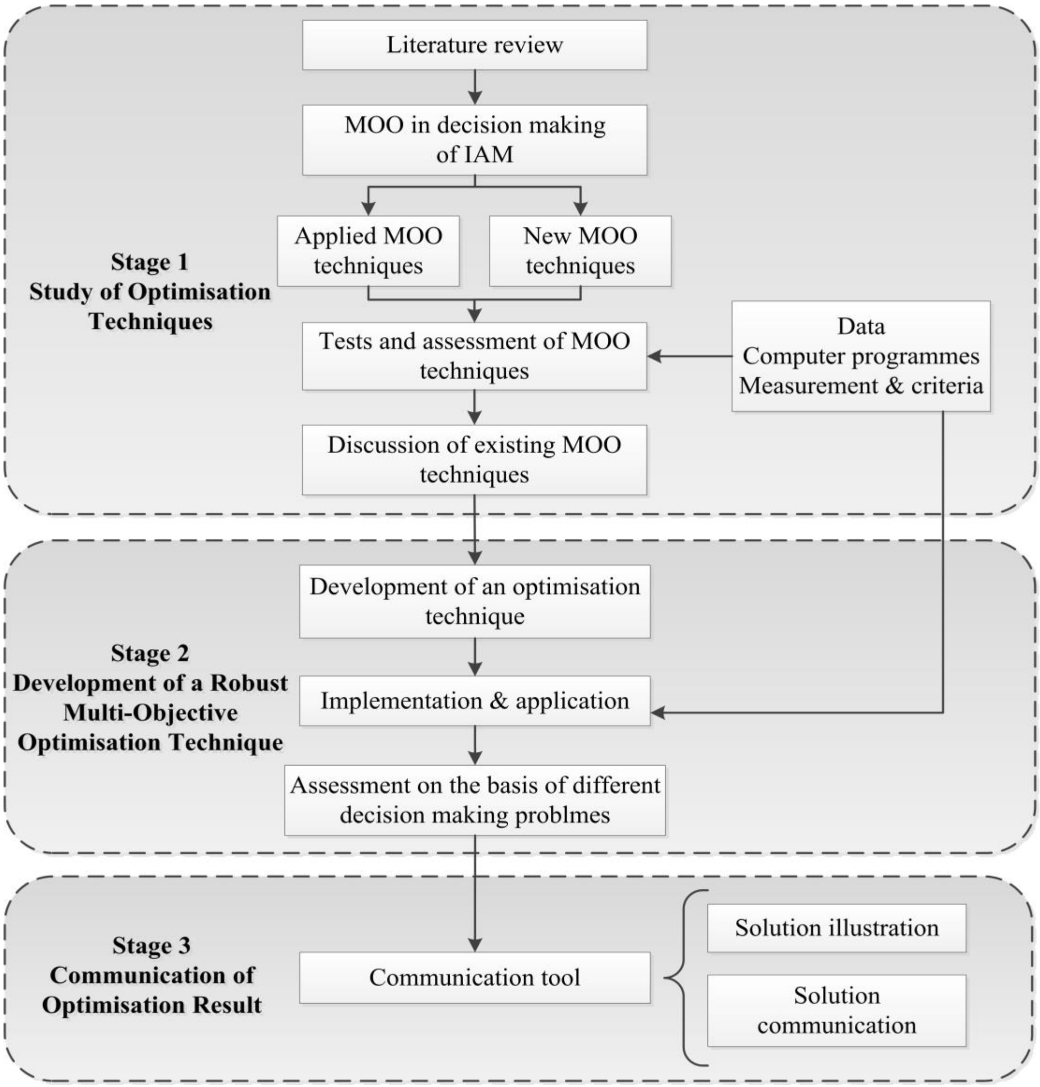


Figure 3.1 Framework of the methodology

To answer these questions, six steps were outlined.

The first step provided the background knowledge of IAM, its decision making and the applications of optimisation techniques. This step was achieved by reviewing literature. More specifically, it explained the concepts of IAM and the decision making and outlined common decision making methods. Then it narrowed its focus down to optimisation and reviewed the previous applications of optimisation techniques in decision making in IAM. Finally, the analysed decision making outcomes in terms of objectives and constraints were surveyed.

The second step introduced fundamental principles when applying MOO in decision making in IAM. Firstly, it helps to understand the function and characteristics of MOO by specifying MOO and its philosophy in the context of decision making in IAM. Then it pointed out the significance and strengths of MOO in decision making in IAM. Finally it described the steps when applying MOO to a decision making problem in IAM.

The third step directly discusses the applied MOO techniques and their abilities in the context of decision making in IAM, which also provides a general status of the previous applications. This step is based on literature review and investigated the MOO techniques that were applied in decision making in IAM. A list is established to record MOO techniques that have potential to deal with long-term and network-level decision making in IAM (thereafter called the MOOT List) for further analysis.

The fourth step introduced new techniques that were not applied in decision making in IAM but had the application potential. This step surveyed other MOO techniques and selected the potential ones for long-term and network-level decision making in IAM. These potential new techniques were supplemented into the MOOT List as applicable choices for further analysis.

The fifth step examined all the techniques in MOOT list in order to provide a direct and comprehensive knowledge of the existing MOO techniques in the context of decision making in IAM. Typical tests were conducted based on practical long-term and network-level decision making in IAM. Databases and computer programmes were prepared for testing. A measurement framework was also established to facilitate the measurement and comparison of MOO techniques from the viewpoint of decision making in IAM.

The final step of Stage 1 further discussed MOO in order to assist the selection and applications of MOO techniques in decision making in IAM. In this step, the benchmark criteria of a robust MOO technique for practical long-term and network-level decision making were outlined, which are used to assess the robustness of the existing MOO techniques. Recommendations on the applications of MOO were provided.

3.2 Stage 2: Development of a Robust Multi-Objective Optimisation Technique

This stage attempted to improve practical long-term and network-level decision making in IAM by developing a robust MOO technique that solved its MOO problems and generated satisfying results. This stage was based on the knowledge obtained in Stage 1. It had three steps.

Stage 2 begins with developing a new MOO technique in decision making in IAM based on the discussion of existing MOO techniques. The developed technique analysed decision making problems and obtained optimisation results that satisfied the benchmark criteria of robust MOO techniques and requirements of practical long-term and network-level decision making in IAM.

In the next step, the effectiveness and efficiency of the developed MOO technique is certified in the context of decision making in IAM. More specifically, the developed technique was measured and assessed with typical tests based on practical long-term and network-level decision making in IAM. Its performance was compared to the existing MOO techniques. Furthermore this step broadened the applications of the developed technique by applying it to target different IAM problems.

The final step aimed at assessing the robustness of the developed technique. According to its performance in the conducted tests and applications to different decision making problems, the developed technique is discussed according to the benchmark criteria of robust MOO techniques.

3.3 Stage 3: Communication of Optimisation Result

The goal of this stage was to improve the applications of MOO techniques in decision making in IAM by interpreting optimisation results. After applying a MOO technique to solve a decision making problem, its optimisation result could be difficult to be understood as many solutions were likely to be obtained, each with a range of outcomes. Thus, a tool was needed to explain the optimisation result in a meaningful and understandable way so as to take the best advantage of optimisation. This communication tool mainly had two abilities:

Solution illustration: The communication tool interpreted the optimisation result and presented the identified solutions. Figures and tables were more beneficial than words when demonstrating obtained solutions. Hence various types of figures and tables were created to illustrate the obtained solutions and their outcomes. Both overall outcomes (e.g. total maintenance cost and total benefit) and yearly outcomes (e.g. yearly condition and yearly level of service) were important information for decision making and needed to be properly presented when illustrating solutions.

Solution communication: The communication tool also helped to examine optimisation results and determine appropriate management decisions. The tool was required to communicate with decision makers so as to explore the management preferences, trade off objectives and refine identified solutions. Dynamic figures enabled decision makers to release the unwanted solutions and focus on the preferred ones. They were adopted when tailoring optimisation results.

CHAPTER 4 FUNDAMENTAL PRINCIPLES WHEN APPLYING MULTI-OBJECTIVE OPTIMISATION IN DECISION MAKING IN INFRASTRUCTURE ASSET MANAGEMENT

This chapter provides the fundamental principles when applying MOO techniques in decision making in IAM with the intention of enhancing the understanding of MOO and facilitating its applications in the context of decision making in IAM. The main achievements of this chapter include:

- Introducing MOO and its philosophy from the viewpoint of decision making in IAM;
- Emphasising the strengths and the significance of MOO techniques when helping with decision making in IAM; and
- Outlining the steps when applying a MOO technique to a practical decision making problem in IAM.

4.1 Some Concepts of Multi-Objective Optimisation

This section introduces the necessary concepts of MOO in the context of decision making in IAM in order to help in understanding the function of MOO and facilitate its applications in decision making in IAM. These concepts are based on (Ehrgott, 2005; Hillier and Lieberman, 2005; Nedjah and Mourelle, 2005):

An optimisation problem: An optimisation problem is a mathematical model that expresses a decision making problem of IAM. It is formulated with formulae and variables. An optimisation technique actually works on these formulae and variables to generate an optimisation result. By adjusting the values of variables, an optimisation method chooses a feasible solution (in terms of constraints) with the best (optimal) objective function value. In other words, an optimisation technique does not directly solve a decision making problem, but solves its corresponding mathematical formulation, the optimisation problem. When the optimisation problem accurately describes a decision making problem, the solution(s) of this optimisation problem can be accepted as management decisions of this decision making problem.

Decision variables: Decision variables represent the decision to be made, which vary and lead to different outcomes. In decision making in IAM, decision variables normally represent the selection of strategies. In this research, binary variables are used as decision variables where variable value 1 indicates a certain strategy is selected and 0 indicates it is not. This is further introduced in Section 4.3.1.

Formulae and formulation: Formulae are used to describe a mathematical model of a decision making problem including goals and requirements, computation of outcomes or required relationships between outcomes. The formulation contains all the formulae required to correctly represent the decision making problem, which can be regarded as the mathematical model of this problem. An example of the formulation is presented in Section 4.3.1.

Objectives and constraints: Objectives and constraints are an important part of an optimisation problem. Generally, objectives describe the goals and targets of a decision making problem and constraints describe its requirements and limitations. Both objectives and constraints need to be expressed using mathematical formulae. The common objectives and constraints used for decision making in IAM are summarised in Section 2.4.

Feasible and infeasible solutions: In decision making in IAM, a solution of an optimisation problem often indicates a selection of strategies for an infrastructure asset network. Feasible solutions satisfy all the constraints of an optimisation problem; and infeasible solutions violate at least one constraint of an optimisation problem. Hence, feasible solutions are the feasible choices of strategy selections in decision making; while infeasible solutions do not satisfy all the decision making requirements. In this research, only feasible solutions are discussed. When defining feasible and infeasible solutions, the objective values of these solutions are not concerned.

Optimal solution: The optimal solution is the best feasible solution of an SOO problem. In the context of decision making in IAM, an optimal solution satisfies all the constraints and achieves the best value on the objective among all feasible solutions for an SOO problem in decision making in IAM.

Non-dominated solutions: Non-dominated solutions exist only when analysing MOO problems. We assume the decision variables are represented as a vector of variables $\boldsymbol{x}$**.** For two feasible solutions $\boldsymbol{x}$ and $\boldsymbol{x}'$, if the value of at least one objective of solution $\boldsymbol{x}$ is better than that of solution $\boldsymbol{x}'$, and the values of all other objectives of solution $\boldsymbol{x}$ are not worse than those of solution $\boldsymbol{x}'$, then we say that $\boldsymbol{x}$ dominates $\boldsymbol{x}'$ and $\boldsymbol{x}'$ is dominated by $\boldsymbol{x}$. Taking Figure 4.1 as an example, considering a MOO with two objectives: minimising cost and minimising risk, solution B dominates solutions E, H, I and J. For a pool of feasible solutions $\boldsymbol{X}$, if a solution $\boldsymbol{x}$ in the pool is not dominated by any other solution in this pool, then solution $\boldsymbol{x}$ is defined as a non-dominated solution in the pool (thereafter a non-dominated solution). In decision making in IAM, non-dominated solutions achieve respective objectives in a better manner than other solutions in the pool. In the example of Figure 4.1, solutions A-D are all non-dominated. Hence, they are the better options of strategy selections than other solutions in the pool.

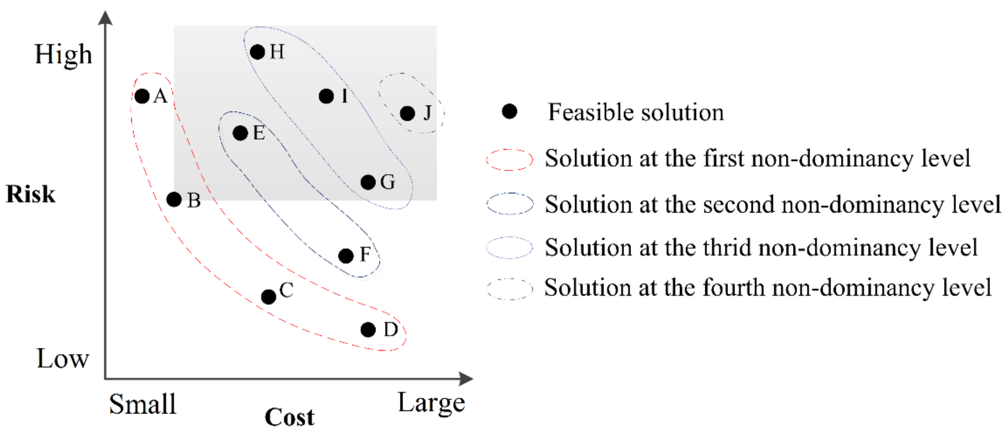


Figure 4.1 An example of non-dominated solutions and non-dominancy

Non-dominancy level: Non-dominancy level is a measurement criterion of the solution goodness when solving MOO problems. It is defined based on the solution non-dominancy. Taking Figure 4.1 as an example, the non-dominated solutions are marked as the first level of non-dominancy, i.e. solutions A-D, and removed from the solution pool. All the infeasible solutions are marked as the last level of non-dominancy and removed from the pool. Then repeatedly, the non-dominated solutions left in the pool are marked as the next level of non-dominancy, i.e. solutions E and F, and removed from the pool. This process stops when all solutions in the pool are removed.

Pareto solutions and Pareto frontier: Pareto solutions, also named efficient solutions, are a set of feasible solutions that cannot be improved in one objective without worsening at least one other objective. In the context of decision making in IAM, Pareto solutions are the best feasible solutions that achieve respective objectives in a manner that cannot be improved upon. It is important to note that, in this research, Pareto solutions and non-dominated solutions may not be the same. Pareto solutions are the best existing solutions; while non-dominated solutions are only the best solutions within a subset of feasible solutions. The Pareto frontier is the frontier of Pareto solutions of an optimisation problem. Every Pareto frontier in this research is the set of all existing Pareto solutions, which may look like a line in the objective space when a large number of Pareto solutions exist.

Solve: In this paper, if a technique obtains at least one feasible solution for a MOO problem of decision making, it is defined that this technique solves this MOO problem even the obtained feasible solutions are not Pareto solutions. All the decision making problems analysed in this research are assumed to be solvable; so their MOO problems have feasible and Pareto solutions.

4.2 Strengths and Significance of Multi-Objective Optimisation in Decision Making in Infrastructure Asset Management

This section outlines the strengths and specifies the significance of MOO in decision making in IAM, therefore explores the attainable support and assistance from MOO. Previous studies claim the effectiveness of MOO in decision making in IAM, however, they normally focus on specific issues. A holistic overview of the strengths of MOO is hardly obtained. This section discusses the importance of MOO in decision making in IAM and introduces the ability of optimisation results that can best assist decision making process.

All MOO techniques aim at identifying Pareto solutions. Some MOO techniques are only able to obtain one Pareto solution. They interact with decision makers, obtain their preference on objectives and then produce one solution based on the preference. When applying these methods, the preference is critical; but in practical decision making in IAM, the correct preference is difficult to be defined as a priori, because:

- Decision makers need to have sufficient knowledge of their decision making problems to determine their preference, which may be difficult especially in the early stage of decision making process.
- The preference is given by a decision maker, which may cause subjectivities and controversies as decision makers may have different opinions on the importance and priority of objectives.

Some MOO techniques are able to obtain a set of Pareto solutions. These methods simultaneously optimise objectives and do not require decision makers' preferences. They release the responsibility of decision makers and better assist decision making by:

- **Providing achievable outcomes.** A set of Pareto solutions presents the best achievable values of objectives for a decision making problem. Figure 4.2 is an example of a decision making problem with two objectives: improving the condition and minimising the cost. Five Pareto solutions exist including the one presenting the lowest necessary funding (solution A) and the best possible condition (solution E).

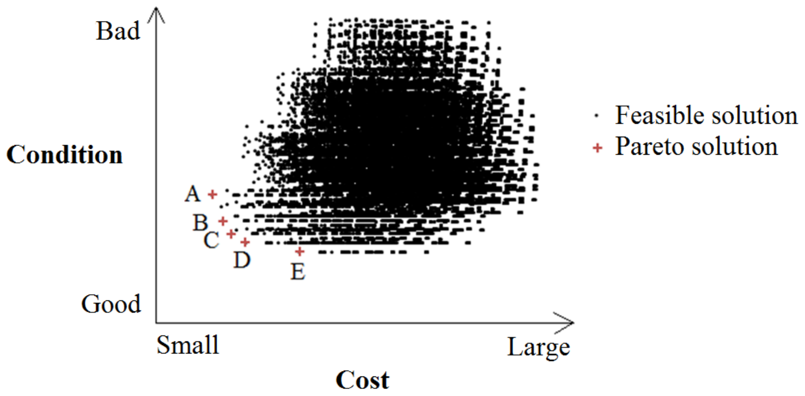


Figure 4.2 Example of Pareto solutions

- **Helping understanding the trade-offs of objectives.** A set of Pareto solutions shows the relationships between objectives for decision making in IAM. When worsening one objective, the return on another can be estimated by moving from one solution to another. For instance, when moving from solution C to D in Figure 4.2, the condition is improved while more funding is needed. Decision makers can trade off the conflicting objectives by balancing the objective improvement and deterioration using Pareto solutions.
- **Simplifying decision making.** Practical decision making may have numerous feasible solutions. After obtaining all Pareto solutions, decision makers only need to focus on

Pareto solutions without considering other possibilities, i.e. only solutions A-E out of over 95,000 feasible solutions in Figure 4.2. This largely releases the workload of decision makers, allows decision makers to put attention on the most relevant alternatives, and thus simplifies decision making process. When more segments are analysed, more feasible solutions may exist, so decision making becomes more difficult without the help of Pareto solutions.

- **Showing the performance of strategies**. A set of Pareto solutions also shows the performance of individual strategies. For example, when the most Pareto solutions select the same strategy, this strategy is highly recommended. Or if the most Pareto solutions apply treatments in a certain period of time, then treatments should be implemented in this period. More information of IAM can be obtained by examining Pareto solutions for a specific decision making problem.
- **Improving the quality of decision making.** These MOO methods directly optimise objectives without the interaction with decision makers. After optimisation, Pareto solutions are obtained to help understanding and simplifying decision making in IAM, so that the management decision can be made based on the optimisation result, which is reasonable and well-grounded.
- **Speeding up decision making process.** MOO techniques can be easily implemented on computers. Because of the cheap computing power, they are able to quickly analyse a large number of segments and strategies and fast obtain solutions, therefore speed up decision making process. When more segments and strategies are analysed, the increase of analysing speed is more important and the need of MOO techniques is more necessary.

According to the statement above, MOO techniques that can obtain a set of Pareto solutions are much more helpful in decision making in IAM than SOO techniques and MOO techniques that only generate one solution. Hence, in this research, only these methods are discussed.

4.3 Process of Applying Multi-Objective Optimisation in Decision Making in Infrastructure Asset Management

This section introduces the process when applying a MOO technique to a specific decision making problem in IAM. Four steps are proposed, including formulation of an optimisation problem, selection of an optimisation technique, implementation of the selected technique, and optimisation and optimisation result.

4.3.1 Formulation of an Optimisation Problem

Before optimisation, a practical decision making problem in IAM needs to be mathematically formulated as an optimisation problem. The optimisation problem should accurately describe a decision making problem with the help of variables and formulae.

More specifically, decision making tries to select appropriate strategies for the segments of an infrastructure asset network, which, in other words, targets at a combination of strategies so that the outcomes of the selected strategies achieve the goals and requirements of IAM. The problems analysed in this research can be categorised as the combinatorial optimisation problem and often formulated as an integer programming problem (IP) that expresses an optimisation problem with integer decision variables and linear formulae of objectives and constraints (Korte and Vygen, 2012). In particular, binary variables given by Equation 4.1 are frequently used as decision variables representing the selection of strategies $\boldsymbol{x} = (x_1, x_2 \cdots, x_N)$. If a decision variable x_i has value 1, strategy i is selected for a segment; otherwise, this strategy is not selected.

$$x_i = \begin{cases} 1 & \text{strategy } i \text{ is selected} \\ 0 & \text{strategy } i \text{ is not selected} \end{cases} \qquad \text{Equation 4.1}$$

After defining decision variables, objectives and constraints are formulated to describe the goals and requirements of IAM using linear formulae. A general formulation of optimisation problems of decision making in IAM is given by Equations 4.2-4.5, where $f_{k,i}$, $g_{l,i}^U$ and $g_{l,i}^L$ represent the corresponding outcomes such as cost and condition index. Objectives can be set up to be maximised or minimised (Equation 4.2); and constraints can be restricted with upper or lower bounds, or both (Equations 4.3 and 4.4). Constraints of Equation 4.5, named one-strategy policy, ensures exactly one strategy is selected for each segment.

$$\text{max/min} \quad \sum_{i=1}^{N} f_{k,i} * x_i, \qquad \text{for } k = 1,2, \dots, K \qquad \text{Equation 4.2}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} g_{l,i}^U * x_i \leq Limit_l^U, \qquad \text{for } l = 1,2, \dots, L_{upper} \qquad \text{Equation 4.3}$$

$$\sum_{i=1}^{N} g_{l,i}^{L} * x_i \geq Limit_l^L, \qquad \text{for } l = 1,2,\ldots, L_{lower} \qquad \text{Equation 4.4}$$

$$\sum_{i \in \mathcal{S}_j} x_i = 1, \qquad \text{for } j = 1,2,\ldots M \qquad \text{Equation 4.5}$$

where, i index of a strategy;

j index of a segment;

k index of an objective;

$f_{k,i}$ value of objective k if strategy i is implemented;

$g_{l,i}^{U}$ value of upper constraint l if strategy i is implemented;

$g_{l,i}^{L}$ value of lower constraint l if strategy i is implemented;

$Limit_l^U$ upper bound of constraint l;

$Limit_l^L$ lower bound of constraint l;

$\mathcal{S}_j$ set of alternative strategies for segment j;

K number of objectives;

L_{upper} number of upper bound constraints;

L_{lower} number of lower bound constraints;

M number of segments; and

N number of strategies;

However, the formulation of decision making problems may vary. For specific decision making problems in IAM, other types of optimisation problems such as non-linear optimisation may be needed to accurately describe the decision making problems and therefore different formulation may be needed.

4.3.2 Selection of an Optimisation Technique

After formulating a decision making problem, an appropriate optimisation technique needs to be selected. Optimisation techniques, following different algorithms, have different performance and produce different optimisation results when solving a same optimisation problem. Hence they should be selected based on the addressed optimisation problem. An appropriate technique can effectively solve the addressed problem and efficiently obtain the best optimisation result, hence it supports decision making in IAM. However, the selection of an appropriate technique can be difficult because of insufficient knowledge of the existing optimisation techniques in the context of decision making in IAM. A main objective of this research is to enhance the knowledge of MOO in the context of decision making in IAM, therefore finding an appropriate MOO technique for long-term and network-level decision making in IAM. This step is further discussed in the later chapters.

4.3.3 Implementation of the Selected Technique

Implementation of the selected optimisation technique is important as it may affect the technique performance. The algorithm of the selected technique could be implemented in different ways based on the implementer's understanding of the technique and implementation preference. A good implementation may help yielding good optimisation results in short time therefore improve the performance of the selected technique.

Implementation is related to many factors and cannot be specified without knowing the algorithm and the addressed optimisation problem. This part only discusses a critical issue of the implementation – decision variables – when implementing exact methods and heuristics, while other implementation issues are discussed later.

Exact methods can be directly implemented after a decision making problem is formulated as IP with binary variables (see Equations 4.2-4.5). When solving MOO problems, an exact method may transform a MOO problem into a set of SOO sub-problems, and solves each SOO sub-problem separately. Many algorithms and software tools can be easily used to solve SOO sub-problems; while their implementation may affects the solution quality and speed of exact methods.

Because of the mechanism of heuristics, in this research, (non-binary) integer variables are used to describe the solutions of MOO problems when applying heuristics. Integer variables $\boldsymbol{y} = (y_1, y_2 \cdots, y_M)$ represent the indices of the strategies selected by a solution. For example, if segment j selects strategy i, then binary variable $x_i = 1$ and integer variable $y_j = i$. Objective and constraint values are calculated by summing the outcomes of the selected strategies. This implementation:

- Significantly reduces the number of decision variables from the number of strategies to the number of segments;
- Naturally satisfies the one-strategy policy so the constraints of Equation 4.5 are released; and
- Simplifies the computation in heuristics.

Therefore, integer variables may improve the performance of heuristics and are adopted when implementing heuristics. However, integer variables cannot ease the implementation of exact methods; hence binary variables are used when implementing exact methods.

4.3.4 Optimisation and Optimisation Result

The final step is to apply the optimisation technique to solve the optimisation problem of decision making in IAM, and collect its result. In practical decision making, a large number of Pareto solutions may exist. Many of them may have similar outcomes and are not helpful for the trade-offs of objectives. Hence, these similar-outcome solutions are not useful for decision making in IAM; and obtaining these solutions wastes much time. Therefore, instead of the whole set of existing Pareto solutions, a set of good representatives of Pareto solutions should be obtained by a MOO technique.

Moreover, solutions of MOO problems may be difficult to be fully understood and compared. These solutions, representing different selections of strategies, are related to many outcomes. It could be complicated to explain all the information associated with the optimisation result, such as the overall and yearly outcomes of individual solutions, achievable outcomes, the outcome relationships, etc. Solution understanding becomes more difficult when many solutions are obtained or a wide range of outcomes are involved. Hence it is important to help decision makers to understand the optimisation results. This is further discussed in Chapter 7.

Then an appropriate decision could be determined by selecting an identified optimisation solution or adjust the solutions based on specific management preference or considerations. Decision making methods such as MCA could be applied to help to explore the management preference and selecting optimisation solutions.

4.4 Summary and Discussion

This chapter provides the fundamental principles of MOO in the context of decision making in IAM, which helps to understand MOO and facilitates its applications in decision making in IAM. According to the analysis in this chapter, MOO has great strengths and can significantly assist decision making in IAM. More specifically, this chapter introduces MOO and its concepts in the context of decision making in IAM. Then it clarifies the strengths of MOO and specifies the advantages of obtaining a set of Pareto solutions when dealing with multiple objectives in decision making in IAM. Finally, this chapter outlines the application process of MOO.

Because of their strengths, MOO techniques are increasingly applied in decision making in IAM. However, the selection and applications of MOO techniques still can be difficult without knowing the existing MOO techniques and their performance. Hence, in the next chapter, a comprehensive study of existing MOO techniques is provided to enhance the knowledge of MOO techniques in the context of decision making in IAM.

CHAPTER 5 INVESTIGATION AND TESTS OF EXISTING MULTI-OBJECTIVE OPTIMISATION TECHNIQUES

This chapter attempts to enhance the understanding of MOO by providing a comprehensive and authentic knowledge of MOO techniques and their performance in the context of decision making in IAM. This knowledge is obtained by investigating and testing the existing MOO techniques based on practical decision making problems arising in IAM. The main points of this chapter include:

- Investigating different types of MOO techniques, and introducing their applications in decision making in IAM;
- Designing typical tests of MOO based on practical long-term and network-level decision making in IAM;
- Establishing a measurement framework with criteria to measure the performance of MOO techniques from the viewpoint of practical decision making in IAM; and
- Testing and assessing existing MOO techniques in the context of decision making in IAM, and making suggestions on their applications.

A broad list of existing MOO techniques is investigated, tested and assessed. Figure 5.1 presents the structure of the studied MOO techniques. These techniques are divided into two groups based on whether they are applied in decision making in IAM:

- **Applied MOO techniques** that are frequently applied in decision making in IAM.
- **New MOO techniques** that have not been applied in decision making in IAM but have application potential.

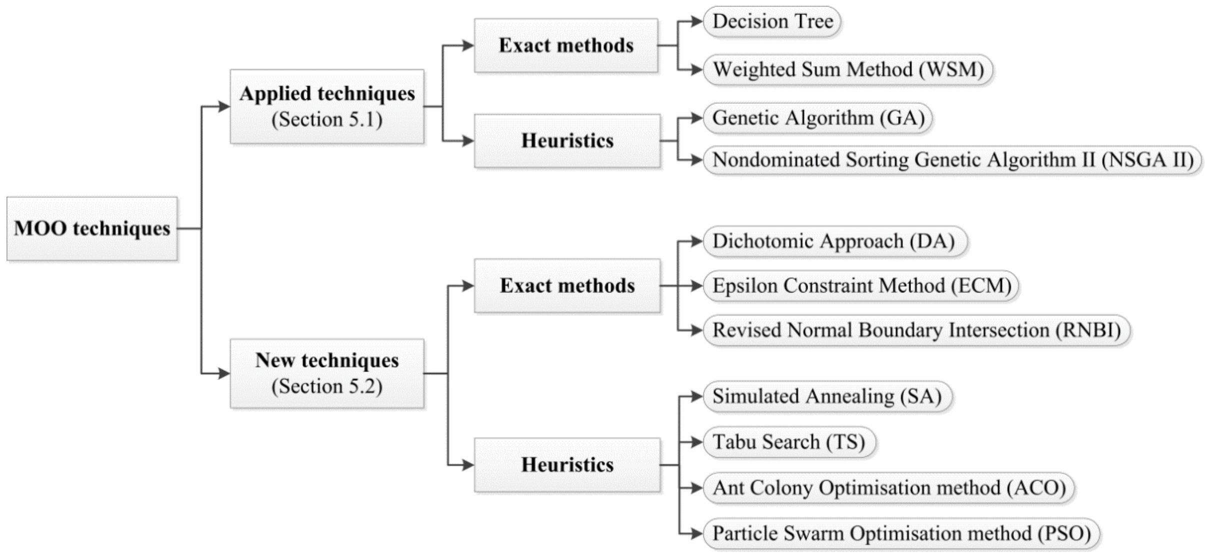


Figure 5.1 Structure of listed MOO techniques

All the listed techniques are examined in this chapter. In detail, this chapter begins with examining those MOO techniques that were previously applied in decision making in IAM. A list of MOO techniques that are potentially applicable to long-term and network-level decision making in IAM (thereafter called the MOOT List) is established by refining the applied techniques. The new MOO techniques are also investigated and added into the MOOT List. After preparing the databases and computer techniques and establishing a measurement framework, all the listed MOO techniques are tested based on two practical long-term and network-level decision making problems in IAM in order to examine their abilities and application potential. Based on the tests, the listed MOO techniques are assessed. This chapter ends with a discussion of the existing MOO techniques when dealing with long-term and network-level decision making in IAM.

5.1 Examination of Applied Multi-Objective Optimisation Techniques

This section attempts to identify effective MOO techniques from the previously applied ones in decision making in IAM. It examines the frequently applied MOO techniques that can generate a set of Pareto solutions, and refines them for long-term and network-level decision making in IAM. According to the literature review in Section 2.3, the frequently applied MOO techniques aiming at a set of Pareto solutions are shown in Table 5.1.

Table 5.1 Summary of the applied MOO techniques

Technique	Abbreviation	Optimisation type	Pareto solutions	Speed
Decision Tree		Exact method	All	Slow
Weighted Sum Method	WSM	Exact method	Some	Fast
Genetic Algorithm	GA	Heuristic	Unsure	Variable
Nondominate Sorting Genetic Algorithm II	NSGA II	Heuristic	Unsure	Variable

The following sections introduce the algorithms of these techniques and discuss their potential of solving long-term and network-level decision making problems in IAM.

5.1.1 Decision Tree

Decision Tree is the most frequently used MOO technique. It is simple but has the ability to obtain all existing Pareto solutions for MOO problems.

It uses a tree-like model to specify all the alternative solutions and measures their outcomes individually. Figure 5.2 is an example of Decision Tree for a network-level decision making problem. It establishes a layout of a tree model by enumerating and combining the alternative strategies; and new solutions are generated by allowing the strategy combination. Then all the solutions are measured by the satisfaction of constraints and the achievement of objectives; and the non-dominated ones are selected as final solutions.

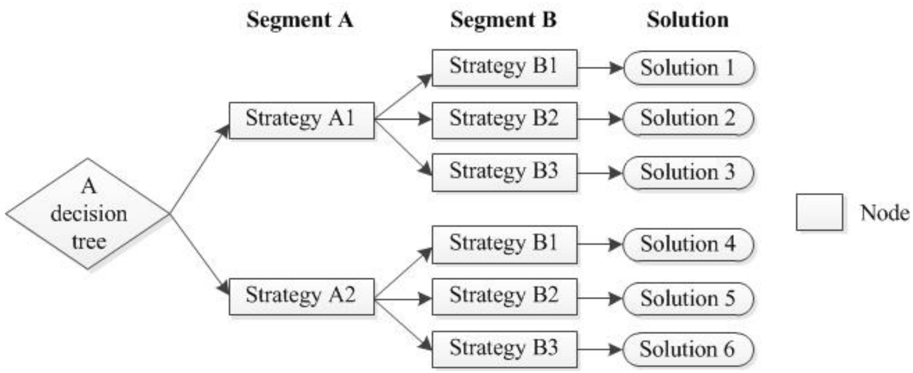


Figure 5.2 An example of the layout of Decision Tree

Decision Tree can effectively handle multiple objectives and constraints, and obtain Pareto solutions. It is also flexible. Each node in the tree model can be split again until all possibilities are explored. In this way, Decision Tree is able to analyse complicated decision making problems (Bonyuet *et al.*, 2002; Anastasopoulos *et al.*, 2014). However, because it enumerates all possibilities, Decision Tree may be time consuming when analysing large decision making problems.

5.1.2 Weighted Sum method

WSM is an efficient MOO exact method proposed by Zadeh (1963). It can quickly analyse a MOO problem and obtain a set of Pareto solutions.

WSM transforms a MOO problem into a set of SOO sub-problems using pre-defined weights. More specifically, for a MOO problem, a new objective can be defined by weighted summing the original objectives with a given weight. Figure 5.3 shows an example of the definition of the new objective. Then a new SOO sub-problem is established by optimising the new objective under the constraints of the original MOO problem. A solution is obtained by solving this SOO sub-problem.

	Original objectives	**Given weight $\boldsymbol{w} = (w_1, w_2)$**	**A new objective**
Objective 1	min *Cost*	w_1	min $w_1 Cost + w_2 Risk$
Objective 2	min Risk	w_2	

Figure 5.3 An example of WSM in bi-objective optimisation

When changing the weights, more SOO sub-problems can be established and more solutions may be obtained. In this research, the weights are defined as equidistance points. Figure 5.4 shows an example of the weight definition. This weight definition equally divides the feasible ranges of all weights into n_s parts, and then defines the combinations of the divided values as weights. Other definitions of weights are introduced by Marler and Arora (2004).

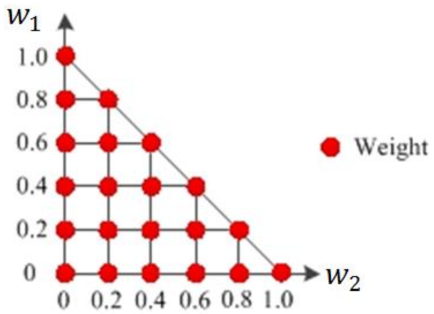


Figure 5.4 Definition of weights

WSM is an easy and effective MOO technique. It can efficiently handle large decision making problems in IAM and identify Pareto solutions. The parameter n_s determines the number of weights. In Figure 5.4, the weight range is divided into 5 parts ($n_s = 5$); so 21 weights are defined in total in this example. When n_s is large, more SOO sub-problems are established; therefore more Pareto solutions may be obtained.

However, even all possible weights are analysed, WSM cannot obtain all the existing Pareto solutions. In detail, it is only able to identify the supported solutions located on the boundary of the convex hull of feasible objective vectors but ignores non-supported solutions located in the interior of the convex hull. Detailed information of the supported and non-supported solutions is introduced by Chen *et al*. (2013). When a MOO problem only has a few Pareto solutions, non-supported solutions may also be needed to contribute a sufficient Pareto solution set. In addition, WSM only transforms a MOO problem into SOO sub-problems; an algorithm or solver is needed to solve the SOO sub-problems, which also affects the optimisation result.

5.1.3 Genetic Algorithms

Genetic Algorithm (GA) is a heuristic that mimics the process of natural selection based on Darwin's theory of evolution (Holland, 1975). According to the literature review in Chapter 2, it is the most frequently used MOO method in decision making in IAM; and many researchers claim GA generates good solutions for their decision making problems.

GA operates on genes, each corresponding to a road segment in the context of IAM. The values of the genes are exchanged and mutated to generate new solutions. Figure 5.5 shows the flowchart of the classic GA.

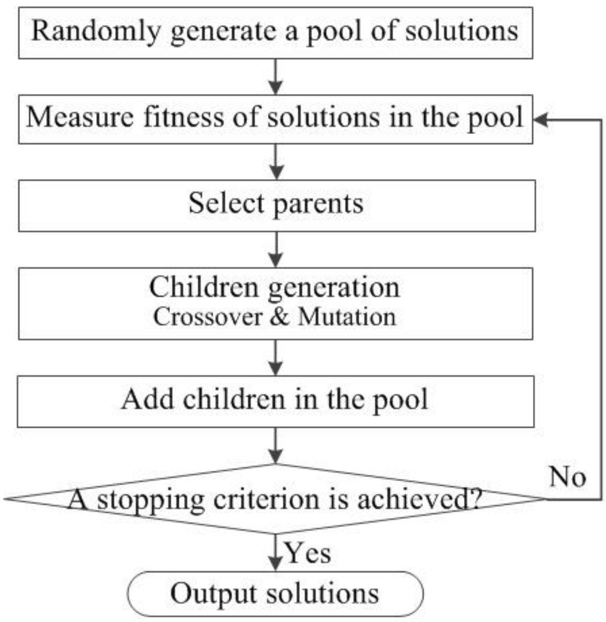


Figure 5.5 Flowchart of GA

Initially, a pool of solutions is randomly generated, and measured by their fitness. The fitness is normally defined according to the achievement of the objective and the satisfaction of constraints. In this research, infeasible solutions have the worst fitness, and the fitness of feasible solutions is measured by the weighted summation of their objective values with random weights. Solutions with the best fitness are selected as parents. For each pair of parents, two new solutions named children are generated by crossover and mutation. Figure 5.6 presents an example of crossover and mutation. Children are generated by exchanging the genes of their parents with a given probability of crossover rate R_c. For every child, its genes are mutated to other feasible values with a given probability of mutation rate R_m. After the crossover and mutation, all children are added into the solution pool. The procedure of parent selection and children generation repeats until a stopping criterion is achieved.

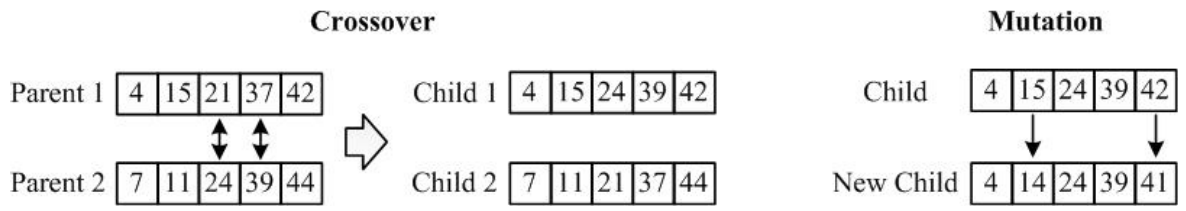


Figure 5.6 Example of crossover and mutation

According to Wang *et al*. (2007), GA is one of the most effective multi-objective heuristics. It has been widely applied in decision making in IAM. An important parameter of GA is the population size determining the number of solutions that are selected as parents. A large population size increases the chance of obtaining good solutions while more time is needed. Other parameters include crossover rate R_c and mutation rate R_m. When they are large, the new solutions inherit fewer genes from their parents so the algorithm is likely to explore broader solution space; otherwise the identified solutions are likely to be improved.

5.1.4 Nondominated Sorting Genetic Algorithm II

NSGA II is based on GA; while it has a superior fitness measurement and therefore it is faster and more effective than the classic GA (Deb *et al.*, 2002).

Similarly with GA, NSGA II also works on genes. Initially, a pool of solutions is randomly generated. In every iteration, solutions are measured by its non-dominancy level (see Section 4.1). Solutions with higher non-dominancy levels are selected as parents. Then the crossover and mutation are applied to the parents, which are same as that of GA. This process repeats until a stopping criterion is satisfied.

According to Bai *et al*. (2012), NSGA II can effectively solve the optimisation problems and help with the trade-offs of objectives in decision making in IAM. Same as GA, NSGA II also has three parameters: one parameter (population size) determines the number of parents, and the other two parameters (crossover rate R_c and mutation rate R_m) determine the inheritance from parents to children.

5.1.5 Discussion of the Applied Multi-Objective Optimisation Techniques

This section introduces the MOO techniques that are frequently applied in decision making in IAM, including Decision Tree, WSM, GA and NSGA II. Many publications certify the applicability of these techniques when handling multiple objectives and obtaining a set of solutions in decision making in IAM. However, the performance of these techniques is different.

Decision Tree enumerates all the possibilities. It can be applied to solve complicated problems and obtain all existing Pareto solutions. On the other hand, it is time consuming and its model may become too complex for large optimisation problems. Hence, Decision Tree is only suitable for short-term or project-level decision making, and not recommended for long-term and network-level decision making in IAM.

WSM transforms a MOO problem into a set of SOO sub-problems. It is simple and fast and guarantees to obtain Pareto solutions for decision making problems in IAM. It can analyse a large number of segments and strategies. Hence, it has the potential to solve long-term and network-level decision making in IAM.

GA and NSGA II iteratively improve their solutions and try to approach Pareto solutions. They can handle various decision making problems including the large and complicated ones. Even though their solutions may not be as good as the solutions of exact methods, they are flexible and could identify good solutions within a given length of time. Hence, they also have the potential to solve long-term and network-level decision making in IAM.

In summary WSM, GA and NSGA II are the potential MOO techniques and added into the MOOT List for further analysis.

5.2 Introduction of New Multi-Objective Optimisation Techniques

A wide range of MOO techniques has been developed to solve various optimisation problems. Little evidence was found that these techniques have been applied in decision making in IAM. This section investigates these new techniques and introduces the ones that have the potential to solve long-term and network-level decision making in IAM. The potential techniques are supplemented into the MOOT List for further study.

According to Figure 5.1, these new MOO techniques contain exact methods and heuristics. The definitions of exact methods and heuristics are introduced in Section 2.3.3. In the following parts, these new techniques and their applications in decision making in IAM are introduced.

5.2.1 Dichotomic Approach

Dichotomic Approach (DA) is an exact method proposed by Cohon *et al*. (1979), Dial (1979) and Cohon (1978) at a similar time. It is developed to solve bi-objective discrete optimisation problems, and can efficiently identify Pareto solutions. DA is applied to bi-objective decision making in IAM and achieves good optimisation results (Chen *et al.*, 2015).

DA is based on WSM but it has a dynamic weighting system where weights are defined using previously identified solutions. An example of its algorithm is shown in Figure 5.7.

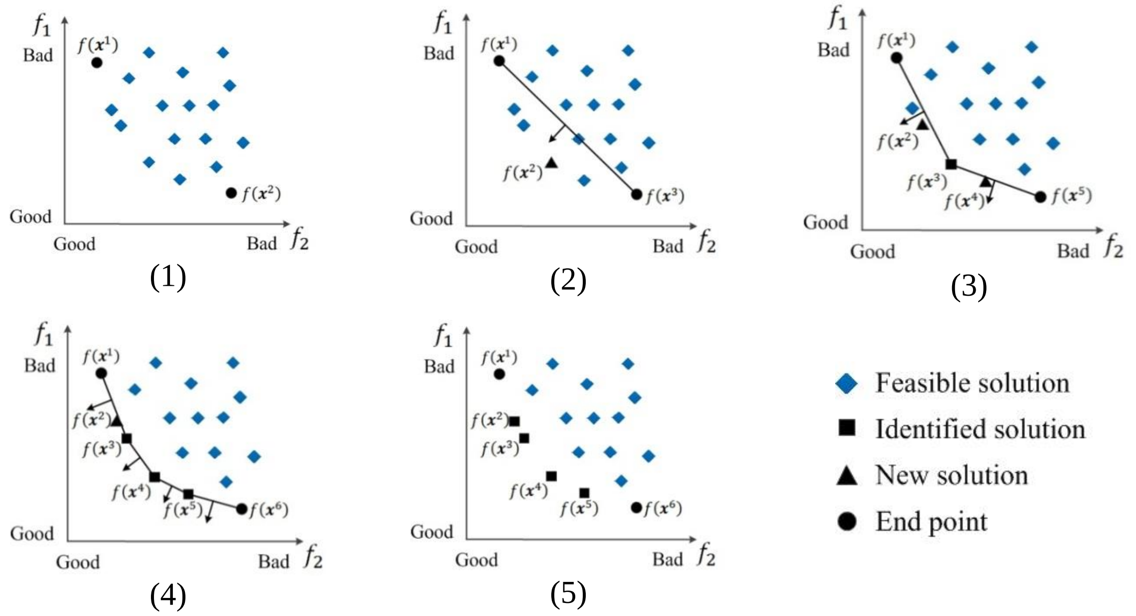


Figure 5.7 Algorithm of DA

Firstly, "end points" are obtained by lexicographically optimising each objective. The end points (see Figure 5.7 (1)) are the best possible solutions measured by their objective values in lexicographic order and indicate the range of Pareto frontier (Isermann, 1982). Iteratively, a pair of consecutive identified solutions is used to produce a weight $\boldsymbol{w} = (w_1,\ w_2)$ using Equations 5.1 and 5.2. Then a new objective is defined with this weight using Equation 5.3. Accordingly, a SOO sub-problem is established by optimising the new objective under the original constraints (Equation 5.4). Figure 5.7 (2), (3) and (4) are examples of iteratively establishing SOO sub-problems. The optimal solutions of SOO sub-problems are the Pareto solutions of the original bi-objective optimisation problem. The algorithm stops when all pairs of consecutive solutions are used to produce weights and define sub-problems. All the identified solutions are Pareto solutions as shown in Figure 5.7 (5).

$$w_1 = f_2(\boldsymbol{x}^{s+1}) - f_2(\boldsymbol{x}^s) \qquad \text{Equation 5.1}$$

$$w_2 = f_1(\boldsymbol{x}^s) - f_1(\boldsymbol{x}^{s+1}) \qquad \text{Equation 5.2}$$

$$\min F = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x}) \qquad \text{Equation 5.3}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \boldsymbol{\Omega} \qquad \text{Equation 5.4}$$

where,

where,	w_1 and w_2	weight of objective 1 and objective 2;
	f_1 and f_2	function of objective 1 and objective 2;
	F	new single objective;
	$\boldsymbol{x}$	a solution;
	$\boldsymbol{x}^s$ and $\boldsymbol{x}^{s+1}$	two consecutive solutions with the index of s and $s+1$; and
	$\boldsymbol{\Omega}$	set of feasible solutions.

Relying on the natural order of Pareto solutions, the classic DA is only applicable to bi-objective optimisation problems. Przybylski *et al*. (2010) introduces a new algorithm which enables DA to optimise three and more objectives. In a weight space as shown in Figure 5.8, each Pareto solution $\boldsymbol{x}$ dominates an area $W\big(f(\boldsymbol{x})\big)$ in which solution $\boldsymbol{x}$ is always the optimal solution when solving the SOO sub-problems established by the weights within the weight set $W\big(f(\boldsymbol{x})\big)$.

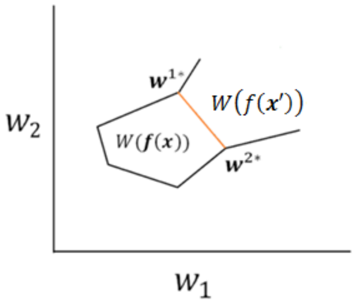


Figure 5.8 Illustration of the weight space

As shown in Figure 5.8, for two consecutive Pareto solutions $\boldsymbol{x}$ and $\boldsymbol{x}'$, their weight areas $W\big(f(\boldsymbol{x})\big)$ and $W\big(f(\boldsymbol{x}')\big)$ have a shared edge between w^{1*} and w^{2*} that when solving the SOO problem established by any weight on the shared edge, both $\boldsymbol{x}$ and $\boldsymbol{x}'$ are the optimal solution. Hence, a new bi-objective sub-problem is established with the shared edge using Equations 5.5-5.7. This bi-objective sub-problem is then solved using the classic DA. Their solutions are the Pareto solutions of the original MOO problem, and are used to define more weights and establish more sub-problems. This algorithm stops when all pairs of identified solutions are analysed and no new solution is obtained.

$$F_1 = \sum_{k=1}^{K} f_k w_k^{1*} \qquad \text{Equation 5.5}$$

$$F_2 = \sum_{k=1}^{K} f_k w_k^{2*} \qquad \text{Equation 5.6}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \boldsymbol{\Omega} \qquad \text{Equation 5.7}$$

where, f_k function of objective k;
F_1 and F_2 function of the new objectives;
w_k^{1*} and w_k^{2*} extreme weight of a shared edge; and
others are same as above.

DA is an effective optimisation technique for bi-objective optimisation, and is able to obtain well-distributed Pareto solutions. With the algorithm developed by Przybylski *et al*. (2010), it can also identify Pareto solutions for three- or more- objective optimisation problems. In addition, the classic DA does not require decision makers to calibrate any parameter; hence its implementation is easy and it can run without decision makers' input. However, DA cannot identify all the existing Pareto solutions. Similar with WSM, it is only able to obtain supported solutions but cannot identify any non-supported solutions. When a MOO problem only has a few Pareto solutions, non-supported solutions may also be needed to produce the set of Pareto solutions.

5.2.2 Epsilon Constraint Method

Epsilon Constraint Method (ECM) is a classic MOO exact method. It is proposed by Haimes *et al*. (1971), which can easily handle multiple objectives and constraints. Hence, ECM is directly applied to various MOO problems (Gearhart, 1979).

ECM handles objectives by converting them into constraints. More specifically, only one main objective is optimised while other objectives are redefined as so-called epsilon constraints. Figure 5.9 illustrates the algorithm of ECM when solving a bi-objective optimisation problem, where objective f_1 is the main objective.

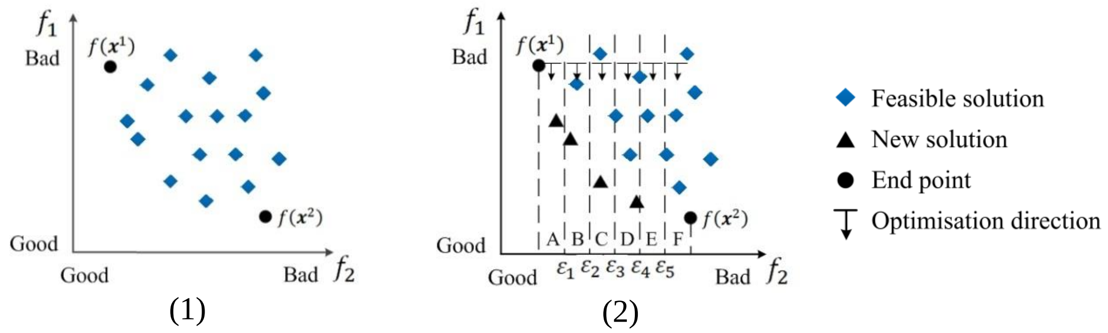


Figure 5.9 Algorithm of ECM for bi-objective optimisation

Firstly, similarly with DA, the end points are identified by lexicographically optimising each objective and present the range of objective values (see Figure 5.9 (1)). Except for the main objective, the feasible ranges of other objectives are equally divided into $(n_s + 1)$ sections, each with an epsilon value (see Figure 5.9 (2)). A SOO sub-problem is established by optimising the main objective f_{main} (Equation 5.8), defining the epsilon constraints of the other objective(s) (Equation 5.9) and subjecting them to the original constraints (Equation 5.10). For example, in Figure 5.9 (2), the feasible range of objective f_2 is equally divided into six ($n_s = 5$) sections, and five epsilon values are defined. For each epsilon, a SOO sub-problem is established. Finally, all the sub-problems are solved and their solutions are also the solutions of the original MOO problem.

$$\min f_{main}(\boldsymbol{x}) \qquad \text{Equation 5.8}$$

s.t. $f_k(\boldsymbol{x}) \leq \varepsilon_k, \;\; k = 1,2,\dots,K, k \neq main$ Equation 5.9

$\boldsymbol{x} \in \boldsymbol{\Omega}$ Equation 5.10

where, f_{main} function of the main objective;

ε_k epsilon value of objective k;

$main$ index of the main objective; and

others are same as above.

ECM can effectively handle multiple objectives and identify Pareto solutions. The critical part of ECM is the definition of epsilon. In this research, equidistant values of epsilon are defined, which is similar to the weight definition of WSM in Figure 5.4. When n_s is larger, more epsilons are generated; so more SOO sub-problems are established and more solutions can be obtained. When n_s is large enough, ECM can identify all existing Pareto solutions for a MOO problem. However, when n_s is large, the computation time may be too long when identifying many solutions.

5.2.3 Revised Normal Boundary Intersection

Revised Normal Boundary Intersection (RNBI) is an exact method that hybridises the normal boundary intersection method and the global shooting method (Shao and Ehrgott, 2007). It is designed to identify good representatives of Pareto solutions for continuous MOO problems. With modifications, this method is applicable to solve MOO problems in decision making in IAM.

RNBI defines reference points and establishes SOO sub-problems by measuring the distance between reference points and feasible solutions. Figure 5.10 demonstrates the algorithm when solving a bi-objective optimisation problem.

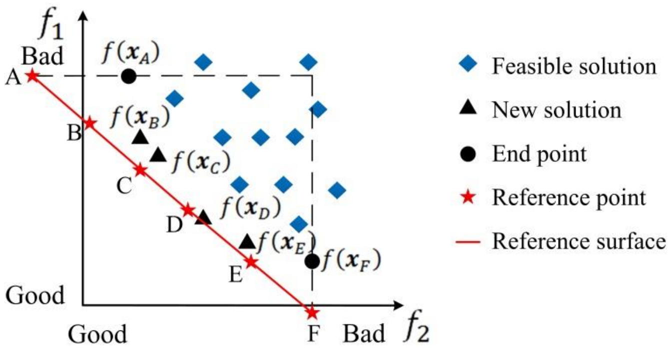


Figure 5.10 Algorithm of RNBI

Firstly, similarly with WSM, end points of the Pareto frontier are identified by lexicographically optimising each objective. Then RNBI establishes a reference surface in the objective space by connecting the end points. This reference surface is a line in a bi-objective optimisation problem (e.g. Figure 5.10), a plane in three-objective optimisation problems or a hyper-plane in four- or more-objective optimisation problems. Next, equidistant points are constructed on the reference surface such as the points A-F in Figure 5.10. SOO sub-problems are established to identify the feasible solution of the original problem that is the closest to each reference point.

In classic RNBI, the distance is measured along the normal direction. However, in decision making in IAM, Pareto solutions are discretely located and may not be in the normal direction of reference points such as the example in Figure 5.11. In this research, Tchebyshev distance given by Equation 5.12 is used to measure the distance between reference points and feasible solutions. In the example of Figure 5.11, a new solution is obtained for the reference point with the minimum Tchebyshev distance of $|f_1(\boldsymbol{x_r}) - f_1(\boldsymbol{x_p})|$. When adopting Tchebyshev distance, all objectives should be normalised into same scale.

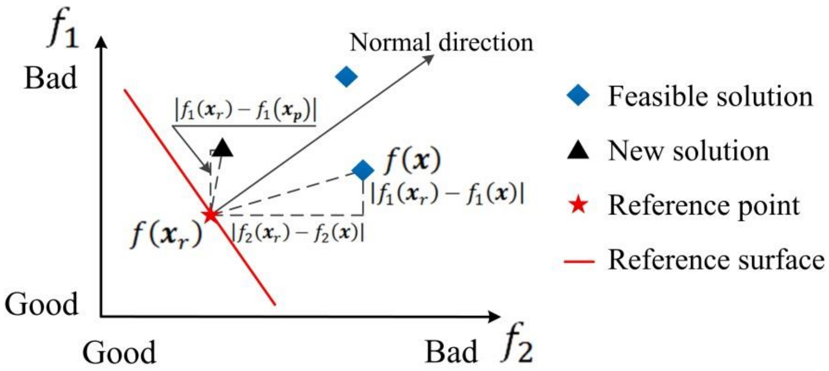


Figure 5.11 Example of distance measurement

An SOO sub-problem is defined for each reference point using Equations 5.11-5.13, and its optimal solution is a potential Pareto solution of the original MOO problem. This algorithm stops once all reference points have been analysed.

$$\min d_{Tchebyshev} \qquad \text{Equation 5.11}$$

$$\text{s.t.} \quad d_{Tchebyshev} = \max_{k=1,2,\cdots,K} (|f_k(\boldsymbol{x}) - f_k(\boldsymbol{x_r})|) \qquad \text{Equation 5.12}$$

$$\boldsymbol{x} \in \boldsymbol{\Omega} \qquad \text{Equation 5.13}$$

where, $d_{Tchebyshev}$ Tchebyshev distance;

$\boldsymbol{x_r}$ reference point; and

others are same as above.

RNBI can identify good representatives of Pareto solutions for MOO problems in decision making in IAM. An important part of RNBI is the density of reference points. When the objective ranges are divided into n_s parts, $(n_s + 1)$ equidistant points are established on each objective vector to locate reference points. This is similar to the weight definition of WSM (see Figure 5.4). When n_s is large, more reference points are established, more SOO sub-problems are established and more solutions could be obtained. However, not all of the SOO sub-problems guarantee a unique Pareto solution.

5.2.4 Simulated Annealing

Simulated Annealing (SA) is a heuristic that mimics the annealing of solids (Kirkpatrick *et al.*, 1983). It is a descent and stochastic algorithm that quickly improves identified solutions (Serafini, 1994; Aarts *et al.*, 2005). SA was initially developed for discrete SOO problems, and later adapted to MOO problems (Czyzżak and Jaszkiewicz, 1998; Suresh and Mohanasundaram, 2006; Antunes *et al.*, 2011).

In this research, a multi-objective version of SA developed by Czyzak and Jaskiewicz (1998) is introduced. The core idea of this algorithm is that a solution corresponds to a solid which modifies its status within the space of possible solutions. Figure 5.12 shows the flowchart of this algorithm.

A solid is in a current status $\boldsymbol{y}$, and has a neighbourhood of solutions $N(\boldsymbol{y})$ that can be reached by making a simple modification from $\boldsymbol{y}$. In this research, the neighbourhood is defined as the set of solutions with at least 99% of the selected strategies identical to those of $\boldsymbol{y}$. The algorithm randomly generates a new solution $\boldsymbol{y}_{new}$ in the neighbourhood $N(\boldsymbol{y})$, and then considers updating the current status from $\boldsymbol{y}$ to $\boldsymbol{y}_{new}$. Figure 5.12 shows the status update. If $\boldsymbol{y}_{new}$ dominates $\boldsymbol{y}$, the status is updated to y_{new}. If $\boldsymbol{y}_{new}$ is dominated by $\boldsymbol{y}$, the status is updated to $\boldsymbol{y}_{new}$ with a given probability p. When $\boldsymbol{y}$ and $\boldsymbol{y}_{new}$ are not dominating each other, the status is updated to $\boldsymbol{y}_{new}$ with probability $P(\boldsymbol{y}, \boldsymbol{y}_{new}, t, w)$ given by Equation 5.14, where the weight w is initially randomly generated and then iteratively updated using Equation 5.15. The temperature of the solid decreases when $\boldsymbol{y}_{new}$ is not dominated by $\boldsymbol{y}$. The procedure of the

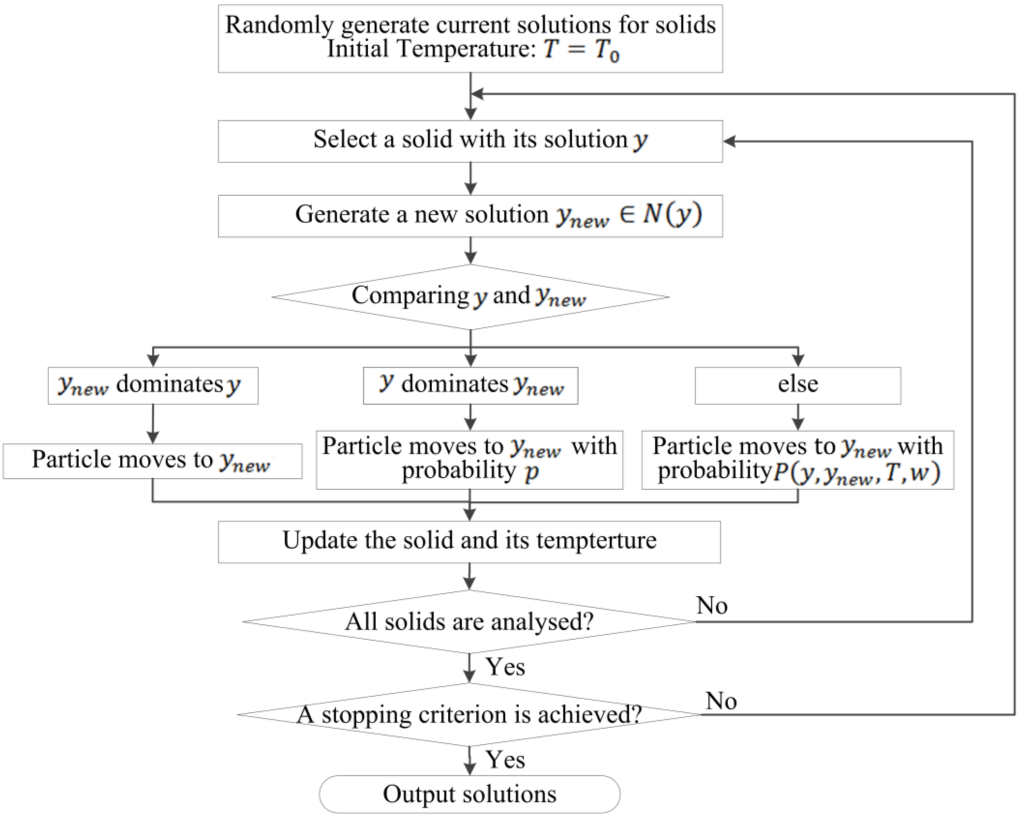


Figure 5.12 Flowchart of SA

status update is performed with each of the solids, and then the solution generation repeats until a stopping criterion is achieved.

$$P = \min\left\{1, \exp\left(\sum_{k=1}^{K} w_k\big(f_k(\boldsymbol{y}) - f_k(\boldsymbol{y}_{new})\big)/t\right)\right\} \qquad \text{Equation 5.14}$$

$$w_k = \begin{cases} \alpha w_k & f_k(\boldsymbol{y}) \textit{ is not worse than } f_k(\boldsymbol{y}_{new}) \\ w_k/\alpha & f_k(\boldsymbol{y}) \textit{ is worse than } f_k(\boldsymbol{y}_{new}) \end{cases} \qquad \text{Equation 5.15}$$

where, P update probability

w_k a weight on objective k;

$\boldsymbol{y}$ and $\boldsymbol{y}_{new}$ two feasible solutions;

t current temperature;

α parameter; and

others are same as above.

A main parameter of SA is a given probability p that controls the chance of accepting non-improved solutions. When the probability p is large, non-improved solutions have higher chance to be accepted as new solid status, so the solution diversity is enhanced; otherwise, good solutions are enhanced. The parameter α ($\alpha = 1.05$) is for weight computing. The parameter of population size determines the number of solids. Other parameters determine the temperature and its reducing.

5.2.5 Tabu Search

Tabu Search (TS) is a local search heuristic that establishes tabu lists to record identified solutions and forces the algorithm to explore more space of feasible solutions. It is said to have outstanding flexibility and good solution coverage (Hertz *et al.*, 1995). The idea of TS was initially proposed in 1977 (Glover) to solve discrete SOO problems. Then it was modified to solve MOO problems (Jaeggi *et al.*, 2008; Ghisu *et al.*, 2010; Stephen and Somasundaram, 2012).

This section introduces a multi-objective version of TS developed by Jaeggi *et al*. (2008). An essential part of this algorithm is a set of lists namely short-term memory (STM), medium-term memory (MTM), long-term memory (LTM) and Intensification Memory (IM). In detail, STM records the recently visited solutions. MTM records the identified non-dominated solutions for intensification. LTM records all the identified solutions for diversification. IM contains the solutions that are not dominated by the current solutions nor selected for the next iteration.

Figure 5.13 shows the flowchart of the algorithm. This algorithm initially generates a set of solutions S as current solutions. For a current solution $\boldsymbol{y} \epsilon S$, a so-called Hooke & Jeeves Move (H&J) is performed. H&J in this research generates a set of new solutions for $\boldsymbol{y}$ from its neighbourhood (same as the neighbourhood of SA). Then one of the new solutions $\boldsymbol{y}_{new}$, which must be feasible, non-dominated and not in STM, is selected to replace $\boldsymbol{y}$ as a new current solution. Once all current solutions are renewed, the four lists are updated based on their definition. This procedure repeats until a stopping criterion is achieved.

This algorithm also performs three enhancements as shown in Figure 5.13. Every $Step_D$ iterations, diversity enhancement replaces current solutions $\boldsymbol{y}$ with random solutions from LTM with the intention of improving solution diversity. Every $Step_I$ iterations, intensity enhancement replaces current solutions with random solutions from IM with the intention of fast improving the quality of identified solutions. Every $Step_r$ iterations, restart enhancement replaces current solutions with random solutions from MTM and updates STM with the intention of improving the algorithm efficiency.

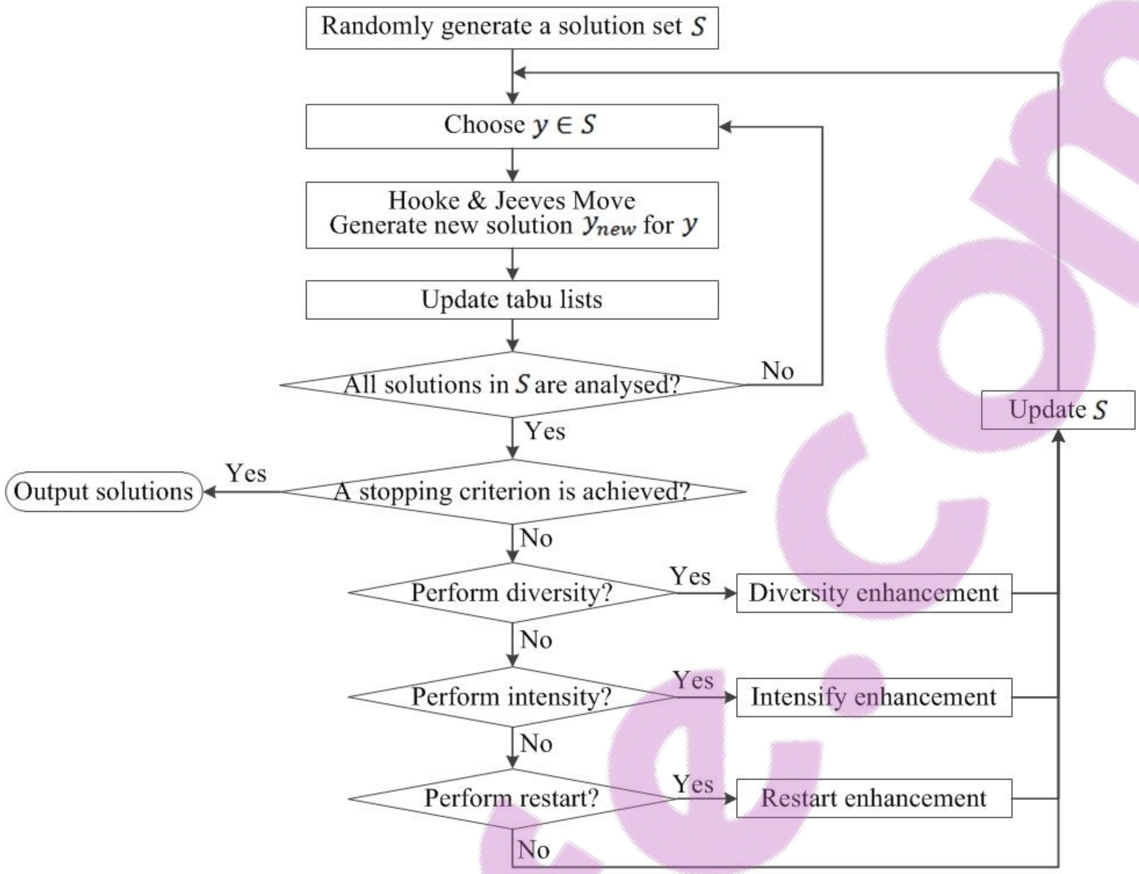


Figure 5.13 Flowchart of TS

There are four main parameters of TS. Parameter $Step_{STM}$ determines the number of iterations that a newly identified solution stays in STM. When it is large, solutions stay in STM longer so the algorithm explores more of the solution space. Parameters $Step_D$, $Step_I$ and $Step_R$ determine the frequency of the diversity, intensity and restart enhancement. When they are large, these enhancements have less frequency. TS also has population size that determines the number of current solutions.

5.2.6 Ant Colony Optimisation Method

Ant Colony Optimization method (ACO) is a heuristic that imitates ants' behaviour when searching for food (Dorigo, 1992). It is a swarm method, where all ants are parallelisable (Dorigo and Socha, 2006). Hence, it can quickly explore the solution space and improve the identified solutions. ACO was originally proposed for discrete SOO problems, including the SOO in decision making in IAM (Lukas and Borrmann, 2011).

In this research, a multi-objective version of ACO developed by Doerner *et al.* (2006) is introduced. Its core idea is that a solution corresponds to an ant that moves around the space of possible solutions. Figure 5.14 shows the flowchart of its algorithm.

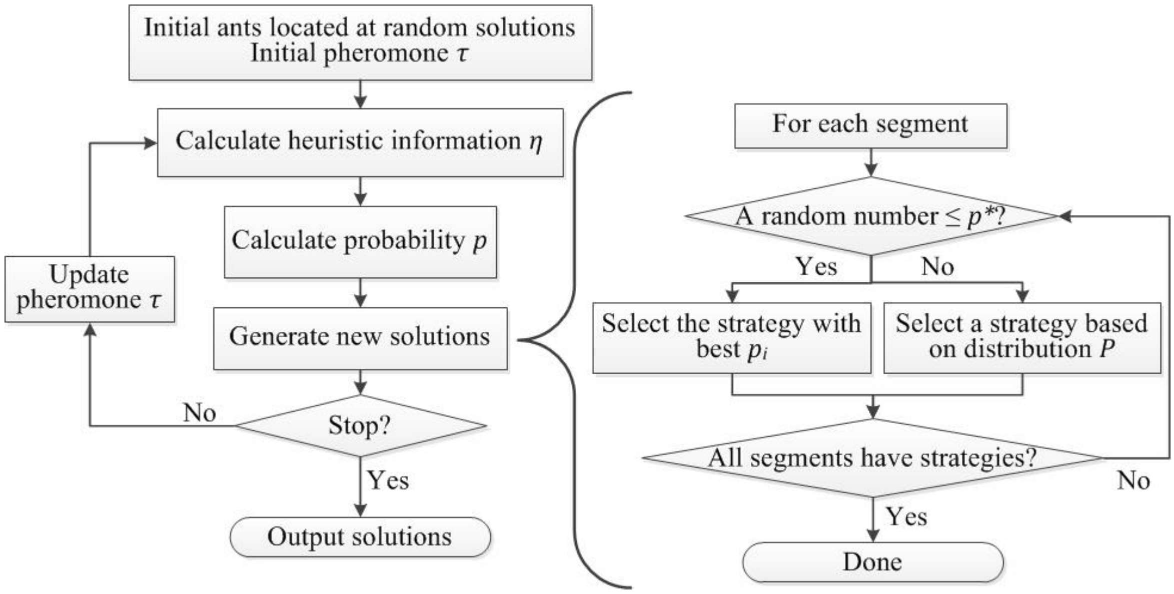


Figure 5.14 Flowchart of ACO

Initially, a set of ants is located at randomly generated solutions with initial pheromone τ. The heuristic information η is measured by weighted summing the objective values of a solution with a randomly generated weight. It is similar to the definition of fitness of multi-objective GA. The performance p_i of alternative strategies of a segment is measured by Equation 5.16. Then an ant moves to a new solution $\boldsymbol{y}_{new}$ by building its path. In the context of decision making in IAM, each segment of an infrastructure asset network is a path section. With a given probability of p^*, the strategy with the best performance is selected for a segment; otherwise, a strategy is selected based on a probability distribution P_i given by Equation 5.17. A path is completed when all segments have corresponding strategies. Then the ant moves to a new solution that contains the selected strategies and updates its pheromone using Equation 5.18. The movement repeats until a stopping criterion is achieved.

$$p_i = \left[\sum_{k=1}^{K}\left(w_k \tau_{k,i}\right)\right]^{\alpha} (\eta_i)^{\beta} \quad \text{Equation 5.16}$$

$$\mathrm{P}_i = \frac{p_i}{\sum_{i\in\delta_j} p_i} \quad \text{Equation 5.17}$$

$$\tau_{k,i} = (1-\rho)\tau_{k,i} + \rho\Delta\tau \qquad \text{Equation 5.18}$$

where,	p_i and η_i	performance and heuristic information of strategy i;
	$\tau_{k,i}$	pheromone of strategy i with respect to objective k;
	w_k	a random weight on objective k;
	α, β, ρ and $\Delta\tau$	parameters; and
	others are same as above.	

There are four main parameters of ACO. The probability p^* affects the selection of strategies. When it is large, the best-performance strategies have a higher chance to be selected for new solutions. Parameters α and β determine the impact of pheromone and heuristic information on the solution performance. When they are large, pheromone and heuristic information have higher impact on the solution performance. Parameter $\Delta\tau$ determines the performance update in Equation 5.18. ACO also has the population size that determines the number of ants.

5.2.7 Particle Swarm Optimisation method

Particle Swarm Optimisation method (PSO) is a heuristic that imitates the social behaviour of birds flocking and fish schooling (Kennedy and Eberhart, 1995). It is a stochastic and population based search method, which is able to continuously improve identified solutions (Poli *et al.*, 2007). PSO was originally developed for continuous SOO problems, and later was modified to solve discrete MOO problems (Ho *et al.*, 2005; Zhao and Suganthan, 2010; Liang *et al.*, 2012). Lertworawanich (2012) applies a multi-objective version of PSO in decision making in IAM; but only one preferred solution is obtained.

In this research, a multi-objective version of PSO developed by Ho *et al*. (2005) is introduced. The core idea of this algorithm is that particles move according to their velocity in the space of possible solutions. To ease the implementation, all solutions are expressed in a binary representation. For example, if a solution selects strategies with indices of 1, 5 and 7, its binary expression is (001, 101, 111). Figure 5.15 shows the flowchart of this algorithm.

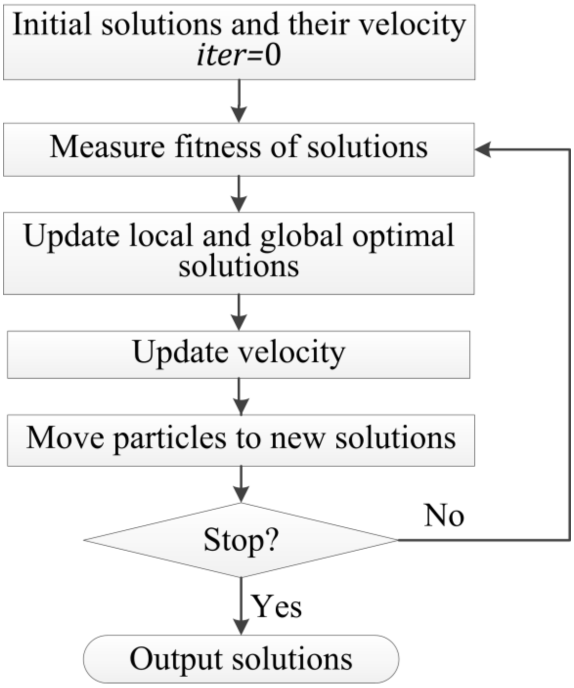


Figure 5.15 Flowchart of PSO

Initially, a set of particles are located at randomly generated solutions and have random velocity. Then all solutions are measured with fitness using Equation 5.19, where strength s^l of a solution $\boldsymbol{y}^l$ is defined as the percentage of the identified solutions that are dominated by $\boldsymbol{y}^l$. This definition of fitness penalises the densely packed particles and improves solution diversity by focusing on the low-strength non-dominated solutions. Then solutions namely local optimal solutions and global optimal solutions are defined to lead all particles. The local optimal solution of a particle is the best-fitness solution that this particle has visited; and the global optimal solution is the best-fitness one of all local optimal solutions. They are updated at every iteration. The velocity of all particles is renewed with the local and global optimal solutions using Equation 5.20; and all particles move to new solutions along their velocity using Equation 5.21. This procedure repeats until a stopping criterion is achieved.

$$f^l = \frac{1}{1 + \sum_{i\ (\boldsymbol{y}^i \text{dominates}\ \boldsymbol{y}^l)} s^i}, \qquad \boldsymbol{y}^i \in \boldsymbol{Y}_{ND} \qquad \text{Equation 5.19}$$

$$v_i^{l,iter} = \omega v_i^{l,iter-1} + c_1 r_1 (p_i^{l,iter} - y_i^{l,iter}) + c_2 r_2 (g_i^{iter} - y_i^{l,iter}) \qquad \text{Equation 5.20}$$

$$y_i^{l,iter+1} = \begin{cases} 1 & r < S\left(v_i^{l,iter}\right) \\ 0 & \text{otherwise} \end{cases} \qquad \text{Equation 5.21}$$

where, $\boldsymbol{Y}_{ND}$ set of non-dominated solutions in the solution pool;

s^i	strength of solution $\boldsymbol{y}^i$;
f^l	fitness of solution $\boldsymbol{y}^l$;
$v_i^{l,iter}$	velocity of element i of Particle l at iteration $iter$;
$p_i^{l,iter}$ and g_i^{iter}	values of element i of local optimal solution of Particle l and global optimal solution at iteration $iter$;
$y_i^{l,iter}$	value of element i of Particle l atiteration $iter$;
$S(\cdot)$	sigmoid function;
$iter$	index of iterations;
ω, c_1 and c_2	parameters; and
r, r_1 and r_2	random numbers between 0 and 1.

There are three main parameters ω, c_1 and c_2 determining the impact of previous velocity, local optimal solutions and global optimal solutions on the new velocity. When they are large, previous velocity, local optimal solutions and global optimal solutions have higher impact on velocity. The population size of PSO determines the number of particles.

5.2.8 Discussion of the New Multi-Objective Optimisation Techniques

Table 5.2 summarises the new techniques introduced in this section. All of them are applicable to solving MOO problems in decision making in IAM and have the ability to handle a large number of segments and strategies. DA is initially developed to optimise two objectives, but can be modified to optimise three or more objectives. RNBI is developed to solve continuous problems, but with Tchebycheff distance, it is applicable to solve optimisation problems in decision making in IAM. The other techniques can be directly applied. In comparison, exact methods, based on mathematical theories, can quickly solve a MOO problem and identify a set of Pareto solutions; and heuristics, based on a specific mechanism, are flexible and may be able to obtain acceptable solutions within a given length of time. All of them have the potential to assist long-term and network-level decision making in IAM. Hence, they are added into the MOOT List for further analysis.

Table 5.2 Summary of the new MOO techniques

Technique	**Class**	**Applicable in decision making in IAM?**	**No. of objectives**	**Pareto solutions?**	**Large-scale problems?**
DA	Exact methods	Yes	2	Yes	Yes
ECM		Yes	2+	Yes	Yes
RNBI		Modified	2+	Yes	Yes
SA	Heuristics	Yes	2+	Unsure	Yes
TS		Yes	2+	Unsure	Yes
ACO		Yes	2+	Unsure	Yes
PSO		Yes	2+	Unsure	Yes

5.3 Preparation of Databases and Computer Techniques

In the last sections, a list of MOO techniques is investigated and needs to be tested. This section prepares the required databases and computer techniques for the tests.

Databases: This research focuses on the long-term and network-level decision making in IAM; hence the testing databases should be based on this type of decision making problems. Furthermore, this research aims at a MOO technique that is able to solve different decision making problems; hence, different databases should be used to test the applicability and scalability of MOO techniques in the context of decision making in IAM.

In this research, two databases of practical decision making problems, City A and B, are selected for experimental tests. A summary of the databases is shown in Table 5.3, where their decision making problems are introduced in Section 5.5.

These two decision making problems are pre-analysed using an IAM software tool named dTIMS CT 8 that is able to generate all the alternative strategies for the segments of an infrastructure asset network and estimate the outcomes of all strategies (Deighton Associates Limited, 2008). The number of alternative strategies for the entire network is shown in Table 5.3.

Table 5.3 Summary of databases

City name	Infrastructure type	Number of segments	Number of strategies	Analysis period (years)	Outcomes
A	Roads	2,789	195,042	20	benefit, cost, benefit-cost-ratio, yearly cost, AADT[1], age, PPI[2], RCI[3], SDI[4], etc.
B	Roads	1,301	78,700	22	benefit, cost, benefit-cost-ratio, yearly cost, AADT[1], age, IRI[5], rutting, SCI[6], SDI[4], etc.

Note: [1] AADT: average annual daily traffic, it is estimated based on the traffic;
[2] PPI: pavement performance index, it is defined based on the specific decision making;
[3] RCI: riding comfort index, it measures the pavement condition from users' perspective;
[4] SDI: surface distress index, it is based on the indication of poor pavement or signs of failure;
[5] IRI: International Roughness Index, it is based on longitudinal road profiles; and
[6] SCI: Structural Condition Index, it measures the pavement structure condition.

Table 5.4 shows an example of the strategies and their outcomes (i.e. cost and condition index). Outcomes are estimated based on the status of the segments and the designed interventions. For example, when implementing intervention I2 in year 3 to segment A, the condition of this segment is improved from 7.76 to 3.00, and a cost of $26,000 is required. Other outcomes can be calculated in a similar way according to their definition. It is important to note that every decision making problem has its own outcome calculation and valuing systems, therefore even the same outcomes (e.g. benefit) may have different calculation and valuing in different decision making problems. Then strategies apply different interventions at different points of

Table 5.4 Example of strategies

Segment Index	Strategy Index	Treatment[1] (cost[2]/condition index)						
		Initial Condition	Year 1	Year 2	Year 3	Year 4	Year 5	Total Cost/ Final Condition
A	A1	7.50	I1 59/1.50	- 0/1.76	- 0/2.01	- 0/2.27	- 0/2.54	59/2.54
A	A2	7.50	- 0/7.50	- 0/7.76	I2 26/3.00	- 0/3.26	- 0/3.52	26/3.52
A	A3	7.50	- 0/7.50	- 0/7.76	- 0/8.02	- 0/8.28	I2 26/3.00	26/3.00
B	B1	5.90	I3 30/1.50	- 0/1.75	- 0/2.00	- 0/2.26	- 0/2.52	30/2.52
B	B2	5.90	- 0/6.15	- 0/6.40	I3 30/1.50	- 0/1.75	- 0/2.00	30/2.00

Note: [1] I1, I2 and I3 represent different interventions; and "-" represents no intervention is applied.
[2] The unit of cost is $1000. All costs shown in the table are actual value in the spending year.

time, therefore generating different outcomes. The databases record all the information including segments, strategies and their outcomes.

Computer techniques: In this research, all the tests are conducted on a computer with Intel (R) Core™ i5 processor, 3.33 GHz Central Processing Unit (CPU), 4.00 GB Random Access Memory (RAM). All the computer programmes are written using Python 2.7.3 (Lutz, 2013) by the author. Gurobi 5.5.0 (Gurobi Optimization Inc, 2012) is used as a SOO solver for the exact methods. All the types of time collected in the case study are measured as CPU time.

5.4 Establishment of a Measurement Framework to Evaluate the Effectiveness and Efficiency of Multi-Objective Optimisation Techniques

This section establishes a measurement framework to evaluate the effectiveness and efficiency of MOO techniques from the viewpoint of practical decision making in IAM, which also eases the assessment of MOO techniques and enhances the understanding of their performance. Although many MOO techniques are applied in decision making in IAM; few publication has been found to document the performance measurement of MOO techniques. In most cases respective optimisation techniques are tested for their applicability to a given problem, without necessarily comparing their performance to other techniques. Some research compares two or three MOO techniques with limited measurement criteria e.g. (Tack and Chou, 2002; Dridi *et al.*, 2008). This comparison is relatively narrow and cannot be used to benchmark and compare different types of MOO techniques. Therefore a measurement framework to rationally and comprehensively describe the performance of MOO techniques is necessary. This section firstly outlines the significance of a comprehensive measurement framework, and then develops a framework with criteria to measure the performance of MOO techniques in decision making in IAM.

5.4.1 Significance of a Comprehensive Measurement Framework

This research has highlighted a vast number of optimisation techniques that are applicable for solving the IAM. Yet, little work has been done to compare or benchmark the performance of optimisation techniques with each other. Even if a variety of criteria are defined to measure MOO techniques (Okabe *et al.*, 2003; Becker and Rauber, 2011); not all of them are effective or meaningful in the context of decision making in IAM. Okabe *et al*. (2003) points out that criteria may "fail to truly reflect the quality of solutions" when misusing them. Hence, a suitable measurement and criteria that provide a "trustworthy, reproducible, and repeatable" result when measuring MOO techniques in the context of decision making in IAM are necessary (Becker and Rauber, 2011). The following paragraphs outline the characteristics of decision making in IAM and its requirements on MOO.

To begin with, practical decision making problems often have a large number of segments and strategies. For example, a city may have over 1,822 segments and over 64,000 twenty-year maintenance strategies. MOO techniques are required to analyse all the segments and strategies, and identify solutions in a reasonable time.

Secondly, MOO problems of practical decision making may have thousands of Pareto solutions. As explained in Section 4.3.4, rather than all the existing Pareto solutions, a set of good representatives are needed to show the achievable outcomes and the relationships of objectives. MOO techniques need to identify good representatives of Pareto solutions for optimisation problems of decision making in IAM.

Thirdly, decision making problems are unique. Practical decision making problems may have different perspectives on MOO (Lacerda *et al.*, 2011). For instance, decision makers may want to identify an expected number of solutions within a preferred time. MOO techniques should be flexible so that specific expectations can be achieved.

According to the statements above, a measurement framework for decision making in IAM should evaluate:

- Solution quality: the goodness of identified solutions,
- Solution distribution: the representativeness of identified solutions,
- Computation time measures: the efficiency of techniques, and
- Implementation considerations: the flexibility and controllability of techniques.

The outline of the measurement framework is shown in Figure 5.16. This framework and its criteria are introduced in the following sections.

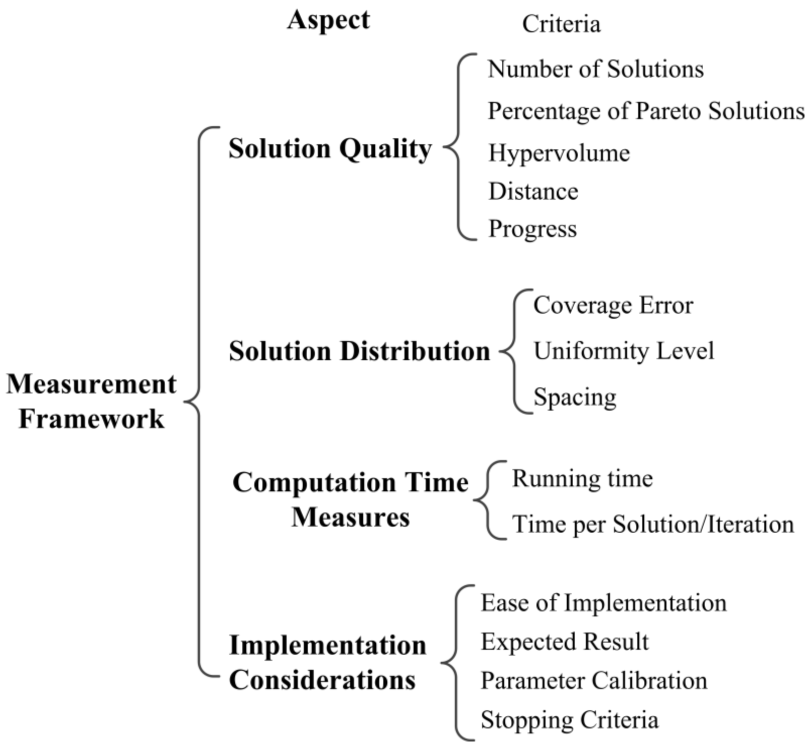


Figure 5.16 Structure of the measurement framework

5.4.2 Solution Quality

MOO techniques are supposed to identify Pareto solutions. However, not all of the techniques guarantee to obtain Pareto solutions. Solution quality measures the number and goodness of identified solutions. It has five criteria: number of solutions, percentage of Pareto solutions, distance, hypervolume and progress.

Number of solutions: This is a cardinality based criterion that measures the number of unique non-dominated solutions identified by a MOO technique (Sayin, 2000). If a technique identifies more unique non-dominated solutions than others under the same circumstance, this technique is more flexible and may perform better from the perspective of solution quality.

When the solutions of all techniques have same goodness, i.e. all solutions are Pareto solutions; this criterion can effectively measure the solution quality. When the solutions have different goodness, the technique identifying more unique solutions has a higher chance to generate a good solution frontier. However, when solution goodness is different, this criterion cannot independently measure the solution quality.

Percentage of Pareto solutions: This is another cardinality based criterion that examines the percentage of Pareto solutions from all identified non-dominated solutions (Sayin, 2000;

Zitzler *et al.*, 2000). If a technique identifies a higher percentage of Pareto solutions, it has a higher chance to generate Pareto solutions when solving similarly MOO problems.

This criterion is an important criterion of solution quality. However, if solutions identified by all the measured algorithm have same solution goodness, i.e. they are all Pareto solutions or no solution is the Pareto solution, this criterion is not informative; and the criterion of distance or hypervolume should be used to measure the solution quality.

Distance: This criterion directly describes the goodness of identified solutions by measuring the distance between Pareto solutions and identified solutions (Sarker and Coello, 2002; Tan *et al.*, 2005). In this research, the distance between two solutions refers to their Euclidean distance in objective space given by Equation 5.22. For an identified solution $\boldsymbol{x}$, its distance $d(\boldsymbol{x})$ is defined as its shortest distance to any existing Pareto solution (see Equation 5.23). Then the distance criterion is defined as the Generational Distance (GD) between the set of identified solutions $\boldsymbol{X}$ and the set of Pareto solutions $\boldsymbol{X_p}$ (see Equation 5.24). To ease the computation, in Equation 5.24, parameter $\gamma\ = 1$; therefore GD is acutally the average of all $d(\boldsymbol{x})$. When the distance criterion is small, averagely the set of identified solutions is close to Pareto frontier and good. In the best scenario, the distance is 0 and this solution is a Pareto solution.

$$d(\boldsymbol{x},\boldsymbol{x}') = \sqrt{\sum_{k=1}^{K}\left(f_k(\boldsymbol{x}) - f_k(\boldsymbol{x}')\right)^2} \qquad \text{Equation 5.22}$$

$$d(\boldsymbol{x}) = \min_{\forall \boldsymbol{x_p}\in \boldsymbol{X_p}} d\left(\boldsymbol{x},\boldsymbol{x_p}\right) \qquad \text{Equation 5.23}$$

$$GD(\boldsymbol{X},\ \boldsymbol{X}_p) = \frac{1}{N_{\boldsymbol{X}}}\sqrt[\gamma]{\sum_{x\in X}\left(d(\boldsymbol{x})\right)^{\gamma}} \qquad \text{Equation 5.24}$$

where,

$d(\boldsymbol{x})$	distance of a solution $\boldsymbol{x}$ to its closest Pareto solution in objective space;
$d(\boldsymbol{x},\boldsymbol{x}')$	Euclidean distance between solutions $\boldsymbol{x}$ and $\boldsymbol{x}'$;
$GD(\boldsymbol{X},\ \boldsymbol{X}_p)$	GD between solution sets $\boldsymbol{X}$ and $\boldsymbol{X}_p$;
$\boldsymbol{x}$ and $\boldsymbol{x}'$	feasible solutions;
$\boldsymbol{X}$	set of identified solutions;
$\boldsymbol{X_p}$	set of all Pareto solutions;
N_X	Number of solutions in $\boldsymbol{X}$;

γ Parameter; and

others are same as above.

This criterion directly and precisely describes the solution quality of MOO techniques. However its computation requires that the set of all existing Pareto solutions $\boldsymbol{X_p}$ is known. When $\boldsymbol{X_p}$ is not be obtainable, this criterion is not applicable and the criterion of hypervolume could be used as a replacement.

Hypervolume: This criterion also directly measures the goodness of identified solutions (Fleischer, 2003; Zitzler *et al.*, 2007). It measures the volume of the area surrounded by an identified solution $\boldsymbol{x}$ and an ideal point in the objective space. Figure 5.17 shows the hypervolume of a bi-objective optimisation example.

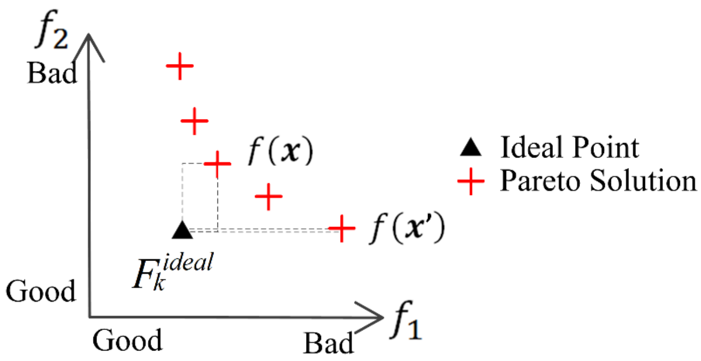


Figure 5.17 Example of hypervolume

The ideal point is a utopian point in objective space, which contains the best value of every objective. If an ideal point is not given, it can be defined as the origin in objective space after normalising all objectives to be minimised. Then the hypervolume of a solution $\boldsymbol{x}$ is calculated by Equation 5.25. The solution quality of a technique is measured by the average hypervolume of all identified solution. When the solutions of a technique are good, they are probably near to the ideal point, so the hypervolume of this technique is small.

$$HV(\boldsymbol{x}) = \prod_{k=1}^{K} \left(f_k(\boldsymbol{x}) - F_k^{ideal}\right) \qquad \text{Equation 5.25}$$

where, $HV(\boldsymbol{x})$ hypervolume of a solution $\boldsymbol{x}$;

F_k^{ideal} value of an ideal point on objective k; and

others are same as above.

Hypervolume only roughly measures the solution goodness. For example, comparing solutions $\boldsymbol{x}$ and $\boldsymbol{x}'$ in Figure 5.17, solution $\boldsymbol{x}$ has larger hypervolume therefore is recognised as a poorer solution. Actually $\boldsymbol{x}$ and $\boldsymbol{x}'$ are both Pareto solutions and have same goodness. Hypervolume is not as accurate as distance but it does not require the set of all Pareto solutions. Hence, it is recommended as a replacement of the criterion of distance.

Progress: This criterion is only a measure applicable to heuristics (Collette and Siarry, 2003). Heuristics improve solutions in an iterative manner. Progress P given by Equation 5.26 describes the solution improvement with iterations.

$$P = \ln \sqrt{\frac{GD(\boldsymbol{X}_0,\ \boldsymbol{X}_{\boldsymbol{p}})}{GD(\boldsymbol{X}_{iter}, \boldsymbol{X}_{\boldsymbol{p}})}} \qquad \text{Equation 5.26}$$

where, P progress;

$\boldsymbol{X}_0$ set of all initial solutions ;

$\boldsymbol{X}_{iter}$ set of solutions identified at iteration $iter$; and

others are same as above.

This criterion directly compares solution goodness of heuristics when solving an optimisation problem. If a heuristic has larger progress, it improves its solutions faster hence has better solution quality. Also, this criterion measures the efficiency of heuristics. Figure 5.18 shows an example of the progress where a heuristic is applied to solve a MOO problem. In the first 200 iterations, its progress increases largely, which means the identified solutions are greatly improved. After 300 iterations, the progress remains at similar values, which means the identified solutions are not obviously improved. In general, if a heuristic can quickly generate high progress, this heuristic is efficient; otherwise, if a heuristic slowly improve its progress, it

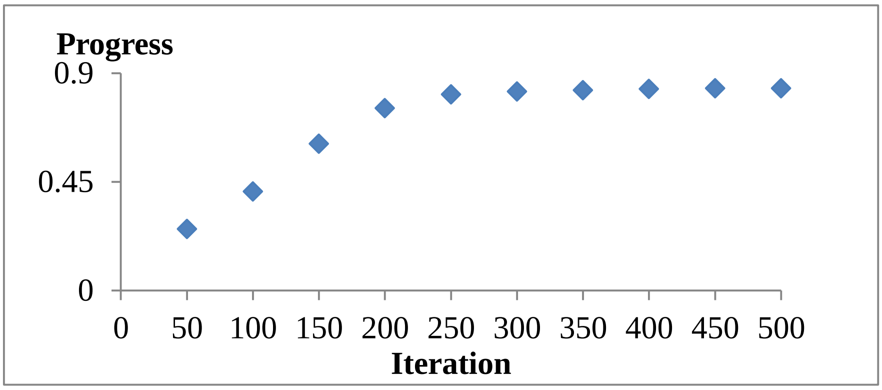


Figure 5.18 Example of progress

is slow and requires many iterations to improve its solutions, and if a heuristic has small final progress value, it cannot effectively improve its solutions and its solutions are poor.

The criterion of progress is based on the distance. Hence, it also requires the set of all Pareto solutions $\boldsymbol{X_p}$. However, in practice, $\boldsymbol{X_p}$ may not be obtainable. In this case the distance can be computed as the GD between initial solutions and solutions at iteration $iter$; and the progress criterion P is simplified to Equation 5.27. This definition of progress is based on an assumption that the applied algorithm keeps improving identified solutions so the newly identified solutions are always on one side of previously identified solutions and getting further away from initial solutions.

$$P = \ln \sqrt{\frac{1}{GD(\boldsymbol{X}_{iter}, \boldsymbol{X_0})}} \qquad \text{Equation 5.27}$$

where, others are same as above.

5.4.3 Solution Distribution

Solution Distribution measures how evenly distributed the identified solutions are and how well they represent the full range of Pareto solutions. MOO techniques may have the same solution quality but their solution distribution may be largely different. For example, in Figure 5.19, both figures identify 9 Pareto solutions but their solution distribution is different. The solutions in Figure 5.19 (1) are closely located at the upper part of Pareto frontier and the lower part is not covered. It is difficult to present the set of all existing Pareto solutions (i.e. Pareto frontier) with the solutions in Figure 5.19 (1). On the contrary, the solutions in Figure 5.19 (2) are well distributed and clearly show the shape and location of the Pareto frontier. Solution distribution has three measurement criteria: coverage error, uniformity level and spacing.

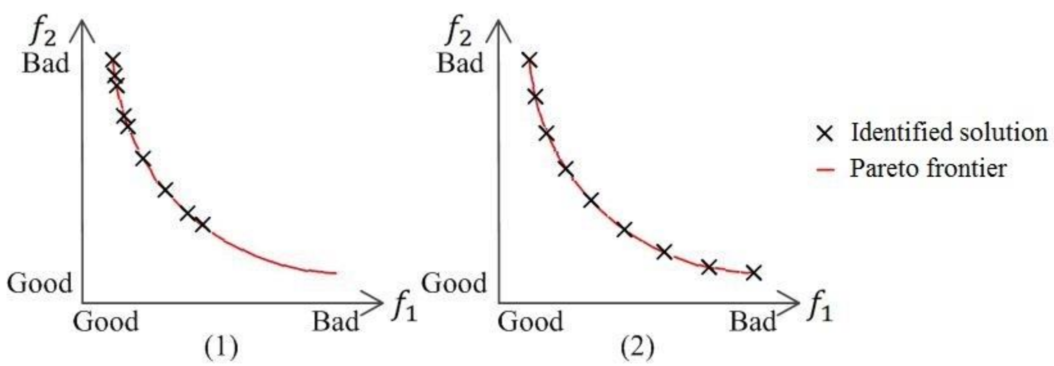


Figure 5.19 Example of solution distribution

Coverage error: This criterion measures the coverage of Pareto frontier by the identified solutions (Sayin, 2000; Sarker and Coello, 2002). Coverage error ϵ is defined using Equation 5.28 as the maximum distance between Pareto solutions and identified solutions.

$$\epsilon = \max_{\forall \boldsymbol{x_p} \in \boldsymbol{X_p}} d(\boldsymbol{x_p}) \qquad \text{Equation 5.28}$$

$$d(\boldsymbol{x_p}) = \min_{\forall \boldsymbol{x} \in \boldsymbol{X}} d(\boldsymbol{x}_p, \boldsymbol{x}) \qquad \text{Equation 5.29}$$

where, ϵ coverage error;

$\boldsymbol{x_p}$ Pareto solution

$d(\boldsymbol{x_p})$ Euclidean distance from a Pareto solution $\boldsymbol{x_p}$ to its closest identified solution in objective space;

others are same as above.

Figure 5.20 shows an example of different solution distribution. Comparing with Figure 5.20 (2), Figure 5.20 (1) has more solutions, but these solutions are far away from an existing Pareto solution $\boldsymbol{x}_p^7$, hence, its coverage error is large and its coverage is poor.

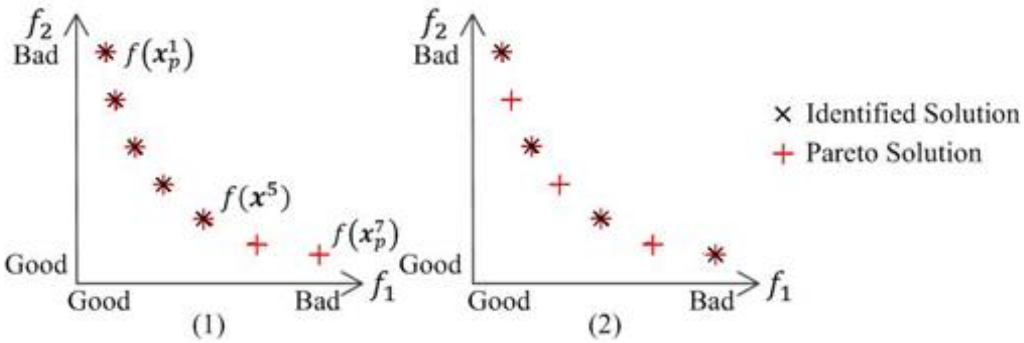


Figure 5.20 Example of coverage error

Coverage error not only describes the coverage of identified solutions, but also roughly indicates the goodness of identified non-Pareto solutions. When it is large, its solutions are more likely to be far from Pareto solutions; so its goodness is poorer.

Coverage error also requires the set of all existing Pareto solutions $\boldsymbol{X_p}$ to be known but $\boldsymbol{X_p}$ may not be obtainable. According to the author’s experience, the solutions of heuristics may be located far from an end of the Pareto frontier. Hence, their coverage error could be estimated only using the end points of Pareto frontier such as $\boldsymbol{x}_p^1$ and $\boldsymbol{x}_p^7$ in Figure 5.20 (1). More specifically, when $\boldsymbol{X_p}$ is unobtainable, the end points are obtained by lexicographically optimising objectives and the set of all end points replaces $\boldsymbol{X_p}$ in Equation 5.28 to calculate coverage error.

Uniformity level: Uniformity level δ measures the spread of identified solutions (Mostaghim and Teich, 2005). It is defined as the minimum distance between two consecutive identified solutions using Equation 5.30. Here the consecutive solution of $\boldsymbol{x}$ is defined as another identified solution that has the shortest Euclidean distance to $\boldsymbol{x}$.

$$\delta = \min_{\substack{\forall \boldsymbol{x},\boldsymbol{x}' \in \boldsymbol{X} \\ \boldsymbol{x} \neq \boldsymbol{x}'}} d(\boldsymbol{x}, \boldsymbol{x}') \qquad \text{Equation 5.30}$$

where, δ uniformity level;

others are same as above.

If identified solutions are not evenly spread over their frontier, some of them must be closer. The closest distance between identified solutions δ is also small. For instance, Figure 5.21 (1) and (2) identify five Pareto solutions to present the set of all existing Pareto solutions (i.e. Pareto frontier). Compared to Figure 5.21 (2), solutions $\boldsymbol{x}^2$ and $\boldsymbol{x}^3$ in Figure 5.21 (1) are very close, so the uniformity level is also small; and the solution distribution of Figure 5.21 (1) is poor. In the best scenario, all solutions are evenly spread out such as Figure 5.21 (2), then the distances between any two consecutive solutions are similar; so δ approaches to the best value: average distance between consecutive solutions and its solution distribution is also good.

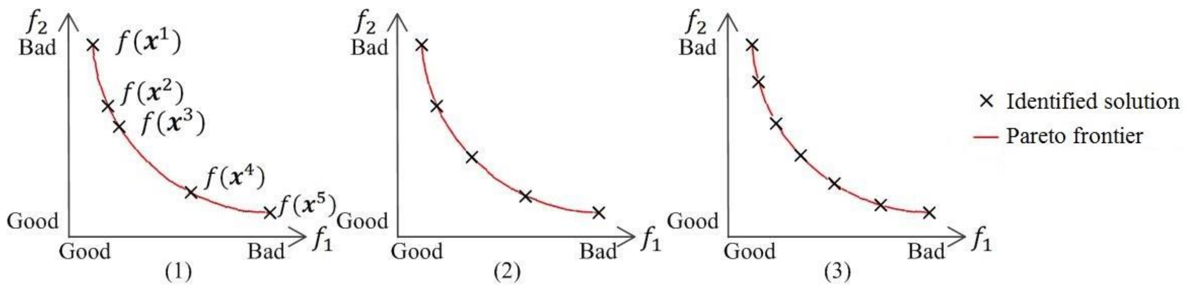


Figure 5.21 Examples of uniformity level

This criterion is an important measurement of solution distribution. It is applicable to measure both Pareto solutions and non-Pareto solutions. This criterion is effective only when techniques have a similar number of solutions. However, when techniques identify different numbers of solutions, this criterion cannot correctly describe the solution distribution. For example, compared to Figure 5.21 (2), Figure 5.21 (3) has more solutions so its solution distance is smaller. Accordingly, its uniformity level is small and its solution distribution is deemed as poor, which is not correct.

In this research, a criterion named uniformity level ratio is introduced to measure the solution uniformity. It is given by Equation 5.31, which reduces the influence of the solution density and therefore measure different sets of solutions. For example, in Figure 5.21 (3), its uniformity level is small and its average distance is also small; hence, its uniformity level ratio is still large and similar with that of Figure 5.21 (2). Therefore, comparing to uniformity level, uniformity level ratio can correctly describe the solution uniformity even when MOO techniques identify different numbers of solutions.

$$r_{UL} = \frac{\delta}{\bar{d}} \qquad \text{Equation 5.31}$$

where, r_{UL} uniformity level ratio;

$\bar{d}$ average distance of all consecutive solutions; and

others are same as above.

Spacing: This criterion measures the uniformity of identified solutions (Raisanen and Whitaker, 2005). Spacing S given by Equation 5.32 measures the dispersion of the distances between consecutive solutions $d'(\boldsymbol{x})$ from their average value. When all identified solutions are evenly spread, the distances between consecutive solutions are similar and approach to their average value; hence, the distance variance is small and the spacing S is also small. Here the consecutive solutions are defined as a pair of identified solutions that have the closest Euclidean distance, which is same as the consecutive solutions of uniformity level. In best scenario, all consecutive solutions have the same distance, and $S = 0$.

$$S = \sqrt{\frac{1}{N_{\boldsymbol{X}} - 1} \sum_{\forall \boldsymbol{x} \in \boldsymbol{X}} \left(\left(\frac{\sum_{\forall \boldsymbol{x} \in \boldsymbol{X}} d'(\boldsymbol{x})}{N_{\boldsymbol{X}}} \right) - d'(\boldsymbol{x}) \right)^2} \qquad \text{Equation 5.32}$$

$$d'(\boldsymbol{x}) = \min_{\substack{\forall \boldsymbol{x}' \in \boldsymbol{X} \\ \boldsymbol{x} \neq \boldsymbol{x}'}} d(\boldsymbol{x}, \boldsymbol{x}') \qquad \text{Equation 5.33}$$

where, S spacing;

$d'(\boldsymbol{x})$ Euclidean distance between solution $\boldsymbol{x}$ and its closest solution in objective space; and

others are same as above.

Spacing effectively describes the solution distribution. For example, compared with Figure 5.21 (2), $d'(\boldsymbol{x})$ in Figure 5.21 (1) varies largely; hence its spacing is large and solution distribution is poor. Furthermore, different from the criterion of uniformity level, spacing is not affected by the number of identified solutions. Comparing Figure 5.21 (2) and (3), even they identify different numbers of solutions, their distances between consecutive solutions are stable and similar to their average distance; so both of them have small spacing and good solution distribution.

The criterion of spacing is important when measuring the solution distribution especially when a few solutions are obtained to illustrate the entire Pareto frontier and when techniques generate different numbers of solutions. It can measure the distribution of both Pareto and non-Pareto solutions. Spacing may also be defined in another way (Tan *et al.*, 2005).

5.4.4 Computation Time Measures

Computation time is an essential measure of the efficiency of MOO techniques within the software environment. In practical decision making, it is important to identify solutions in a reasonable time, especially when analysing a large number of segments and strategies. In this framework, two criteria, running time and time per solution, are used to measure the computation time.

It is important to note that the computation time measures are related to the solution quality and distribution. It is not fair to measure computation time of MOO techniques with different solution quality or distribution; because the techniques identifying poor solutions may generate better solutions with longer time.

Running time: This criterion measures the overall running time of MOO techniques when solving an optimisation problem in decision making in IAM (Tormos and Lova, 2001). When a technique solves a problem in less time, this technique is faster.

When using this criterion, it is recommended that exact methods are fully completed or heuristics stop when their solutions are not obviously improved in recent iterations. When all techniques are required to solve a problem using the same time, this criterion is not effective and the criterion of time per solution/iteration should be used.

Time per solution/iteration: This criterion measures the productivity of MOO techniques. It is defined as the average running time per unique non-dominated solution. If a MOO technique spends less time to identify a solution, this technique is deemed to be more productive and performs better on the aspect of computation time measures.

Heuristics that solve optimisation problems in an iterative manner can also be measured using the criterion of time per iteration. It is the average running time per iteration. When a heuristic completes an iteration faster, it performs better in the aspect of computation time measures. This criterion is useful when the assessed heuristics have similar running time and population size. However, the definition of iterations may be different when applying different heuristics. Hence, when assessing this criterion, the compared heuristics should define iterations in a similar way.

5.4.5 Implementation Considerations

Implementation considerations evaluate the issues when applying MOO techniques to an optimisation problem in decision making in IAM, including the flexibility and controllability. Four criteria are defined to measure the implementation: ease of implementation, expected result, parameter calibration and stopping criteria, which are specified in the following paragraphs. However, the measurement of implementation is based on the implementer's preference and viewpoint. Hence, different implementers may differently appraise a same implementation of a technique.

Ease of implementation: This criterion measures the ease of implementing a technique to solve optimisation problems in decision making in IAM. It is a qualitative and subjective measure based on three considerations:

- **Number of objectives:** Is a technique applicable to solve optimisation problems with different numbers of objectives? Some techniques, such as the classic DA, can only solve bi-objective optimisation problems and cannot directly optimise three or more objectives. These techniques are deemed as inflexible and make the implementation more difficult.
- **Extra package and tool:** Is any extra package or tool needed for the implementation? For example exact methods may require a single-objective solver during optimisation. The extra packages and tools may affect the performance of these

techniques and make the implementation more difficult. However, effective packages and tools may accelerate MOO techniques and improve their performance.

- **Applicability:** Does the technique solve different types of optimisation problems? Decision making problems may be expressed as different types of optimisation problems such as nonlinear or continuous optimisation problems. MOO techniques that are capable of solving different types of optimisation problems are preferred.

Expected result: This criterion measures the controllability of MOO techniques. Decision making in IAM may have specific perspectives on MOO techniques and their results, such as a preferred number of solutions and acceptable running time. These perspectives are often achieved by defining stopping criteria. If a perspective is achieved, the corresponding stopping criterion is triggered and a technique stops. However, techniques may not be sensitive to some perspectives. For example, it is difficult to generate a certain number of non-dominated solutions using heuristics. If a technique can achieve more perspectives, it is more controllable and has better implementation.

Parameter calibration: Parameters affect the performance of MOO techniques, and need to be calibrated based on the addressed problem. Some parameters are sensitive to the optimisation, including solution quality and distribution, computation time, etc. It is difficult to calibrate these parameters in a way that results in consistently best possible performance. If a MOO technique has more sensitive parameters, this technique is more difficult to be implemented.

Stopping criteria: Stopping criteria determine the termination point of a MOO technique when solving an optimisation problem. There are two types of stopping criteria: one type is defined to achieve specific perspectives of a decision making problem as mentioned with the criterion of expected result. The other type is required by the techniques themselves. This criterion evaluates the second type. All techniques have at least one stopping criterion. Some of them have self-defined stopping criteria. For example, RNBI completes when all reference points are analysed. Decision makers do not have to define a stopping criterion when applying these techniques. However, some techniques such as heuristics need at least one pre-defined stopping criterion that has to be decided by a decision maker. To properly define stopping criteria, decision makers should have sufficient knowledge of the applied techniques and give the techniques enough time to obtain good solutions. Therefore, the techniques requiring pre-defined stopping criteria are less preferred in the aspect of implementation.

5.4.6 Scoring System of the Measurement Framework

Sections 5.4.1-5.4.5 introduce a measurement framework to compare and assess MOO techniques when solving optimisation problems for decision making in IAM. Four aspects can be individually assessed or used together to describe the overall performance of MOO techniques. When measuring techniques, a scoring system may be needed to describe the performance of MOO techniques with numbers to ease their comparison. This section introduces a scoring system for the measurement framework.

Initially, objective values of the identified solutions are normalised to $[0, 1]$ where 0 (1) represents the worst (best) achievable value of an objective (Hansen and Jaszkiewicz, 1998). This is because when objectives have different scales, the distance-based criteria, including distance, hypervolume, progress, coverage error, uniformity level and spacing are heavily affected by the large-scale objectives and may ignore the small-scale objectives. After normalisation, objectives have approximately the same influence on the criteria.

Next, criteria are calculated and scored. Most criteria can be calculated based on their definition and linearly scored. A score is assigned to indicate the achievement of a technique on a criterion, where a score of 10 (0) means the best (worst) value of this criterion. The criterion of ease of implementation has three considerations. The satisfaction of each consideration adds 3.33 score on this criterion. The criteria of expected result, parameter calibration and stopping criteria have to be scored by decision makers. This framework includes many criteria but not all of them are applicable to every MOO technique. For example, the criterion of progress is only applicable to heuristics; and only one of the criteria of distance and hypervolume should be used to avoid repetition.

Thirdly, a weighting system is adopted. In this research, all defined criteria have the same importance; hence they have same weights. If an aspect considers NC criteria, the weight of every considered criterion is $1/NC$. Then the score of a measurement aspect is calculated using Equation 5.34, where each criterion is an element.

$$score = \sum_{i=1}^{NC} w_i s_i \qquad \text{Equation 5.34}$$

where, w_i weight of element i;

s_i score of element i; and

NC number of element.

Finally, the overall performance score of a technique is calculated by weighted summing the scores of all aspects using Equation 5.34; where each aspect is an element. In this research, the weights of the four aspects, solution quality, solution distribution, computation time measures and implementation considerations, are same (0.25). When an aspect is more important, its weight can be higher. When a technique has higher overall performance score, this technique is recognised to be better.

5.4.7 Summary of the Measurement Framework

This section establishes a measurement framework to evaluate the effectiveness and efficiency of MOO techniques in the context of decision making in IAM. It helps rationally and comprehensively scoring and comparing different types of MOO techniques, and therefore enhances the understanding of the performance of MOO techniques in the context of decision making in IAM. This framework measures the MOO techniques on four aspects: solution quality, solution distribution, computation time measures and implementation considerations, each with corresponding criteria. This measurement framework will be used to assess the MOO techniques in the experimental tests.

5.5 Experimental Tests of the Multi-Objective Optimisation Techniques Based on Practical Decision Making

This section aims at providing an explicit and experimental knowledge of performance of MOO techniques in the context of decision making in IAM. More specifically, this section conducts typical experimental tests based on practical long-term and network-level decision making in IAM, and then evaluates and compares the techniques in the MOOT List.

Listed MOO techniques:

According to the completed analysis, a variety of MOO techniques including the applied techniques (Section 5.1) and the new techniques (Section 5.2) are added to the MOOT List. They are summarised in Table 5.5.

Table 5.5 MOOT List: summary of the techniques

Type	Technique	Abbreviation	Type	Introduction
Exact method	Weighted Sum Method	WSM	Applied	Section 5.1.2
	Dichotomic Approach	DA	New	Section 5.2.1
	Epsilon Constraint Method	ECM	New	Section 5.2.2
	Revised Normal Boundary Intersection	RNBI	New	Section 5.2.3
Heuristic	Genetic Algorithm	GA	Applied	Section 5.1.3
	Nondominated Sorting Genetic Algorithm II	NSGA II	Applied	Section 5.1.4
	Simulated Annealing	SA	New	Section 5.2.4
	Tabu List	TS	New	Section 5.2.5
	Ant Colony Optimisation method	ACO	New	Section 5.2.6
	Particle Swarm Optimisation method	PSO	New	Section 5.2.7

Tests:

In this section, two groups of tests are designed based on practical decision making problems. One group deals with bi-objective optimisation and the other deals with three-objective optimisation. The main considerations of the test design are:

Firstly, all the tests are based on the data from the two practical long-term and network-level decision making problems in IAM. Hence, the test result is able to demonstrate the performance of MOO techniques when dealing with this type of decision making problems.

Secondly, according to Zitzler *et al.* (2000), two or three objectives are sufficient to model most engineering problems. Tests with more objectives may not be necessary for testing purposes.

Finally, the performance of MOO techniques may be largely different when optimising two or three objectives. The techniques generating strong results for bi-objective optimisation problems may not generate strong results for three-objective optimisation problems. Hence, both bi- and three- objective optimisation should be tested in order to obtain a completed evaluation of MOO techniques. An indication of the increase of problem difficulty with increasing number of objectives may also be obtained.

5.5.1 Evaluation of the Performance and Scalability of the Multi-Objective Optimisation Techniques for Bi-Objective Optimisation Problems

5.5.1.1 Introduction of the Tested Decision Making Problem: City A

City A is a North American city that wants to efficiently keep its road network in acceptable condition for at least twenty years (2009-2028). In detail, this decision making problem is to make a twenty-year management decision that purses financial benefit and cost and keeps the road network in acceptable condition.

The road network of City A has 16 main roads and 15 country roads, (1,483.26 km in total). It is divided into 3,640 segments, where 1,822 segments are analysed here. Strategies are generated using dTIMS CT 8 (Deighton Associates Limited, 2008); of which 61,936 strategies including do-nothing strategies are feasible and selectable (committed strategies are not considered in the test). Corresponding outcomes are also estimated using dTIMS CT 8, including:

- Yearly treatment cost: actual maintenance cost in a year, if a strategy is applied;
- Cost: strategy expense in present value (Year 2009);
- Benefit: measured by condition improvement on network segments; and
- Pavement performance index: composite index measuring the performance of network segments, where 0 (5) is the best (worst) condition.

The corresponding requirements of this decision making problem include:

- Annual budget: annual budget for every year; and
- Acceptable pavement performance index: the acceptable worst pavement performance index for every year.

According to the decision making goals and the available outcomes, a bi-objective optimisation problem is established, which is to obtain maximising benefit (Equation 5.35) and minimising cost (Equation 5.36) under the constraints of the acceptable average annual condition (Equation 5.37) and annual budget (Equation 5.38). Equation 5.39 represents the constraints of one-strategy policy (introduced in Section 4.3).

$$\max \sum_{i=1}^{N} B_i x_i \qquad \text{Equation 5.35}$$

$$\min \sum_{i=1}^{N} C_i x_i \qquad \text{Equation 5.36}$$

$$\sum_{i=1}^{N} PPI_{t,i} x_i \leq APPI_t, \qquad t = 1,2,\cdots,20 \qquad \text{Equation 5.37}$$

$$\sum_{i=1}^{N} YC_{t,i} x_i \leq AB_t, \qquad t = 1,2,\cdots,20 \qquad \text{Equation 5.38}$$

$$\sum_{i \in \mathcal{S}_j} x_i = 1, \quad j = 1,2,\cdots,M \qquad \text{Equation 5.39}$$

where, C_i expense of strategy i in present value;

B_i value measured by the condition improvement of the corresponding segment when applying strategy i;

$YC_{t,i}$ actual maintenance cost in year t, if strategy i is applied;

$PPI_{t,i}$ composite index measuring the performance of a segment in year t, if strategy i is applied;

AB_t annual budget for year t;

$APPI_t$ the acceptable worst pavement performance index in year t; and others are same as above.

5.5.1.2 Tests of the Bi-Objective Optimisation Techniques (City A)

For this decision making problem, three tests were designed to analyse a part of the network or the entire network of this decision making problem in order to evaluate the scalability of MOO techniques when helping with decision making with different sizes. A summary of these tests is shown in Table 5.6. The annual budget and the acceptable average condition index are set for testing purposes. Different budget levels are defined and tested for testing purposes, while their performance is similar to the performance of the budget levels in Table 5.6.

Table 5.6 Summary of the tests of the decision making of City A

Test index	Number of segments	Number of strategies	Annual budget (million)	Acceptable average performance index
A1	100	3,718	5	3
A2	1,000	33,300	50	3
A3	1,822	61,936	80	3

Parameters of the techniques are calibrated according to the author's experience and shown in Table 5.7. For exact methods, parameters are calibrated in a way that all exact methods generate similar numbers of SOO sub-problems for a MOO problem. For heuristics, their parameters are calibrated by testing different parameter calibrations with a small decision making problem. However, it is worth noting that this parameter calibration may not generate the best solutions in all tests.

Table 5.7 Parameter calibration of the listed MOO techniques

Type	Technique	Parameters		Stopping criteria
Exact method	WSM	$n_s = 15$		14 SOO sub-problems
	DA			
	ECM	$n_s = 15$		
	RNBI	$n_s = 15$		
Heuristic	GA	$R_c = 0.7$ and $R_m = 0.4$	Population size: 200 in Test A1, 300 in Test A2, 400 in Test A3	500 iterations in Test A1 800 iterations in Test A2 1000 iterations in Test A3
	NSGA II	$R_c = 0.7$ and $R_m = 0.4$		
	SA	$p = 0.4$		
	TS	$Step_{STM} = 20$, $Step_D = 20$, $Step_I = 30$ and $Step_R = 50$.		
	ACO	$\alpha = 1.5$, $\beta = 1.0$, $p^* = 0.6$ and $\Delta = 5$		
	PSO	$c_1 = 0.5$ and $c_2 = 0.5$		

5.5.1.3 Test Results and Discussion for Bi-Objective Optimisation

All the tests in Table 5.6 are solved by the techniques in the MOOT List and their results are presented in Appendix B. This section evaluates and compares the listed MOO techniques when solving bi-objective optimisation problems in decision making in IAM from the four aspects of the measurement framework developed in Section 5.4, including solution quality, solution distribution, computation time measures and implementation considerations. It is important to point out that this evaluation is based on the author's implementation and the technique performance may be different with other implementation.

Solution quality:

All the listed MOO techniques are able to obtain feasible solutions in the tests in Table 5.6. Generally, the exact methods identify better solutions than the heuristics with the calibrated parameters and defined stopping criteria. Moreover, the solution quality of the exact methods is stable while the solution quality of the heuristics is affected by many factors including problem size and restrictiveness of constraints.

Figure 5.22 illustrates the solution quality of the MOO techniques in Test A2, where the solution goodness is measured by distance criterion. The solution quality of the listed techniques in Tests A1 and A3 is similar with that in Test A2, which is not shown here.

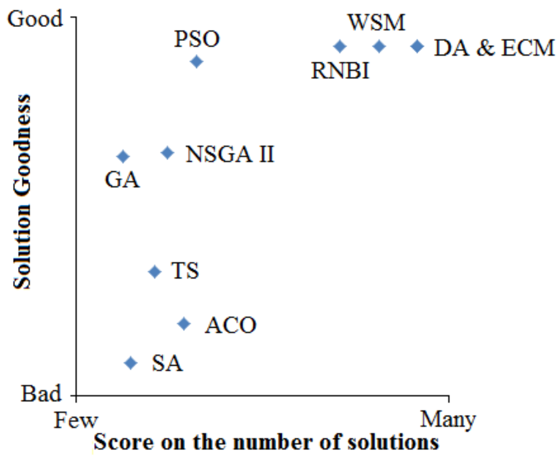


Figure 5.22 Solution quality of the listed techniques in Test A2

All the exact methods obtain the best solutions (Pareto solutions); and their main difference lies at the quantity. DA and ECM are able to obtain a unique Pareto solution for each SOO sub-problem. Hence, their solution quality is the best. WSM and RNBI obtain 14 and 12 unique Pareto solutions including two end points with 14 sub-problems in Test A2. Their solution quality follows DA and ECM.

The solution quality of heuristics is more complex. None of the listed heuristics obtains Pareto solutions in the tests. Comparing with the other heuristics, solutions of PSO are the closest to the Pareto frontier, closely followed by NSGA II. Because NSGA II averagely obtains 71 more unique non-dominated solutions (within the solution pool) than PSO with a population of 300 in Test A2, its overall solution quality is better. ACO also has good solution quality. The solution quality of TS and GA is not good in this test because they only generate a few unique non-dominated solutions that not near to the Pareto solutions. However, TS may be able to generate more solutions in other tests. SA generates the poorest solutions measuring by quantity and quality.

When applying heuristics, the solution quality is affected by problem size. Normally, when more segments and strategies are analysed, the distance between the identified solutions and Pareto solutions is increasing, i.e. heuristics perform worse with increasing problem size. Figure 5.23 shows the solution goodness (measured by distance criterion) with the number of the analysed segments. When the number of segments increases from 100 to 1,000, the solution goodness of the heuristics dramatically reduces; while the reduction slows down when the segment number increases from 1,000 to 1,822. By comparison, NSGA II, PSO and ACO are relatively stable, as their reduction of solution goodness here is less than the reduction of the others. The other listed heuristics have similar reduction rates on the solution goodness with the growth of the number of analysed segments.

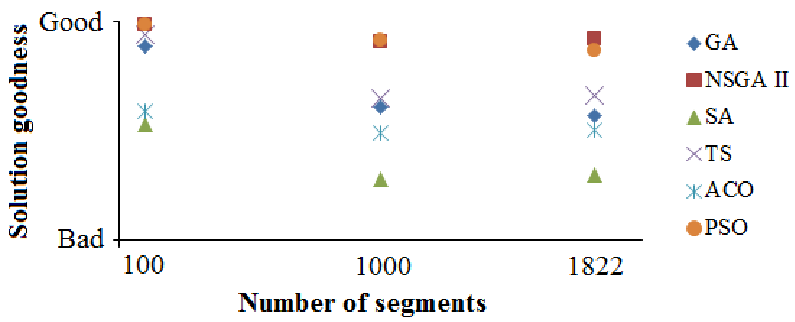


Figure 5.23 Comparing solution goodness and segment number (City A)

The constraint restrictiveness may also affect the solution quality of the heuristics. For example, in Test A2, when the annual budget reduces to $30 million, all the listed heuristics cannot initially generate feasible solutions. Table 5.8 shows when the heuristics begin to yield feasible solutions based on Test A2. According to the table, NSGA II and PSO are always able to yield feasible solutions in the first 25 iterations and GA has 96.7 % chance to generate feasible solutions in the first 25 iterations. These three heuristics are more effective than the others. However, when applying ACO, there is only half chance to yield feasible solutions in the first 25 iterations; while 30% chance to yield feasible solutions after 50 iterations. SA mainly generates feasible solutions in the first 50 iterations while it may begin generating feasible solutions after 100 iterations. TS is the least effective in this test. It only has 26.7%

chance to begin generating feasible solutions in the first 25 iterations, which is the least effective. After identifying feasible solutions, all the listed heuristics continuously improve their solutions until a stopping criterion is achieved.

Table 5.8 Percentage of the runs/trials required to yield feasible solutions (for 30 runs/trials)

Heuristic technique	**Number of iterations beginning to yield feasible solutions**			
	<25	25-50	50-100	>100
GA	96.7%	3.3%		
NSGA II	100%			
PSO	100%			
SA	40%	40%	13.3%	6.7%
TS	26.7%	33.3%	20%	20%
ACO	50%	20%	13.3%	16.7%

A low annual budget was also adopted in Tests A1 (3 million) and A3 (60 million). Their performance is similar to Test A2. The listed heuristics only identify infeasible solutions at the beginning while they manage to obtain feasible solutions before the completion of the algorithms.

Different from the heuristics, the problem size and the restrictiveness of constraints do not affect the solution quality of the listed exact methods. All the listed exact methods successfully identify Pareto solutions when problem size grows or the annual budget reduces. Moreover, they are able to identify Pareto solutions when solving other problems if Pareto solutions exist. Hence their solution quality is stable.

Solution distribution:

Solution distribution varies based on the applied techniques. Figures B.1-B.3 demonstrate the identified solutions of the listed MOO techniques. According to these figures, solutions of the exact methods produce good frontiers that are able to represent the entire Pareto frontier; while the solutions of the heuristics are confined to a narrower range. All the MOO techniques perform similarly on solution distribution in the three tests. Figure 5.24 shows their solution distribution in Test A2.

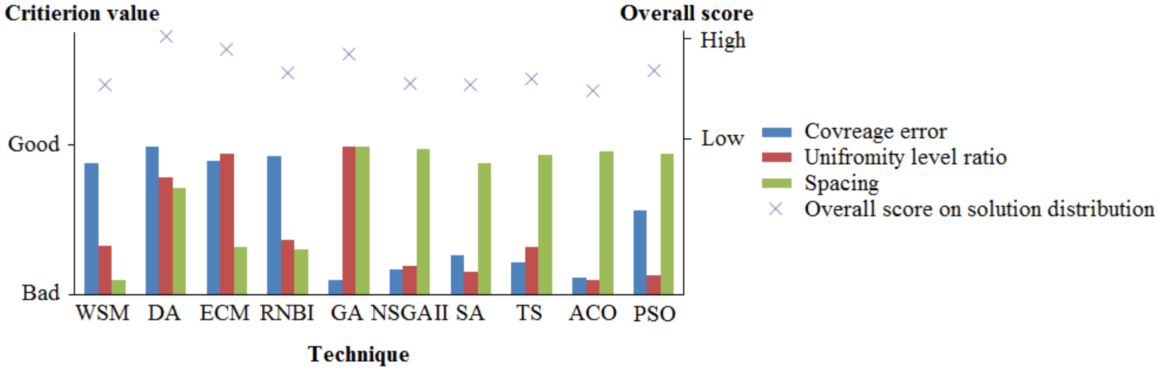


Figure 5.24 Solution distribution of MOO techniques in Test A2

When solving bi-objective optimisation problems, the solutions of DA are the best distributed. Its solutions cover and evenly spread to the entire Pareto frontier; hence, its solution distribution is the best. ECM also has good solution distribution. Its solutions are uniformly located on the Pareto frontier. WSM and RNBI obtain less unique solutions and have a poor uniformity level ratio while their solutions are still able the show the shape and location of the Pareto frontier in the test.

The listed heuristics have small spacing. This is because their solutions are closely located in the objective space. Their solution coverage is much poorer than that of the exact methods hence their solutions only cover a small part of the Pareto frontier. GA has a good uniformity level ratio, so its solutions are evenly distributed on its solution frontier. However, its solutions fail to present the Pareto frontier. The solution distribution of the other heuristics is poor. In summary, the solution frontiers of the heuristics fail to present the shape and location of the entire Pareto frontier in the test.

Computation time measures:

Figure 5.25 shows the computation time measures of the listed MOO techniques in Test A2, which is similar with that in Tests A1 and A3. In general, the computation time of the exact methods is similar while the computation time of the heuristics varies largely. Because exact methods and heuristics solve a MOO problem in a different manner; their computation time is separately scored and discussed.

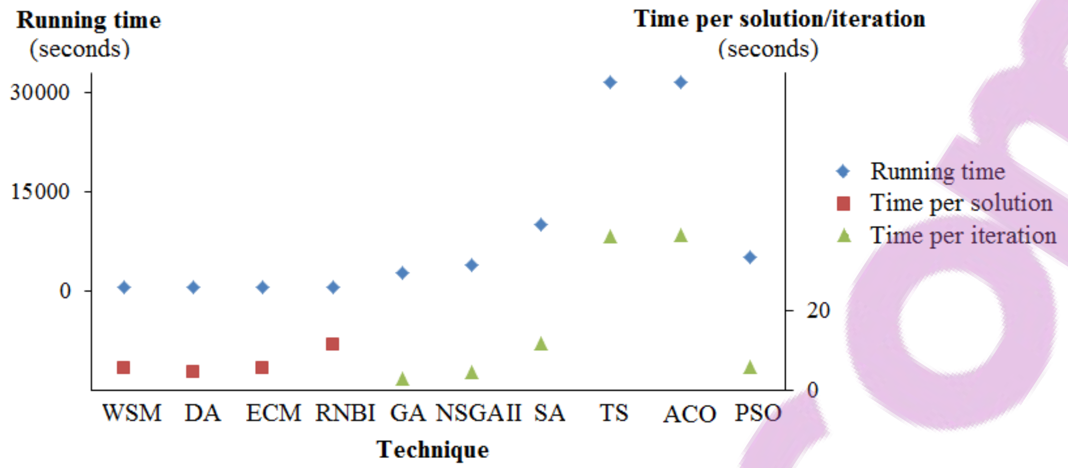


Figure 5.25 Computation time of the listed MOO techniques in Test A2

The computation time of the exact methods mainly depends on the number of sub-problems and the time of solving the sub-problems. In these tests, 14 SOO sub-problems are established excluding lexicographically optimising all objective when applying the exact methods. Therefore, their computation time is mainly determined by the time of solving the SOO sub-problems.

DA is the fastest technique, closely followed by the WSM. This is because they simply weighted sum the original objectives under the original constraints. Their SOO sub-problems are simpler and need less time to be solved. ECM needs more time as it adds extra constraints (epsilon constraints) when constructing SOO sub-problems. RNBI is the slowest exact method. It transfers a discrete MOO problem into continuous SOO sub-problems with extra constraints. Its sub-problems need much time to be solved by the adopted solver. In addition, RNBI obtains less unique solutions than the other exact methods; hence, its time per solutions is also long.

The computation time of the heuristics mainly depends on the stopping criteria and the duration of an iteration. Because the stopping criterion of the heuristics is identical (the maximum number of iterations), in the tests their computation time is determined by the average time of an iteration.

GA is the fastest. Its algorithm is simple and requires 2,330.43 seconds to analyse 800 iterations in Test A2. NSGA II with a more complicated solution measurement spends 1,142.73 more seconds than GA. PSO is at the middle level and doubles the running time of GA in this test.

SA is slow and can be even slower when constraints are restrictive. TS and ACO are the slowest in the tests. Their computation time is much longer than the others.

The computation time is affected by the problem size. Figure 5.26 shows the relationship of running time and the problem size. The running time of the listed MOO techniques grows when more segments are analysed.

For exact methods, because of the growth of the problem size, the MOO problems have more decision variables and more constraints; hence more time is needed. Averagely, their running time increases to around 8 times, when the segment number grows from 100 to 1,000, and is doubled when the segment number grows from 1,000 to 1,822.

For the heuristics, when more segments are analysed, their running time increases significantly because more decision variables and more iterations are analysed. Averagely, their running time increases to over 30 times (26 for PSO) when the segment number grows from 100 to 1,000 and around 3.5-5 times when the segment number grows from 1,000 to 1,822.

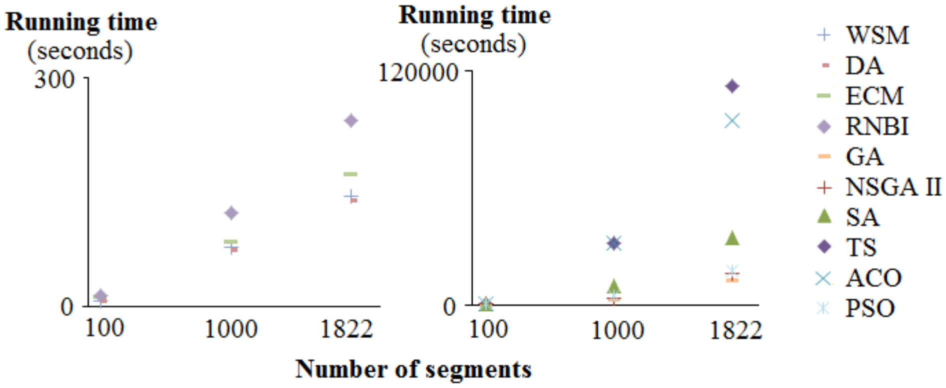
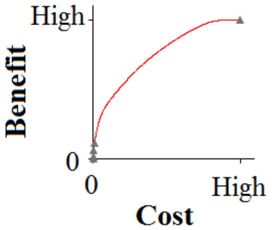
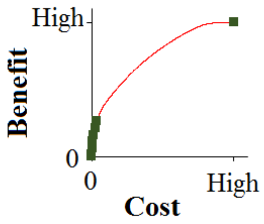
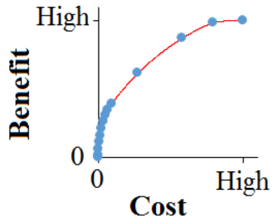
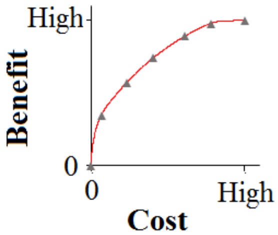
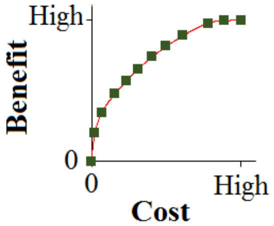
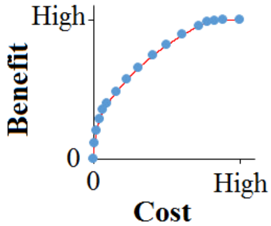
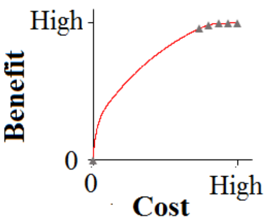
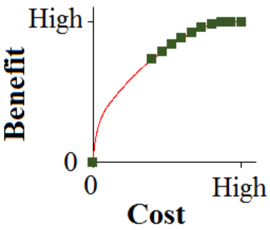
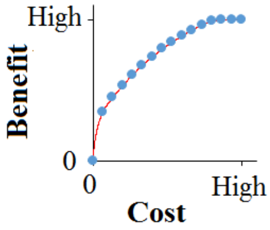
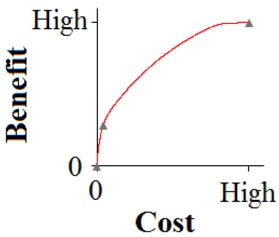
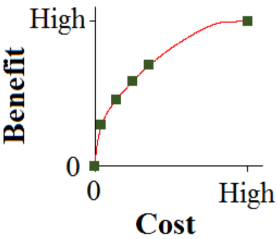
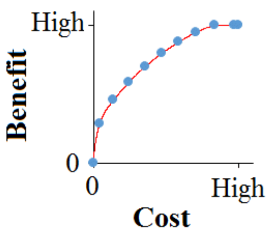


Figure 5.26 Comparing running time and number of segments (City A)

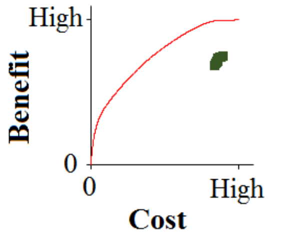
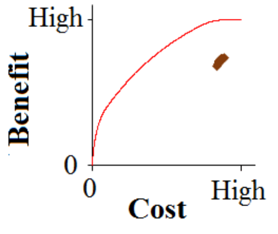
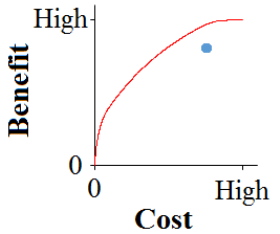
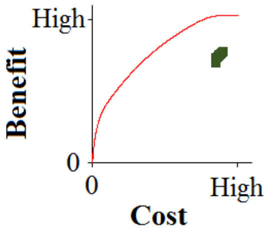
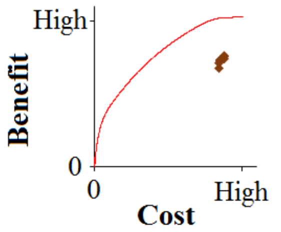
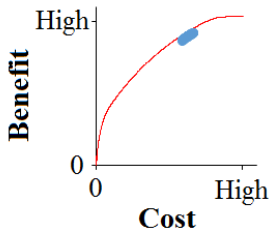
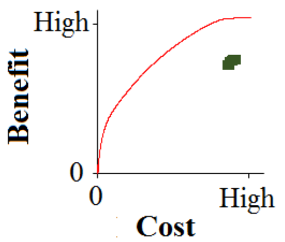
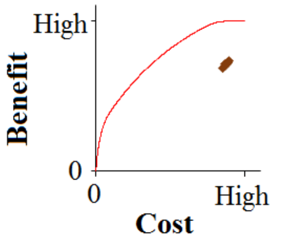
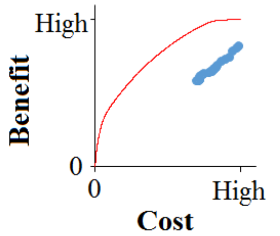
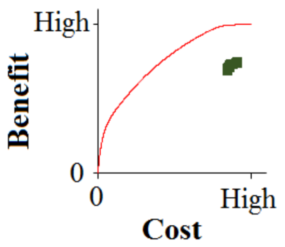
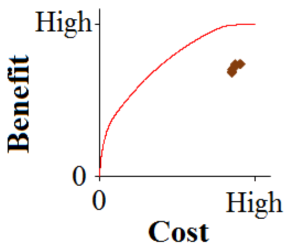
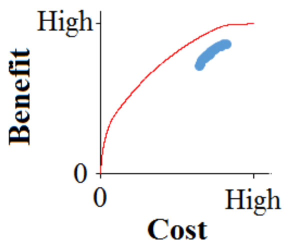
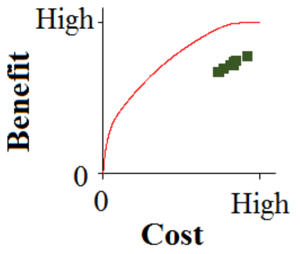
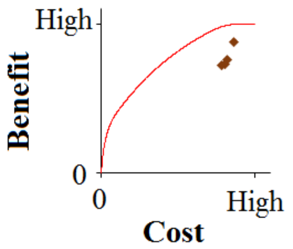
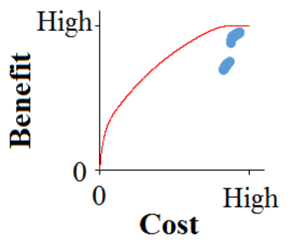
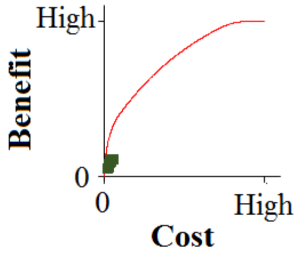
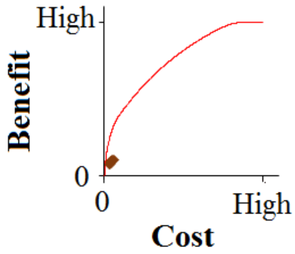
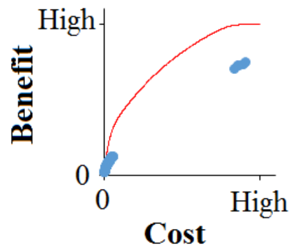
The computation time may affect solution quality and distribution. In the tests, optimisation results are collected after the techniques are fully completed. When insufficient time is allowed, the performance of the MOO techniques is weakened. Table 5.9 shows the solutions identified within different lengths of time based on Test A2. Those figures present how MOO techniques identify new solutions with time. It is important to note that the Pareto frontier in this table is a large set of discrete but very closely located Pareto solutions, which looks like a line.

Table 5.9 Number and distribution of solutions obtained within different lengths of time

Technique	Solutions		
	25 seconds	50 seconds	Unlimited
WSM			
DA			
ECM			
RNBI			

▲ solutions obtained in 25 seconds
■ solutions obtained in 50 seconds
◆ solutions obtained in 100 seconds
● final solutions
— Pareto froniter

Table 5.9 (Continuous)

Technique	Solutions		
	50 seconds	100 seconds	Unlimited
GA			
NSGA II			
SA			
TS			
ACO			
PSO			

For the exact methods, the insufficient time only weakens their solution distribution. WSM, ECM and RNBI obtain new solutions based on the definition of weights (WSM), epsilon values (ECM) and reference points (RNBI). In this research, they identify solutions from one side to the other. Therefore, when insufficient time is allowed, their solutions only spread to a part of the Pareto frontier. However, it is important to note that when these techniques are differently implemented, their solution distribution may be improved. Sufficient running time is critical for these techniques to produce a good solution frontier. DA, on the other hand, is much better than the other exact methods. It obtains new solutions that fill the gap between the identified ones. Hence, when insufficient time is allowed, its solutions are looser but still evenly located along the Pareto frontier.

For the heuristics, the insufficient running time mainly weakens their solution quality and also affects their solution distribution. According to the figures in Table 5.9, the heuristics improve their solutions with time. When 50 seconds are allowed, all the heuristics only start to improve their solutions. Hence, their difference in solution quality and distribution is not big. When 100 seconds are allowed, effective heuristics such as GA, NSGA II and PSO obviously improve their solutions; and some heuristics such as TS and ACO begin to spread their solutions in the objective space. Because TS and ACO are slow; their solution quality does not obviously improved within 100 seconds. SA is faster than TS and ACO but it is not effective in this test; therefore its solutions are the worst comparing with the others.

Implementation considerations:

The implementation of the MOO techniques depends on their algorithms. It is important to notice that an algorithm could be implemented in different ways. In this section, the implementation is measured based on the author’s implementation and scored based on the authors’ viewpoints, which may be different from others.

Figure 5.27 shows the author’s scores on the implementation criteria, which is same in the three tests. Generally, exact methods have the higher overall score on implementation considerations than the heuristics. The following paragraphs discuss the performance of the techniques on each implementation criterion.

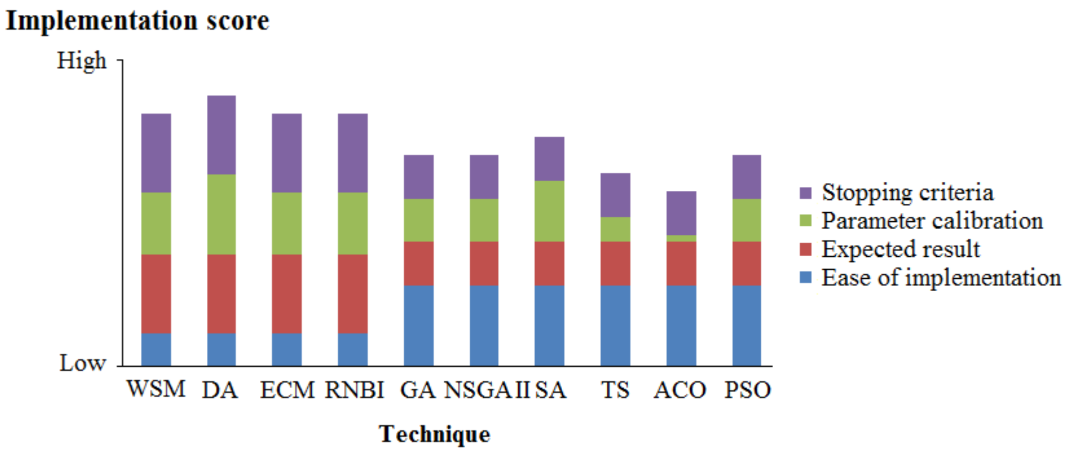


Figure 5.27 Scores of the criteria of implementation (City A)

The criterion of ease of implementation: The heuristics are easier on the implementation. They can directly and flexibly deal with various objectives and constraints. However, when applying the listed exact methods, mathematical background and a SOO solver is needed; thus, their implementation is harder than the heuristics.

The criterion of expected solutions: The listed exact methods perform better on this criterion than the heuristics. When applying the exact methods to solve a bi-objective optimisation problem, a decision maker can easily estimate their solutions and therefore obtain the expected optimisation result. However, when applying heuristics, their solutions are not predictable, hence uncontrollable.

The criterion of parameter calibration: The parameters are introduced with the algorithms and calibrated as shown in Table 5.7. DA, of which no parameter has to be calibrated by a decision maker, is the best on this criterion. Other exact methods only have one main parameter that controls the density of solutions, therefore also have a high score on this criterion. The heuristics often have more parameters that are sensitive to their solutions. More specifically, there are four main parameters needed to be calibrated when applying ACO and TS, while the parameters of TS are not sensitive to its solutions; hence it has a higher score than ACO on this criterion. There are 1-2 parameters when applying the other listed heuristics.

The criterion of stopping criteria: When applying the exact methods, a stopping criterion is not necessary (but can be applied, for example to limit the runtime). When applying the heuristics, at least one stopping criterion has to be given by a decision maker. Therefore, the exact methods have a better score on this criterion than the heuristics.

5.5.2 Evaluation of the Performance and Scalability of the Multi-Objective Optimisation Techniques for Three-Objective Optimisation Problems

5.5.2.1 Introduction of the Tested Decision Making Problem: City B

City B is another city in North America that tries to allocate insufficient maintenance budget to different sections of its road network in the next 22 years (2012 to 2033) in order to obtain the largest return on the investment. More specifically, its road network is divided into three sub-networks and this decision making needs to decide the amounts of funding that should be spent on maintaining each sub-network under the budget for the whole network so that the greatest investment return is obtained.

In detail, the road network in City B is 8,510.22 km long, which is divided into 1,944 segments and 1,301 segments are analysed in this decision making problem. Similar to the decision making problem of City A, strategies are generated using dTIMS CT 8 (Deighton Associates Limited, 2008); where 72,562 strategies including do-nothing strategies are feasible and selectable (committed strategies are not considered in the tests). Corresponding outcomes are also estimated using dTIMS CT 8, including:

- Benefit: measured by condition improvement of network segments; and
- Cost: strategy expense in present value (Year 2012).

The corresponding requirements of this decision making problem include:

- Total budget: total budget for the entire project in present value (Year 2012).

According to the goals of this decision making problem and the available outcomes, a three-objective optimisation problem is established, where objectives are to obtain the maximising benefit of each of the three sub-networks (Equations 5.40-5.42) under the constraint of the total budget for the whole network (Equation 5.43). Equation 5.44 ensures one strategy is selected per segment (introduced in Section 4.3.1).

$$\max \sum_{i=1}^{N} B_i x_i G_i^1 \qquad \text{Equation 5.40}$$

$$\max \sum_{i=1}^{N} B_i x_i G_i^2 \qquad \text{Equation 5.41}$$

$$\max \sum_{i=1}^{N} B_i x_i G_i^3 \qquad \text{Equation 5.42}$$

$$\sum_{i=1}^{N} C_i x_i \leq TB \qquad \text{Equation 5.43}$$

$$\sum_{i \in S_j} x_i = 1, \quad j = 1,2,\cdots,M \qquad \text{Equation 5.44}$$

where, G_i^1 indicator of the segments in sub-network 1. If the segment of strategy i is in sub-network 1, $G_i^1 = 1$; otherwise, $G_i^1 = 0$;

G_i^2 indicator of the segments in sub-network 2. If the segment of strategy i is in sub-network 2, $G_i^2 = 1$; otherwise, $G_i^2 = 0$;

G_i^3 indicator of the segments in sub-network 3. If the segment of strategy i is in sub-network 3, $G_i^3 = 1$; otherwise, $G_i^3 = 0$;

TB total budget; and

others are same as above.

5.5.2.2 Tests of the Three-Objective Optimisation Techniques (City B)

Similar to the tests based on City A, three tests are designed to analyse a part of the network or the entire network in order to test the scalability of MOO techniques when dealing with three-objective decision making with different sizes. A summary of these tests is given in Table 5.10. In these tests, the sub-networks are defined for the testing purpose. Different budget levels are defined for testing purposes, while their performance is same with the performance of the budget level in Table 5.10.

Table 5.10 Summary of the tests of the decision making of City B

Test index	**Number of segments (sub-network 1/ sub-network 2/ sub-network 3)**	**Number of strategies (sub-network 1/ sub-network 2/ sub-network 3)**	**Budget (million)**
B1	100 (20 / 40 / 40)	4,127 (1,025 / 1,213 / 1,889)	1
B2	600 (200 / 200 / 200)	30,359 (7,232 / 13,419 / 9,708)	4
B3	1,301 (400 / 400 / 501)	72,562 (20,651 / 19,519 / 32,392)	8

Parameters are calibrated as same as those in the tests of City A (see Table 5.7). The stopping criterion of the heuristics is 500 iterations in Test B1, 800 iterations in Test B2 and 1000 iterations in Test B3. However, it is worth noting that this parameter calibration may not generate the best solutions in the tests of this decision making problem.

5.5.2.3 Test Results and Discussion for Three-Objective Optimisation

All the tests in Table 5.10 are solved by the techniques in the MOOT List. Because the budget is very low, none of the listed heuristics finds feasible solutions in any test of this decision making problem. Therefore, the heuristics are not discussed here. Appendix C presents the optimisation result of the listed exact methods.

This section evaluates and compares the listed exact methods when solving three-objective optimisation problems in decision making in IAM. In the following paragraphs, these exact methods are measured from the four aspects of the measurement framework developed in Section 5.4 including solution quality, solution distribution, computation time measures and implementation considerations. It is important to note that this evaluation is based on the author's implementation; and the technique performance may be different with other implementation.

Solution quality:

All the listed exact methods are able to obtain Pareto solutions for the three-objective optimisation problems in decision making in IAM. Hence their solution goodness is always the best. Figure 5.28 shows the solution quality in Test B2. Their main difference is the number of unique solutions.

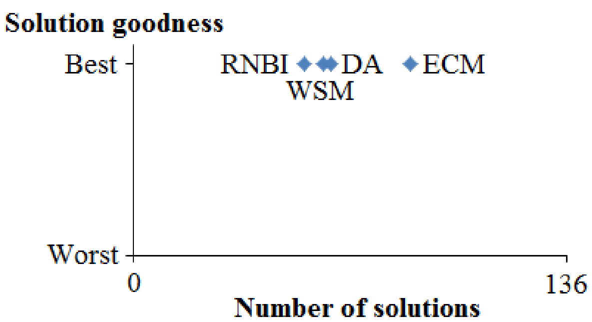


Figure 5.28 Solution quality of the listed techniques in Test B2

According to Figure 5.28, ECM identifies the most unique Pareto solutions; hence its solution quality is the best. WSM and DA obtain around 60 unique solutions in Test B2. RNBI only identifies 54 unique solutions in Test B2, which is the least among the listed exact methods. Hence, its solution quality is the worst. The solution quality of the listed exact methods in Tests B1 and B3 is similar with that in Test B2 and is not discussed again.

Solution distribution:

The solution distribution of the exact methods when solving three-objective optimisation problems is largely different from their solution distribution when solving bi-objective optimisation problems in decision making in IAM. Figures C.1-C.3 demonstrate the solutions of the listed exact methods. According to the figures, the solutions of WSM, ECM and RNBI are able to show the shape and location of the Pareto frontier in the three tests; but DA only presents a general Pareto frontier in Test B1 and obtains poorly distributed solutions in the other two tests.

Figure 5.29 shows the solution distribution of the exact methods in Test B2. In general, DA has the worst solution distribution; while the others perform similarly on solution distribution.

ECM has good uniformity level ratio and spacing, which means its solutions are uniformly distributed along the Pareto frontier. It has the highest score on the solution distribution in Test B2. WSM also has good solution distribution. Its uniformity level ratio and spacing are not as good as those of ECM; while it has better coverage error. Hence, its overall score on solution distribution closely follows ECM. The solutions of RNBI often have the best coverage error, but their uniformity and spacing is poor. Accordingly, its score on the solution distribution is also low. Compared with the other listed exact methods, DA only has a good solution

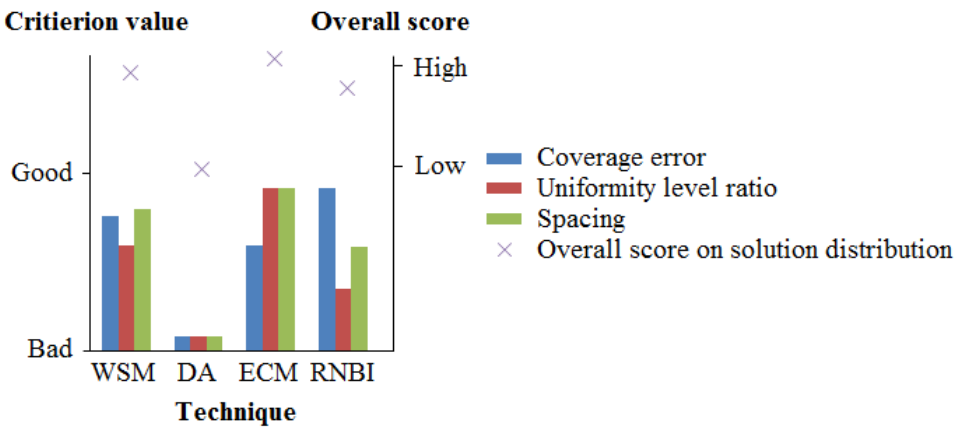


Figure 5.29 Solution distribution of the exact methods in Test B2

distribution in Test B1; but in Tests B2 and B3 its solution distribution is the poorest and its solutions are poorly distributed and fail to present the Pareto solutions.

Computation time measures:

Figure 5.30 shows the computation time of the listed exact methods in Test B2 as an example. Generally, their computation time varies depending on their algorithms.

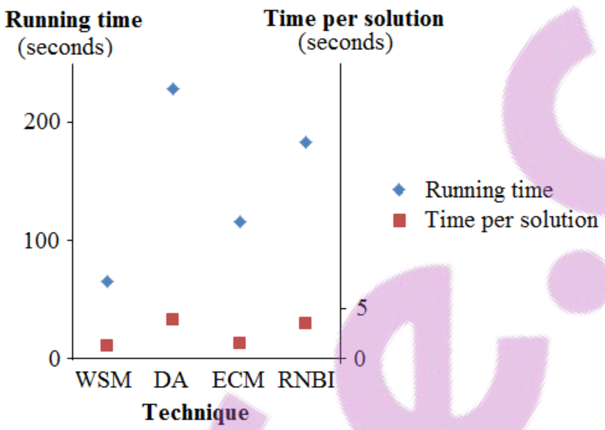


Figure 5.30 Computation time of the listed techniques in Test B2

WSM is the fastest among the listed exact methods. It only spends 63.66 seconds in total and 1.06 seconds per unique solution in Test B2. ECM almost doubles the running time of WSM. RNBI requires 67.77 more seconds than ECM to solve the same number of sub-problems (133) in Test B2. DA is the slowest, which spends 3.65 seconds for a unique solution.

The computation time of the listed exact methods increases with the growth of the problem size. Figure 5.31 shows the relationship between the running time and the problem size. With the growth of the analysed segments, the running time of ECM largely increases

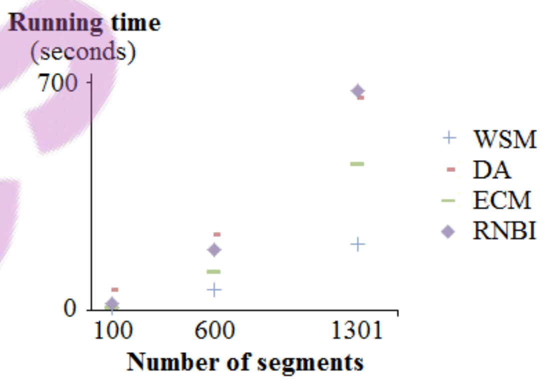


Figure 5.31 Comparing running time and number of segments (City B)

from 9.36 to 114.38 and 433.99 seconds when the segment number grows from 100 to 600 and 1,301. On the contrary, DA has the lowest increase rate on the running time when the analysed segments grow. The running time of WSM and RNBI increases to around 8 times and 3 times when the segment number grows.

Similar to the previous tests, **insufficient running time affects the solution distribution of the listed exact methods**. Figure 5.32 shows the solutions obtained within a given length of time (50 seconds).

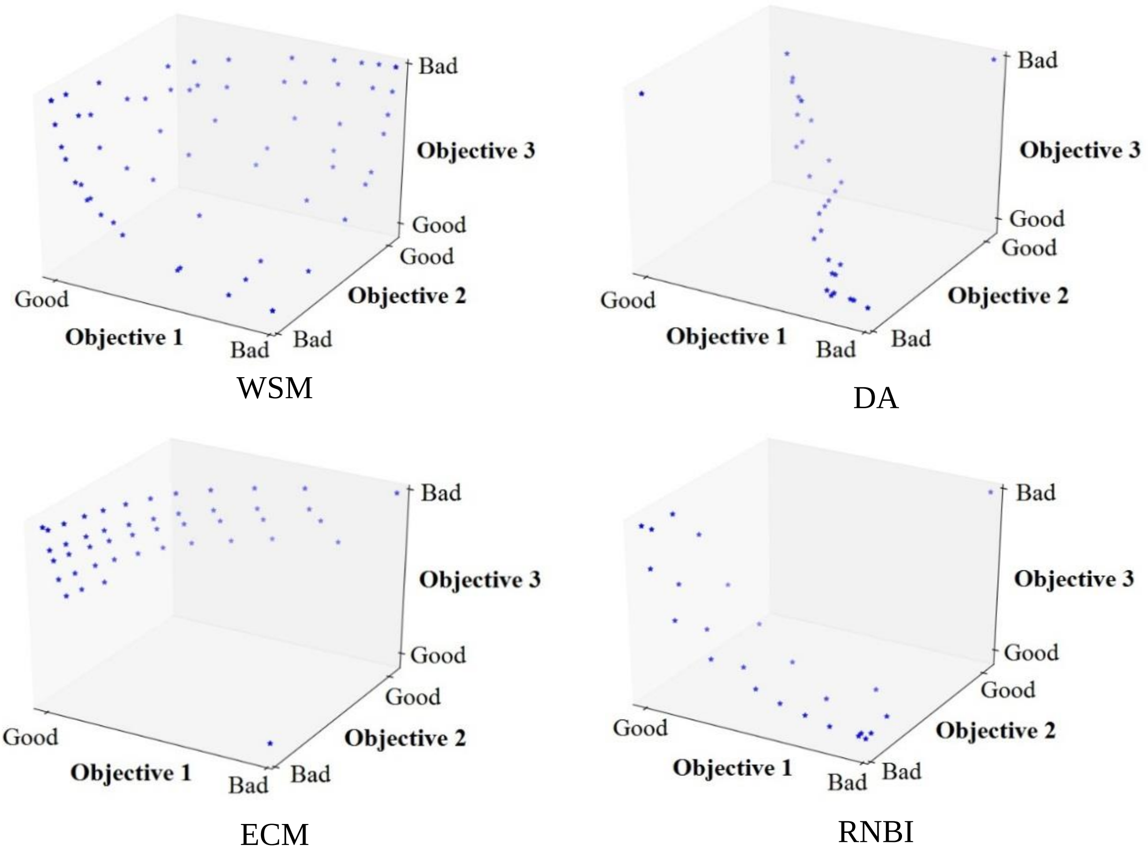


Figure 5.32 Solutions obtained in 50 seconds

ECM and RNBI obtain solutions from one side to another according to the definition of epsilon values (ECM) or reference points (RNBI). Hence, when insufficient time is allowed, their solutions may only spread to a part of the Pareto frontier. WSM may also spread to a part of the Pareto frontier according to the definition of weights within the insufficient time. However, WSM is fast, so in the example of Figure 5.32, it almost identifies all the unique Pareto solutions within 50 seconds (total running time is 63.66 seconds). Hence, its solution distribution is better than ECM and RNBI. DA cannot produce a good frontier with the given stopping criterion. When the insufficient time is allowed, less unique solutions are obtained and its solution distribution is poorer. However, it is important to note that when these techniques are differently implemented, their solution distribution may be improved.

Implementation considerations:

Again, the implementation of the MOO techniques depends on their algorithms and an algorithm could be implemented in different ways. In this section, the implementation is measured based on the author's implementation and scored based on the author's viewpoint, which may be different from others.

Figure 5.33 shows the author's scores on the implementation criteria, which is same for all the tests of this decision making. Compared with the tests of bi-objective optimisation (City A), the exact methods have a lower score on the criterion of expected result in these tests. This is because the three-objective optimisation problems are difficult, and their optimisation results cannot be easily controlled by predicting possible solutions. DA also has a lower score on the criterion of ease of implementation when optimising three objectives than the score when optimising two objectives because it cannot be directly applied to the three-objective optimisation problem. In general, DA has the highest overall score on implementation considerations mainly because it does not have parameters.

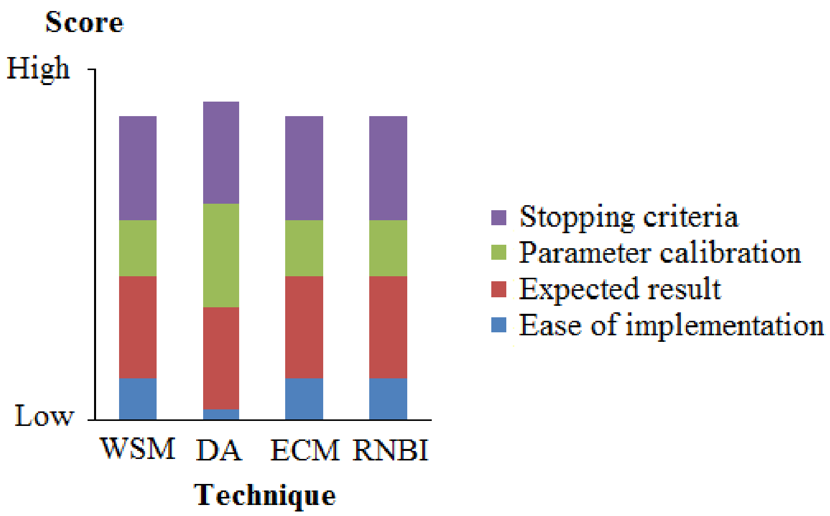


Figure 5.33 Implementation of the listed exact methods in Test B2

5.6 Assessment of the Listed Multi-Objective Optimisation Techniques Based on Practical Decision Making in Infrastructure Asset Management

This section provides a decision-making-oriented education of the existing MOO techniques through an assessment of the techniques in the MOOT List in the context of practical long-term and network-level decision making in IAM. According to the discussion in this chapter, decision making in IAM requires a MOO technique that can fast solve and be easily applied to practical long-term and network-level decision making problems and obtain good solutions. If a MOO technique can satisfy this requirement, it is a robust technique for long-term and network-level decision making in IAM. According to this requirements, eight benchmark criteria are proposed. Figure 5.34 shows the measures of these benchmark criteria. If a MOO technique can satisfy all these benchmark criteria, this technique can be regarded as a robust technique for long-term and network-level decision making in IAM. In the following paragraphs, these benchmark criteria are specified and used to assess the listed techniques.

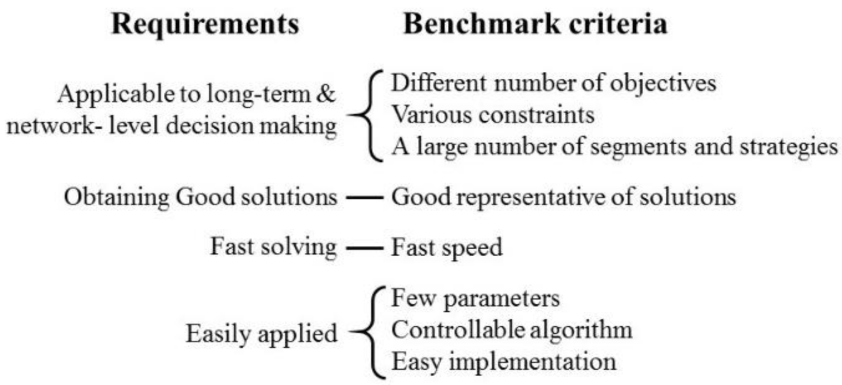


Figure 5.34 Benchmark criteria of robust MOO techniques

Different numbers of objectives: Practical decision making problems in IAM can be modelled as MOO problems where different numbers of objectives may be optimised. For instance, problems with both two and three objectives are defined in this chapter for different decision making problems. More objectives may be needed in other decision making problems. A robust MOO technique should be able to handle optimisation problems with different numbers of objectives.

Both exact methods and heuristics can handle multiple objectives. Generally speaking, heuristics are more flexible than exact methods. All the listed heuristics can directly optimise different numbers of objectives. Hence, these heuristics can be directly applied to handle different decision making problems even they have different objective numbers. Exact methods are more sensitive to the number of objectives. When decision making problems are expressed with three or more objectives, the optimisation process of exact methods may become complicated and time consuming (i.e. WSM, ECM and RNBI) or even ineffective (i.e. DA). Modifications may be needed to enable exact methods to deal with different decision making problems effectively (i.e. DA), while the performance of these exact methods may be affected by the modifications.

Various constraints: Constraints expressed the decision making requirements, the outcome relationships, outcome restrictions, etc. Decision making problems in IAM may involve different types of constraints. A common type of constraints is the one-strategy policy which ensures only one strategy is selected for a segment (see Section 4.3.1). It is determined by the number of analysed segments. Another type of constraints is annual constraints, such as the annual budget condition, which depends on the analysis period. The third type of constraints is overall constraints, such as the total budget, which vary from case to case. There may be other types of constraints such as the constraints of required level of service (an example in Section 6.4.1). A robust MOO technique should be able to handle various constraints and identify feasible solutions, therefore help with different decision making problems.

All the listed MOO techniques can handle constraints, while they do it in different ways. Exact methods guarantee to identify feasible solutions. When more restrictive constraints are analysed, exact methods may spend more time but always identify feasible solutions. Hence, when applying the exact methods, decision makers do not need to worry about the solutions even the constraints are hard. On the contrary, the heuristics may fail to generate feasible solutions when constraints become restrictive as they cannot avoid infeasible solutions. GA, NSGA II and ACO penalise infeasible solutions so infeasible solutions have less chance to be involved in later iterations. PSO guides the new solutions with identified solutions so the feasible solutions have a high chance to be generated. TS and SA attempt to select feasible solutions from the neighbourhoods of identified solutions. However, they only reduce the chance of generating infeasible solutions but cannot guarantee the solution feasibility. Hence, when applying the heuristics, decision makers may not get feasible solutions even feasible solutions exist for their decision making problems. When constraints are too restrictive, the heuristics may need more iterations to generate feasible solutions or worse, cannot identify feasible solutions at all. For example, in the tests of City B, all the heuristics failed to generate feasible solutions, therefore this decision making problem cannot be solved by these heuristics. In general, the listed exact methods are able to handle various constraints and therefore help with the decision making problems with these constraints; however, the heuristics may be inapplicable to the decision making problems with restrictive constraints.

A large number of segments and strategies: Long-term and network-level decision making normally involves a large number of segments and strategies. A robust MOO technique is required to analyse all the segments and strategies in reasonable time.

According to the experimental tests, the listed techniques can handle a large number of segments and strategies for decision making in IAM, while their efficiency is different. In general, heuristics are more sensitive to the problem size than the exact methods. When decision making problems have more segments and strategies, the exact methods only require more time but the heuristics not only require much more time but may also suffer from weakened solution quality. For example, in the problem of City A, when the number of segments grows from 100 to 1,000, the computation time increases to 7-9 times when applying exact methods and over 25 times when applying heuristics. Additionally, the solutions of the exact methods are still guaranteed to be the best solutions; while the solutions of the heuristics approximately doubled their distance to the Pareto solutions. Hence, when analysing decision making problems with different problem size, the exact methods still can guarantee their solution quality while the heuristics may only obtain poor solutions and their analysing time varies largely. Therefore, from the viewpoint of analysing a large number of segments and strategies, exact methods are preferred to the heuristics.

Good representatives of solutions: A MOO problem in practical decision making in IAM may have a large number of Pareto solutions. As stated in Section 4.3.4, too many Pareto solutions may not simplify the decision making process, but waste the decision makers' time. Only a subset of solutions that can correctly represent the existing Pareto solutions is required. A robust MOO technique should identify good representatives of Pareto solutions for MOO problems in decision making in IAM so that these representatives can accurately describe the achievable outcomes, outcome relationships and alternative trade-offs, so as to help to balance objectives. In this research, solution representativeness is measured by solution quality and distribution. A robust MOO technique needs to identify solutions with good quality and distribution.

According to the experimental tests, exact methods produce better representations than the heuristics. Taking advantage of analytical properties, exact methods guarantee to identify Pareto solutions that cover the entire Pareto frontier so they are able to show the complete relationship and trade-offs of objective outcomes. With the exception of DA, other exact methods successfully present the shape and location of the Pareto frontiers in bi- and three-objective optimisation; therefore they are capable of providing correct information of the analysed outcomes to help with different decision making problems. On the contrary, heuristics cannot guarantee their solutions and their solutions are affected by many factors including problem size, restrictiveness of constraints and stopping criteria. In all the tests, solutions obtained by the heuristics fail to present the entire Pareto frontier. Hence, even the heuristics are able to generate feasible solutions for a decision making problem, their solutions may not be able to provide accurate information of the decision making outcomes to help with the decision making process. Therefore, the exact methods that yield good solution distribution are preferred.

Analysis speed: Speed, measured by computation time, is an important measure in practical decision making in IAM. Decision makers may not want to wait for too long to obtain the optimisation result. Hence, a robust MOO technique is required to efficiently solve optimisation problems.

The computation time is related to many factors. It is unfair to measure computation time without considering obtained solutions. When more time is allowed, exact methods may identify more Pareto solutions to improve their solution distribution and therefore better present the outcomes and the outcome relationship. When applying the exact methods, it is important to allow their algorithms to be completed so that a satisfying solution distribution can be achieved to correctly describe the outcome relationship and outcome trade-offs. The heuristics are likely to improve their solution quality with time especially in the early iterations. Hence when applying the heuristics, sufficient time is required to obtain acceptable solutions and therefore provide feasible selections of strategies for decision making problems. According to the experimental tests, GA, TS and NSGA II are faster than PSO, SA and ACO. In general, the listed exact methods can analyse decision making problems and obtain better solutions within less time than the heuristics. Hence, the exact methods are preferred.

Few parameters: Parameters are an important part of MOO techniques. Their calibration affects optimisation results and makes the implementation hard when applying MOO techniques to decision making problems. Hence, a robust MOO technique should have fewer or no parameters.

DA has no parameters, hence when applying it, parameter calibration is not needed. It is the most preferred. The other listed exact methods including WSM, ECM and RNBI only have one parameter that determines the solution density and therefore affects the solution distribution and computation time. However their parameters do not affect the solution goodness. Hence, when applying these methods, their parameter calibration is relatively easy. The listed heuristics have at least one parameter and these parameters are sensitive to the solution quality and computation time, and affect solution distribution. Furthermore, when applying heuristics, proper parameter calibration may largely vary when solving different MOO problems in decision making in IAM. It is impossible to calibrate the heuristic parameters in a way that consistently generates good solutions. When applying the heuristics to decision making problems, their parameters should be carefully calibrated by the decision makers, so their implementation is relatively hard. Therefore, from the parameter calibration point of view, DA is the most preferred technique, followed by WSM, ECM and RNBI. When applying heuristics, GA and NSGA II are preferred as they have fewer parameters.

Controllable algorithm: Decision making in IAM may have specific perspectives on MOO techniques, such as a preferred number of solutions, expected running time, etc. A robust MOO technique should be controllable so that specific perspectives of decision making can be satisfied.

Compared with the heuristics, the exact methods can be more easily controlled as their solutions are more predictable. They are able to consider many perspectives of decision making and generate good solutions satisfying these perspectives. Moreover, DA can identify well distributed solutions under the many decision making perspectives when solving bi-objective optimisation problems, while the solution distribution of WSM, ECM and RNBI is affected when achieving some specific perspectives. When applying heuristics, some perspectives of decision making may be difficult to be achieved such as the preferred number of solutions because of the uncertainties within heuristics. Overall, the exact methods are more controllable and therefore produce more expected results for decision making in IAM than the heuristics.

Flexible implementation: Before applying an optimisation technique to decision making problems, it should be properly implemented. This research aims at a general MOO technique for long-term and network-level decision making in IAM. Its implementation should be flexible so that once implemented this technique can be easily and directly applied to different decision making problems.

All the listed MOO techniques are successfully implemented using Python. Most of them are able to be implemented in a way that both bi- and three- objective optimisation problems of decision making can be directly solved without modification. However, DA requires modifications when optimising three or more objectives when helping with decision making in IAM.

According to the author's viewpoint, the implementation of heuristics is easier than that of the exact methods when solving different decision making problems. The listed exact methods transfer a MOO problem into SOO sub-problems. Thus, a SOO algorithm or tool is needed to solve the sub-problems, which affects the performance of the exact methods. Also, the exact methods are based on mathematical analytics. A mathematical background is necessary for the implementation. However, the heuristics can be easily implemented without any tools or mathematical background. Computer packages and tools of the heuristic algorithms are available, which can be directly invoked to solve optimisation problems. Some techniques such as WSM and DA are effective when decision making problems are expressed as linear optimisation problems and may not be effective when decision making problems are expressed as other types of optimisation problems. Overall, the heuristics are more flexible in their implementation than exact methods when solving MOO problems in decision making in IAM.

5.7 Refinement of Multi-Objective Optimisation Techniques in the Context of Decision Making in Infrastructure Asset Management

This section aims at a further understanding of MOO techniques in the context of long-term and network-level decision making in IAM. A list of MOO techniques is tested and assessed in this chapter. Based on the analysis completed, this section discusses and refines the MOO techniques to guide the development of a robust MOO technique for long-term and network-level decision making in IAM.

5.7.1 Comparing Exact Methods and Heuristics

This research studies two classes of MOO techniques: exact methods and heuristics. MOO techniques are selected from both classes and assessed with experimental tests based on practical decision making in IAM. According to the assessment in Section 5.6, both exact methods and heuristics are applicable to helping with decision making in IAM; however, in general exact methods produce better optimisation result when dealing with MOO problems in decision making in IAM.

The most outstanding strength of exact methods is their high-quality solutions. With an effective SOO algorithm or tool such as Gurobi (Gurobi Optimization Inc, 2012), exact methods are able to analyse different decision making problems and yield Pareto solutions in an acceptable time even for large and complicated decision making problems in IAM. Their solution quality is not affected by problem size and restrictiveness of constraints of the analysed decision making problems. Also, they only have at most one parameter and good controllability.

Heuristics also have advantages. Firstly, they are applicable to solve hard decision making problems, even the decision making problems are expressed as non-linear or fuzzy optimisation problems. However, the decision making problems studied in this research can be expressed as IPs, which are not hard optimisation problems. Hence, this advantage of heuristics is not beneficial for the decision making problems discussed in this research. Secondly, their implementation is easy. It does not require a mathematical background and can be intuitively understood. Yet their applications are difficult because their parameter calibration is difficult when solving specific decision making problems.

In summary, exact methods generate better optimisation results for decision making in IAM. Heuristics, as a more flexible choice, can be applied when the optimisation problems are too hard to be solved by exact methods. This research aims at a robust MOO technique for decision making in IAM; therefore, exact methods are regarded as the main focus.

5.7.2 Comparing Exact Methods

Many exact methods are introduced in this research, including the applied techniques and the new techniques. All of them successfully generate Pareto solutions in a reasonable time in the experimental tests; while none of them achieves satisfying results when solving MOO problems in decision making in IAM.

WSM can easily handle multiple objectives and quickly solve a MOO problem. However, it is not efficient as it may identify the same solutions with different sub-problems when solving a practical decision making problem. According to the tests, around 87.5% of the identified solutions were unique solutions in the two-objective tests (City A); and this number reduces to 14.0%-48.5% in the three-objective tests (City B). According to the author's experience, its efficiency may be worse when analysing other decision making problems of IAM.

The performance of **DA** is very good when solving decision making problems with two objectives. Pareto solutions are efficiently obtained and always well distributed. However, the classic DA can only optimise two objectives. Even when a new algorithm was adopted, which enables optimising three objectives using DA, it fails to obtain good representatives of Pareto solutions with the implementation in this paper; so its solutions cannot correctly present the decision making outcomes and outcome relationship.

ECM and **RNBI** can easily solve bi- and more-objective optimisation problems of decision making in IAM, and obtain Pareto solutions. With their implementation adopted in this research, they identify solutions from one side to another. When fully completed, their solutions are well distributed and able to correctly present the decision making outcomes and outcome relationship. However, if a stopping criterion is achieved before analysing all weights (ECM) or reference points (RNBI), their solutions may only spread over a part of the Pareto frontier (see Table 5.9 and Figure 5.32). Thus, they are not able to generate well spread solutions and describe decision making outcomes under different circumstances. In addition, their computation time is also longer than the other two exact methods. Another weakness of RNBI is it identifies the least unique solutions compared with other listed exact methods.

To be a robust MOO technique, it not only needs to identify good solutions for long-term and network-level decision making in IAM but efficiently and flexibly obtain well distributed solutions under the specific perspectives so that these solutions are able to correctly and completely present the achievable decision making outcomes and outcome relationship. None of the listed MOO techniques can meet all the benchmark criteria of the robust MOO techniques; therefore, it was recommended on the basis of findings in this chapter to redevelop a MOO technique.

CHAPTER 6 DEVELOPMENT AND APPLICATION OF AN OPTIMISATION TECHNIQUE

This chapter aims at improving decision making in IAM by developing a robust MOO technique for long-term and network-level decision making in IAM. According to the discussion completed, none of the listed techniques provides a satisfying optimisation result for long-term and network-level decision making in IAM. Hence, on the basis of the existing MOO techniques, a robust technique is developed to effectively and efficiently solve MOO problems and obtain a satisfying result for the decision making problems in IAM. The main points of this chapter include:

- Developing a new MOO technique for long-term and network-level decision making in IAM;
- Certifying the robustness of the developed MOO technique; and
- Trialling different types of practical decision making problems in IAM with the developed MOO technique.

6.1 Proposed Multi-Objective Optimisation Technique

6.1.1 Bases of the Technique Development

This section specifies the basis of the development of a robust MOO technique. According to the discussion of the existing MOO techniques in Section 5.7, the preference was to develop an exact method rather than a heuristic. The main reasons were:

- More solutions that are guaranteed to be Pareto optimal are identified by the exact methods compared to the solutions of the heuristics;
- The solution quality of the exact methods is not affected by other factors such as problem size and restrictiveness of constraints;

- Solutions of the exact methods have a better distribution covering the entire Pareto frontier;
- The application/implementation of exact methods are easy as they have fewer parameters and are more controllable than the heuristics; and
- The heuristics have some advantages that do not quite outweigh their limitations.

As indicated in the literature review, exact methods can solve a MOO problem by enumerating possible solutions or transforming the MOO problem into a set of SOO sub-problems. Compared to the enumeration based methods, the transformation based methods can efficiently handle a large number of segments and strategies; hence, they can be applied to help with long-term and network-level decision making in IAM. All the listed exact methods solve MOO problems through transformation. Hence, the new MOO technique should be based on transformation. The critical issue is how to transform a MOO problem into SOO sub-problems.

Four exact methods, namely WSM, DA, ECM and RNBI, were examined in Chapter 5. WSM and DA construct a SOO sub-problem by weighted summing all objectives with a pre-defined weight under the original constraints. They are simple and fast. When changing the weights, more SOO sub-problems are constructed and more solutions may be obtained. In this research, WSM defines equidistant weights and DA defines new weights based on the solutions identified in earlier iterations. Both methods generate good results for bi-objective optimisation problems. However, when three or more objectives are optimised, the definition of weights becomes much more complicated; therefore the performance of WSM and DA is weakened with the implementation used in the tests, especially that of DA. RNBI establishes SOO sub-problems by minimising the distance between feasible solutions and reference points. It easily handles multiple objectives and has a good solution distribution. However, compared with the other exact methods, RNBI is not effective as it may identify the same solutions several times and its sub-problems require much time to be solved. ECM constructs a SOO sub-problem by optimising one main objective and converting other objectives into constraints. In particular, it splits the feasible objective space by defining epsilon constraints and searches for solutions within the split objective areas. This transformation is simple and effective, which enables ECM to optimise different numbers of objectives. In the classic ECM, the values of epsilons are equidistant. To achieve well distributed solutions, all the pre-defined values of epsilon need to be analysed. If ECM is not fully completed, for instance a stopping criterion is achieved before analysing all the pre-defined values of epsilon, its solution distribution may be poor. This is the main weakness of ECM. However, if the objective space is split and the epsilon is defined in a better manner, the algorithm can be improved and a better result could be obtained. Thus, in this research, a MOO technique is proposed based on ECM but with a sound way of splitting objective space and defining epsilon.

6.1.2 Analysis Algorithm

This part introduces a MOO technique named Dynamic Epsilon Constraint method (DECM) for long-term and network-level decision making in IAM. The proposed MOO technique solves a MOO problem by transforming it into SOO sub-problems. It is based on ECM, but splits the objective space and defines the epsilon in a better manner; so that the strength of ECM is inherited and its weaknesses are improved when solving MOO problems in decision making in IAM.

Similar with ECM, the new technique constructs SOO sub-problems by optimising one main objective and converting other objectives into epsilon constraints. Borrowing the idea from DA, DECM uses a dynamic way to define the epsilon. More specifically, it splits an objective space and defines epsilon according to the solutions identified in the earlier iterations. Once a new solution is obtained by solving a SOO sub-problem this new solution is used to split its objective space, generate more epsilon values and construct more new SOO sub-problems. This procedure repeats until the entire objective space is explored or a stopping criterion is satisfied.

To simplify the description of the proposed technique, terms and assumptions are specified in the following paragraphs. Figure 6.1 illustrates some terms with an example of a three-objective optimisation problem of decision making in IAM, where f_1 is considered as the main objective and f_2 and f_3 are the other objectives that are converted to epsilon constraints.

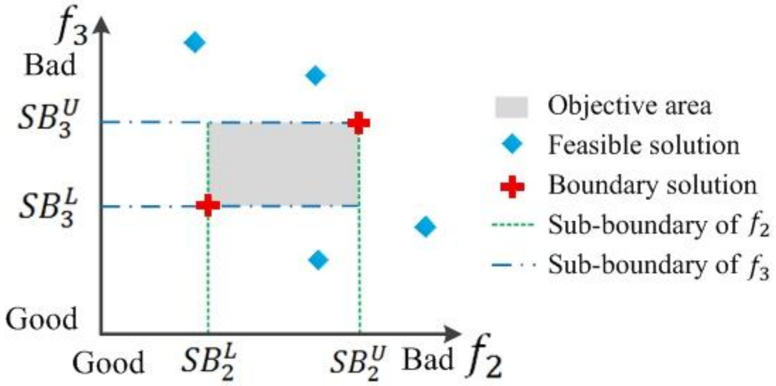


Figure 6.1 Example of an objective area

The terms used in this chapter include:

- The "other objectives" are all the objectives excluding the main objective;
- The "objective space" represents the entire area covered by all the feasible solutions in the space of all objectives;
- An "objective area" represents a split area surrounded by boundary solutions in objective space;
- "Boundary solutions" are feasible solutions located at two corners of an objective area;
- A "sub-boundary" of objective k is defined with its upper bound (SB_k^U) and lower bound (SB_k^L);
- A "boundary" is the set of all sub-boundaries of an objective area; and
- "Epsilon" $\boldsymbol{\varepsilon}$ is a set of values which is used to define epsilon constraints when constructing a SOO sub-problem. It is similar with the epsilon of ECM.

The assumptions used in this chapter include:

- The MOO problem of a decision making problem is formulated as IP, where all objectives are to be minimised;
- Pareto solutions of a MOO problem exist; and
- The main objective of a MOO problem is the first objective f_1.

Figure 6.2 contains a flow chart of the developed technique. Figure 6.3 intuitively describe its steps when dealing with a bi-objective optimisation problem.

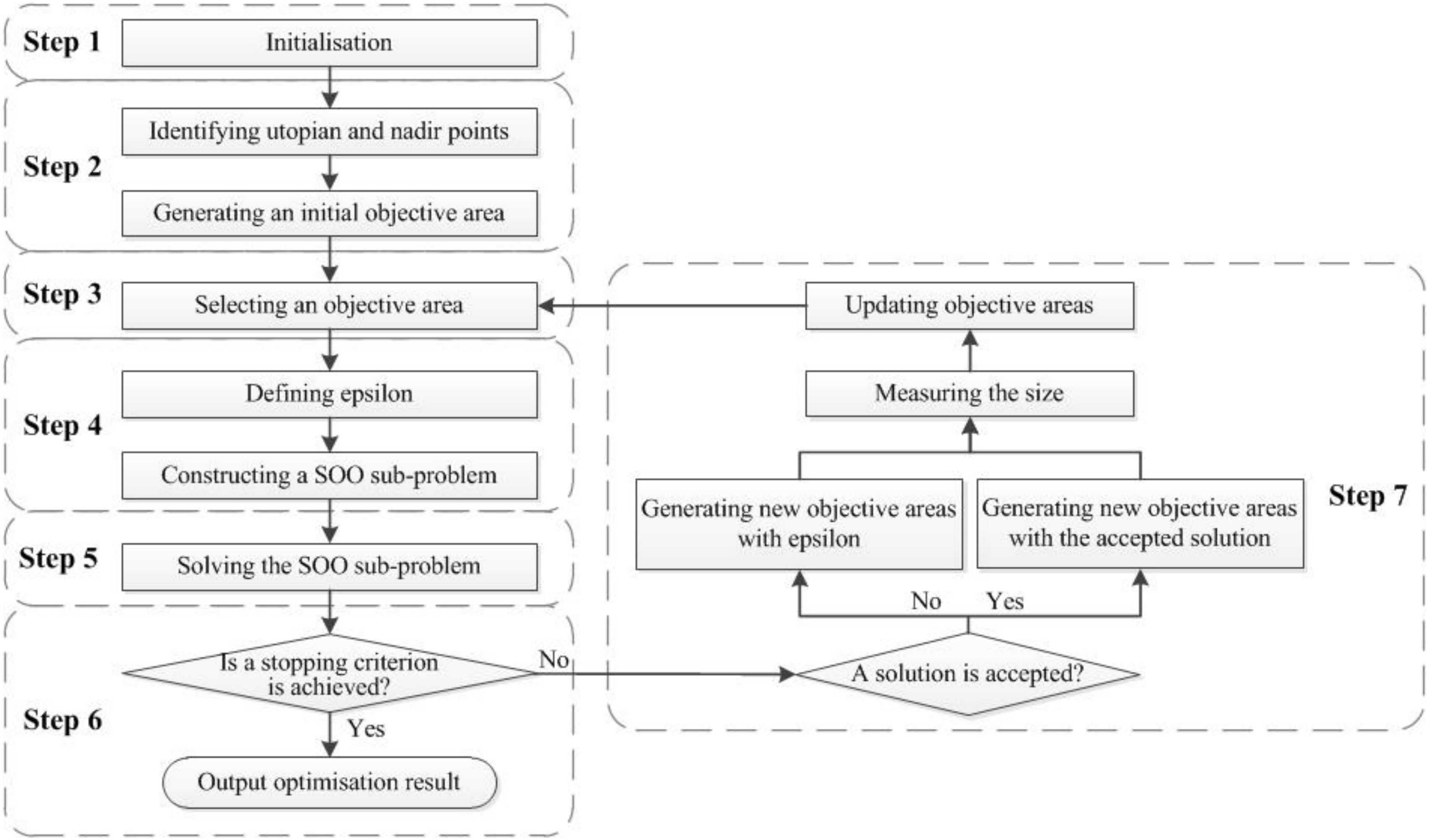


Figure 6.2 Flowchart of DECM

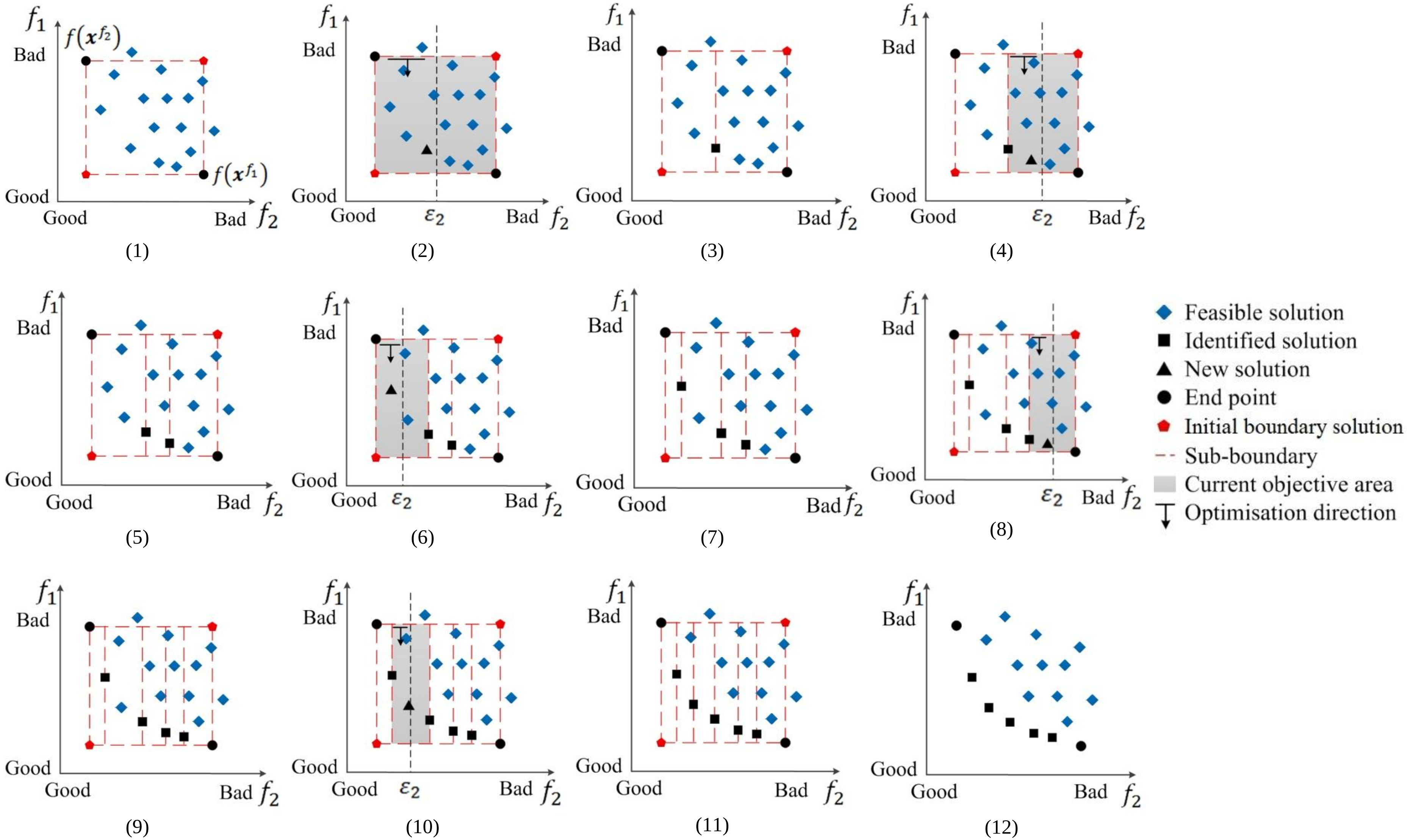


Figure 6.3 Steps of DECM

The steps involved in DECM are:

Step 1: Initialisation. This step initialises all the variables and lists, including the set of identified solutions ($\boldsymbol{X} = \emptyset$) and the list of all objective areas ($\boldsymbol{Area} = \emptyset$). Specific perspectives and requirements of decision making in terms of stopping criteria should also be defined if they exist.

Step 2: Initial objective area. This step generates an initial objective area and its boundary using end points. In detail, every objective is lexicographically optimised and generates an end point. Then two initial boundary solutions are generated, where one is defined as the best values of objective vectors that all the end points are achieved and the other is defined as the worst values of all objective vectors that all the end points are achieved. An example is shown in Figure 6.3 (1).

The initial objective area is surrounded by the utopian and nadir points as shown in Figure 6.3 (1). In other words, utopian and nadir points are the boundary solutions of the initial objective area. The initial objective area is added into $\boldsymbol{Area}$.

Step 3: Selection of an objective area. This step measures the size of all the objective areas in $\boldsymbol{Area}$ using Equation 6.1 and selects the one with the largest size as the current objective area for further analysis. If an acceptable smallest size ($Size_{limit}$) is defined, the objective areas that are smaller than $Size_{limit}$ are abandoned.

$$size = \prod_{k=2}^{K} \frac{f_k(\boldsymbol{x}) - f_k(\boldsymbol{x}')}{R_k} \qquad \text{Equation 6.1}$$

$$R_k = N_k - U_k \qquad \text{Equation 6.2}$$

where, $size$ size of an objective area;

R_k range of objective k;

N_k and U_k utopian and nadir value of objective k; and

others are same as above.

Step 4: Construction of a SOO sub-problem. This step constructs a SOO sub-problem within the current objective area. More specifically, only the main objective (f_1) is optimised (Equation 6.3) under the epsilon constraints of the other objectives (Equation 6.4) and the original constraints of the MOO problem (Equation 6.6).

$$\min f_1(\boldsymbol{x}) \qquad \text{Equation 6.3}$$

$$\text{s.t.} \quad f_k(\boldsymbol{x}) \le \varepsilon_k, \quad k = 2,3,\cdots,K \qquad \text{Equation 6.4}$$

$$\varepsilon_k = \frac{1}{2}(SB_k^U + SB_k^L) \qquad \text{Equation 6.5}$$

$$\boldsymbol{x} \in \boldsymbol{\Omega} \qquad \text{Equation 6.6}$$

where, SB_k^U and SB_k^L upper and lower bound of the sub-boundary on objective k; and others are same as above.

Here, the epsilon, given by Equation 6.5, is defined as the average of the upper and lower bound of the sub-boundaries of the current objective area (see Figure 6.3 (2), (4), (6), (8) and (10)). MOO problems of practical decision making in IAM normally have a large number of Pareto solutions that spread to the entire Pareto frontier. This definition of epsilon attempts to obtain a solution that is located in the middle of an objective area, and therefore fills the gap between the solutions identified in the earlier iterations. Hence, this definition of epsilon improves the distribution of identified solutions.

Step 5: Solving. This step solves the SOO sub-problems constructed in Step 4. Many algorithms and software tools are applicable. In this research, Gurobi is used to identify the optimal solutions of SOO sub-problems.

When no Pareto solution exists in the current objective area, the solution of the SOO sub-problem is only a feasible solution of the original MOO problem. Hence once a feasible solution is obtained, its non-dominancy is checked. If this solution is dominated by identified solutions, this solution is not a Pareto solution and is abandoned; otherwise this solution is accepted and added into the solution set $\boldsymbol{X}$. It is important to note that even this solution is not dominated by all the previously identified solutions it still could be non-Pareto solutions. However, this is rare because MOO problems of practical decision making in IAM often have a large number of Pareto solutions covering all the space of the objective areas.

Step 6: Completion check. This step checks the completion of the algorithm. If any stopping criterion is achieved, this algorithm stops and outputs the optimisation result. Otherwise, the algorithm goes to Step 7.

Step 7: Generation of objective areas. This step splits the current objective area and generates smaller objective areas. This step has two branches according to whether a solution is accepted in Step 5:

If a solution is accepted in Step 5; this solution is used to split the current objective area according to its location. Figure 6.4 shows an example of the generation of new objective areas with an identified solution. A sub-boundary is divided into two smaller parts by inserting the corresponding objective value of the accepted solution. After dividing the sub-boundaries of all the other objectives, the new objective areas are generated by combing the divided sub-boundaries. For a bi-objective optimisation problem, an objective area is split into two new areas (see Figure 6.3 (3), (5), (7), (9) and (11)); and for a three-objective optimisation problem, an objective area is split into four new areas (see Figure 6.4). When more objectives are optimised, more objective areas are generated with one current objective area.

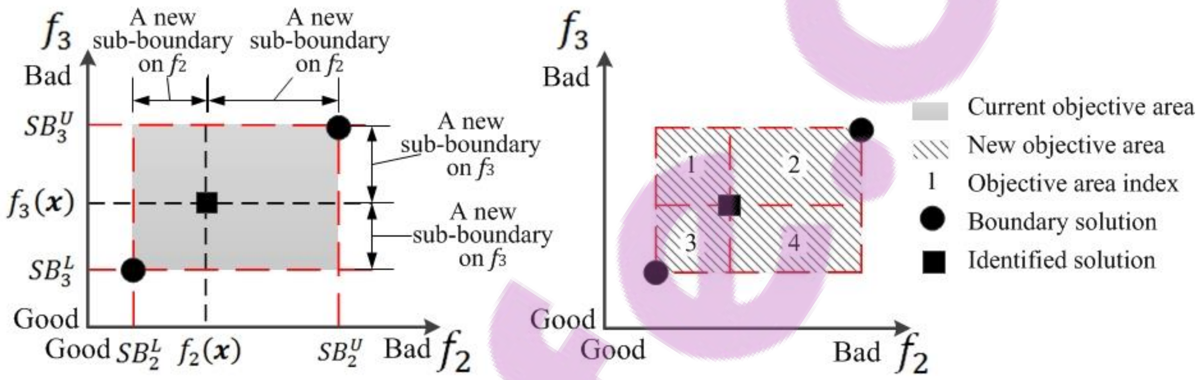


Figure 6.4 Generation of objective area with an identified solution

If no solution is accepted in Step 5, new objective areas are generated by allowing the non-analysed parts of the current objective area. DECM only searches for solutions within a part of the current objective area as shown in Figure 6.5; but Pareto solutions may be located in the other part, such as solution A in Figure 6.5. Therefore, when no solution is accepted in Step 5; the algorithm defines new objective areas with the non-analysed parts of the current objective area in order to avoid losing potential Pareto solutions.

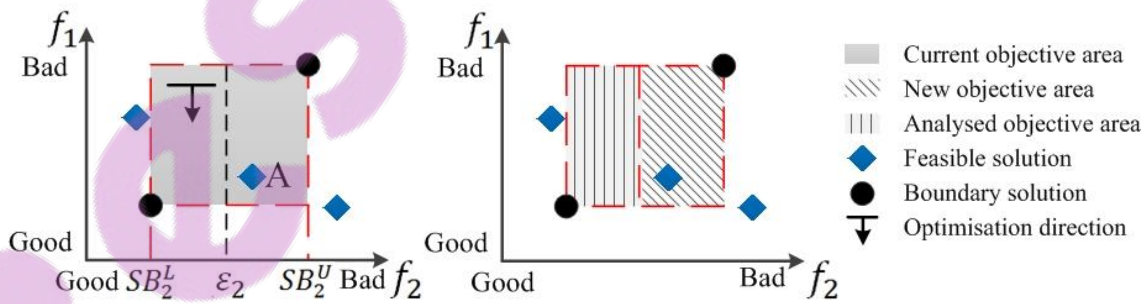


Figure 6.5 Generation of objective area with epsilon

Finally, all the new objective areas are added into $\boldsymbol{Area}$. The algorithm goes back to Step 3 until all the objective areas in $\boldsymbol{Area}$ are analysed.

DECM not only inherits the strengths of ECM, but also improves the weaknesses of the classical ECM. Similarly with ECM, it easily handles multiple objectives and obtain Pareto solutions; while it is more flexible than ECM and does not require pre-defining epsilon, which eases the applications of DECM. Furthermore, it has good solution distribution even if a stopping criterion (i.e. a given length of time) is satisfied before analysing all pre-defined epsilon.

Other MOO methods are also developed based on the classic ECM, i.e. an adaptive ECM (Laumanns *et al.*, 2005), a hybrid ECM (Becerra and Coello, 2006), box method (Hamacher *et al.*, 2007), an improved ECM (Ehrgott and Ruzika, 2008). Comparing with these methods, DECM has a more suitable definition and measurement of objective areas when solving MOO problems of decision making in IAM. As stated earlier, MOO problems of decision making in IAM often have a large number of Pareto solutions existing and only good representatives of Pareto solutions are needed. DECM is able to obtain new Pareto solutions that fill the gap between the identified ones through dynamically splitting objective areas. Hence it has a higher chance to yield the good representatives of solutions with the same number of SOO sub-problems or same computation time when solving MOO problems of decision making in IAM than the other ECM based method. On the other hand, if a MOO problem only has a few Pareto solutions existing, DECM is able to obtain all the existing Pareto solutions that are unique in the objective space between the end points; while other ECM based methods such as a hybrid ECM (Becerra and Coello, 2006) may not. Finally, DECM is flexible when solving optimisation problems in decision making in IAM.

6.1.3 Implementation of the Developed Technique

This section discusses the developed technique by specifying its computer implementation. More specifically, the developed technique is implemented in a way that its computer programmes can be easily invoked to solve different MOO problems for decision making in IAM. The pseudo codes of the developed technique including Algorithms 1-5 are provided, where variables are in italics and functions are in bold and italics. To clarify the implementation, the assumptions and terms stated in Section 6.1.2 are also applied here.

Comment is added on the right side of the pseudo codes beginning with “#”. Gurobi is used as a SOO solver; thus, the proper installation of Gurobi is required (Gurobi Optimization Inc, 2012).

Algorithm 1: *Main*

this programme is the main program of DECM

$\boldsymbol{InputData}()$	# Input data.
$solutionList = []$	# a list of identified solutions.
$objectiveAreaList = []$	# a list of objective areas.
FOR each obj in $objectives$:	# $objectives$: objective matrix, where $objectives[k]$ is the list of the corresponding outcome value of all strategies on objective k and $objectives[k][i]$ is the corresponding outcome of objective k if strategy i is applied.
$\boldsymbol{x} = \boldsymbol{LexOptimising}(obj)$	# Return the optimal solution when lexicographically optimising objective obj under original constraints.
add $\boldsymbol{x}$ into $solutionList$	
ENDFOR	
$initialBoundarySolution = \boldsymbol{Solution}(solutionList)$	# Return the initial two boundary solutions
$objectiveAreaList = \boldsymbol{Intialisation}(initialBoundarySolution)$	# Return the initial objective area defined by the two initial boundary solutions.
WHILE $objectiveAreaList$ is not empty:	
$index = \boldsymbol{MaxSize}(objectiveAreaList)$	# Return the index of the largest-sized objective area in $objectiveAreaList$.
$currentObjectiveArea = objectiveAreaList[index]$	
delete $objectiveAreaList[index]$ from its list	
$boundary = \boldsymbol{Boundary}(currentObjectiveArea)$	# Return the boundary of the current objective area, where a boundary is a set of upper and lower bounds on all objectives.
$epsilon = \boldsymbol{Epsilon}(currentObjectiveArea)$	# Return the epsilon of the current objective area.

$\boldsymbol{x} = \boldsymbol{OptWithEpsilon}(epsilon)$ # Return optimal solution of a SOO sub-problem established by $epsilon$ (see Algorithm 2).

IF $\boldsymbol{x}$ is a non-dominated solution and x not in $solutionList$:

  $accept$ = True

  add $\boldsymbol{x}$ into $solutionList$

ELSE:

  $accept$ = False

ENDIF

IF a stopping criterion is satisfied:

  Output $solutionList$

ENDIF

IF $accept$ is True:

  $newObjectiveAreas = \boldsymbol{GenerateNewObjectiveAreasWithSolution}(currentObjectiveArea, \boldsymbol{x})$

  # Generate new objective areas based on $currentObjectiveArea$ and solution $\boldsymbol{x}$ (see Algorithm 3).

ELSE:

  $newObjectiveAreas = \boldsymbol{GenerateNewObjectiveAreasWithEpsilon}(currentObjectiveArea, epsilon)$

  # Generate new objective areas with $currentObjectiveArea$ and epsilon $epsilon$ (see Algorithm 3).

ENDIF

Add $newObjectiveAreas$ into $objectiveAreaList$

ENDWHILE

Algorithm 2: $\boldsymbol{OptWithEpsilon}(epsilon)$

This algorithm constructs a SOO problem with epsilon and solves it. Gurobi is needed.

$newObjective = objectives[1]$	# Defining the first objective as the main objective.
$newConstraints = constraints$	# $constraints$: original constraint matrixe, where each row is the coefficients of a constraint
$newConstraintLimits = constraintLimits$	# $constraintLimits$: a list of the limitation of the original constraints
FOR k = 2 to $numObj$:	$numObj$: the number of objectives
add $objective[k]$ into $newConstraints$	
add $epsilon[k]$ into $newConstraintLimit$	
ENDFOR	

$\boldsymbol{x} = \boldsymbol{Optimising}(newObjective, newConstraints, newConstraintLimits)$

Optimising the new objective $newObjective$ under new constraints $newConstraints$.

return $\boldsymbol{x}$

Algorithm 3: $\boldsymbol{GenerateNewObjectiveAreasWithSolution}(objectiveArea,\ aSolution)$

This algorithm generates new objective areas with current objective area and an identified solution.

```
boundValue = Boundary(objectiveArea)      # Return the boundary of the current objective area.
obj = ObjectiveValue(aSolution)           # Return the objective values of a solution.
FOR k = 1 to numObj:
    add obj[k] into boundValue[k]         # boundValue: new values of sub-boundary of objective k.
ENDFOR
return BoundGeneration(boundValue)        # Return a list of boundaries (see Algorithm 5).
```

Algorithm 4: $\boldsymbol{GenerateNewObjectiveAreasWithEpsilon}(objectiveArea,\ aEpsilon)$

This algorithm generates new objective areas with current objective area and a set of epsilon.

```
boundValue = Boundary(objectiveArea)      # Return the boundary of the current objective area.
FOR k = 1 to numObj:
    Add aEpsilon[k] into boundValue[k]
ENDFOR
return BoundGeneration(boundValue)        # Return a list of boundaries (see Algorithm 5).
```

Algorithm 5: *BoundGeneration*($boundValue$)

This algorithm generates new objective areas.

$newboundaryValue = []$ # $newboundaryValue$: a matrix of available sub-boundaries, where a row contains the available values of sub-boundaries of an objective.

FOR k = 2 to $numObj$:

  Sort the order of the values in $boundValue[k]$

  FOR $index$ = 2 to element number in $boundValue[k]$:

    $aSubBoundary = [boundValue[k][index - 1],\ boundValue[k][index]]$ # Generating a sub-boundary of objective k

    Add $aSubBoundary$ into $newboundaryValue[k]$

  ENDFOR

ENDFOR

$newboundaryList = \boldsymbol{Combination}(newboundaryValue)$ # Each row of the matrix $newboundaryValue$ contains alternative bounds on an objective. This step combines the bounds of all objectives and returns all the possible combinations.

return $\boldsymbol{ObjectiveAreaGeneration}(newboundaryList)$ # Return the new objective areas that are defined based on the boundaries in $newboundaryList$.

6.2 Discussion of the Algorithm

This section further explores the characteristics of DECM. DECM is flexible and can be easily controlled to achieve specific perspectives and requirements of decision making in IAM. The flexibility and controllability is managed by defining and calibrating parameters. This section discusses these parameters and their impacts. However, if a decision making problem does not have any specific requirement, these parameters are set as default values, and the algorithm obtains all the existing Pareto solutions that are unique in the objective space between end points.

6.2.1 Size Measurement

Size measures the gap between identified solutions and decides the priority of objective areas; therefore its measurement affects the solution distribution. A proper size measurement can improve the solution distribution, and produce a good frontier with fewer solutions.

Originally, the size is measured by the product of the edges of an objective area using Equation 6.1. According to the author's experience, this size measurement generates good optimisation results for many decision making problems. However, other size measurements are also investigated. For instance, the size can also be measured by Tchebychev distance of the boundary solutions using Equation 6.7. With this measurement, the size is determined by the longest edge of objective areas which is suitable when the shape of the objective space is narrow. Other size measurements may be adopted in specific IAM decision making problems

$$size = \max\left\{\frac{f_k(x)-f_k(x\prime)}{R_k}\middle| k = 2,3,\cdots,K\right\} \qquad \text{Equation 6.7}$$

$$R_k = N_k - U_k \qquad \text{Equation 6.8}$$

where, all are same as above.

6.2.2 Soft Epsilon Constraints

Soft epsilon constraints broaden the searching space and therefore may improve the efficiency of DECM. The developed technique defines the epsilon constraints as Equation 6.4. When a large number of Pareto solutions exist, this definition of epsilon constraints is able to obtain new solutions located in the middle of the objective areas so that improves the solution distribution and representativeness. However, when a MOO problem only has a small number of Pareto solutions existing, such as decision making problems with a very low budget, it is important to quickly obtain existing Pareto solutions in a short time, and soft epsilon constraints should be used and make an optimisation problem easier to be solved. Soft constraints are introduced for example by Meseguer *et al.* (2006). For DECM, instead of Equations 6.3-6.6, SOO sub-problems are established using Equations 6.9-6.12, where a slack variable sc is added to relax the original epsilon constraints (Equation 6.4).

$$\min f_1(\boldsymbol{x}) + w_{sc} * sc \qquad \text{Equation 6.9}$$

$$\text{s.t.} \quad f_k(\boldsymbol{x}) \leq \varepsilon_k + sc, \quad k = 2,3,\cdots,K \qquad \text{Equation 6.10}$$

$$\varepsilon_k = \frac{1}{2}(SB_k^U + SB_k^L) \qquad \text{Equation 6.11}$$

$$\boldsymbol{x} \in \boldsymbol{\Omega}, \; sc \geq 0 \qquad \text{Equation 6.12}$$

where, sc slack variable;

w_{sc} penalty variable of soft constraints; and

others are same as above.

Figure 6.6 shows an example of soft epsilon constraints. When applying the original epsilon constraints, the epsilon constraint line is located in the middle of an objective area and only the left half part of the objective area is examined. Hence further analysis is needed to obtain solution A. However, when the soft epsilon constraints are used the epsilon constraint line moves to the right so that more of the objective area is examined and the algorithm has more chance to obtain Pareto solutions such as solution A. However, if solution A is located on the left side of the middle line, it is still able to obtain solution A with soft epsilon constraints and in this case, $sc = 0$.

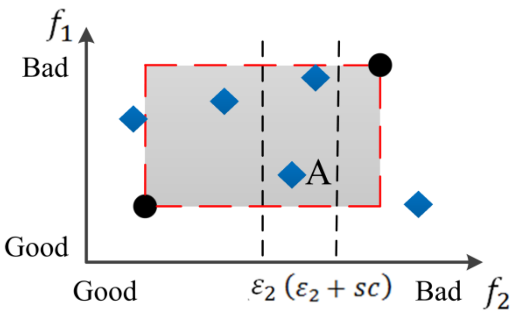


Figure 6.6 Example of epsilon constraints and soft epsilon constraints

In general, soft epsilon constraints enable the algorithms to obtain Pareto solutions with fewer SOO sub-problems, therefore improving their efficiency especially when a small number of Pareto solutions exist.

6.2.3 Stopping Criteria

Stopping criteria are defined to achieve specific requirements of decision making in IAM. The following paragraphs discuss some common requirements and their stopping criteria. In DECM, all the stopping criteria are checked at Step 6. If any of the stopping criteria is satisfied, the algorithm stops and outputs the obtained optimisation result.

Expected number of solutions: In practical decision making in IAM, a large number of Pareto solutions may exist. Some decision makers may only want a small number of solutions to represent the entire relationship between outcomes and alternative trade-offs and a stopping criterion for the number of solutions is defined. In Step 6, if the number of identified solutions equals the expected number of solutions then the algorithm stops. Because the algorithm analyses one objective area in an iteration, the exact expected number of solutions can be obtained.

Preferred computation time: In long-term and network-level decision making in IAM, it may require a long time to completely solve its MOO problem. Decision makers may only want to spend a short time on the optimisation; therefore a stopping criterion of the computation time is needed. In Step 6 once the algorithm spent the preferred length of time, the algorithm stops. Because the computation time is checked once in every iteration; the real computation time is likely to be a little longer than the preferred one.

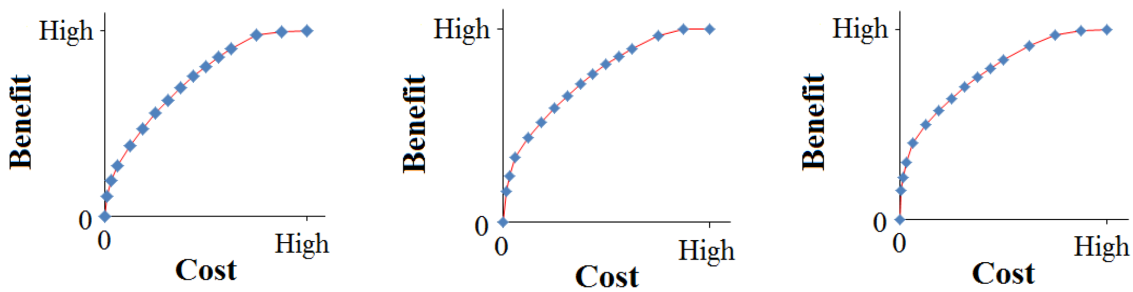
Preferred density of solutions: This is another way to control the distribution of identified solutions. In practical decision making in IAM, some Pareto solutions have similar outcomes and are not helpful with the trade-offs. These solutions are closely located in the objective space; hence they can be avoided by controlling the density of solutions. To achieve this, a stopping criterion for the minimum acceptable size of objective areas is defined. If solutions are too close, the size of the objective area defined by these two solutions is small. When the size of an objective area is smaller than the minimum acceptable size, this objective area is abandoned so that no more solutions will be obtained between these two solutions.

6.3 Comparison of the Dynamic Epsilon Constraint Method with Other Existing Techniques

The aim of this section is to benchmark the performance of the developed MOO technique through evaluating and comparing the developed technique with the other MOO techniques. More specifically, the experimental tests designed in Section 5.5 are repeated with the developed technique and its performance is discussed and compared to the techniques in the MOOT List.

6.3.1 Tests of Bi-Objective Problem (City A)

The decision making of City A is introduced in Section 5.5.1, which attempts to keep its road network in acceptable condition for at least twenty years. A bi-objective optimisation problem, given by Equations 5.35-5.39, is established for this decision making problem, which maximises the benefit and minimises the cost under the constraints of acceptable yearly condition and annual budget. Three tests are designed to analyse a part of the road network or the entire road network (see Table 5.6). These three tests are solved by DECM. Similarly to the other exact methods, 14 SOO sub-problems are constructed (exclude the lexicographical optimisation for end points) by DECM in each test. The cost and benefit of the identified solutions are illustrated in Figure 6.7; and the performance is shown in Table 6.1. It is important to note that the Pareto frontier in this table is a large set of discrete but closely located Pareto solutions, which looks like a line.



(1) Test A1 (2) Test A2 (3) Test A3

Figure 6.7 Identified solutions in the tests of bi-objective problem (City A)

Table 6.1 Criterion values of DECM in the experimental tests of bi-objective optimisation (City A)

Test index	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time		Implementation			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
A1	16	100	0	0.0699	0.8018	0.0176	13.80	0.86				
A2	16	100	0	0.0602	0.7602	0.0251	104.69	6.54	3.3	Most	None	N
A3	16	100	0	0.0619	0.6732	0.0273	203.85	12.74				

Note: "Most" means most common expectations can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need a pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

Compared with the listed MOO techniques, for which optimisation results are shown in Appendix B, DECM effectively solves the tested MOO problems for City A and obtains good optimisation results in all the three tests. The following paragraphs discuss the performance of the developed MOO technique.

Solution quality: As an exact method, DECM identifies Pareto solutions; hence, its solution quality is better and more stable than the solutions of the heuristics. Compared with the listed exact methods, DECM is able to obtain a unique Pareto solution with each sub-problem. As shown in Figure 6.8, DECM is one of the techniques that generate the most unique Pareto solutions in Test A2, as well as in other tests. Overall, DECM is one of the MOO techniques that have the best solution quality.

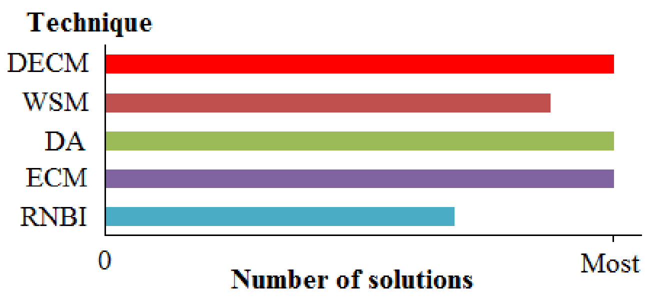


Figure 6.8 Number of identified unique Pareto solutions in Test A2

Solution distribution: DECM produces the best solution distribution of the listed MOO techniques. As shown in Figure 6.7, the solutions of DECM evenly spread to and completely cover the entire Pareto frontier.

Compared with the listed heuristics, the solutions of DECM correctly demonstrate the shape and location of the Pareto frontier; whereas the solutions of the heuristics fail to show the entire Pareto frontier. Hence, DECM has much better solution distribution than the heuristics.

Compared with the listed exact methods, DECM also has better solution distribution. In detail, the solution distribution is measured by the three criteria: coverage error, uniformity level ratio and spacing. According to Figure 6.9, DECM performs better on every criterion than the listed exact methods in Test A2. In the other two tests, DECM also performs very well on these solution distribution criteria. In general, DECM achieves the best solution distribution in the three tests of bi-objective optimisation.

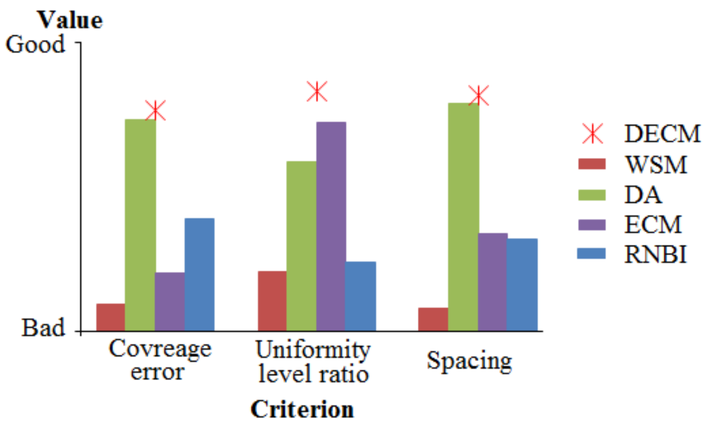


Figure 6.9 Solution distribution in Test A2

Computation time measures: The computation time of DECM is not the shortest but is acceptable. DECM is based on ECM but needs more time than ECM as it uses a dynamic way of splitting objective areas and defining epsilon. However, it only spends 203.85 seconds to analyse an experimental test of two-objective decision making problems with 1,822 segments and 61,936 strategies, and averagely obtains a unique Pareto solution in 12.74 seconds in this test. This decision making problem is based on twenty-year and only needs to be solved once before the management. Comparing with the analysis period, its computation time is not long.

The computation time of DECM increases with the growth of the problem size as shown in Figure 6.10. The computation time grows to around 8 times when the analysed segments increase from 100 to 1,000 and is doubled when the analysed segments increase from 1,000 to 1,822. This is similar to the other listed exact methods and better than the listed heuristics. More time is needed when analysing more segments and strategies.

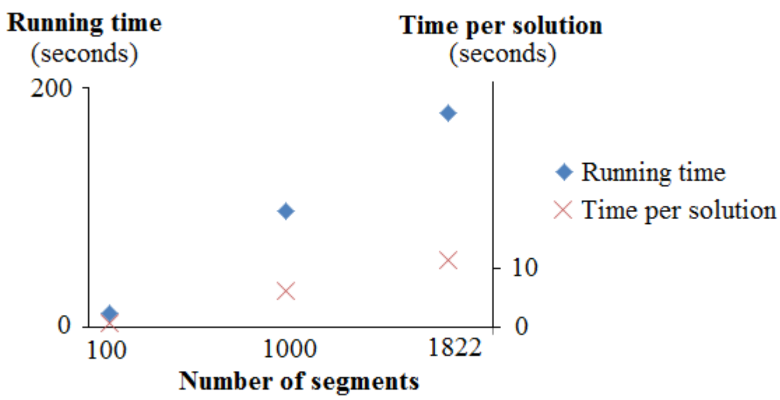


Figure 6.10 Increase in computation time with the growth of problem size (City A)

Furthermore, the computation time of DECM only has a small effect on its solutions measuring by solution goodness and distribution. In respect of solution goodness, DECM always obtains Pareto solutions, so its solution goodness is always the best. This is much better than the heuristics, of which insufficient time may largely weaken the solution quality. In respect of solution distribution, when insufficient time is allowed, DECM obtains fewer solutions so the solution density is smaller and the solution distribution is poorer. However, its impact is small comparing with the other listed exact methods. Figure 6.11 shows the solutions identified within different lengths of time in Test A2. Compared with the other techniques, the solutions of DECM are still evenly distributed and present the entire Pareto frontier with insufficient time. This is because DECM identifies new solutions that fill the gap between the identified solutions; therefore, even though a few solutions are obtained, these solutions still have good distribution and able to show the entire outcome relationship and trade-offs. However, the solutions of the other techniques including WSM, ECM and RNBI may only cover a part of Pareto frontier within insufficient time.

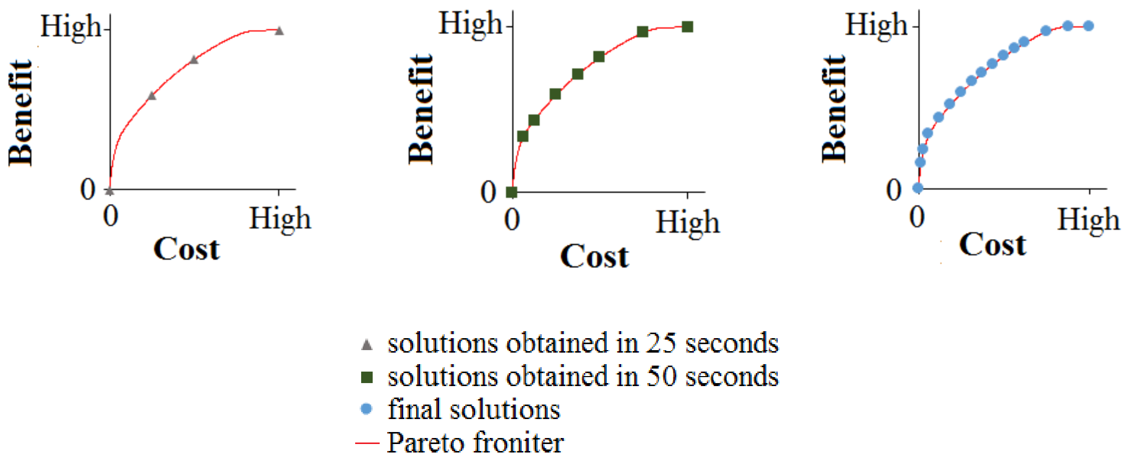


Figure 6.11 Solutions obtained in different lengths of time for Test A2

Implementation considerations: DECM performs well on the implementation considerations. According to Figure 6.12, it achieves the highest score on every implementation criterion compared with the listed exact methods. Similarly with the exact methods, DECM also requires a SOO algorithm or solver such as Gurobi, and may not be applicable to other types of optimisation problems such as non-linear optimisation problems. Hence, a score of 3.3 is given to the criterion of ease of implementation. However, it obtains the highest score on the other implementation criteria because (1) it can be easily controlled and achieve various stopping criteria; (2) it does not have parameters but specific parameters can be defined to adjust optimisation result; and (3) a pre-defined stopping criterion is not necessary.

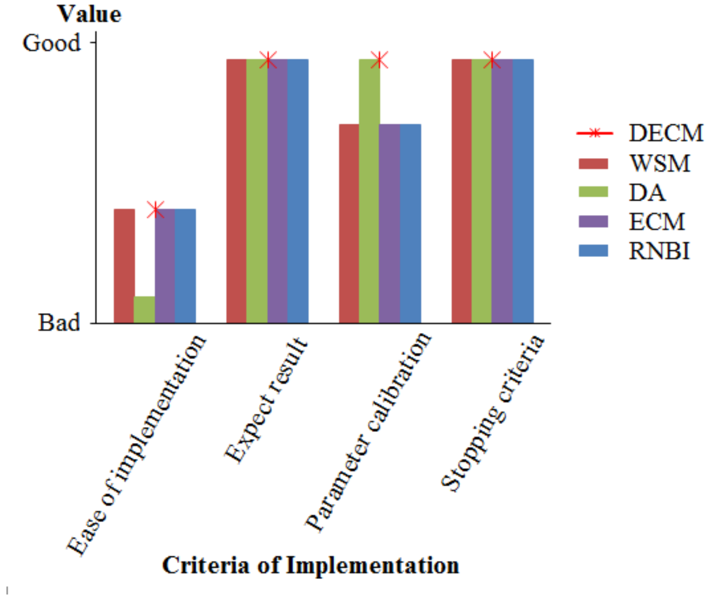
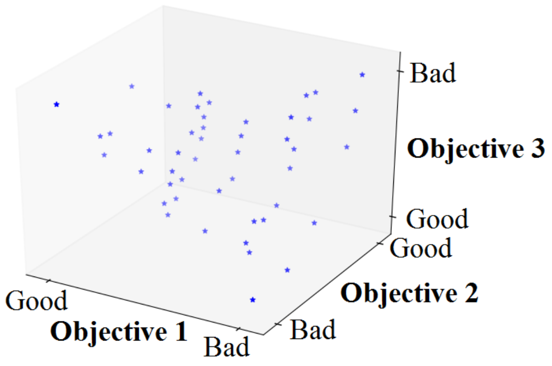


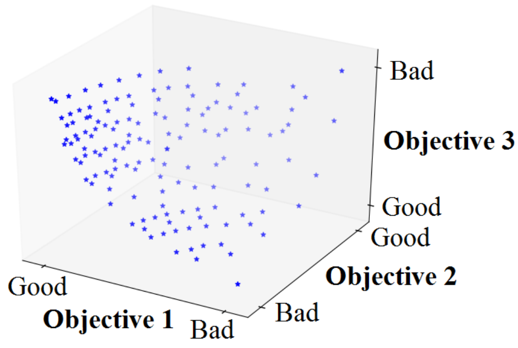
Figure 6.12 Implementation of DECM compared with the other exact methods

6.3.2 Tests of Three-Objective Problem (City B)

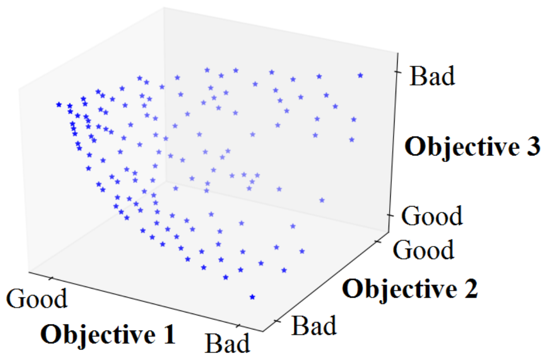
The decision making problem of City B is introduced in Section 5.5.2, which attempts to allocate an insufficient maintenance budget to the three sub-networks of its pavement network in the next 22 years. A three-objective optimisation problem given by Equations 5.40-5.44 is established for this decision making problem to maximise the benefit of each sub-network under the constraints of the insufficient budget of the whole network. Here benefit is also calculated based on the condition improvement. Three experimental tests are designed to analyse a part of the road network or the entire road network (see Table 5.10). These three tests are solved by DECM. Similarly with the other exact methods, a number of 133 SOO sub-problems are constructed by DECM. The objective values of the identified solutions are illustrated in Figure 6.13; and its performance is shown in Table 6.2.



(1) Test B1



(1) Test B2



Test B3

Figure 6.13 Identified solutions in the tests of three-objective optimisation (City B)

Table 6.2 Criterion values of DECM in the experimental tests of three-objective optimisation (City B)

Test index	**Aspects and Criteria**											
	Solution Quality			Solution Distribution			Computation Time		Implementation			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
B1	47	100	0	0.3518	0.4756	0.0796	19.61	0.42	3.3	Most	None	N
B2	129	100	0	0.3242	0.3489	0.0346	171.16	1.32				
B3	139	100	0	-	0.4109	0.0348	513.51	3.69				

Note: "Most" means most common expectations can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.
"-" means this criterion is not applicable.

Compared with the other MOO techniques, of which the results are shown in Appendix C, DECM effectively solves the MOO problem of this decision making problem and produces good optimisation results. Because the listed heuristics fail to generate feasible solutions in the tests, they are not compared to DECM.

Solution quality: DECM identifies Pareto solutions; and its solution goodness is always the best. Compared with the other exact methods, it is more effective. According to Figure 6.14, DECM obtains the most unique Pareto solutions in Test B2 with 133 sub-problems. It also obtains the most unique Pareto solutions in the other two tests. Therefore, DECM has a better solution quality than the listed exact methods.

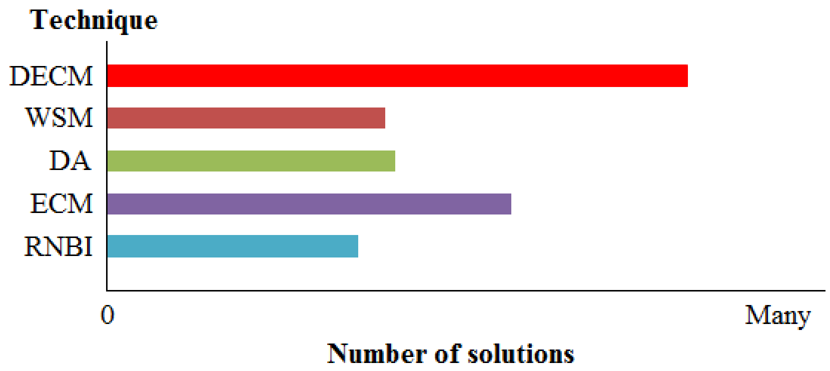


Figure 6.14 Number of identified unique Pareto solutions in Test B2

Solution distribution: The solution distribution of DECM is also good when dealing with three-objective optimisation problems in decision making in IAM. According to Figure 6.13, the solutions of DECM spread to the entire Pareto frontier, which clearly illustrate the shape and location of the Pareto frontier.

Figure 6.15 demonstrates the solution distribution of DECM in Test B2. Compared with the other exact methods, even though DECM does not have the best criterion of coverage error, it has higher uniformity level ratio and spacing than all the other exact methods. Therefore, in general, DECM produces the best solution distribution. Hence, its solutions are evenly distributed and well spread to the entire Pareto frontier. DECM also had good solution distribution in the other two tests as presented in Table 6.2.

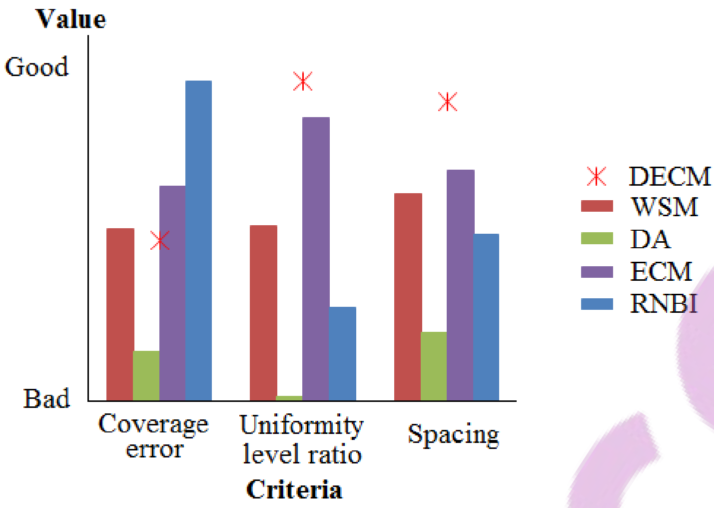


Figure 6.15 Solution distribution in Test B2

Computation time measures: DECM is not the fastest among the listed exact methods. However, it only spends 513.51 seconds in total and 3.69 seconds per unique solution when analysing a three-objective decision making problem with 1,301 segments and 72,562 strategies. Comparing with the size of the analysed problem, its computation time is not long.

Its computation time increases with the growth of the problem size. According to Figure 6.16, when the number of analysed segments increases from 100 to 600, the running time increases to 6.9 times which is at the average level among all the listed exact methods. When the analysed segments increase from 600 to 1,301, the running time is only doubled, which is at the low level among all the listed exact methods.

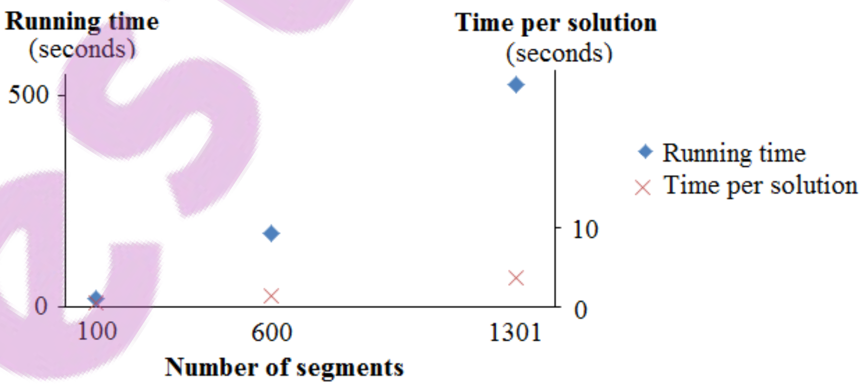
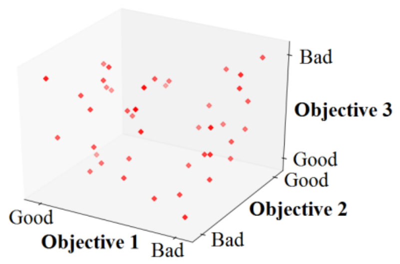


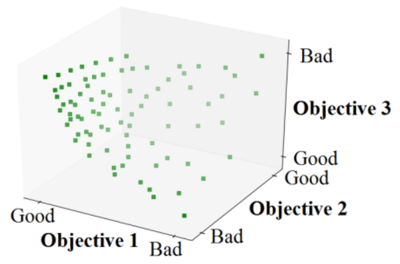
Figure 6.16 Increase on computation time with the growth of problem size (City B)

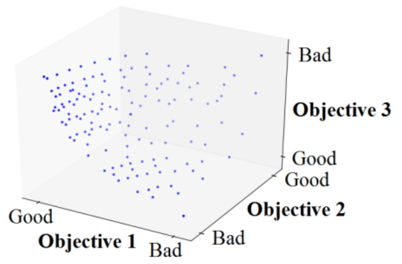
The insufficient computation time also impacts the solution distribution of DECM, but this impact is much smaller than that of the listed exact methods. Table 6.3 shows the solutions identified within different lengths of time in Test B2. Firstly, its identified solutions are always Pareto solutions even with insufficient computation time. Secondly, because of the insufficient time, DECM obtains fewer solutions; so its solution density is looser and the solution distribution is poorer. However, these solutions are still evenly distributed and spread to the entire Pareto frontier; hence they are able to correctly describe the shape and location of the Pareto frontier and therefore correctly present the complete outcome relationship and trade-offs. Compared with the other exact methods (see Figure 5.27), DECM produces the best frontier within the same length of insufficient time.

Table 6.3 Solutions obtained in different lengths of time in Test B2

Identified solutions		
25 seconds	50 seconds	Unlimited







Implementation considerations: The implementation of DECM is also good. According to Figure 6.17, DECM has the highest score on every implementation criterion compared with the listed exact methods. DECM is flexible and controllable. It can easily deal with the three-objective optimisation problems in decision making in IAM. Modification is not needed when addressing optimisation problem with more objectives. In addition, DECM is able to obtain expected Pareto solutions based on specific perspectives from decision makers, and does not necessitate calibrating parameters or defining a stopping criterion.

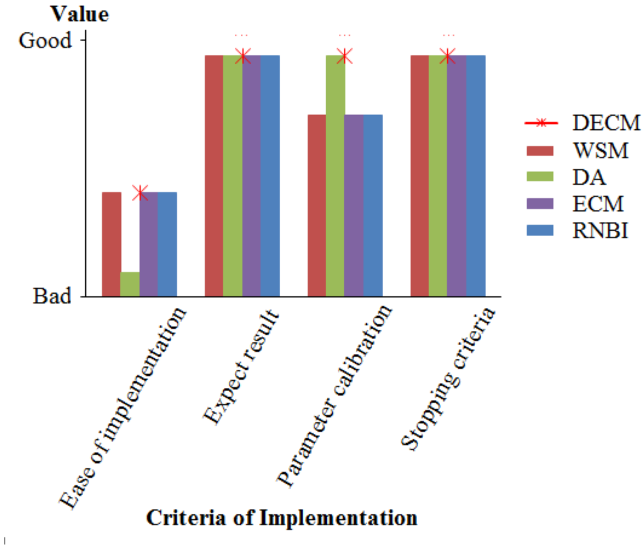


Figure 6.17 Implementation of DECM compared with the other exact methods

6.4 Tests of the Ability of Multi-Objective Optimisation to Solve Specific Infrastructure Asset Management Questions

This section investigates the versatility of MOO to answer specific IAM questions. In the previous sections, two types of decision making problems (City A and City B) were introduced and analysed. However, IAM may have various questions which also need to be answered by MOO. This section discusses specific IAM questions, and verifies how MOO is able to handle them.

Because the optimisation process is similar when applying DECM after properly formulating practical decision making problems as solvable MOO problems, this section mainly explains the formulation of practical decision making problems and the definition of objectives and constraints.

6.4.1 Target of Specific Level of Service Expectations

The level of service expressed by the condition is a very important criterion of decision making in IAM. The condition of an infrastructure asset network is often described by the condition index. However, the average condition index may be insufficient as it cannot correctly describe the level of service.

More specifically, the condition index is a composite index that may consider rutting, roughness, age, etc. and the network condition is defined as the average of the condition indices of all the segments of a network. However, it ignores the condition of individual segments. Even though the average condition index of an infrastructure asset network is acceptable; this network may still have too many poor segments to provide a preferred level of service. Therefore another condition measure is needed to describe the level of service.

Condition levels describe the condition of a segment or a network in a descriptive way. They can be defined using descriptive criteria, infrastructure texture and/or expert experience; hence this condition measurement is more flexible than the numerical condition index. Table 6.4 shows an example of condition levels that are defined based on the physical condition, deterioration, function, etc.

Table 6.4 Example of the definition of condition levels (NAMS, 2011)

Condition level		**Description**
No.	**Name**	
1	Excellent	Plant in robust physical condition designed to meet current standards. Asset likely to perform adequately with routine maintenance for the medium term.
2	Good	Acceptable physical condition but not designed to current standards or showing minor wear. Deterioration has minimal impact on asset performance. Minimal short-term failure risk but potential for deterioration or reduced performance in the medium term.
3	Fair	Functionally robust plant, but showing some wear with minor failures and some diminished efficiency. Assets may need replacement or repair now but asset still functions safely. Deterioration beginning to be reflected in performance and higher attendance for maintenance.
4	Poor	Plant functions but requires a high level of maintenance to remain operational. Likely to cause a marked deterioration in performance in the short-term. No immediate risk to safety but works required in short-term to ensure asset mains safe.
5	Very poor	Failed or failure imminent. Plant effective life exceeded and excessive maintenance costs incurred. High risk of breakdown with a serious impact on performance. Safety hazards exist.

All the segments of an infrastructure asset network are measured based on the definition of condition levels and classified into the corresponding condition levels. The network condition is described by the percentages/numbers/lengths of the network segments at different condition levels. If a network has a higher percentage of segments or more/longer segments in good condition levels, this network is better. More importantly, by analysing condition levels, decision making can control the poor segments and enhance the good segments therefore ensuring a required level of service is provided.

For example, City A (introduced in Section 5.5.1.1) may want to efficiently provide a required level of service with its road network. In this case, the segment condition is classified into four levels: excellent condition (level 1), good condition (level 2), fair condition (level 3) and poor condition (level 4). It is assumed that this required level of service can be achieved when in every year at least 10% of its road segments are at excellent condition level (level 1), 50% at good or better condition level (level 2) and 80% at fair or better condition level (level 3). Then a bi-objective optimisation problem is established for this decision making problem to maximise the benefit (Equation 6.13) and minimise the cost (Equation 6.14) under the constraints of the annual budget (Equation 6.15) and condition level requirements (Equations 6.16-6.18). Equation 6.19 is the constraints of one-strategy policy introduced in Section 4.3.1.

$$\max \sum_{i=1}^{N} B_i x_i \qquad \text{Equation 6.13}$$

$$\min \sum_{i=1}^{N} C_i x_i \qquad \text{Equation 6.14}$$

$$\sum_{i=1}^{N} YC_{t,i} x_i \leq AB_t, \qquad t = 1,2,\cdots,20 \qquad \text{Equation 6.15}$$

$$\text{Level 1} \qquad \sum_{i=0}^{N} CL_{t,i}^{1} x_i \geq 10\% \times M, \quad t = 1,2,\cdots,20 \qquad \text{Equation 6.16}$$

$$\text{Level 2 or better} \qquad \sum_{l=1}^{2} \sum_{i=0}^{N} CL_{t,i}^{l} x_i \geq 50\% \times M, \quad t = 1,2,\cdots,20 \qquad \text{Equation 6.17}$$

$$\text{Level 3 or better} \qquad \sum_{l=1}^{3} \sum_{i=0}^{N} CL_{t,i}^{l} x_i \geq 80\% \times M, \quad t = 1,2,\cdots,20 \qquad \text{Equation 6.18}$$

$$\sum_{i \in S_j} x_i = 1, \quad j = 1,2,\cdots,M \qquad \text{Equation 6.19}$$

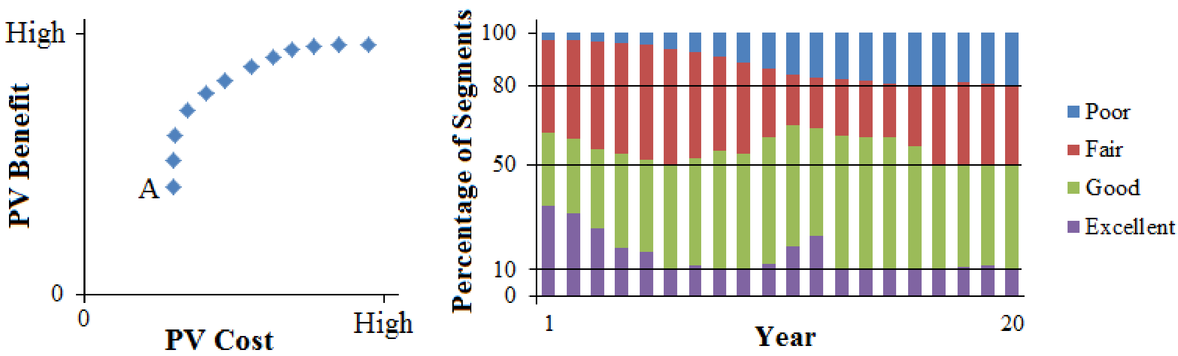
where, $CL_{t,i}^{l}$ condition level of a segment in year t if strategy i is applied;

others are same as above.

The constraints of the condition level are formulated with a condition level matrix ($\boldsymbol{CL^l}$), given by Equation 6.20, which records the condition levels of all the network segments in every year when applying different strategies. Parameter $CL_{i,t}^{l}$ is binary variable indicating whether the condition of a segment in year t is at level l if strategy i is applied. When if a segment is at condition level l in year t if strategy i is applied, $CL^l_{i,t} = 1$; otherwise, $CL^l_{i,t} = 0$. For instance, after applying strategy i, if the condition of its corresponding segment in year t is at good condition (level 2), then $CL^2_{i,t} = 1$ and $CL^1_{i,t} = CL^3_{i,t} = CL^4_{i,t} = 0$. This condition matrix is added into the formulae of the condition level constraints (see Equations 6.16-6.18).

$$\boldsymbol{CL}^l = \begin{bmatrix} CL^l_{1,1} & \cdots & CL^l_{N,1} \\ \vdots & \ddots & \vdots \\ CL^l_{1,T} & \cdots & CL^l_{N,T} \end{bmatrix} \qquad \text{Equation 6.20}$$

This optimisation problem is solved using DECM. Pareto solutions are obtained and shown in Figure 6.18. These solutions indicate the best achievable benefit and cost and their relationship and trade-offs. Even though they generate different benefit and cost, all solutions are guaranteed to satisfy all the constraints including the constraints on condition levels. The network condition of solution A is specified in Figure 6.18 (2) as an example. According to this figure, solution A successfully achieves the condition requirements so as to provide required level of service in every year.



(1) Identified solutions (2) Network condition of solution A

Figure 6.18 Solutions of the decision making with condition level constraints

If a specific requirement on a certain condition level is proposed, for example at least 50% of segments must be at the fair level (level 3) in year 10; it can be formulated as Equation 6.21 based on the condition level matrix introduced in Equation 6.20.

$$\sum_{i=1}^{N} CL^3_{10,i} x_i \geq 50\% \times M \qquad \text{Equation 6.21}$$

where, $CL^3_{10,i}$ fair condition matrix of a segment in year 10 if strategy i is applied; and others are same as above.

The level of service can also be described on the basis of the length of infrastructure assets. For example, the total length of the analysed segments of City A is 661.92 km. It is assumed that in every year at least 100 km of its roads should be at excellent condition level (level 1) and at least 300 km of its roads should be at good or better condition level (level 2). In this example, a length parameter L_i should be added into the condition level constraints (Equations 6.22 and 6.23). The formulation of objectives and other constraints and the optimisation process is similar to the other decision making problems when applying DECM.

Level 1 $$\sum_{i=0}^{N} CL_{t,i}^{1} L_i x_i \geq 100, \quad t = 1,2,\cdots,20$$ Equation 6.22

Level 2 or better $$\sum_{l=1}^{2}\sum_{i=0}^{N} CL_{t,i}^{l} L_i x_i \geq 300, \quad t = 1,2,\cdots,20$$ Equation 6.23

where, L_i length of the corresponding segment to which strategy i is applied; and others are same as above.

6.4.2 Target of an Annual Investment Level

In most decision making in IAM, an annual investment level is given as the annual budget. Decision makers need to predict the expense on the asset management in order to set a reasonable investment level. However, sometimes an investment level is hard to be predicted because:

- Decision makers may not have adequate knowledge to give a proper investment level. They may want to know the amount of expense needed in every year to achieve a satisfying infrastructure asset network or provide required level of service.
- Decision makers may want to balance the investment level and its return when making a management decision. A small increase in the investment may generate much more return. If the investment level is manually given, the opportunity of generating more return is lost. The understanding the trade-offs of investment level and its return can help in efficiently managing the infrastructure assets.

To handle these issues, instead of the annual budget, an objective of minimising the annual investment level is required. For example it is assuming that City A (Section 5.5.1) wants to efficiently use its annual investment and keep its road network in acceptable condition in the next twenty years. This decision making process requires balancing the investment and its return. Hence, a bi-objective optimisation problem is established to maximise benefit and minimise the annual investment under the constraints of condition requirements. For the test purpose, the acceptable condition is defined as same as the condition level requirements (see Equations 6.16-6.18).

An annual investment level is a certain number throughout the management period; hence it should be at least as same as the required maintenance cost in every year. In other words, it depends on the largest yearly cost. Therefore, the objective of minimising annual investment level is converted to the objective of minimising the largest yearly cost during the analysis period. In this research, a variable LYC is introduced to measure the largest yearly cost. Then the objective of minimising the largest yearly cost can be expressed using Equations 6.25 and 6.26. The objective of maximising benefit (Equation 6.24) and the condition constraints (Equations 6.16-6.18) remain unchanged.

$$\max \sum_{i=1}^{N} B_i x_i \qquad \text{Equation 6.24}$$

$$\min LYC \qquad \text{Equation 6.25}$$

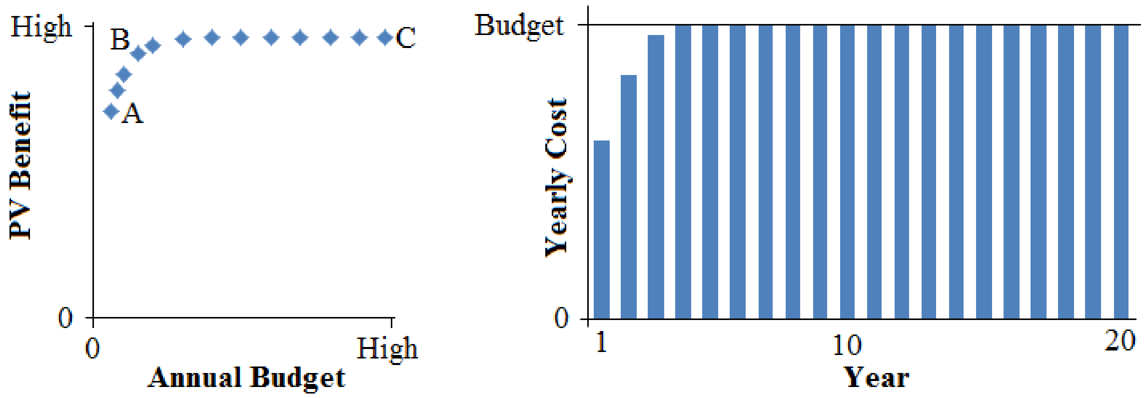
$$LYC \geq YC_{t,i} x_i, \quad t = 1, 2, \cdots, T \qquad \text{Equation 6.26}$$

where, LYC largest yearly cost;

T number of analysed years; and

others are same as above.

This problem is solved by DECM, and the solutions are shown in Figure 6.19. This figure not only shows the minimum annual budget (solution A) to achieve the satisfying network but also indicates the required annual budget to obtain the greatest benefit (solution C). Even if more funding is spent, it is impossible to obtain more benefit than solution C with the analysed strategies.



(1) Identified solutions (2) Yearly expense of solution A

Figure 6.19 Solutions of the decision making of annual budget

Figure 6.19 (2) shows the required yearly funding of solution A as an example. In the first four years, the network is in good condition so the funding is not fully spent. From year 4, the network condition degenerates and many interventions are needed; hence, the budget is fully spent. Then decision makers can trade off the annual budget and its corresponding benefit with the identified solutions. In the example of Figure 6.19, with the growth of annual budget, the benefit significantly increases until solution B. After a certain level, the increase on annual budget only brings slight growth on the benefit. Therefore, solutions located on the right side of solution B are not recommended. A specific annual investment level could be determined based on practical considerations such as available funding.

6.4.3 Cross asset optimisation

Different types of infrastructure assets may exist in one infrastructure asset network. Because the outcome measurements are different these outcomes are not commensurable, such as the benefit of road maintenance and the benefit of bridge maintenance. Thus different types of infrastructure assets should be individually analysed. Moreover, decision makers may have goals and requirements on a specific type of infrastructure assets, which also requires individually analysing these types of infrastructure assets.

To explain this, City C is analysed as an example. City C has 50 bridges and 801 segments of roads. It attempts to allocate the annual budget of \$300 million on bridge maintenance and road maintenance. For this decision making problem, a bi-objective optimisation problem is established, which tries to obtain the maximum benefit of bridge maintenance and road maintenance under the annual budget of the whole network. There are two ways of formulating this decision making problem.

When the strategies for different types of assets are generated and recorded together, group indicators can be used to distinguish the strategies for different assets. Then the group indicators are added into the formulae of objectives and constraints. The example of group indicators and the formulation is the same as introduced in the tests of City B (Section 5.5.2), where three sub-networks of roads are analysed.

When the strategies of each type of infrastructure asset are separately generated and recorded, these assets should be individually formulated. Equations 6.27-6.31 show the formulation of the decision making problem of City C, where objectives are to obtain the maximising benefit of bridge maintenance (Equation 6.27) and the maximising benefit of road maintenance (Equation 6.28) under the constraints of an annual budget of the whole network (Equation 6.29). Equations 6.30 and 6.31 are the constraints of one-strategy policy.

$$\max \sum_{b=1}^{N^B} B_b^B x_b^B \qquad \text{Equation 6.27}$$

$$\max \sum_{r=1}^{N^R} B_r^R x_r^R \qquad \text{Equation 6.28}$$

$$\sum_{b=1}^{N^B} YC_{b,t}^B x_b^B + \sum_{r=1}^{N^R} YC_{r,t}^R x_r^R \leq AB_t, \quad t = 1,2,\cdots,T \qquad \text{Equation 6.29}$$

$$\sum_{b \in \mathcal{S}_j^B} x_b^B = 1, \quad j = 1,2,\cdots,M^B \qquad \text{Equation 6.30}$$

$$\sum_{r \in \mathcal{S}_j^R} x_r^R = 1, \quad j = 1,2,\cdots,M^R \qquad \text{Equation 6.31}$$

where,
- B_b^B benefit of strategy b when maintaining a bridge;
- B_r^R benefit of strategy r when maintaining a road segment;
- $YC_{b,t}^B$ maintenance cost of strategy b in year t when maintaining a bridge;
- $YC_{r,t}^R$ maintenance cost of strategy r in year t when maintaining a road segment;
- x_b^B decision variable of bridge maintenance strategy b;
- x_r^R decision variable of road maintenance strategy r;
- $\mathcal{S}_j^B$ set of available strategies for bridge j;
- $\mathcal{S}_j^R$ set of available strategies for road segment j;

M^B number of bridges;

M^R number of road segments;

N^B number of strategies for bridges;

N^R number of strategies for road segments; and

others are same as above.

Then this optimisation problem is solved with the DECM. Pareto solutions are identified and shown in Figure 6.20.

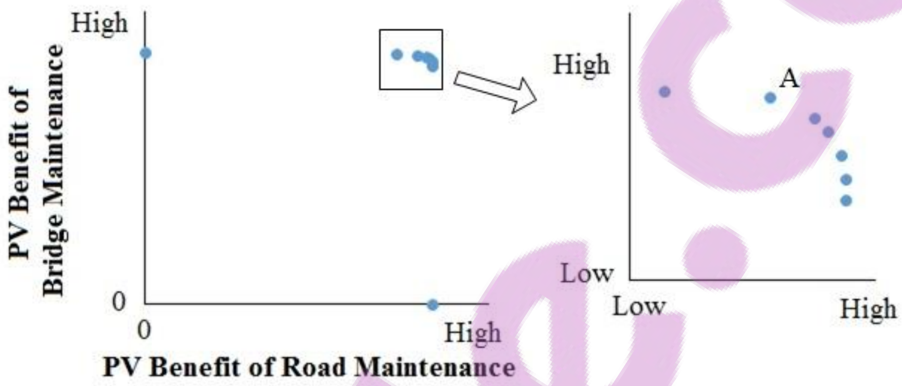


Figure 6.20 Solutions of budget allocations

Each solution in Figure 6.20 indicates a way of allocating the annual budget to bridge maintenance and road maintenance. These solutions indicate the highest benefit that can be obtained on bridge maintenance and that on road maintenance. Also, these solutions present the trade-offs of the two types of benefits when allocating the annual budget differently. Figure 6.21 shows the budget allocation of solution A. According to the figure, higher investment is allocated to road maintenance especially in years 9 and 11 to 15 to achieve the outcomes of solution A.

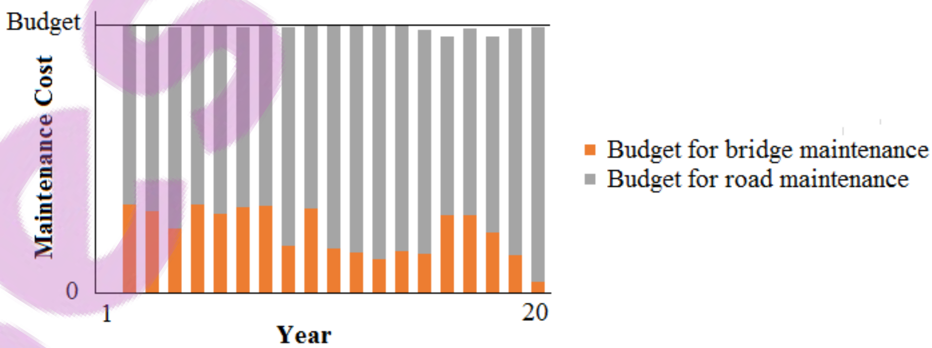


Figure 6.21 Budget allocation of solution A

In this example the budget allocation fluctuates in every year. The fluctuation may make the implementation complicated. When the same budget allocation is expected throughout the analysis period; Equation 6.29 should be replaced by Equations 6.32-6.34 and the other objectives and constraints remain unchanged. This optimisation problem can also be solved by DECM. However, compared with the fluctuated budget allocation, the benefit generated with the same budget allocation is smaller.

$$AB^{B} + AB^{R} \leq AB_{t} \qquad \text{Equation 6.32}$$

$$\sum_{b=1}^{N^B} YC_{b,t}^{B} x_{b}^{B} \leq AB^{B}, \quad t = 1,2,\cdots,T \qquad \text{Equation 6.33}$$

$$\sum_{r=1}^{N^R} YC_{r,t}^{R} x_{r}^{R} \leq AB^{R}, \quad t = 1,2,\cdots,T \qquad \text{Equation 6.34}$$

where, AB^{B} annual budget for bridge maintenance;

AB^{R} annual budget for road maintenance; and

others are same as above.

6.4.4 Discussion of Analysing Decision Making in IAM with Multi-Objective Optimisation

This section discusses three different IAM questions, and handles them with DECM. In practice, other questions may arise in IAM. According to the author's experience, the critical issue on the applications of MOO in decision making in IAM is how to describe a practical problem as a solvable optimisation problem. In this research, decision making problems are formulated as IP. Extra variables and matrices, such as the largest yearly cost (LYC) in Equation 6.26, may be added to model specific considerations. Once a MOO problem is established, an effective MOO technique, such as DECM, is applied to solve this MOO problem. Finally, a decision maker needs to select one identified solution or adjust the solutions based on the optimisation results and practical considerations.

6.5 Assessment of the Dynamic Epsilon Constraint Method

This section aims at enhancing the understanding of DECM and certifying its robustness. In detail, Section 5.6 outlines eight benchmark criteria of robust MOO techniques for practical long-term and network-level decision making in IAM. This section assesses DECM based on the eight benchmark criteria.

Different numbers of objectives: DECM is able to handle MOO problems with different numbers of objectives. It was demonstrated in the computational experiments that DECM successfully optimises problems with two and three objectives under a series of constraints, and yields good optimisation results. More objectives can be optimised using DECM. After a proper implementation, the proposed technique can be directly applied to solve different MOO problems without modifications.

Various constraints: DECM is able to deal with various types of constraints. In Sections 6.3 and 6.4, DECM is applied to handle yearly constraints (i.e. the acceptable yearly condition index), overall constraints (i.e. total budget) and other specific constraints of IAM. Other constraints can also be handled as long as they can be properly modelled within the IP framework.

A large number of segments and strategies: DECM can analyse a large number of segments and strategies. In Section 6.2, MOO problems with 1,822 segments (City A) and 72,564 alternative strategies (City B) are solved, and Pareto solutions are obtained in an acceptable time. More segments and strategies can be analysed. However, with the growth of problem size, the computation time is also increasing.

Good representatives of solutions: The solution representativeness is measured by the solution quality and distribution. DECM is able to identify well distributed Pareto solutions; hence its solution representativeness is good.

In terms of solution quality, DECM always identifies Pareto solutions, so that its solution quality is much better and more reliable than that of the heuristics. Theoretically, DECM may obtain non-Pareto solutions; however, according to the experimental tests, all of its solutions are Pareto solutions. This is because practical decision making in IAM normally has a large number of Pareto solutions. Even small objective areas contain Pareto solutions and DECM can identify a Pareto solution for each objective area if it exists. Compared with the other exact methods, DECM identifies the most unique Pareto solutions when a fixed number of SOO sub-problems is solved. Hence its solution quality, measured by the solution goodness and quantity, is better than the listed exact methods.

In terms of solution distribution, DECM generates solutions that are well distributed on the Pareto frontier. Firstly, its solutions cover the entire range of the Pareto frontier, which is much better than the solutions of the heuristics. Secondly, compared with the other exact methods, DECM has high scores on all the distribution criteria, which means its solutions are uniformly distributed and spread to the entire Pareto frontier. Finally, the solution distribution of DECM is good, even when the algorithm is not fully completed. DECM identifies new solutions that fill the gap between the identified solutions. Therefore, when a stopping criterion is achieved, such as insufficient computation time, fewer solutions may be obtained; but these solutions still cover and are uniformly distributed on the Pareto frontier.

Analysis speed: DECM is not the fastest among the exact methods but its computation time is acceptable. It spends 203.85 seconds to analyse a bi-objective problem with 1,822 segments and 61,936 strategies and 513.51 seconds to analyse a three-objective problem with 1,301 segments and 72,562 strategies. Because these decision making problems attempt to make long-term management decisions; and the problems are only solved at the beginning of the IAM process. Hence, comparing the length of the analysis period, the computation time of this technique is acceptable. Furthermore, it obtains more unique Pareto solutions than the other exact methods, especially in three-objective optimisation; hence, the average time per unique solution is not long.

Few parameters: DECM does not have any parameter that has to be calibrated by decision makers. It searches for new solutions based on the identified ones. Its algorithm can dynamically and independently proceed and does not require any input from decision makers. Some parameters and stopping criteria can be added to control the optimisation result. When no parameter is defined, this technique is able to identify all the Pareto solutions that are unique in the objective space between the end points for a MOO problem in decision making in IAM; while some Pareto solutions may locate outside the objective space between the end points and cannot be obtained.

Controllable algorithm: DECM is adaptable and can be easily controlled by a decision maker. Different parameters and stopping criteria can be defined, which enable it to yield satisfying results under their perspectives. More discussion on the parameters and stopping criteria is in Section 6.2.

Easy implementation: DECM can be flexibly implemented. In this research, it is implemented using Python (see Section 6.1.3), and can be directly applied to assist many decision making problems. Modification is not needed for its applications. However, a SOO algorithm or solver is required to solve the SOO sub-problems.

In summary, DECM can satisfy the eight benchmark criteria of robust MOO techniques proposed from the viewpoint of long-term and network-level decision making in IAM; therefore it can be regarded as a robust MOO technique for this decision making.

6.6 Summary and Discussion

This chapter improves decision making in IAM by developing a robust MOO technique named DECM for long-term and network-level decision making in IAM. According to the analysis completed, this technique can effectively and efficiently solve the MOO problems and obtain satisfying optimisation results for different types of decision making in IAM.

More specifically, according to the discussion of the existing MOO techniques, DECM is proposed to solve the MOO problems in long-term and network-level decision making in IAM. It is proposed based on ECM, but explores objective space and defines epsilon in a dynamical manner. Then its characteristics are explored and its performance is tested with experimental tests based on practical long-term and network-level decision making in IAM. Compared with the techniques in the MOOT List (see Table 5.5), DECM produces well distributed Pareto solutions in acceptable time. Its application and implementation are also easy and flexible, and it can be applied to solve specific IAM questions. According to the assessment in Section 6.5, DECM is able to achieve the eight benchmark criteria of robust MOO techniques in decision making in IAM. Hence, it can be regarded as a robust MOO technique for long-term and network-level decision making in IAM.

However, a MOO technique only assists decision making in IAM, but does not make the management decision. It solves the corresponding MOO problems in decision making process, and provides optimisation results that simplify and clarify decision making problems. Then a decision maker can directly select an identified solution as the management decision or adjust the solutions to create new management decisions. Both of the choices are based on the optimisation result, hence the understanding of the optimisation result is important and is heavily affected by the interpretation of optimisation results. The accurate and appropriate interpretation of optimisation results can help decision makers to read the identified solutions, understand the outcomes and their relationships and collect expected information; therefore the appropriate management decision can be made easier. Because of the importance of the interpretation of optimisation results, in the next chapter a communication tool is developed to interpret optimisation results in an understandable and meaningful way.

CHAPTER 7 COMMUNICATION OF OPTIMISATION RESULTS

This chapter promotes the applications of MOO in decision making in IAM by enhancing the understanding of optimisation results. It helps decision makers to accurately and comprehensively understand the optimisation results in order to make appropriate management decisions. More specifically, this chapter firstly outlines the significance of the communication of optimisation results and discusses different types of solution representation. Then it introduces a prototype communication tool to communicate the optimisation results with decision makers.

7.1 Significance of the Communication of Optimisation Results

After applying a MOO technique in decision making in IAM, it is important to interpret the optimisation results in an understandable and meaningful way. This section outlines the significance of the result communication, including:

Improving the understanding of optimisation results.

Decision making in IAM attempts to select appropriate strategies for an infrastructure asset network. Only one solution indicating a strategy selection is needed at the end of the decision making process. However, in the context of MOO, a set of Pareto solutions exists and each of them corresponds to a wide range of outcomes related to objectives and constraints. Decision makers need to make the final management decisions based on the identified solutions. If they cannot understand the outcomes of these solutions and outcome relationship, they are not able to make proper decisions. In essence the different optimisation solutions become a starting point for a dialog that will decide which objectives would be considered as the most important ones without necessarily compromising any of the other objectives. The communication of optimisation results can accurately interpret the identified solutions and help decision makers to understand the achievable outcomes and their relationships.

Explaining the optimisation result in multiple dimensions.

When optimising multiple objectives, the understanding of optimisation results could be difficult because of its high-dimensional objective space. Each objective corresponds to a dimension in its objective space; hence the solutions of MOO are located in a multi-dimension objective space. According to Korhonen and Wallenius (2008), the visual representation for humans is limited to two dimensions. When more than two objectives are directly presented, it may cause “a significant cognitive burden” (Gettinger *et al.*, 2013). In decision making of City B, solutions are presented in three-dimensional objective space, which requires decision makers having good ability to imagine their locations in the objective space. When more objectives are optimised, even solutions are obtained; it is difficult to read these solutions by visually presenting these solutions. The communication of optimisation results should present solutions with multiple objectives and enable decision makers to trade off multiple outcomes thus assisting decision makers in making appropriate management decisions.

Exploring management preference on its decision.

Each decision making problem in IAM is a unique problem with its own preference on management decisions. It is important to trade off the objectives and explore the preferences. The communication of optimisation results is an interactive process that the optimisation result is interpreted to specify the management preference, and the preference in turn is used to customise the result interpretation. This interactive process helps decision makers to take the best advantage of the optimisation results, gradually understand the decision making problem at hand and finally make an appropriate and reasonable management decision.

Facilitating the decision making process.

After MOO, a set of Pareto solutions is provided in decision making in IAM. These solutions are necessary at the early stage of the decision making process as they provide useful information including the achievable outcomes, the relationship of outcomes, etc. However, with a deeper and clearer understanding of the decision making problem at hand, some solutions may be not useful anymore. Hence decision makers may want to refine the identified solutions and focus on the preferred solutions. The communication of optimisation results provides a dynamic process that enables communicating with decision makers and adjusting the optimisation results accordingly. At the early stage, general information of the optimisation results is provided to draw a big picture of a decision making problem. Then solutions are refined and detailed information is provided to narrow down the focus and deepen the understanding of the decision making problem. It helps decision makers to efficiently and easily obtain the expected information from the optimisation results.

In summary, the communication of optimisation results is important and beneficial in decision making in IAM. It not only correctly interprets the optimisation results of MOO, but also interacts with decision makers and helps understanding decision making problems, exploring management preference and refining solutions. It attempts to take the best advantage of the optimisation results so that an appropriate management decision can be made easily and efficiently.

7.2 Discussion of Solution Representation

This section provides background knowledge of the interpretation of optimisation results. The results interpretation requires proper representation of the identified solutions and their outcomes. According to the previous research on cognitive fit (Vessey, 1991; Umanath and Vessey, 1994), there are two main types of solution representation: tabular and graphical representation. Tabular representation uses tables to directly show the outcomes of solutions. It has high accuracy (Gettinger *et al.*, 2013). Graphical representation uses graphs to illustrate solutions. According to Huysmans *et al*. (2011), the graphical representation is more effective especially when comparing spatial information. Taking the advantage of multi-colour, graphically represented solutions may be more understandable and more identical. In summary, solutions could be understood faster with the graphical representation, while more accurate responses could be achieved with tabular representation (Umanath and Vessey, 1994). Hence, both types of representation are used in the communication of optimisation results, where the graphical representation is adopted as the main representation to illustrate solutions and the tabular representation is adopted to enumerate outcomes of solutions.

7.3 Development of a Communication Tool

According to the completed discussion, a communication tool is necessary to communicate the optimisation results by:

- properly interpreting the optimisation results and improving the understanding of decision making in IAM;
- accurately presenting identified solutions with multiple outcomes and helping with understanding the trade-offs of objectives;
- interacting with decision makers and exploring the preference on management decisions; and
- dynamically refining solutions and providing required information based on a decision maker's preferences.

In this research, a communication tool was developed with three user interfaces to gradually interpret the optimisation result for decision making in IAM. More specifically, **the first user interface** provides a "big picture" of the decision making problem at hand. Achievable outcomes are presented and decision makers can abandon undesirable solutions. Then the reserved solutions are sent to **the second user interface**, which deepens the understanding of the optimisation result by accurately displaying and further comparing the reserved solutions. Finally, **the third user interface** provides the detailed information of preferred solutions including overall outcomes and yearly outcomes, presents the performance of the entire network as well as individual segments and therefore helps to refine the identified solutions.

7.3.1 First User Interface

The first user interface aims at providing general achievements of the decision making problems. After optimisation, all the identified solutions are collected and sent to the communication tool. Then the communication tool builds the first user interface with;

(1) two types of graphs to demonstrate the achievable objective values and their distribution and relationships, and
(2) dynamic scroll bars to enable decision makers to adjust the preferred range of objectives and refine the solutions.

Figure 7.1 shows an example of the first user interface of a three-objective optimisation problem in decision making in IAM, where objectives are maximising benefit, minimising cost and minimising the average condition index.

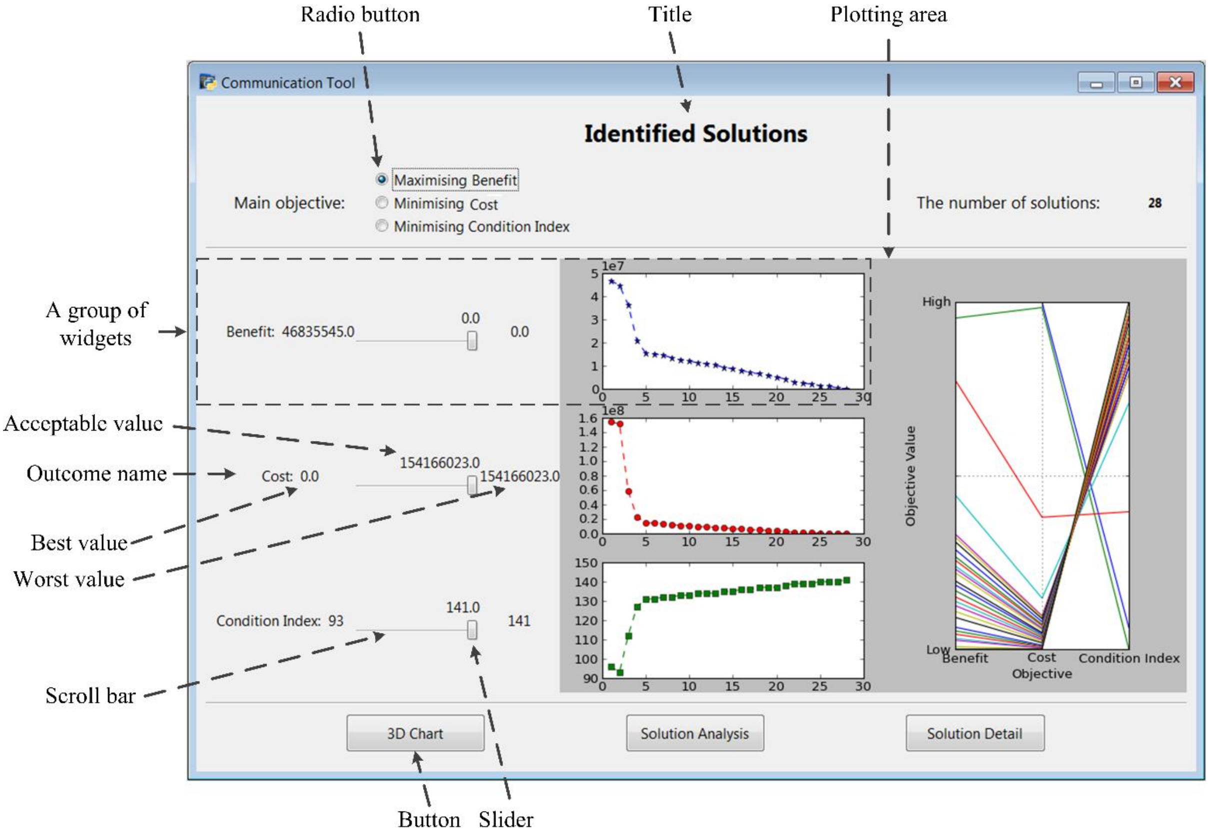


Figure 7.1 Example of the first user interface

The top part of the first user interface introduces the problem at hand. The title “Identified Solutions” is located on the top-middle of the interface. The names of all objectives and the number of identified solutions are located on the left and right side below the title. Each objective has a radio button. The main objective can be selected by clicking the radio button, which decides the representation order of the identified solutions on the graphs located in the middle. Decision makers can change the main objective at any time by clicking the radio button.

The middle part is the main section of the first user interface, which shows the achievement of the objectives, and allows decision makers to adjust the preferred objective ranges and abandon poor solutions. The achievable value of each objective is presented by a group of widgets including a scroll bar, a figure and labels. As shown in Figure 7.1, the best (worst) value of an objective is shown at the left (right) end of its scroll bar. The objective values of the identified solutions are illustrated with a scatter plot in the centre of the interface, where the horizontal axis presents the solution index and vertical axis presents the values of this objective. All the identified solutions are ordered according to the values of the main objective. For instance, in Figure 7.1 “Maximising Benefit” is the main objective, so the solutions are ordered from the greatest-benefit solution to the least-benefit solution. For a three-objective optimisation problem, three groups of widgets are generated. When more objectives are optimised, more groups can be created.

The scatter plots in this interface clearly show the objective values that can be achieved by the identified solutions and their relationships. When sacrificing one objective, the return on other objectives can be easily estimated with these figures, which helps to balance the objectives.

An "objective graph" is located on the right side of the middle part, which shows the achievable objective values and indicates their distribution. In particular, the horizontal (vertical) axis of this graph presents the objectives (objective value), and each line corresponds to a solution. When the lines are closely located such as the lines Figure 7.1, many solutions have similar objective values and are not helpful for trading off. Hence, these solutions could be refined so that decision makers can focus on the more useful solutions.

The first user interface can also help decision makers to set a preferred range of an objective by moving the slider on its scroll bar. Figure 7.2 is an example where the expected cost is under 60.32 million and the expected condition index is smaller than 135.2. The solutions that cannot achieve these expectations are eliminated. Accordingly, the number of identified solutions and all graphs in this interface are automatically updated. With the help of the dynamic figures, decision makers can freely investigate the objectives and narrow down their focus to their preferred solutions.

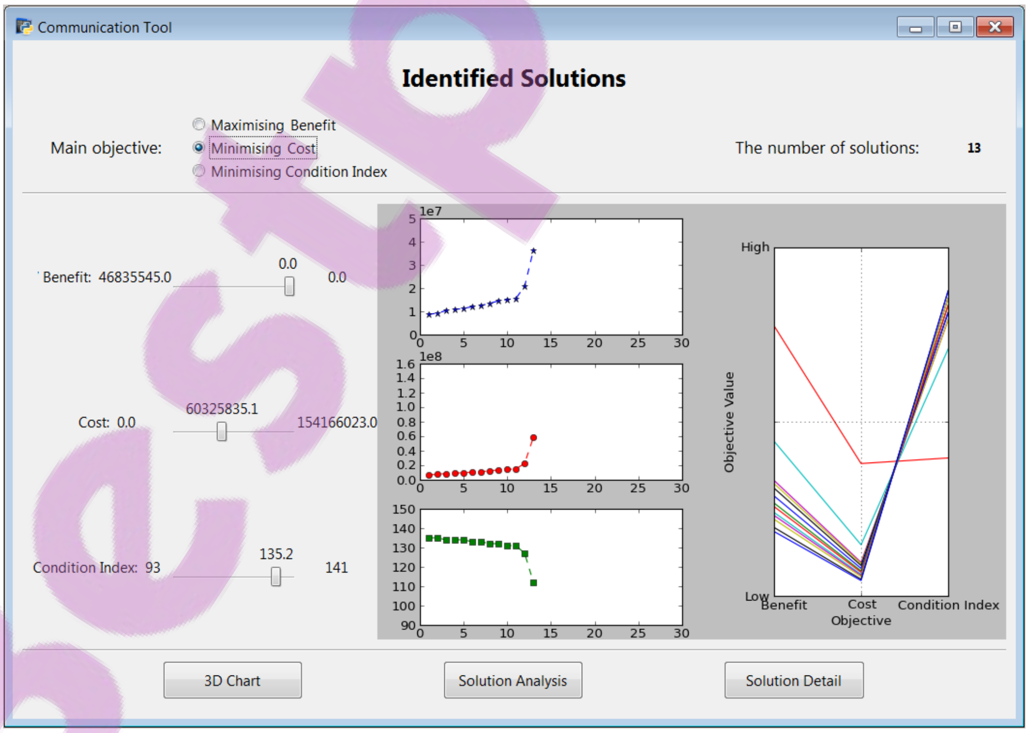


Figure 7.2 Example of the first user interface with preferred ranges of objectives

At the bottom of the first user interface, there are three buttons: “3D Chart”, “Output Solutions” and “Solution Detail”.

The button of “3D Chart” directs the communication tool to a 3D figure, which demonstrates the identified solutions in three dimensions. Figure 7.3 is an example of 3D chart, where each objective is represented on one axis. This figure provides an intuitive sense of these solutions and their frontier. Also this figure can be rotated so that decision makers can examine the solutions from different viewpoints which helps understand trade-offs in the 3D chart. Furthermore, this figure distinguishes the reserved and abandoned solutions, so that decision makers can visually recognise their reserved solutions. This 3D figure only shows three objectives at once.

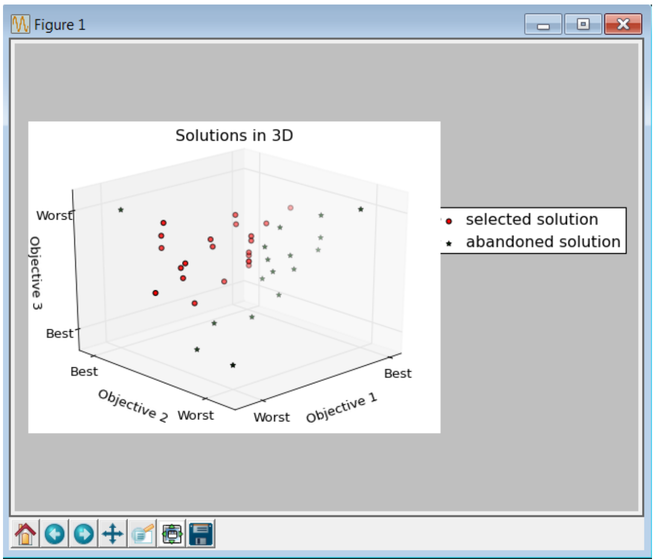


Figure 7.3 An example of 3D chart

When clicking “Output Solution” or “Solution Detail”, the communication tool goes to the second or third user interface with the reserved solutions.

7.3.2 Second User Interface

The second user interface attempts to intuitively enhance the understanding of optimisation results and help decision makers explore their preference on decisions. It enumerates the reserved solutions using tabular solution representation and visually compares them using graphical representation. Figure 7.4 shows an example of the second user interface.

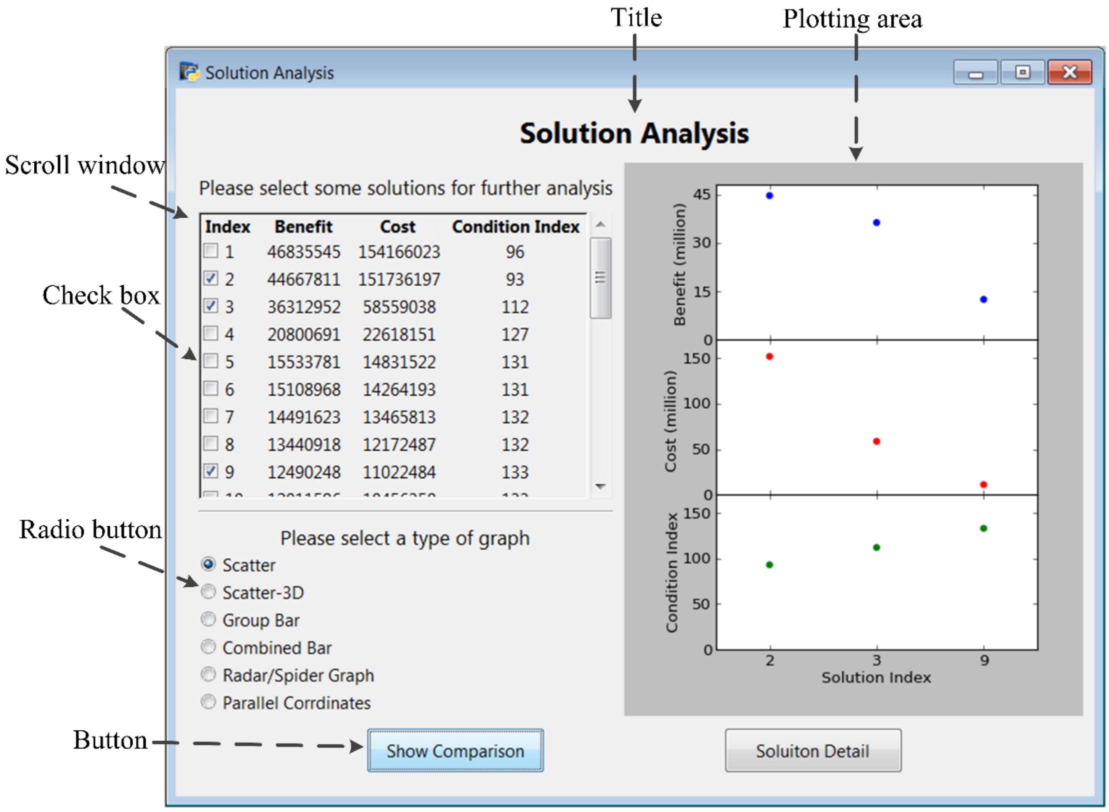
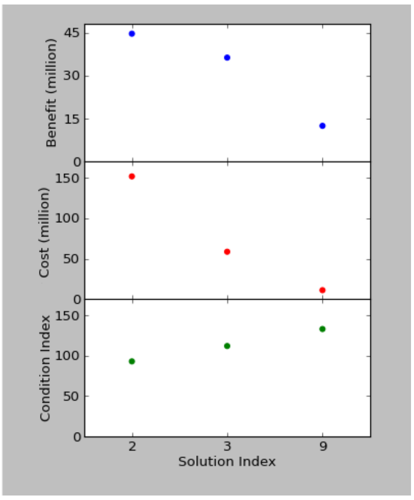


Figure 7.4 Example of the second user interface

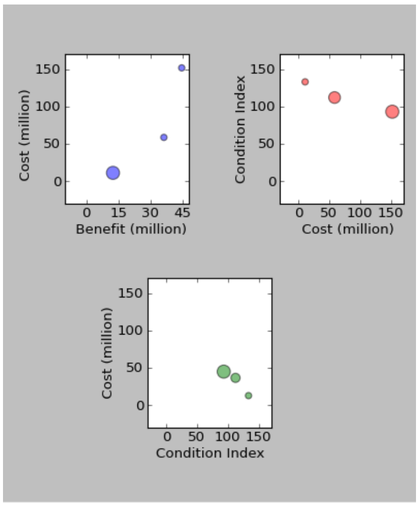
At the top of the second user interface the title “Solution Analysis” is stated. Under the title, objective values of the reserved solutions are enumerated on the left side within a scroll window, which provides the exact outcomes of these solutions. Based on this information, a decision maker can select some solutions for further comparison by clicking the check boxes in front of these solutions.

Under the scroll window, there are a set of radio buttons, each corresponds to a type of graph that will be shown in the plotting area on the right. For example, in Figure 7.4, solutions with indices of 2, 3 and 9 are selected to be shown using the “Scatter” graph. After clicking the button “Show Comparison”, the corresponding graph is drawn in the plotting area. A decision maker is allowed to change the compared solutions and the graphs at any time and then update the plotting area by clicking the “Show Comparison” button.

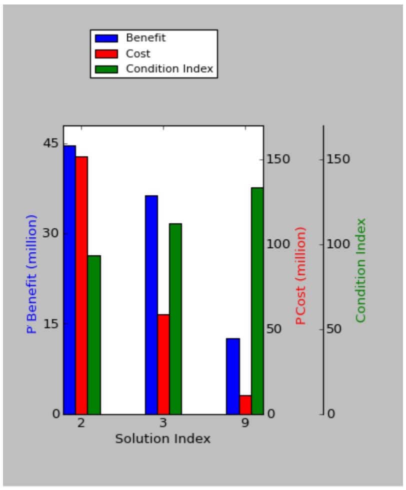
In this user interface, six types of graphs are provided, namely “Scatter”, “Scatter-3D”, “Group Bar”, “Combined Bar”, “Radar/Spider Graph”, and “Parallel Coordinates”, each with different characteristics. Figure 7.5 is an example of these types of graphs, where the selected solutions are solutions 2, 3 and 9.



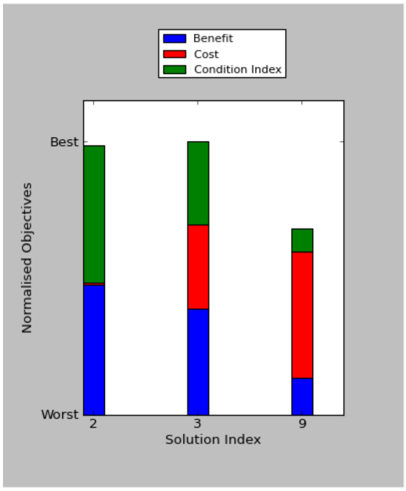
(1)



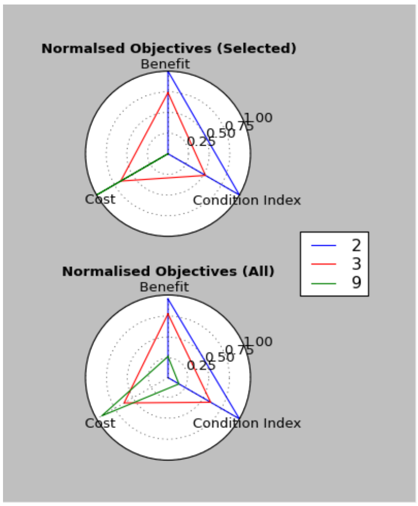
(2)



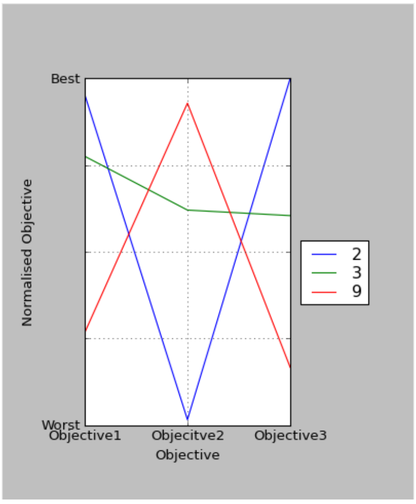
(3)



(4)



(5)



(6)

Figure 7.5 Example of graphs in the second user interface

Scatter: This graph evaluates the selected solutions by comparing their outcomes on individual objectives. As shown in Figure 7.5 (1), for a three-objective optimisation problem three figures are created; each presents one objective. All the figures share a horizontal axis indicating the indices of the selected solutions. According to Figure 7.5 (1), solution 2 has the highest benefit and the lowest condition index but also requires the most cost. When more objectives are analysed, more figures can be created.

Scatter-3D: This graph directly and visually compares three objectives of the selected solutions in one figure. According to Figure 7.5 (2), the values of two objectives are given on the horizontal and vertical axis and the value of the third objective is represented by the dot size. This graph is useful when trading off three or more objectives. It is also applicable to analyse other outcomes.

Group Bar: This graph uses a group of bars to visually present the solution outcomes. It can be used to compare the individual objectives and the overall achievement of the solutions. As shown in Figure 7.5 (3), solution index is given on the horizontal axis and the height of the bar indicates the objective value, where objectives are differentiated by the colours. Because objectives are differently measured every objective has its own vertical axis. When examining a specific objective, the solutions are compared to the height of the bars with the associated colour. The overall achievement of a solution can be measured by its bar group. For example, compared with solution 9 in Figure 7.5 (3), even though Solution 2 spends more cost it generates much more benefit and results in a good condition; hence it may be preferred. When more outcomes are analysed, this graph can establish more vertical axes.

Combined Bar: This graph clearly demonstrates the overall performance of solutions and their outcome variation. As shown in Figure 7.5 (4), it represents a solution using one bar, where objectives are distinguished by their colours. Because objectives have different scales, it is necessary to normalise them into the same scale. In this research, objectives are normalised into [0, 1], where 0 (1) represents the worst (best) objective value of all the reserved solutions. The achievement of a solution is presented by the bar height. When a bar is higher, its solution may perform better at a holistic level such as solutions 2 and 3 in Figure 7.5 (4). This graph also shows the objective variation of solutions. For instance, the bar sections of solution 3 have similar height; hence this solution performs similarly on all the individual objectives. On the contrary, solution 2 with one short bar section and two high bar sections, is not evenly performed. With this graph, decision makers can easily recognise these evenly performed solutions.

Radar/Spider Graph: This graph can visually display multiple objectives and outcomes using a two-dimensional figure. It not only shows the solution achievement on individual objectives, but also presents the overall performance of the solutions. An example of this graph is shown in Figure 7.5 (5), where each objective or outcome is given on a polar axis and each solution corresponds to a closed line. For both figures, normalised objectives are used; therefore when a closed line is located further to the centre, its associated solution performs better.

The figure named “Normalised Objectives (Selected)” compares the selected solutions where the centre (edge) stands for the worst (best) objective value obtained by the selected solutions. If the closed line of a solution is located nearer to the edge, this solution is likely to have a better performance than the other selected solutions. For instance, the closed line of Solution 2 reaches the edge of the axis of benefit so this solution generates best value on this objective among the three selected solutions.

The figure named “Normalised Objectives (All)” compares the selected solutions with all the reserved solutions. Its centre (edge) stands for the worst (best) objective values obtained by all the reserved solutions. If a closed line is located near to the edge, this solution is likely to have a good performance among all the reserved solutions including the selected and non-selected ones.

Comparing the two figures, Normalised Objectives (Selected) amplifies the performance of the selected solutions in order to ease comparing their performance; while Normalised Objectives (All) clarifies the status of the selected solutions among the reserved solutions

Parallel Coordinates: This graph shows the solution objective values and their variation. It is similar to the objective graph in the first user interface, but this graph compares the selected solutions in a clearer and more detailed way. As shown in Figure 7.5 (6), the vertical and horizontal axis of this graph represents the normalised objective values and solution indices, and a line represents a solution. If a line is located higher, its solution has better performance. For example in Figure 7.5 (6) solution 2 generates good benefit and condition index. Moreover, this graph also indicates the objective variation of the solutions. When the solution line is gradual such as solution 3, this solution has similar performance on all the objectives.

On the bottom, there is another button named “Solution Detail”, which directs the communication tool to the third user interface with the reserved solutions.

7.3.3 Third User Interface

The third user interface presents a wide range of outcomes of a whole network and individual segments; so that decision makers can take an exhaustive look at these solutions, and therefore tailor them. After clicking the "Solution Detail" button in the first or the second user interface, the communication tool directs to the third user interface with the reserved solutions. An example of the third user interface is shown in Figure 7.6. On the top, the title "Solution Detail" of the third user interface is located. Below the title, there is a scroll window on the left and a plotting area on the right.

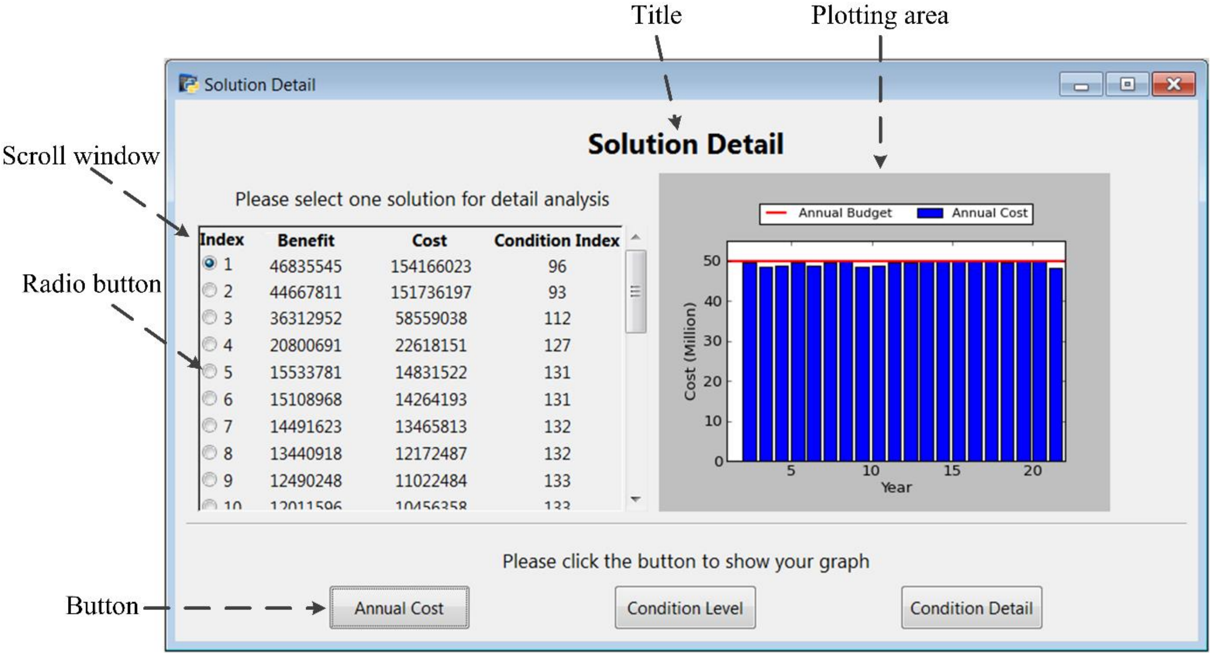


Figure 7.6 Example of the third user interface

The scroll window again lists all the exact objective values of the reserved solutions. A radio button is provided to each solution; so decision makers can select a solution such as solution 1 in Figure 7.6 for further information. The decision maker can examine another solution at any time by clicking its radio button.

The plotting area depicts a selected solution with different types of graphs. It presents the solution outcomes on an annual basis, where both the whole network and individual segments are shown. Three types of graphs are provided namely "Annual Cost", "Condition Level" and "Condition Detail", each corresponding to a button at the bottom of the third user interface.

Annual Cost: This graph presents the yearly cost of a selected solution using a bar graph, where the height of a bar represents the funding needed in a year (see Figure 7.6). The annual budget is highlighted with a red line to specify the consumption of the annual budget. When the gap between the line and a bar is large, the funding in this year is not fully spent. In addition, the variation of the bar height demonstrates the fluctuation of the yearly cost of a solution. When the variation is small, the yearly cost is stable so the interventions of this solution are successive such as solution 1 in Figure 7.6. This graph can also show other annual outcomes such as yearly condition index.

Condition Level: This graph presents the yearly condition of an infrastructure asset network. Figure 7.7 is an example of this graph, where four condition levels are defined for a 500-segment decision making problem. Each condition level has a special colour; and the height of a bar section represents the number of segments that are at the condition level of this colour. If the bar section is higher, this network has more segments at its corresponding condition level. The overall height of all bars is the total number of the segments. This graph not only shows the proportion of the segments at each condition level, but also indicates the variation of these proportions over time. According to Figure 7.7, if applying solution 1, the very-good-condition segments are obviously reducing in the first five years, and the poor-condition segments are increasing from year 3 to 10. In year 11 around half of the segments are in good condition. Other outcomes such as budget with different budget categories can also be analysed with this graph.

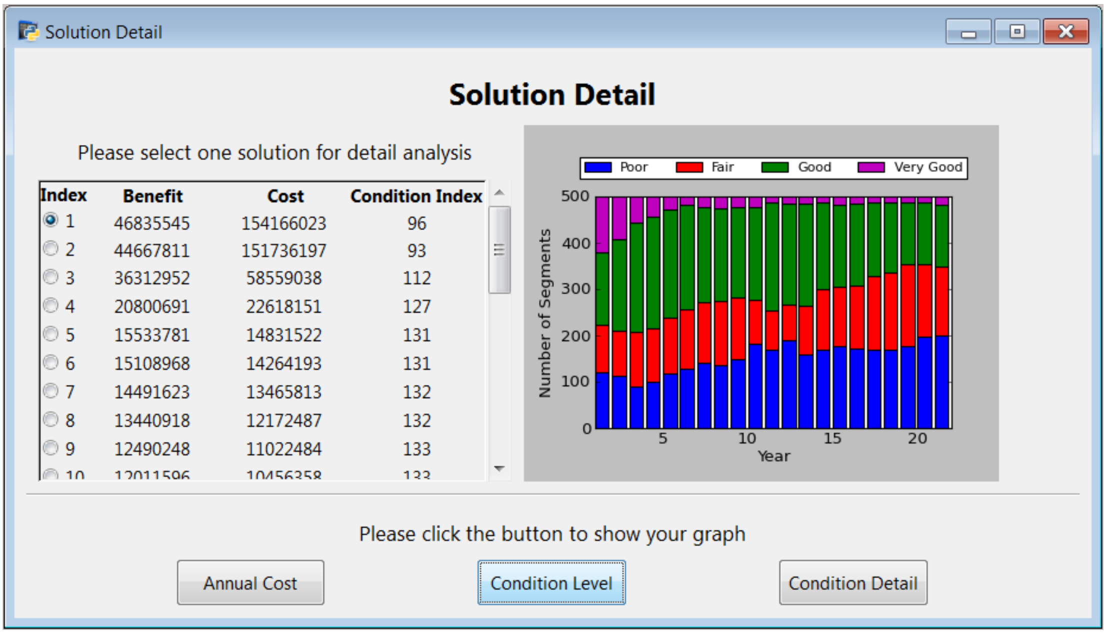


Figure 7.7 Example of the figure of condition levels

Condition Detail: This graph shows the condition of the entire infrastructure network and individual segments. It allows decision makers to examine their condition improvement. After clicking "Condition Detail" button, a scroll window is embedded on the right side of the third interface (see Figure 7.8). This scroll window has a list of radio buttons, where the first one is named "Average" and each of the others correspond to a segment. Initially, the "Average" radio button is active; hence the average condition index of the network is depicted by a red line in the plotting area. For the comparison purpose, the network average condition index when applying do-nothing strategies to all the segments is depicted by a black line. The condition improvement of a solution is clearly demonstrated by the gap between the red and black lines. When the gap is big, the selected solution largely improves the network condition.

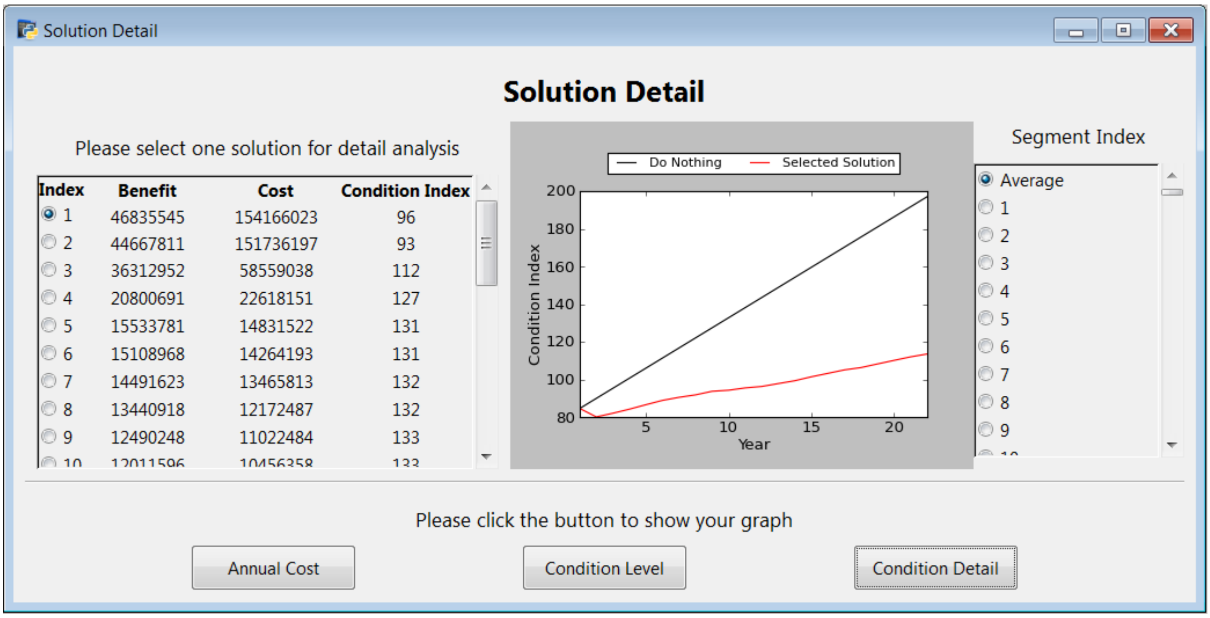


Figure 7.8 Example of the third user interface of condition detail

Decision makers can also examine the condition and condition improvement of a specific segment by clicking its radio button. Then the plotting area is automatically updated to show the condition of this segment. For example, Figure 7.9 shows the condition of segment 467, where the red (black) line represents the yearly condition index of segment 467 if solution 1 (do-nothing strategy) is implemented. In this example, the red line has one big drop between year 8 and 9 and one small drop between year 17 and 18. This indicates that solution 1 implements a major treatment in year 8 and a minor treatment in year 17 on segment 467. Other outcomes such as yearly rutting can also be analysed with this type of graph.

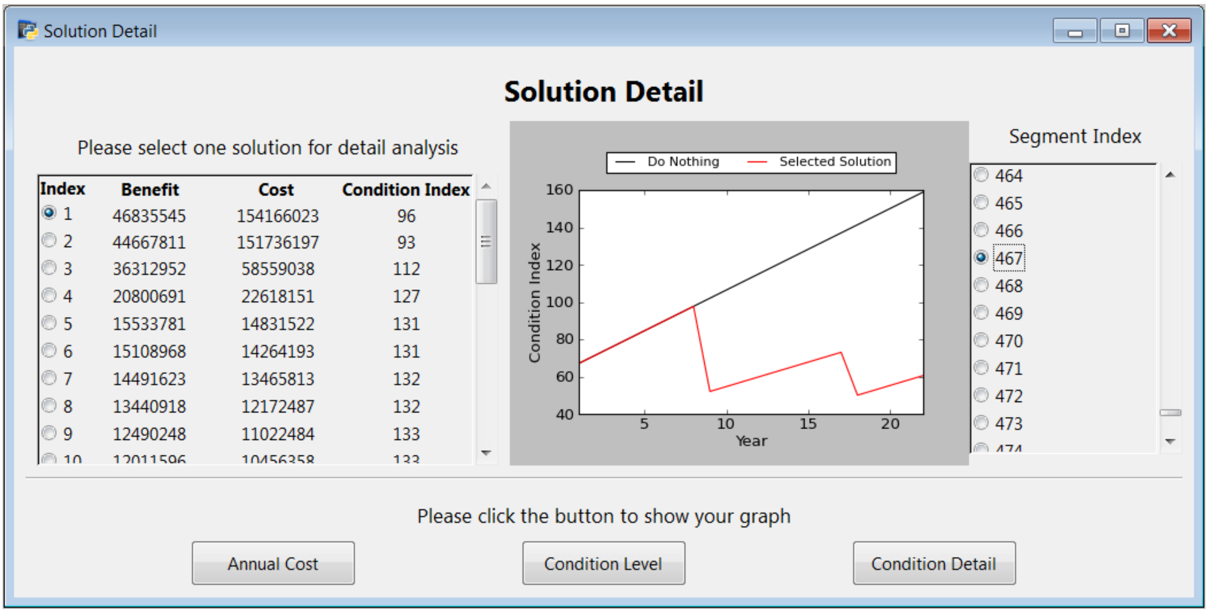


Figure 7.9 Example of condition detail of a segment

In the examples of Figures 7.8 and 7.9, a smaller condition index represents better condition. Hence, the red line follows the black line or is below it. When condition index is differently defined, for example higher index presents better asset condition, the red line may be located above the black line. However, the gap between them always indicates the condition improvement of a solution.

7.3.4 Required Software Tools

To develop the communication tool, Python is used as the programming language and some software tools are used to realise the functions of the communication tool, as outlined in this section.

PyGTK: PyGTK is used to establish the interfaces of the communication tool, which can be easily invoked using Python. All the three user interfaces, including windows, plotting areas, scroll windows, buttons, radio buttons, check boxes and labels, are created using PyGTK 2.22.5 (Finlay, 2005).

Matplotlib: Matplotlib is able to produce various high-quality figures. It can work independently, and also can be adopted in other interactive environments, including PyGTK. In this research, all the figures are drawn using Matplotlib 1.3.0 (Tosi, 2009).

xlrd and **xlwt**: xlrd and xlwt are two packages that enable Python read and write Microsoft Excel files (Withers, 2009). Package xlrd is for data reading and xlwt is for data writing. The outcomes of all identified solutions are recorded using Microsoft Excel spreadsheets; hence, these two packages are necessary.

This communication tool also has dynamic graphs, which are instantly adjusted based on the decision makers' input. The dynamic graphs need the cooperation of PyGTK and Matplotlib. Once a change is recognised by PyGTK, an updating comment is raised; and accordingly Matplotlib modifies the current figures or draws new figures. Finally, the plotting area is updated by PyGTK. When the update only changes figures such as the figure updates in the first user interface, it can be completed immediately; however, when the update requires drawing new figures such as the figure updates of "Condition Detail" in the third user interface, the response time is longer (1 to 2 seconds).

7.4 Summary

This chapter helps the applications of MOO in decision making in IAM by developing a communication tool to interpret optimisation results, explore decision maker's preferences and refine the solutions. More specifically, the developed communication tool has three user interfaces. The first user interface provides the general achievement of the identified solutions. Undesirable solutions are abandoned. The reserved solutions are listed and compared using different types of graphs in the second user interface. Finally, detailed information of the solutions is shown in the third user interface including overall outcomes and annual outcomes of a network and its individual segments. The developed communication tool dynamically and comprehensively illustrates the optimisation result, and enables decision makers to examine, compare and refine solutions so that decision makers can take the best advantage of optimisation results and make appropriate management decisions.

CHAPTER 8 CONCLUSIONS AND FUTURE RESEARCH

8.1 Conclusions

This research aims at improving decision making in IAM by specifying MOO and introducing a robust MOO technique to help with long-term and network-level decision making in IAM. The objectives were to:

(1) Enhance the knowledge of MOO in the context of decision making in IAM;
(2) Examine the existing MOO techniques that have been applied to help with decision making in IAM, recognising the research status quo of the applications;
(3) Investigate other MOO techniques that have not been applied to decision making in IAM but have application potential, therefore extending the knowledge of MOO and providing more choices for decision making in IAM;
(4) Measure, compare and assess the existing MOO techniques in the context of decision making in IAM based on the typical tests of practical decision making problems and a prototype measurement framework;
(5) Introduce a robust MOO technique to handle multiple objectives, solve optimisation problems and provide satisfying optimisation result for practical long-term and network-level decision making in IAM; and
(6) Develop a communication tool to interpret optimisation results in an understandable and meaningful way, and enable decision makers to explore their management preferences and tailor the results.

This research specified MOO and its philosophy from the viewpoint of decision making in IAM and emphasised the strengths and significance of MOO techniques in practical decision making in IAM.

This research comprehensively studied MOO and outlined the related concepts in the context of decision making in IAM. Also, this research stated the status and significance of MOO in decision making in IAM. It concludes that MOO is necessary for advanced decision making. Because of the challenges of decision making in IAM and the strengths of MOO, the role of MOO becomes more important. More specifically, MOO mathematically describes practical decision making problems, optimises multiple objectives and provides optimisation results. It can clarify decision making problems and simplify decision making process.

This research also suggested the application process of MOO in practical decision making in IAM. Four steps are necessary when handling practical decision making problems in IAM with MOO techniques.

Through a comprehensive literature review, this research investigated the status quo of the applications of MOO techniques in decision making in IAM and studied the commonly used techniques in this regard.

This research reviewed, summarised and compared the MOO techniques that had been applied in decision making in IAM. According to the literature review, it is concluded that a wide range of MOO techniques was applied in decision making in IAM. Many studies certified their effectiveness and helpfulness; however, the previous research focused on the application and improvement of individual techniques while the comparison and discussion of different MOO techniques were hardly discussed. Therefore the comprehensive research status of MOO applications in decision making in IAM was unknown.

According to the analysis completed, this research pointed out that a set of Pareto solutions is needed in the decision making with multiple objectives as they provide much information and can help to clarify and simplify the decision making problem at hand. According to the literature review, this research found that many applications only generated a preferred solution for a MOO problem in decision making in IAM, which was not as helpful as a set of Pareto solutions.

Also, this research summarised the frequently applied MOO techniques in decision making in IAM, which is able to generate a set of Pareto solutions, including Decision Tree, WSM, GA and NSGA II. However it also revealed that most previous applications mainly deal with small decision making problems or impractical data.

This research examined other MOO techniques that were new in decision making in IAM but had the application potential and discussed their applications in decision making in IAM.

This research suggested existing MOO techniques, including exact methods and heuristics, which are not applied in decision making in IAM but have application potential. Because they were new in decision making in IAM, their performance was not clear. This research investigated these new MOO techniques and their characteristics. Experimental tests were conducted based on two practical decision making problem in IAM to compare and assess these techniques.

Based on the analysis completed, this research concluded that all the new MOO techniques were able to analyse MOO problems of decision making in IAM. Because each MOO technique has its own algorithm, their performance in decision making in IAM is different. This research outlined the applications and characteristics of these new MOO techniques. It also pointed out that the selection of an appropriate technique should be based on the decision making problem at hand and the knowledge of these MOO techniques.

A list of existing MOO techniques was discussed and assessed with typical tests based on practical decision making problems in IAM, and a measurement framework was established to measure the performance of MOO techniques from the viewpoint of practical decision making in IAM.

This research enhanced the knowledge of MOO in decision making in IAM by comparing and assessing existing MOO techniques. A list of existing MOO techniques including the applied techniques and new techniques were investigated and assessed. Typical tests were conducted based on two practical decision making problems in IAM to understand the performance of the listed MOO techniques when dealing with MOO problems in decision making in IAM.

According to the tests, this research found that each technique had its own strengths and weaknesses. In general, exact methods generated solutions with better quality and distribution, while heuristics were more flexible and easy on implementation. Therefore, this research suggested that an exact method should be firstly considered when solving MOO problems of decision making in IAM, and the heuristics, as a more flexible choice, could be applied when exact methods cannot be easily applied.

According to the assessment of the existing MOO techniques, this research concluded that none of the studied MOO techniques could obtain a satisfying optimisation result for long-term and network-level decision making in IAM. Eight benchmark criteria of robust MOO techniques were proposed from the viewpoint of practical decision making in IAM. A robust MOO technique should satisfy all these benchmark criteria to effectively and efficiently assist decision making in IAM. However, none of the listed MOO techniques can satisfy the eight benchmark criteria. In this regard, the research confirmed that a new MOO technique is needed for practical long-term and network-level decision making in IAM.

In addition, this research argued that even though a wide range of criteria was proposed to measure MOO techniques, a rational and comprehensive measurement framework still lacked in the context of decision making in IAM. This research established a prototype measurement framework with criteria which is able to describe the performance of different types of MOO techniques when dealing with MOO problems in decision making in IAM. Thus, the comparison and measurement of MOO techniques in decision making in IAM become easier and more reasonable.

A robust MOO technique named DECM was developed in this research to solve the MOO problems of long-term and network-level decision making in IAM.

Based on the analysis completed, a MOO technique named DECM was developed based on ECM but has a dynamic way to split the objective space and define epsilon that is well suited to solving MOO problems in the context of IAM. It is flexible and can be easily controlled by decision makers so as to obtain satisfying optimisation results under the specific perspectives from decision making.

Through the experimental tests of practical decision making in IAM, this research concluded that DECM could effectively solve practical MOO problems and obtained satisfying optimisation results for long-term and network-level decision making in IAM. Compared with the other MOO techniques, DECM produced well distributed Pareto solutions in acceptable time and can be easily applied and implemented. According to the assessment, this research pointed out that DECM was able to satisfy all the benchmark criteria of robust MOO techniques proposed from practical decision making in IAM, therefore it can be regarded as a robust MOO technique for long-term and network-level decision making in IAM.

This research also certified that the developed technique is able to handle different issues in IAM. Other decision making problems in IAM were illustrated and successfully solved. This research claimed that a critical issue of the applications of MOO was how to model a practical decision making problem as an optimisation problem. Once a solvable MOO problem is established, it can be handled by the proposed technique and the optimisation process is similar.

This research established a communication tool to properly interpret the optimisation result.

According to the discussion completed, it was concluded that a communication tool was necessary to properly interpret the optimisation result so that decision makers understand the result and collect the expected information. This research developed a communication tool that not only provides understandable and meaningful explanations of the optimisation results but also enables a decision maker to explore optimisation results to help clarify management preferences and refining solutions.

This communication tool uses three dynamic user interfaces to present solutions and their outcomes. This research set forth the great benefit of this communication tool which takes the best advantage of the optimisation result and helps with the management decision.

Based on the conclusions achieved in this research, this research can be applied to help with different types of decision making in IAM especially the long-term and network-level decision making problems. Furthermore, this research is supported by Deighton Associate Limited. Besides the academic achievements, the main achievements of this paper will be implemented in a commercial IAM software dTIMS and help decision makers to handle practical decision making problems in IAM.

8.2 Lessons Learnt from This Research

This research significantly enhances the knowledge of MOO in decision making in IAM. Some critical lessons are learnt based on its findings, including:

- Many decision making problems in IAM can be expressed and formulated as so-called integer linear programmes (IP). Different types of practical decision making problems are discussed in this research and all of them are successfully formulated as IP. When formulating a decision making problem as another type of optimisation problem, the optimisation process may be different and other MOO techniques may be needed;

- The optimisation techniques should be selected based on the decision making problem at hand. Some decision making problems only need a SOO technique to produce an optimal solution. However, when objectives of a decision making problem are incommensurable or conflicting, a MOO technique is needed to obtain a set of Pareto solutions. One preferred Pareto solution cannot help understand the trade-offs of objectives;
- For the most MOO techniques, optimisation constraints are not the main concern. Once being formulated into the required form, all constraints can be analysed. However, heuristics may not identify feasible solutions when the constraints are restrictive. This research also finds that the computation time is generally increasing when the constraints become more restrictive.
- When applying exact methods, mathematical assumptions may exist and have to be satisfied by the established MOO problems of decision making in IAM;
- Heuristics are not suitable for long-term and network-level decision making in IAM as their solution quality could be poor. When exact methods are not applicable, a heuristic can be recognised as an applicable option, but it is recommended to check its effectiveness and efficiency;
- Many MOO techniques are applicable to solving the MOO problems of decision making in IAM and identify Pareto solutions. In practice, a MOO problem of practical decision making in IAM often has a large number of Pareto solutions. Not all of the existing Pareto solutions must be obtained. A technique that generates good representatives of the Pareto solutions is preferred;
- Parameters are an important part of optimisation techniques. They affect the optimisation result, especially when applying heuristics. The understanding of the applied technique and the addressed problem are important for the proper parameter calibration and can be a barrier to the successful application of methods;
- Stopping criteria are recommended but may weaken the optimisation result. They can help manage the optimisation process and identified solutions. However, they may also result in insufficient running time so that a MOO technique may not have enough time to obtain good optimisation results, especially when applying heuristics.

- In practical decision making in IAM, the interpretation of optimisation results is important, especially when many outcomes are considered. DECM can optimise multiple objectives. However, the interpretation of optimisation results can be extremely complicated when too many objectives are optimised;
- MOO and its techniques cannot make a management decision for decision making in IAM, but instead provide useful information that can be used to clarify and simplify the decision making problem at hand.

8.3 Recommendations on Multi-Objective Optimisation in Decision Making in Infrastructure Asset Management

On the basis of the findings, this research recommends to:

- Apply a MOO technique to assist decision making in IAM when objectives are incommensurable or conflicting. A MOO technique solves MOO problems of decision making and identifies a set of Pareto solutions, which could simplify and clarify the decision making so that the management decision can be made in a rational and well-grounded way;
- Accurately describe a decision making problem as an optimisation problem that can be solved by existing optimisation techniques. When an optimisation formulation poorly expresses a decision making problem, even the optimisation problem can be solved and solutions are obtained, these solutions cannot correctly represent the outcomes of the decision making problem and may lead decision makers in a wrong direction. On the other hand, if the formulated optimisation problem cannot be solved with an existing optimisation technique, even it correctly describes the addressed decision making problem, solutions cannot be obtained to help with decision making. Hence, an accurate and solvable optimisation problem is critical;
- Only obtain good representatives of solutions when applying MOO in decision making in IAM. Again MOO problems of practical decision making in IAM often have a large number of Pareto solutions, and excessive solutions do not help with decision making but waste time. Stopping criteria are highly recommended to avoid solution abundance and improve the optimisation efficiency. However, this requires the applied MOO technique to be flexible and able to achieve good optimisation results when stopping criteria are present;

- Select a MOO technique based on the decision making problem at hand. This research investigates different types of MOO techniques and develops a robust MOO technique for decision making in IAM; however, none of them is a panacea. The selection of a technique should be based on the characteristics of the decision making problem;
- Implement a MOO technique based on its algorithm and the addressed optimisation problem. Good implementation can improve the identified solutions and optimisation efficiency. When specific requirements are proposed, adjustment on implementation may be needed; and
- Read optimisation results with multiple types of solution representation. MOO solutions often involve many outcomes and the outcome relationships are complex. When only one type of solution representation is adopted, some information may be neglected. Therefore, different types of solution representation are recommended to help to present solutions in different manners and reduce the chance of missing information.

8.4 Further Work

This research discusses MOO in decision making in IAM. Important conclusions are achieved. These conclusions not only help with the academic research in this regard, but also assist in solving practical decision making problems through implementing these conclusions in the existing IAM software dTIMS. However, owing to the limitation and scope of this research, further work is still needed on the following subjects.

Intervention based decision making: The decision making discussed in this research is named strategy based decision making, which is based on alternative management strategies. The strategy based decision making may cause two issues:

- All possible alternative strategies have to be obtained. Long-term and network-level decision making often has a large number of alternative strategies. However, the management decision only selects one strategy for a segment and the most strategies are abandoned after the decision has been made. The generation of all the alternative strategies makes decision making complicated and time consuming. Specific software tools may be needed for the strategy generation.
- It may ignore interventions. Decision making in IAM may have requirements on its interventions. For instance, decision making may want to maintain the connected

segments of a road together in order to ease the implementation of maintenance treatments and avoid the movement of resources and machinery. This decision making may need to schedule the maintenance treatments, and generate particular strategies. This could be difficult to be handled with strategy based decision making.

To deal with these issues, intervention based decision making should be used. Intervention based decision making directly schedules interventions to each network segment. Hence, pre-defining all alternative strategies is not needed, which eases and speeds up the decision making process. Furthermore, because intervention based decision making directly appoints interventions to the segments, it is more flexible and can handle the requirements on interventions.

However, intervention based decision making may be difficult to be solved, as it analyses more information including the intervention scheduling and interaction between interventions and segments. In addition, intervention based decision making needs to estimate the outcomes of segments after applying an intervention, such as condition improvement after applying overlay treatment on a road segment. Thus, the prediction of impacts of interventions is necessary, which results in more computation. Some outcomes are non-linear, such as road condition that follows Markov chain. Hence, non-linear optimisation may be needed in the intervention based decision making in IAM.

Formulation of decision making problems: Every decision making problem in IAM is a unique problem with specific goals and requirements. In this research, the decision making problems are strategy based and formulated as IP. However, in practical decision making, other types of formulations, such as non-linear ones, may describe some decision making problems in a more accurate way. For example, practical decision making is filled with uncertainties and risks. Instead of integer variables, fuzzy variables may be a better choice to describe the uncertainties and risks. For another example, some decision making requirements may be preferred but not obligatory, such as a decision making problem that may prefer keeping the maintenance impact on the environment to a low level but is not compulsory. This preference can be described using soft constraints. Other decision making problems in IAM may require other formulations. Research on the formulation of practical decision making problems is needed.

Feedback model: IAM is based on historical experience and predicted data but focuses on the long-term future. According to the Ministry of Transport in New Zealand (Lyons *et al.*, 2015), IAM is an evolving process so the current predictions of future are not likely to be exactly the same as the future reality. The management decision that is appropriate today may not be appropriate in the future. This requires the decision making process to be adjustable according to the reality during the IAM process.

Therefore a feedback model is needed in decision making in IAM. After a management decision is made based on the current data, the monitoring is continuous and the feedback of the reality is collected. Based on the feedback, the decision making model should be able to adjust the previous management decision. In addition, the feedback model could also help learning the behaviour of infrastructure assets so improving the prediction quality and the management decision.

However, a feedback model requires interaction with the data, which makes the decision making process more complicated. An effective MOO technique is needed to analyse this model.

Other MOO techniques: MOO is a developing research area. This research introduces and examines 13 MOO techniques while more new techniques are proposed and may produce good optimisation results for decision making in IAM. Studies are needed to introduce new MOO techniques in decision making in IAM. Moreover, technique hybridisation is also a promising research area. When hybridising different MOO techniques, the optimisation result may be significantly improved. This research only focuses on the classic MOO algorithms and does not discuss the hybridised techniques.

Outcomes measurement: Outcomes measurement is a critical part of decision making in IAM. When the outcomes are inaccurate, the optimisation and analysis may lead decision makers to the wrong answer. Hence, the outcomes need to be accurately measured. In this research, all the outcomes are estimated using a software tool named dTIMS CT 8. Research is needed to improve outcome measurement, especially of the intangible outcomes.

Applications of MOO in the real world: According to this research, the research literature certifies the effectiveness of MOO and recommends applying MOO in decision making in IAM. Typical tests based on practical decision making in IAM were analysed in this research and others. However, currently the applications of MOO in decision making in IAM are still in the phase of researching. When making decisions in the real world, the traditional decision making methods are more popular than optimisation.

Of the applications of MOO in decision making in IAM, the author thinks one of the most challenging things is introducing MOO to decision makers. Researchers have to let the decision makers be aware of the strengths of MOO but tell them that MOO is not utopian that is impractical, but an applicable assistant that really supports in their decision making process in IAM.

The author was not surprised at the issues raised when applying MOO in real world decision making in IAM. The gap between mathematical models and real world problems cannot be avoided. Yet with feedbacks from decision makers, researchers can improve the applications of MOO and provide more practical optimisation results. This is also the goal of the author.

APPENDIX A: SUMMARY OF APPLIED OPTIMISATION TECHNIQUES IN DECISION MAKING

Author	Infrastructure Type	No. of objectives	Constraints[1]	Level[2]	Algorithm	Solution set	Practical Case?[3]	Note
Magee (1964)	Infrastructure	2		P	Decision tree	Pareto solutions		Certifying of the effectiveness of this method
Meyer (1973)	Infrastructure	1	Y	P	Enumeration based method	an optimal solution		Applying a mixed-integer programming when providing strategic decisions.
Abelson & Flowerdew (1975)	Pavement	1	Y	NW	Linear programming	an optimal solution		Certifying the importance of road maintenance and effectiveness of optimisation
Terrell (1975)	Infrastructure	1	Y	P	Linear programming Simplex tableau	an optimal solution		Verifying the feasibility of this method and introducing different constraints.
Friesz & Fernandez (1979)	Pavement	1	Y	P	Dynamic optimisation model	an optimal solution		Scheduling interventions of maintenance investments
Bell (1982)	Infrastructure	1+		P	Utility analysis method	a preferred solution		Trading off financial return, and guiding decision makers.

Kemp (1983)	Pavement	1	Y	NW	Costing schemes	an optimal solution	Y	Optimising national resources with stimulated cost consciousness.
Moghtaderi-Zadeh & Kiureghian (1983)	Infrastructure	1+		NW	An improved step-by-step procedure	a preferred solution	Y	Discussing the reliability of existing lifeline networks.
Cook (1984)	Pavement	1	Y	NW	A two-phase approach	an optimal solution	Y	Allocating funds to competing pavement projects based on serviceability levels.
Way (1985)	Pavement	1	Y	NW	A dynamic and probabilistic algorithm	an optimal solution	Y	Developing a decision making tool to a preserve road network.
Fwa *et al*. (1988)	Pavement		Y	P	A prioritisation method	an optimal solution	Y	Developing a procedure to schedule maintenance activities at the network level and incorporating it into an existing pavement management system.
Jiang & Sinha (1989)	Bridge		Y	NW	Linear programming	an optimal solution	Y	Providing bridge managers with tools for making consistent and cost-effective decision.
Male *et al.* (1990)	Pipe			P	A simulation based method	an optimal solution	Y	Assisting in planning for the rehabilitation of a water distribution system
Jacobs (1992)	Bridge	1		NW	A simulation based method	an optimal solution	Y	Determining bridge deck replacement and rehabilitation throughout the planning horizon.
Lansey *et al*. (1992)	Pipe	1	Y	NW	Generalized Reduced Gradient Program	an optimal solution		Determining the major piping management alternatives.
Li & Haimes (1992)	Pipe	1		P	Semi-Markovian model	an optimal solution		Determining the optimal replacement/repair decision at various deteriorating stages.
Stewart (1992)	Electricity	1+		P	Goal programming	a preferred solution		Suggesting robustly and effectively and non-expert MCDM approaches.
Ben-Akiva *et al*. (1993)	Pavement	1	Y	P	Markov chain theory Decision tree	an optimal solution		Analysing infrastructure performance and planning management activities
Fwa & Chan (1993)	Pavement	3		NW	NN	a preferred solution		Certifying the feasibility of neural network models for priority assessment of highway pavement maintenance.
Grivas *et al*. (1993)	Pavement	1		P	State increment method	an optimal solution	Y	Specifying treatment alternatives for each state and analysing life-cycle cost decision making.

Tzeng & Teng (1993)	Pavement	1+	Y	P	Weighting method	an optimal solution		Applying fuzzy set theory to transportation investment planning.
Augusti *et al.* (1994)	Pavement	1	Y	NW	Dynamic programming	an optimal solution		Presenting a procedure to increase the reliability of vulnerable elements of the network.
Chan *et al.* (1994)	Pavement	1	Y	NW	GA	an optimal solution		Introducing GA in pavement management
Farid *et al.* (1994)	Bridge	1	Y	NW	Incremental Benefit-Cost programme	an optimal solution	Y	Selecting the near-optimal bridge improvement alternatives under several levels of budget.
Fwa *et al.* (1994)	Bridge	1	Y	NW	GA	an optimal solution		Introducing GA in pavement management
Kim & Mays (1994)	Pipe	1	Y	NW	Implicit enumeration scheme	an optimal solution	Y	Selecting the pipes to be rehabilitated and/or replaced in an existing water distribution system.
Arulraj & Rao (1995)	Pipe	1		NW	Linear programming	an optimal solution		Defining significance index for pipe measurement.
Frangopol (1995)	Infrastructure	2	Y	P	Multi-criteria reliability-based optimisation	a preferred solution		Finding the best possible solution without compromising structural reliability
Mohamed *et al.* (1995)	Bridge	1	Y	NW	NN	an optimal solution	Y	Allocating resources to bridge maintenance projects.
Mohammadi *et al.* (1995)	Bridge	1	Y	P	Linear programming	an optimal solution		Enabling rational decisions that best suits a bridge's needs with constraints.
Ravirala & Grivas (1995)	Infrastructure	1+	Y	NW	Goal programming	a preferred solution	Y	Presenting a method for integrating the decisions involved in the development of annual pavement and bridge programmes.
Flintsch *et al.* (1996)	Pavement	3		NW	NN	an optimal solution		Describing NN used to develop an automatic procedure for pavement management.
Fwa *et al.* (1996)	Pavement	3	Y	NW	GA	a preferred solution		Describing a computer model to solve the pavement maintenance-rehabilitation trade-off problem at the network level.
Harper (1996)	Infrastructure	2	Y	NW	Multi-year PMS/BSMS optimisation	a preferred solution	Y	Maximising quality given the available resources and focusing the global maintenance decisions.

Li *et al.* (1996)	Pavement	1		P	Nonhomogeneous Markov Probabilistic Modelling Program	an optimal solution	Y	Determining activities for each element on the basis of a reliability analysis and a Monte Carlo simulation technique.
Liu & Wang (1996)	Pavement	1	Y	NW	Linear programming	an optimal solution	Y	Simulating a possible scenario by effectively using a given budget in the planning period.
Mbwana & Turnquist (1996)	Pavement	2	Y	NW	Weighting method	a preferred solution	Y	Developing a pavement management system for large-scale network.
Ravirala *et al.* (1996)	Bridge	2	Y	NW	Weighting method Goal programming	a preferred solution	Y	Analysing small and medium-size bridges or a population of spans of a large bridge.
Razaqpur *et al.* (1996)	Bridge	1	Y	NW	Dynamic programming NN	an optimal solution	Y	Allocating a limited budget to bridge maintenance projects.
Tam & Stiemer (1996)	Infrastructure	1	Y	NW	A dynamic programming approach	an optimal solution	Y	Discussing the relationship between maintenance procedures and cost
Wang & Zaniewski (1996)	Pavement	1	Y	NW	Linear programming	an optimal solution	Y	Performing an optimisation method across the entire network of pavements in the state.
Bellehumeur *et al.* (1997)	Pipe	1+		P	Weighting method Fuzzy set theory	a preferred solution	Y	Comparing methods when resolving a problem of sewage pipe management
Frangopol *et al.* (1997)	Pavement	1	Y	NW	Decision tree	an optimal solution	Y	Developing a reliability-based lifetime approach to decide an optimum repair strategy with cost savings and improved efficiency.
Fwa *et al.* (1997)	Pavement	2	Y	Both	NN	a preferred solution		Developing a decision analysis framework based on past experience of maintenance.
Halhal *et al.* (1997)	Pipe	2	Y	NW	Structured Messy GA	Pareto solutions		Describing an approach using capital cost and benefit and enabling varying cost to be derived.
Hicks *et al.* (1997)	Pavement	1		NW	A pavement serviceability approach	an optimal solution		Selecting the proper maintenance strategies for different distress types in asphalt pavements, depending on traffic level and environment.
Hsieh & Liu (1997)	Infrastructure	1+	Y	P	Staged heuristic	a preferred solution		Introducing an approach to tackle the investment problem.

Itoh *et al*. (1997)	Bridge	1	Y	NW	GA	an optimal solution	Y	Developing a bridge life-cycle management system in a user-friendly environment.
Li *et al.* (1997)	Pavement	1	Y	NW	A priority programming	an optimal solution	Y	Focusing on an integrated approach to pavement network preservation programming
Liu *et al*. (1997)	Bridge	2		NW	GA	Pareto solutions	Y	Comparing GAs for the deck rehabilitation plan of network-level bridges.
Sirajuddin (1997)	Pavement	1	Y	P	Tabulated manual procedure	an optimal solution		Facilitating the allocation of limited funding to management projects.
Augusti & Ciampoli (1998)	Pavement	1+	Y	NW	Weighting method	a preferred solution		Presenting a procedure for the optimal allocation of resources with special reference to the case of road networks.
Augusti *et al.* (1998)	Bridge	1	Y	Both	Dynamic programming	an optimal solution		Planning interventions on bridges included in a highway network, taking into account their deterioration and available economic resources.
Fwa & Shanmugam (1998)	Pavement	1	Y	P	A simulation based method	an optimal solution		Developing a fuzzy logic-based system of pavement network.
Kleiner *et al*. (1998)	Pipe	1	Y	NW	M-PRAWDS	an optimal solution		Proposing an approach in which the water distribution network economics and hydraulic capacity are analysed over an analysis period.
Kleiner *et al.* (1998)	Pipe	1	Y	NW	M-PRAWDS	an optimal solution		Implementing a method that facilitates the optimal rehabilitation strategy.
Abaza & Ashur (1999)	Pavement	1	Y	P	Penalty function method	an optimal solution	Y	Optimising pavement condition under constrained budgets.
Das (1999)	Bridge	1	Y	Both	A prioritisation model	a preferred solution		Prioritising bridge maintenance activities based on the needs.
Guignier & Madanat (1999)	Infrastructure	1	Y	NW	Markov decision model	an optimal solution	Y	Presenting an approach for the joint optimisation of maintenance and improvements of the components of a network of infrastructure.

Halhal *et al.* (1999)	Pipe	2	Y	NW	Structured Messy GA	Pareto solutions		Presenting the optimal design and scheduling of investment for the rehabilitation of water distribution networks.
Kerali & Mannisto (1999)	Pavement	3	Y	P	HDM-III	a preferred solution		Strategic planning the short- or long- term road development and preservation under various budgetary and economic scenarios.
Kiyota *et al.* (1999)	Pavement	1	Y	NW	Multistage Optimisation	an optimal solution		Determining the road segments treated during road widening programmes.
Abdelrahim & George (2000)	Pavement	1+		P	Genetic Adaptive Neural Network Training Algorithm	an optimal solution		Considering various situations for rehabilitation of a deteriorated pavement section.
Avineri *et al*. (2000)	Pavement	1+		P	Weighting method	a preferred solution	Y	Presenting an efficient technique for the selection of transportation projects using fuzzy sets theory.
Bender & Simonovic (2000)	Pipe	1+		P	Fuzzy compromise approach	a preferred solution		Discussing various uncertainties and providing a flexible form of group decision support.
Boulos *et al*. (2000)	Pipe	1	Y	NW	GA	an optimal solution		Designing the rehabilitation of municipal water distribution piping systems.
Demetriades & Mamuneas (2000)	Infrastructure	1	Y	P	A dynamic model	an optimal solution	Y	Studying the effects of public infrastructure capital on output supply and input demands in 12 countries.
Frangopol *et al*. (2000)	Bridge	1		NW	Modified decision tree	an optimal solution		Optimising the resource allocation for the management of deteriorating bridges.
Frangopol *et al*. (2000)	Bridge	2		P	User-based model condition-based model corrective model	a preferred solution		Developing a reliability- and cost-oriented process for optimal bridge maintenance planning.
Fwa *et al.* (2000)	Pavement	2-3	Y	NW	GA	Pareto solutions		Comparing two- and three- objective solutions with practical considerations
Mamlouk *et al.* (2000)	Pavement	1	Y	P	Mechanistic Multilayer Elastic System Model	an optimal solution	Y	Developing a method that leads to cost-effective pavement management.

Miyamoto *et al.* (2000)	Bridge	2		NW	GA	a preferred solution		Developing a bridge management system for deteriorated concrete bridges.
Shekharan (2000)	Infrastructure	1		P	GA	an optimal solution		Verifying the feasibility of this method.
Smilowitz & Madanat (2000)	Pavement	1	Y	NW	Latent Markov model	an optimal solution		Presenting formulations of the network-level LMDP.
Abaza *et al.* (2001)	Pavement	1	Y	P	AASHTO	an optimal solution		Applying an effective optimum decision policy and presenting serviceability index.
Attoh-Okine & Gibbons (2001)	Infrastructure	1+		P	Decision tree	a preferred solution		Constructing a hierarchical network for decision making considering a range of factors.
Chan *et al*. (2001)	Pavement	1+	Y	NW	GA	an optimal solution	Y	Proposing a computationally efficient method based on prioritised allocation of resources.
Dandy & Engelhardt (2001)	Pipe	1	Y	P	GA	an optimal solution	Y	Introducing GA to find a near-optimal schedule for the replacement of the water supply pipes.
Dogaki *et al*. (2001)	Pavement	2	Y	NW	GA	a preferred solution		Explaining how to appropriately decide the repair order for deteriorating decks.
Frangopol *et al.* (2001)	Bridge	1	Y	P	Monte Carlo simulation	an optimal solution		Discussing life-cycle management of highway bridges.
Jha *et al*. (2001)	Pavement	1	Y	P	GA	an optimal solution	Y	Considering intangible parameters for highway network management.
Kleiner (2001)	Infrastructure	2		P	A probability based method	an optimal solution		Introducing a decision framework to assist to optimise decisions regarding the renewal of large infrastructure.
Kleiner *et al.* (2001)	Pipe	1	Y	NW	Dynamic programming	an optimal solution	Y	Developing a method to identify an optimal rehabilitation strategy considering the asset deterioration.
Mamlouk & Zaniewski (2001)	Pavement	1	Y	P	Cost-effective preservation approach	an optimal solution		Presenting a useful tool for highway engineers and superintendents to develop a preventive maintenance programme.

Morley *et al.* (2001)	Pipe	1	Y	NW	GA	an optimal solution		Developing of evolution programmes with technical applications.
Najafi & Valerie (2001)	Pavement	1		NW	A simplified five-step priority procedure	an optimal solution	Y	Developing a simple method for the highway maintenance and the allocation of funding.
Smadi (2001)	Pavement	1	Y	P	Dynamic programing	an optimal solution		Using knowledge based expert systems to develop a comprehensive pavement management system.
Abaza (2002)	Pavement	1		NW	AASHTO	an optimal solution		Evaluating potential maintenance and rehabilitation plans with an optimisation process.
Bonyuet *et al*. (2002)	Infrastructure	1	Y	NW	Decision tree	an optimal solution		Discussing a road-bridge rehabilitation model.
Chien *et al.* (2002)	Pavement	1	Y	NW	An optimization procedure	an optimal solution		Determining construction and maintenance activities on two-lane two-way highways.
Durango (2002)	Pavement	1	Y	P	Temporal-Difference Learning Methods	an optimal solution		Describing Reinforcement Learning methods used to address the problem of developing maintenance and repair policies.
Evdorides *et al*. (2002)	Pavement	1	Y	NW	A priority based method	an optimal solution		Dealing with optimal timing and spatial location of road infrastructure maintenance projects with engineering-driven criteria.
Ferreira *et al.* (2002)	Pavement	1	Y	NW	GA	an optimal solution	Y	Developing a model for pavement management systems regarding the maintenance and rehabilitation of road pavements.
Ferreira *et al.* (2002)	Pavement	1	Y	NW	GENETIPAV-D	a preferred solution	Y	Presenting a segment-linked optimisation model for deterministic pavement management systems.
Petersen (2002)	Pavement	1	Y	NW	Dual optimisation Lagrangian relaxation method Dynamic programming	an optimal solution	Y	Calculating an optimal investment plan for a highway corridor or number of corridors, subject to budget constraints

Tack & Chou (2002)	Pavement	1		NW	Preconstrained GA Simple GA Dynamic programming	an optimal solution		Implementing and comparing two GAs and a dynamic programming approach to determine multiyear repair schedules.
Akgül and Frangopo (2003)	Bridge	1	Y	NW	Weighting method	an optimal solution	Y	Describing a reliability-based lifetime evaluation of existing bridges within a bridge network.
Chan *et al.* (2003)	Pavement	1+	Y	Both	Two-stage three-region GA	Pareto solutions		Allocating the funds to the district/regional agencies.
Hassanain & Loov (2003)	Bridge	1	Y	P	Many algorithms	different		Encouraging bridge engineers to move towards the increased use of advanced analysis and design optimisation methods
Kong & Frangopol (2003)	Infrastructure	1		P	Decision tree	an optimal solution		Evaluating life-cycle maintenance cost of deteriorating structures considering uncertainties.
Kong & Frangopol (2003)	Bridge	1	Y	Both	A ranking method	an optimal solution		Considering the reliability and uncertainties of deteriorating structures in decision making.
Streicher & Rackwitz (2003)	Infrastructure	1+	Y	P	Weighting method	a preferred solution		Proposing a well-known renewal model and covering the majority of cases in practice.
Tsunokawa & Ul-Islam (2003)	Pavement	1	Y	P	HDM-IV	a preferred solution		Investigating the relationship between optimal pavement design and maintenance strategy and the level of economic development.
Wang *et al.* (2003)	Pavement	2	Y	NW	Weighting method	Pareto solutions		Developing a model to select a set of candidate projects for pavement management.
Yang *et al.* (2003)	Pavement	3		NW	NN	a preferred solution	Y	Verifying the feasibility of ANN models in pavement management.
Yi *et al.* (2003)	Bridge	1	Y	P	An optimisation algorithm	a preferred solution		Presenting a general formulation of life-cycle cost models for steel bridge management.
Abaza *et al.* (2004)	Pavement	1+	Y	P	A penalty function method simultaneous search	an optimal solution		Designing an integrated pavement management system for pavement maintenance and rehabilitation work.

Adey *et al.* (2004)	Bridge	1	Y	NW	Decision tree	an optimal solution		Examining the effect of bridge failures on indirect costs of the system at the network level.
Cai *et al.* (2004)	Pipe	1+		P	Weighting method	a preferred solution		Combining techniques to support plan generation and evaluation, preference elicitation, and negotiation for a consensus plan.
Dicdican (2004)	Infrastructure	3	Y	P	Weighting method	a preferred solution		Developing a systemic risk-based method to maintain highway infrastructure systems.
Frangopol & Neves (2004)	Bridge	3	Y	P	GA	a preferred solution	Y	Presenting a model including interaction between structural safety analysis, visual inspections and non-destructive tests.
Furuta *et al.* (2004)	Infrastructure	3		NW	GA	Pareto solutions		Discussing Life-Cycle Cost, extension of service life and the target safety level.
Hsieh & Liu (2004)	Infrastructure	3	Y	NW	GA	Pareto solutions		Verifying the feasibility of GA.
Kaliszewski (2004)	Infrastructure	1+		Both	Weighting method Reference point method	a preferred solution		Comparing and discussing the two methods.
Kulkarni *et al*. (2004)	Pavement	1+		Both	A multi-attribute need method	a preferred solution		Presenting a need-based method to prioritise and select highway projects for improvement.
Liu & Frangopol (2004)	Bridge	3	Y	P	GA	Pareto solutions		Confining uncertainties to define the selected computational models and their effects.
Noortwijk & Frangopol (2004)	Infrastructure	1	Y	P	Condition-based model Reliability-based model	a preferred solution		Comparing maintenance models for deteriorating civil infrastructures.
Nunoo & Mrawira (2004)	Infrastructure	1	Y	NW	Shuffled Complex Evolution Procedure	an optimal solution		Proposing a procedure for solving network level infrastructure works programming problems.
Picado-Santos *et al.* (2004)	Pavement	1	Y	NW	GENENTIPAV-D	an optimal solution	Y	Describing the process leading to the creation of the PMS and the activities.
Stewart *et al*. (2004)	Bridge	1	Y	P	Event-based simulation procedure	an optimal solution	Y	Investigating the effect of limit state selection on bridge deck life-cycle costs and thus on optimal repair strategies.

Tan *et al.* (2004)	Pavement	1	Y	NW	GA	a preferred solution		Developing a two-step fund allocation approach using GA for pavement maintenance.
Yoo (2004)	Pavement	1	Y	NW	Hybrid algorithm	an optimal solution		Selecting an M&R activity for each pavement section in each period of a specified extended planning horizon.
Zayed (2004)	Bridge	1	Y	P	Dynamic programming Greedy heuristic	an optimal solution	Y	Making intelligent decisions as to which projects will be funded and the degree of funding
Bako & Ambrus-Somogyi (2005)	Infrastructure	1	Y	NW	Network Level Multi-Stage PMS-Model	an optimal solution		Allocating available budget for the rehabilitation and maintenance which are not enough for holding the system in a certain condition level its whole lifetime.
Chassiakos *et al.* (2005)	Pavement	1	Y	P	Weighting method	a preferred solution		Presenting a knowledge-based system used for maintenance planning of highway concrete bridges.
Elazouni & Metwally (2005)	Infrastructure	2	Y	P	GA	Pareto solutions		Developing finance-based scheduling to reduce project indirect costs and financing costs.
Fang *et al*. (2005)	Infrastructure	2	Y	NW	Weighting method	one preferred		Proposing a bi-objective mixed asset portfolio selection model.
Herabat & Tangphaisankun (2005)	Pavement	2	Y	P	Constraint-Based GA Optimization Model	Pareto solutions	Y	Developing a MOO model to support the multi-year decision making process of the highway maintenance management.
Hiep & Tsunokawa (2005)	Pavement	1	Y	P	HDM-4	an optimal solution	Y	Presenting a systematic approach for pavement management systems.
Hugo *et al.* (2005)	Pipe	2	Y	P	A generic optimisation-based model	Pareto solutions		Presenting a model for the strategic long-range investment planning.
Kong & Frangopol (2005)	Infrastructure	1		P	B&B Probabilistic optimisation	an optimal solution		Analysing time-varying uncertainties associated with reliability deterioration of structures and effects of maintenance interventions on the system reliability
Kuhn & Madanat (2005)	Infrastructure	1	Y	NW	Robust optimisation based on Markov chain	an optimal solution		Contrasting the expected costs incurred with uncertainties using robust optimisation.

Lee *et al.* (2005)	Pavement	1	Y	Both	CA4PRS	an optimal solution	Y	Designing a software to balance rehabilitation productivity, traffic inconvenience, and agency cost.
Liu & Frangopol (2005)	Bridge	2	Y	NW	GA	Pareto solutions		Solving network-level bridge maintenance planning problem to select and allocate maintenance interventions.
Liu & Frangopol (2005)	Bridge	3		P	GA	Pareto solutions	Y	Locating maintenance scenarios that exhibit an optimised trade-off of conflicting objectives.
Liu & Frangopol (2005)	Bridge	3	Y	P	GA	Pareto solutions		Presenting a procedure to prioritise maintenance for deteriorating reinforced concrete bridge.
Liu & Frangopol (2005)	Bridge	3		Both	GA	Pareto solutions		Trading off objectives when making optimal maintenance planning for bridge networks.
Morcous & Lounis (2005)	Infrastructure	1	Y	NW	GA Markov-chain models	an optimal solution		Determining the optimal set of maintenance alternatives for an infrastructure network.
Neves & Frangopol (2005)	Bridge	3		NW	Monte-Carlo simulation	a preferred solution		Analysing the evolution in time of probabilistic performance indicators of existing structures.
Parke *et al.* (2005)	Bridge	2	Y	Both	A combinatorial and evolutionary optimization technique	a preferred solution		Finding optimal maintenance management solutions that meet bridge managers' specific requirements on lifetime bridge performance under budgetary constraints.
Pelet *et al.* (2005)	Electricity	2	Y	P	Queuing multi-objective optimiser	Pareto solutions		Dealing with complex systems in which the synergy between the various components.
Singh & Tiong (2005)	Pavement	1	Y	P	Stepwise procedure	an optimal solution	Y	Discussing cost and statistical factors while assessing the life-cycle cost of a highway management.
Abaza (2006)	Pavement	1	Y	NW	The method of Hooke and Jeeves	an optimal solution		Incorporating both the pavement deterioration rates and improvement rates.
Bucher & Frangopol (2006)	Infrastructure	1	Y	NW	A smoothing procedure	an optimal solution		Discussing time-based maintenance and performance-based maintenance under uncertainties and risk
Chen *et al*. (2006)	Infrastructure	2	Y	NW	PSO	a preferred solution		Studying the constrained portfolio selection problem.

Chootinan *et al.* (2006)	Pavement	2	Y	NW	Simulation-based GA	an optimal solution		Introducing a multi-year pavement maintenance method with uncertainty.
Dandy & Engelhardt (2006)	Pipe	2		NW	GA	a preferred solution	Y	Demonstrating GA to generate trade-off curves between cost and reliability.
Durango-Cohen & Tadepalli (2006)	Pavement	1		Both	Kalman filter algorithm	an optimal solution		Presenting a framework to support investment decisions in maintenance and repair of transportation infrastructure.
Elbehairy *et al.* (2006)	Bridge	1	Y	Both	GA Shuffled Frog Leaping	an optimal solution		Introducing an integrated model for bridge deck repairs with life-cycle costs of network-level and project-level decisions.
Furuta *et al.* (2006)	Pavement	2		NW	GA	a preferred solution		Providing a framework for optimal allocation of cost for increasing the seismic performance of road networks.
Gabriel *et al*. (2006)	Infrastructure	2	Y	P	Weighting method	Pareto solutions		Analysing the cost and the priority rank of each project considering probabilistic constraints related to budget.
Jha & Abdullah (2006)	Pavement	1		NW	Decision tree GA	an optimal solution		Considering optimal maintenance strategies for roads over a given planning horizon.
Liu & Frangopol (2006)	Bridge	3	Y	NW	GA	Pareto solutions	Y	Discussing network-level bridge maintenance management.
Liu *et al.* (2006)	Infrastructure	2		NW	GA	Pareto solutions	Y	Dealing with an empirical application for the rehabilitation planning of bridges at a network level
Lounis (2006)	Bridge	3	Y	NW	Bogdanoff's cumulative damage model	a preferred solution		Reducing the risk of failure due to bridge deck deterioration and maintenance activities.
Madanat *et al.* (2006)	Infrastructure	1	Y	P	Feedback control approach	an optimal solution		Presenting systematic probing for selecting optimal MR&R policies for infrastructure under uncertainty.
Neves *et al.* (2006)	Bridge	3	Y	NW	GA	Pareto solutions		Developing a probabilistic approach to bridge maintenance considering maintenance types.

Neves *et al.* (2006)	Bridge	3	Y	NW	GA	Pareto solutions		Using MOO under uncertainty to find the best combinations of condition, safety, and cost of deteriorating bridges.
Neves *et al.* (2006)	Bridge	3		P	GA	Pareto solutions		Using probabilistic MOO over time and defining performance of existing bridges.
Robelin & Madanat (2006)	Bridge	1+		P	Dynamic programming	an optimal solution		Developing a reliability-based optimisation model of bridge maintenance and replacement decisions.
Thompson (2006)	Bridge	1+	Y	NW	Weighting method	a preferred solution	Y	Developing a framework to optimise bridge level and network level programmes.
Tsunokawa *et al.* (2006)	Pavement	1+		P	What-if analysis model	an optimal solution		Finding the true optimal without requiring exogenously specified alternatives.
Yang *et al.* (2006)	Bridge	2	Y	P	Optimising one objective	a preferred solution	Y	Analysing deterioration and effect of maintenance considering the performance of existing infrastructure.
Zecchin *et al.* (2006)	Pipe	1	Y	NW	Max-Min Ant System	Pareto solutions	Y	Applying optimisation methods to minimise the costs associated with such infrastructure.
Zhang (2006)	Infrastructure	1	Y	Both	Markov decision model	an optimal solution		Selecting strategies for deteriorating infrastructure.
Abaza (2007)	Pavement	1	Y	NW	Global Network Optimisation Model	an optimal solution		Global network optimisation of the pavement management problem.
Abaza & Murad (2007)	Pavement	1	Y	NW	Dynamic probabilistic-based approach	an optimal solution		Identifying of a long-term restoration strategy.
Babani (2007)	Pipe	1	Y	NW	Tranches generation	an optimal solution	Y	Finding optimal rehabilitation strategies for sewer pipe networks.
Benati & Rizzi (2007)	Infrastructure	1		NW	Value-at-Risk optimisation	an optimal solution	Y	Considering an extension of the Markov model, in which Value-at-Risk is used.
Brownlee *et al*. (2007)	Infrastructure	1	Y	NW	TOPSIS	a preferred solution		Presenting a method for identifying potential pavement maintenance schemes.
Carbonell *et al.* (2007)	Pavement	1	Y	NW	A heuristic optimisation	an optimal solution		Dealing with the economic optimisation of roads and with the optimisation of bridge sections.

Dashti *et al.* (2007)	Infrastructure	2		P	PSO	a preferred solution		Constructing an optimal risky portfolio for an optimal combination of their investments among different financial assets.
Deb *et al.*(2007)	Electricity	2	Y	P	NSGA II	Pareto solutions		Modifying NSGA-II in tracking a Pareto optimal front.
Frangopol & Liu (2007)	Bridge	3	Y	NW	Two-phase stochastic dynamic programming	a preferred solution		Presenting a procedure for bridge network maintenance planning considering remaining service lifetimes and reliability.
Frangopol & Liu (2007)	Infrastructure	3	Y	Both	GA	Pareto solutions	Y	Using optimisation for bridge maintenance considering often competing criteria.
Hajdin & Lindenmann (2007)	Pavement	1	Y	NW	A directed graph	an optimal solution		Presenting a method for road agencies.
Heywood *et al.* (2007)	Pipe	1	Y	NW	Decision tree	an optimal solution		Describing an approach to assessing the investment requirements for sewerage systems
Lee & Kim (2007)	Bridge	3	Y	NW	GA	a preferred solution		Suggesting an algorithm to prioritise bridge maintenance activities at the network level.
Maji & Jha (2007)	Pavement	1	Y	NW	GA	an optimal solution	Y	Evaluating the condition considering budget, and suggesting the optimal maintenance schedule.
Morcous (2007)	Bridge	2		NW	GA	Pareto solutions		Solving infrastructure maintenance optimisation problems involving criteria.
Ouyang (2007)	Pavement	1	Y	NW	A multidimensional dynamic programming	an optimal solution		Presenting a framework for planning pavement resurfacing activities on highway networks.
Archondo-Callao (2008)	Pavement	1	Y	NW	HDM-4	an optimal solution		Presenting the experience applying HDM-4 to road network strategic planning.
Bako *et al.* (2008)	Pavement	2	Y	NW	An Optimal Quality Management algorithm	a preferred solution		Handling pavement maintenance in the multi-period, long time model.
Castanier & Yeung (2008)	Pavement	1	Y	P	Markov semi-renewal theory	an optimal solution		Utilising a combination of a Poisson and gamma process to account for uncertainty.
Dridi *et al.* (2008)	Pipe	1	Y	NW	NSGA II	an optimal solution		Using NSGA-II to optimise large water distribution systems.

Gao & Zhang (2008)	Pavement	1	Y	P	Stochastic programming	an optimal solution		Considering uncertainties in pavement maintenance management.
Goktepe *et al.* (2008)	Pavement	1	Y	NW	Fuzzy inference methodology	an optimal solution		Presenting the applicability of a fuzzy rule-based system for earthwork optimisation.
Halfawy *et al.* (2008)	Pipe	3	Y	NW	GA	Pareto solutions	Y	Identifying and selecting the most suitable renewal technologies
Kong *et al.* (2008)	Pipe	1	Y	NW	A priority based method	an optimal solution	Y	Evaluating optimal rehabilitation scheduling considering the characteristics of pipelines.
Miyamoto & Uchino (2008)	Bridge	2		NW	GA	a preferred solution		Developing a comprehensive decision support system with/without annual budget limitations.
Šelih *et al.* (2008)	Pavement	1+		P	Weighting method AHP	a preferred solution		Developing a decision support system for priority ranking of asset rehabilitation projects.
Tang & Chien (2008)	Pavement	1	Y	P	GA	an optimal solution	Y	Improving efficiency of maintenance work through optimising work schedules.
Yare *et al.* (2008)	Electricity	2	Y	NW	Discrete PSO	a preferred solution	Y	Generating optimal preventive maintenance schedule considering economical and reliable operation of a power system.
Yoo & Garcia-Diaz (2008)	Pavement	1	Y	NW	Sub-gradient optimisation procedure	an optimal solution	Y	Determining the cost-effective maintenance and rehabilitation for a highway pavement network.
Abaza & Ashur (2009)	Pavement	1	Y	NW	An age-based optimisation model	an optimal solution	Y	Using a microscopic approach to yield optimum pavement conditions for a given pavement system.
Abiri-Jahromi *et al.* (2009)	Electricity	1	Y	NW	Mixed integer linear programming	an optimal solution	Y	Long-term maintenance schedule of overhead lines.
Amador-Jimenez & Mrawira (2009)	Pavement	2	Y	NW	LINDO, MOSEK, CPLEX, etc	a preferred solution	Y	Supporting trade-off and optimisation analyses in a road management system.
Barker & Haimes (2009)	Electricity	1+	Y	P	Inoperability Input–Output model	a preferred solution		Measuring the sensitivity of extreme event consequences to uncertainties.
Dridi *et al.* (2009)	Pipe	2	Y	NW	NSGA II	Pareto solutions		Establishing the optimal replacement schedule for a water distribution network.

Farhan & Fwa (2009)	Pavement	3	Y	P	AHP	a preferred solution		Comparing different AHP for the prioritisation of pavement maintenance activities
Fan *et al.* (2009)	Pavement	1	Y	NW	A two-stage stochastic programming model	an optimal solution	Y	Optimising retrofit decision for highway systems and minimising damage.
Ferreira *et al.* (2009)	Pavement	1	Y	Both	A decision-aid tool	an optimal solution	Y	Developing a new decision-aid tool in the pavement management system
Ferreira *et al.* (2009)	Pavement	1	Y	NW	A deterministic segment-linked optimisation model	an optimal solution	Y	Aiming at a GIS-based pavement management system.
Frangopol & Okasha (2009)	Bridge	3	Y	P	NSGA II	Pareto solutions		Multi-criteria optimisation of life-cycle performance of systems under uncertainty.
Krueger & Garza (2009)	Pavement	1		P	Linear programming	an optimal solution	Y	Reporting on the sensitivity of the Cost/Benefit ratio in pavement management.
Li (2009)	Pavement	1	Y	NW	Solution Algorithm	an optimal solution		Addressing budget uncertainty in highway investment decision making.
Li & Madanu (2009)	Infrastructure	1	Y	NW	A LaGrangian relaxation based technique	an optimal solution		Introducing an uncertainty-based method for highway project-level life-cycle benefit/cost analysis.
Li & Sinha (2009)	Pavement	1		NW	Shackle's model	an optimal solution		Proposing a method for highway investment decision making under uncertainty.
Nafi & Kleiner (2009)	Pipe	2	Y	P	NSGA II	Pareto solutions	Y	Considering economies and harmonisation with other known infrastructure works.
Orcesi & Cremona (2009)	Bridge	1	Y	NW	Markov decision model	an optimal solution	Y	Increasing safety levels and reducing budgets for highway bridge management.
Rogers & Grigg (2009)	Pipe	1+		NW	a Multicriteria Decision Analysis Module	an optimal solution		Using data found in utilities to assist with pipe renewal decisions.
Santos *et al.* (2009)	Pavement	1+	Y	NW	Weighting method AHP	a preferred solution	Y	Helping policy makers in long-term strategic reflections of a national or regional network.
Wu & Flintsch (2009)	Pavement	2	Y	NW	Weighting method	Pareto solutions		Proposing an approach for pavement preservation programming.

Zhang (2009)	Pavement	1	Y	NW	Mean-Variance Optimal Portfolio	an optimal solution		Providing a systematic tool to study statistical properties and characteristics of risk and performance measures.
Afshar (2010)	Pipe	1	Y	NW	Continuous ACO	an optimal solution		Applying a new method to optimal design of sewer networks
Fallah-Fini (2010)	Pavement	1	Y	NW	A priority based method	an optimal solution	Y	Providing a blueprint for designing optimal highway maintenance practices.
Gao *et al.* (2010)	Pavement	2		NW	A parametric method	Pareto solutions	Y	Optimising the pavement conditions and budget utilisation.
Ge (2010)	Infrastructure	2	Y	P	PSO	a preferred solution		Studying the impact of maintenance toward system reliability and economic cost.
Golroo *et al.* (2010)	Pavement	1	Y	NW	A prioritisation algorithm	an optimal solution	Y	Optimising resource allocation based on reliability of transport infrastructure
Gopalakrishnan & Khaitan (2010)	Pavement	1		Both	ANN	an optimal solution		Searching for the optimal combination of pavement treatments.
Jang *et al.* (2010)	Pavement	1	Y	Both	An overall heuristic algorithm	an optimal solution	Y	Integrating road maintenance planning decisions.
Jbara & Amman-Jordan (2010)	Pipe	3	Y	NW	2D-3D Continuous Ant Colony Approach	a preferred solution		Solving pipe distribution network problems in 3D continuous search spaces.
Lukas *et al.* (2010)	Infrastructure	2	Y	NW	ACO	a preferred solution		Optimal planning of maintenance schedules for urban road systems.
Martí & González-Vidosa (2010)	Bridge	1	Y	NW	Annealing and Threshold Optimisation procedure	an optimal solution		Dealing with the economic optimisation of bridges in public works.
Meneses & Ferreira (2010)	Infrastructure	3	Y	NW	WSM	Pareto solutions	Y	Presenting the development and implementation of a Multi-Objective Decision-Aid Tool for Pavement Management System.
Mouratidis & Papageorgiou (2010)	Pavement	1+		NW	A prioritisation algorithm	a preferred solution		Introducing significant factors to the processing algorithm designating the optimal intervention.
Okasha & Frangopol (2010)	Bridge	3		P	GA	a preferred solution		Models reflecting the separate or combined effects of preventive maintenance.

Orcesi & Cremona (2010)	Bridge	2	Y	NW	Decision tree GA	Pareto solutions		Proposing a network-level bridge management system considering the position of bridges in the network
Orcesi & Cremona (2010)	Bridge	1+	Y	P	Markov decision model	a preferred solution	Y	Selecting maintenance strategies and keeping bridges into an acceptable level of service and safety.
Orcesi *et al.* (2010)	Bridge	1	Y	P	GA	an optimal solution		Showing a global approach when determining optimal maintenance strategies associated with several bridge limit states.
Ozbek *et al.* (2010)	Bridge	3		P	Data Envelopment Analysis	a preferred solution	Y	Providing a method dealing with issues faced during the maintenance of bridges.
Petreska & Kolemisevska-Gugulovska (2010)	Infrastructure	1+	Y	P	A fuzzy logic method Monte Carlo Simulation	a preferred solution		Managing assets against given liability and risk estimation of different portfolio structures.
Scheinberg & Anastasopoulos (2010)	Pavement	1	Y	NW	A flexible multi-year multi-constraint optimisation technique	an optimal solution	Y	Identifying a set of recommended strategies applied to a set of individual road sections
Seidl & Cypra (2010)	Pavement	1+		NW	A simulation based method	a preferred solution		Introducing a platform for the decisions of the maintenance as comprehensive decision-making support.
Shahata & Zayed (2010)	Infrastructure	1+		P	Decision tree GA	a preferred solution		Establishing a method to facilitate decision making process and ensures reliable and optimum decision.
Sharma (2010)	Infrastructure	2	Y	NW	GA AHP	a preferred solution		Developing the framework for Levels of Service based decision support systems for infrastructure network investment.
Sirvio & Hollmén (2010)	Pavement	1	Y	NW	GA Variable Neighbourhood Search	an optimal solution	Y	Presenting a problem of road maintenance programming as a large-scale optimisation problem.
Thompson *et al.* (2010)	Pavement	2	Y	NW	Weighting method Evolutionary algorithm	Pareto solutions	Y	Handling difficulties associated with sediment reduction, objectives preferences.
Weber & Allen (2010)	Pavement	1	Y	NW	A rank-based optimisation	an optimal solution		Considering a broad landscape context based on green infrastructure ranking.

Zhu & Lu (2010)	Pavement	1	Y	P	AHP	an optimal solution		Establishing a highway intersection safety maintenance evaluation system from the demand of highway safety maintenance management.
Augeri *et al.* (2011)	Pavement	1+	Y	P	Dominance-Based Rough Set Approach	a preferred solution	Y	Presenting a decision-support system for the management of highway assets.
Bocchini & Frangopol (2011)	Bridge	2	Y	NW	GA	Pareto solutions		Considering probabilistic of bridge condition when bridge maintenance.
Bocchini & Frangopol (2011)	Bridge	2	Y	NW	GA	Pareto solutions		Presenting a probabilistic computational framework for preventive maintenance of highway bridges.
Cafiso & Graziano (2011)	Pavement	1	Y	NW	Total enumeration and effective gradients method	an optimal solution		Examining cost versus safety trade-offs for a series of alternative projects.
Enevoldsen (2011)	Bridge	1	Y	NW	Probability-based assessment	an optimal solution	Y	Achieving higher load ratings for the bridges than those resulted from traditional deterministic analysis.
Ferreira *et al.* (2011)	Pavement	1	Y	P	Many methods	an optimal solution	Y	Comparing different pavement performance models
Furuta *et al.* (2011)	Infrastructure	3	Y	NW	GA	Pareto solutions	Y	Life-cycle cost design and methods with emphasis on bridges and road networks
Gao *et al.* (2011)	Pavement	1	Y	NW	Decomposition algorithm	an optimal solution	Y	Discussing road maintenance problem and the road expansion problem.
Jesus *et al.* (2011)	Pavement	1+	Y	NW	Weighting method	a preferred solution		A decision-making tool for Microsoft Office Excel.
Lee *et al.* (2011)	Bridge	2		P	NSGA II	Pareto solutions	Y	Determining optimal schedule of maintenance actions for deteriorating bridges.
Li *et al.* (2011)	Pavement	2	Y	NW	ACO	a preferred solution	Y	Determining the optimal set of alternatives for highway infrastructure.
Lin & Lin (2011)	Pavement	2	Y	NW	Utility analysis method	a preferred solution	Y	Allocating a pavement maintenance budget involving pavement quality and traffic.
Lukas & Borrmann (2011)	Pavement	1	Y	NW	ACO	an optimal solution	Y	Considering traffic flow when maintaining road networks.

McDonald & Madanat (2011)	Pavement	1	Y	NW	M-E Design Optimisation	an optimal solution		Presenting an optimisation method for mechanistic-empirical pavement maintenance.
Ng *et al.* (2011)	Pavement	1	Y	NW	Linear programming	an optimal solution	Y	Presenting a method to account for uncertainties in decision making.
Oh *et al.* (2011)	Pavement	2	Y	NW	GA	Pareto solutions	Y	Scheduling the maintenance projects for different zone.
Orabi & El-Rayes (2011)	Pavement	2	Y	NW	Multi-objective optimisation model	Pareto solutions		Presenting the development of a MOO model for planning highway rehabilitation efforts.
Orabi & El-Rayes (2011)	Pavement	2	Y	NW	NSGA II	Pareto solutions		Planning and optimising aging highway rehabilitation programs.
Orcesi & Frangopol (2011)	Bridge	3	Y	P	NSGA II	Pareto solutions	Y	Considering multiple criteria optimisation of bridge maintenance strategies.
Orcesi & Frangopol (2011)	Bridge	2	Y	P	NSGA II	Pareto solutions	Y	Developing models both in space and time by using non-destructive testing methods.
Orcesi & Frangopol (2011)	Bridge	3	Y	P	NSGA II	Pareto solutions	Y	Considering uncertainties in deterioration, assessment and maintenance processes.
Qin *et al.* (2011)	Pavement	1		P	An analysis method	an optimal solution		Analysing road-region ecosystem and its environmental features.
Santos & Ferreira (2011)	Pavement	1	Y	NW	GA	an optimal solution	Y	Considering performance, costs, the residual value and preventive maintenance and rehabilitation interventions.
Stewart (2011)	Infrastructure	2		P	A decision support analysis	a preferred solution		Discussing risk and measurement in decision support analysis.
Tee & Li (2011)	Infrastructure	1	Y	P	An overall computational procedure	an optimal solution		Developing a maintenance strategy based on risk-cost optimisation during the whole life.
Xue-zhen *et al.* (2011)	Pavement	1+		Both	Road Performance Assessment Optimisation Module	a preferred solution		Selecting protective maintenance modules for road.
Yang & Kumaraswamy (2011)	Bridge	1	Y	P	A systematic approach	an optimal solution	Y	Presenting approaches towards improving infrastructure maintenance strategies, models and practices.

Adey (2012)	Pavement	1	Y	P	An enumeration based method	an optimal solution	Y	Evaluating the total benefits of road preservation interventions
Bocchini & Frangopol (2012)	Bridge	2	Y	P	GA	Pareto solutions	Y	Optimal resilience- and cost-based prioritisation of interventions on highway bridges.
Chou & Wang (2012)	Pavement	1	Y	NW	CPLEX 12.1	an optimal solution		Determining the best network condition state achievable with a given budget.
Essahli & Madanat (2012)	Bridge	1	Y	NW	A priority method	an optimal solution		Prioritising the maintenance of bridges most significant to the functionality of the entire network.
Farran & Zayed (2012)	Infrastructure	1	Y	P	GA	an optimal solution		Providing a broader infrastructure management system and capital budgeting problems.
Fwa & Farhan (2012)	Pavement	1-2	Y	NW	GA	Pareto solutions	Y	Allocating budget to multiple types of infrastructure networks, under individual and overall objectives
Han *et al.* (2012)	Electricity	2	Y	P	WSM	Pareto solutions		Presenting a model in planning sustainable electricity infrastructure under uncertainty.
Ibrahim *et al.* (2012)	Pavement	3	Y	P	Weighting method	a preferred solution		Applying reliability analysis for optimising the safety of highway cross-sections.
Jorge & Ferreira (2012)	Pavement	1	Y	P	GENEPAV-HDM4	an optimal solution	Y	Developing a model to integrate the pavement management system
Lertworawanich (2012)	Pavement	2		NW	PSO	a preferred solution	Y	Presenting the sequential highway network restoration decision model when budgets and resources are unknown.
Maier *et al.* (2012)	Bridge	2	Y	NW	A holistic integrated approach	a preferred solution		Evaluating steel-composite bridges with a complete design of realistic case studies.
Medury & Madanat (2012)	Pavement	2	Y	NW	Bottom-up method	Pareto solutions		Discussing the impact of network-based constraints on the decision-making process.
Ward & Savic (2012)	Pipe	3		NW	GA	a preferred solution	Y	Exploring MOO tools to assist engineers with the specification of optimal rehabilitation
Xie (2012)	Pavement	1	Y	NW	Linear programming	an optimal solution	Y	Discussing safety and efficiency of rural highway.

Yadollahi & Zin (2012)	Pavement	1+	Y	NW	GA	a preferred solution		Presenting a multi-strategy decision support system for rehabilitation budget allocation.
Zhang & Gao (2012)	Pavement	1	Y	NW	Semi-Markov process	a preferred solution		Obtaining an optimal maintenance policy for deteriorating roads.
Almeida *et al.* (2013)	Pavement	2	Y	NW	GA	Pareto solutions		Discussing different Degradation model and costs for long- and medium- term analysis.
Amador-Jiménez & Afghari (2013)	Pavement	M2	Y	NW	A large-scale decision tree	Pareto solutions	Y	Trading off between safety and condition.
Barone *et al.* (2013)	Bridge	2	Y	NW	Decision tree GA	Pareto solutions	Y	Developing a life-cycle optimisation technique to manage bridge systems considering uncertainties and several budgetary and safety constraints.
Gao & Zhang (2013)	Pavement	2	Y	NW	WSM	Pareto solutions	Y	Developing a multi-objective Markov-based model.
Gao *et al.* (2013)	Infrastructure	1	Y	NW	Augmented Lagrangian decomposition approach	an optimal solution	Y	Scheduling maintenance and rehabilitation.
Kleiner (2013)	Infrastructure	1	Y	NW	GA	an optimal solution		Optimising the maintenance of water mains under paved deterioration roads.
Lambert *et al.* (2013)	Pavement	1+		P	Weighting method	a preferred solution	Y	Extending a scenario-based multi-criteria decision framework that assists in allocating limited resources.
Liang & Wey (2013)	Pavement	1+	Y	NW	WSM	Pareto solutions		Evaluating and prioritising project implementation difficulty levels based on the resource.
Marzouk & Omar (2013)	Pipe	2	Y	P	GA	a preferred solution	Y	Life-cycle maintenance planning of deteriorating sewer network.
Meneses & Ferreira (2013)	Pavement	2	Y	NW	Normal-Boundary Intersection method	Pareto solutions	Y	Developing a Multi-Objective Decision-Aid Tool in Pavement Management System.
Santos & Ferreira (2013)	Pavement	1	Y	P	OPTIPAV system	an optimal solution	Y	Choosing the best pavement structure for a road or a highway.

Adey *et al.* (2014)	Pavement	1	Y	P	MINLP model	an optimal solution	Y	Investigating optimisation models to determine the optimal intervention strategy for a road link.
Anastasopoulos *et al.* (2014)	Pavement	1	Y	NW	Decision tree	an optimal solution		Investigating public and private partnerships in pavement preservation programming.
Charmpis *et al.* (2014)	Bridge	1		NW	B&B	an optimal solution	Y	Decision making on a large-scale realistic database from the highway system
Chien & Tang (2014)	Pavement	1		NW	GA	an optimal solution		Considering benefits of responsible agencies, contractors and roadway users.
Galenko *et al.* (2014)	Pavement	1		NW	A single SOS Integer Programming	an optimal solution	Y	Introducing a state-of-the-art planning tool for managing highway network.
Khan *et al.* (2014)	Pavement	2	Y	NW	A probabilistic road deterioration model	a preferred solution	Y	Highlighting existing pavement management systems in Australia.
Lethanh *et al.* (2014)	Infrastructure	1	Y	NW	GA	a preferred solution		Determining optimal interventions strategies under different deterioration process.
Liu & Madanat (2014)	Bridge	1	Y	NW	Open-loop feedback control	an optimal solution		Improving maintenance, rehabilitation and reconstruction decisions and reducing system costs.
Lwambuka & Mtenga (2014)	Bridge	3		NW	Weighting method	a preferred solution	Y	Presenting a practical approach for prioritisation of bridge maintenance within a given bridge network.
Rashedi & Hegazy (2014)	Infrastructure	1	Y	NW	Mathematical optimisation tools	an optimal solution		Allocating limited renewal funds among numerous asset components.
Zhang & Wang (2014)	Infrastructure	1	Y	P	Simulation Algorithm	an optimal solution		Using subjective expert knowledge and information gathered for only a small sample of assets.
Chen *et al.* (2015)	Pavement	2	Y	NW	DA	Pareto solutions		Developing a MOO method for decision making of pavement management.
Fallah-Fini *et al.* (2015)	Pavement	1+	Y	P	System Dynamics technique	an optimal solution	Y	Evaluating the performance of highway maintenance policies.
Famurewa *et al.* (2015)	Railway	1	Y	P	FORTRAN	a preferred solution		Optimising a schedule in railway infrastructure management.

Farran & Zayed (2015)	Infrastructure	2	Y	P	GA	a preferred solution		Providing decision-makers a set of optimal rehabilitation trade-offs over a desired analysis period.
Jovanovic & Bozovic (2015)	Railway	1	Y	P	Condition-based approach	an optimal solution	Y	Describing the optimal structure and manner of implementing a railway maintenance management system.
Osman (2015)	Infrastructure	1+		P	Goal programming	a preferred solution	Y	Enabling temporal coordination of water, sewer and road intervention activities.
Yepes *et al.* (2015)	Bridge	2	Y	P	Glow worm swarm optimisation algorithm	an optimal solution		Optimising cost and CO2 emission for road bridges

Note: [1] "Y" means this decision making problem has a constraint or constraints on outcomes;
[2] "P" means this decision making problem is project-level and "N" means this decision making problem is network-level; and
[3] "Y" means this decision making problem applies a practical case.

APPENDIX B: OPTIMISATION RESULTS OF THE TESTS OF BI-OBJECTIVE OPTIMISATION (CITY A)

This appendix demonstrates the optimisation results of the tests of the decision making of City A. A summary of these tests is shown in Table B.1; and the optimisation results are shown in the following figures and tables. It is important to note that the Pareto frontier in this table is a large set of discrete but closely located Pareto solutions.

Table B.1 Summary of the tests of bi-objective optimisation (City A)

Test index	Number of segments	Number of strategies	Annual budget (million)	Acceptable average condition index
A1	100	3,718	5	3
A2	1,000	33,300	50	3
A3	1,822	61,936	80	3

In particular, heuristics may generate different solutions when solving a same problem with multiple times; hence, they are applied thirty times, and the tables are based on their average performance. The MOO techniques are measured based on the adopted implementation and the measurement framework introduced in Section 5.4, where a score of 0 (10) represents the worst (best) criterion value achieved by the tested techniques. Because the exact methods and the heuristics solve a MOO problem in different manners, their computation time is differently measured (time per solution for exact methods and time per iteration for heuristics). Accordingly, their computation time is separately scored. The implementation is scored based on the author's viewpoint.

B.1 Optimisation result of Test A1

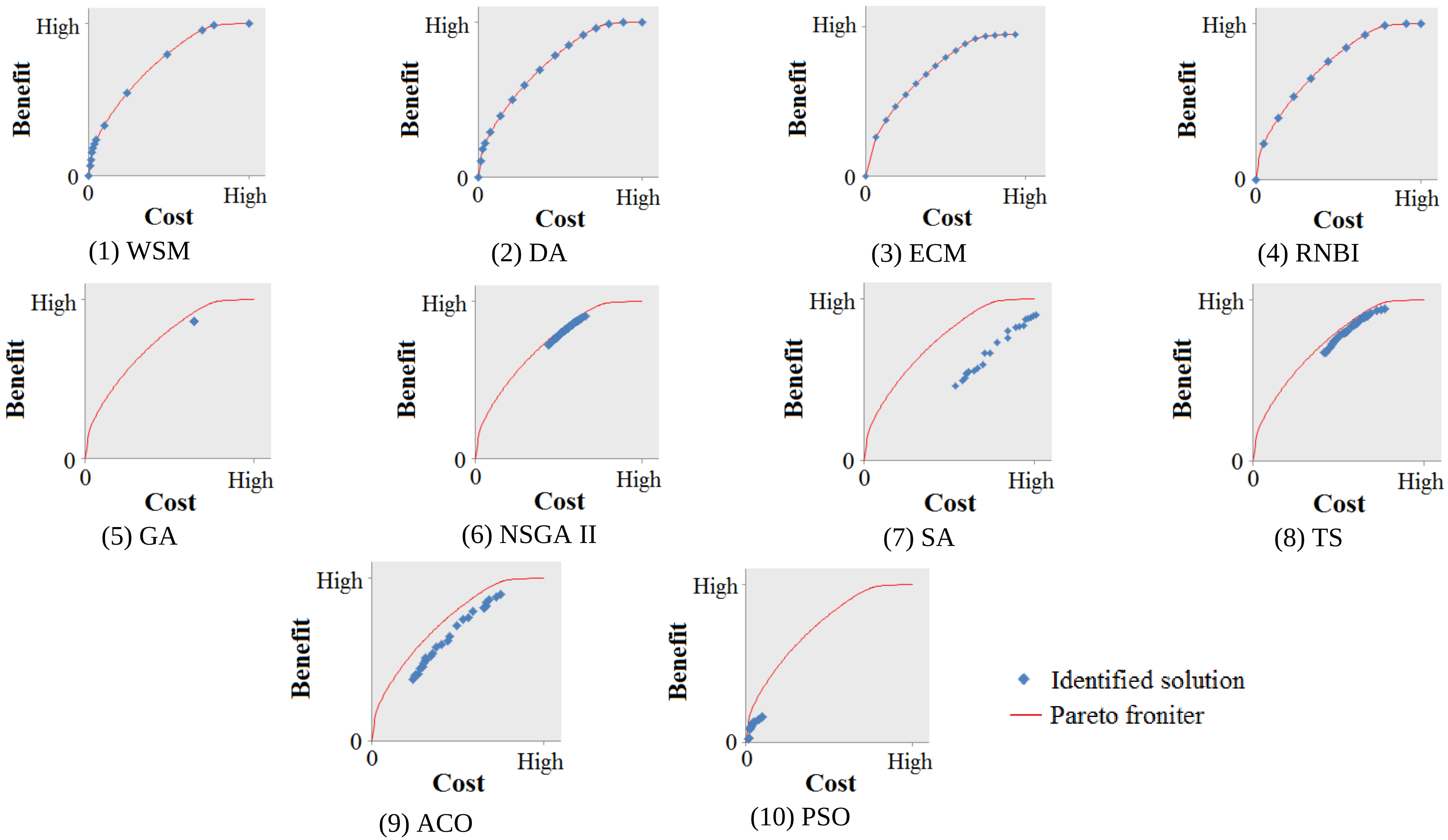


Figure B.1 Identified solutions in Test A1

Table B.2 Criterion values in Test A1

Algorithm	**Aspects and Criteria**											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	13.00	100	0	0.1547	0.0460	0.0862	7.86	0.56	3.3	Most	1 main parameter	N
DA	16.00	100	0	0.0603	0.4527	0.0326	7.70	0.45	0	Most	None	N
ECM	16.00	100	0	0.1292	0.0394	0.0606	12.46	0.78	3.3	Most	1 main parameter	N
RNBI	12.00	100	0	0.1068	0.0267	0.0681	13.23	1.10	3.3	Most	1 main parameter	N
GA	10.70	0	0.0451	1.0734	0.6434	0.0001	46.80	4.37	10	Some	2 main parameter	Y
NSGA II	116.30	0	0.0036	0.8617	0.0564	0.0024	113.32	0.97	10	Some	2 main parameter	Y

Table B.2 (Continuous)

Algorithm	**Aspects and Criteria**											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution /iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
SA	20.60	0	0.1874	0.6956	0.2483	0.0207	153.56	7.45	10	Some	1 main parameter	Y
TS	63.70	0	0.0215	0.7113	0.1033	0.0061	534.93	8.40	10	Some	4 main parameter	Y
ACO	28.60	0	0.1639	0.4133	0.2205	0.01381	532.68	18.63	10	Some	4 main parameter	Y
PSO	16.80	0	0.0039	1.0509	0.2162	0.0075	174.68	4.76	10	Some	2 main parameter	Y

Note: “Most” means most common perspectives can be easily achieved;
“Some” means only some common perspectives can be easily achieved;
“N” means this algorithm does not need pre-defined stopping criterion; and
“Y” means this algorithm needs at least one pre-defined stopping criterion.

Table B.3 Criterion scores in Test A1

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	8.67	10	10	**9.56**	9.07	9.69	10	**9.59**	9.71	8.33	**9.02**	3.3	10	7.5	10	**7.70**
DA	10	10	10	**10**	10	3.09	3.78	**5.62**	10	10	**10.00**	0	10	10	10	**7.50**
ECM	10	10	10	**10**	9.32	9.79	7.03	**8.72**	1.40	4.99	**3.19**	3.3	10	7.5	10	**7.70**
RNBI	8.00	10	10	**9.33**	9.54	10	7.91	**9.15**	0	0	**0.00**	3.3	10	7.5	10	**7.70**
GA	0	0	7.59	**2.53**	0	10	10	**6.67**	10	7.80	**8.90**	10	5	5	5	**6.25**
NSGA II	10	0	9.81	**6.60**	2.07	0.48	9.72	**4.09**	8.64	9.66	**9.15**	10	5	5	5	**6.25**

Table B.3 (Continuous)

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
SA	0.94	0	0	**0.31**	3.73	3.59	7.60	**4.98**	7.81	6.11	**6.96**	10	5	7.5	5	**6.88**
TS	5.02	0	2.89	**4.62**	3.57	1.24	9.30	**4.71**	0	5.60	**2.80**	10	5	2.5	5	**5.63**
ACO	1.70	0	1.26	**0.98**	6.52	3.14	8.40	**6.02**	0.05	0	**0.02**	10	5	0	5	**5.00**
PSO	0.58	0.8	9.53	**1.94**	0.22	3.07	9.14	**4.14**	7.38	10	**8.69**	10	5	5	5	**6.25**

B.2 Optimisation result in Test A2

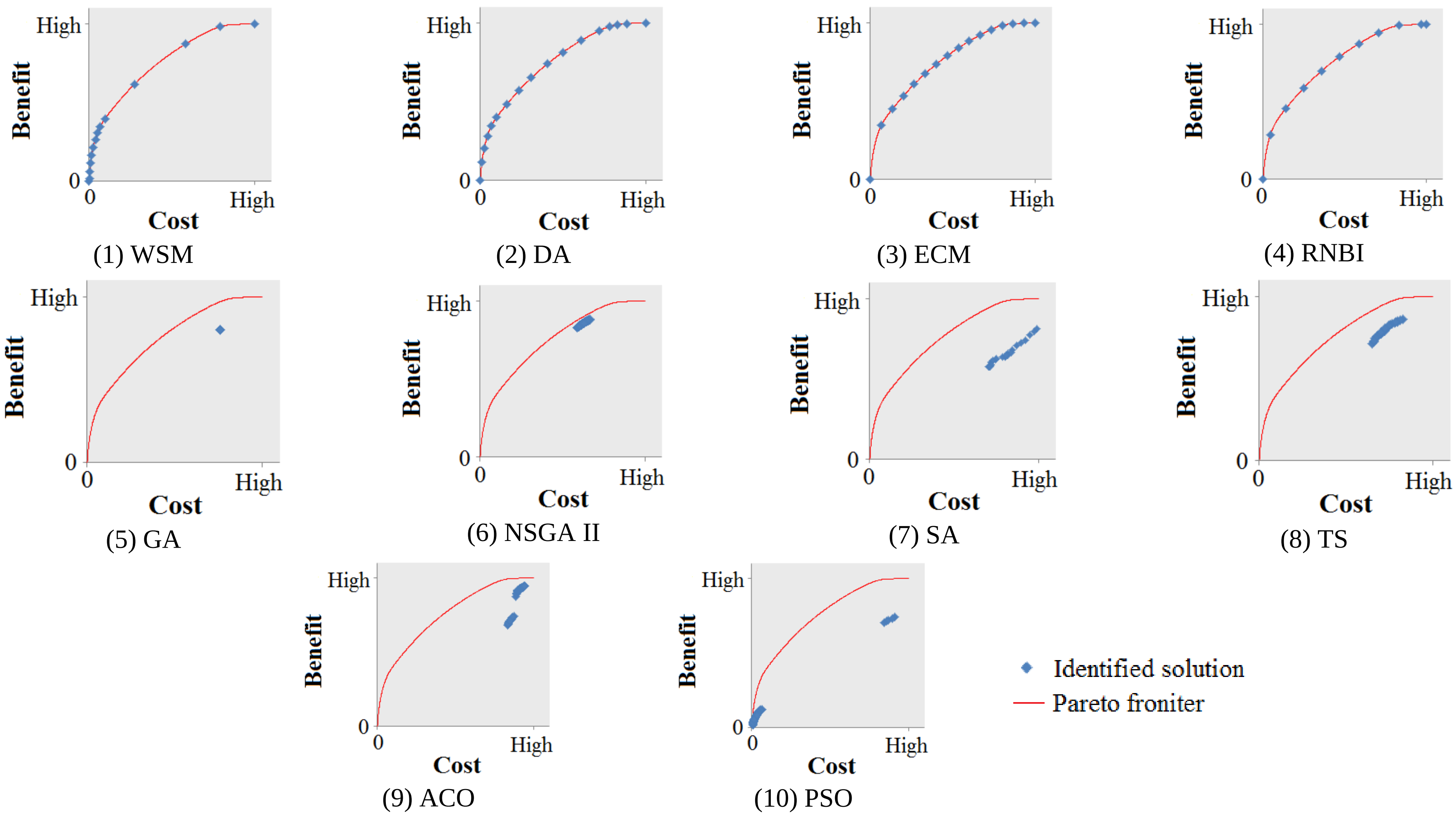


Figure B.2 Identified solutions in Test A2

Table B.4 Criterion values in Test A2

Algorithm	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	14	100	0	0.1828	0.1898	0.0925	76.89	5.49	3.3	Most	1 main parameter	N
DA	16	100	0	0.0661	0.5395	0.0276	75.06	4.42	0	Most	None	N
ECM	16	100	0	0.1628	0.6621	0.0690	85.39	5.34	3.3	Most	1 main parameter	N
RNBI	11	100	0	0.1268	0.2194	0.0708	114.32	10.39	3.3	Most	1 main parameter	N
GA	17.2	0	0.1548	1.1041	0.6901	0.0001	2330.43	2.91	10	Some	2 main parameter	Y
NSGA II	175.8	0	0.0362	1.0140	0.0809	0.0005	3473.16	4.34	10	Some	2 main parameter	Y

Table B.4 (Continuous)

Algorithm	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
SA	23.9	0	0.2889	0.9096	0.0545	0.0101	9321.47	11.65	10	Some	1 main parameter	Y
TS	27.1	0	0.1395	0.9654	0.1805	0.0050	30289.33	37.86	10	Some	4 main parameter	Y
ACO	102.6	0	0.2030	1.0780	0.0084	0.0026	30310.21	37.89	10	Some	4 main parameter	Y
PSO	104.9	0	0.0345	0.5550	0.0336	0.0035	4586.58	5.73	10	Some	2 main parameter	Y

Note: "Most" means most common perspectives can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

Table B.5 Criterion scores in Test A2

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	8.7	10	10	**9.58**	8.88	2.66	0.00	**3.85**	9.53	8.20	**8.87**	3.3	10	7.5	10	**7.70**
DA	10	10	10	**10.00**	10.00	7.79	7.01	**8.27**	10.00	10.00	**10.00**	0	10	10	10	**7.50**
ECM	10	10	10	**10.00**	9.07	9.59	2.54	**7.07**	7.37	8.46	**7.91**	3.3	10	7.5	10	**7.70**
RNBI	6.8	10	10	**8.96**	9.41	3.10	2.34	**4.95**	0	0	**0.00**	3.3	10	7.5	10	**7.70**
GA	0	0	4.64	**1.55**	0	10	10	**6.67**	10	10	**10.00**	10	5	5	5	**6.25**
NSGA II	10	0	8.75	**6.25**	0.87	1.06	9.95	**3.96**	9.18	9.59	**9.39**	10	5	5	5	**6.25**

Table B.5 (Continuous)

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
SA	0.37	0	0	**0.13**	1.87	0.68	8.90	**3.82**	8.33	7.50	**7.92**	10	5	7.5	5	**6.88**
TS	0.63	0	5.17	**1.94**	1.34	2.53	9.46	**4.44**	0.84	0.01	**0.42**	10	5	2.5	5	**5.63**
ACO	5.37	0	2.97	**2.78**	0.25	0.00	9.72	**3.32**	0.00	0.00	**0.00**	10	5	0	5	**5.00**
PSO	5.50	0	8.80	**4.77**	5.29	0.37	9.62	**5.09**	8.39	9.19	**8.79**	10	5	5	5	**6.25**

B.3 Optimisation result in Test A3

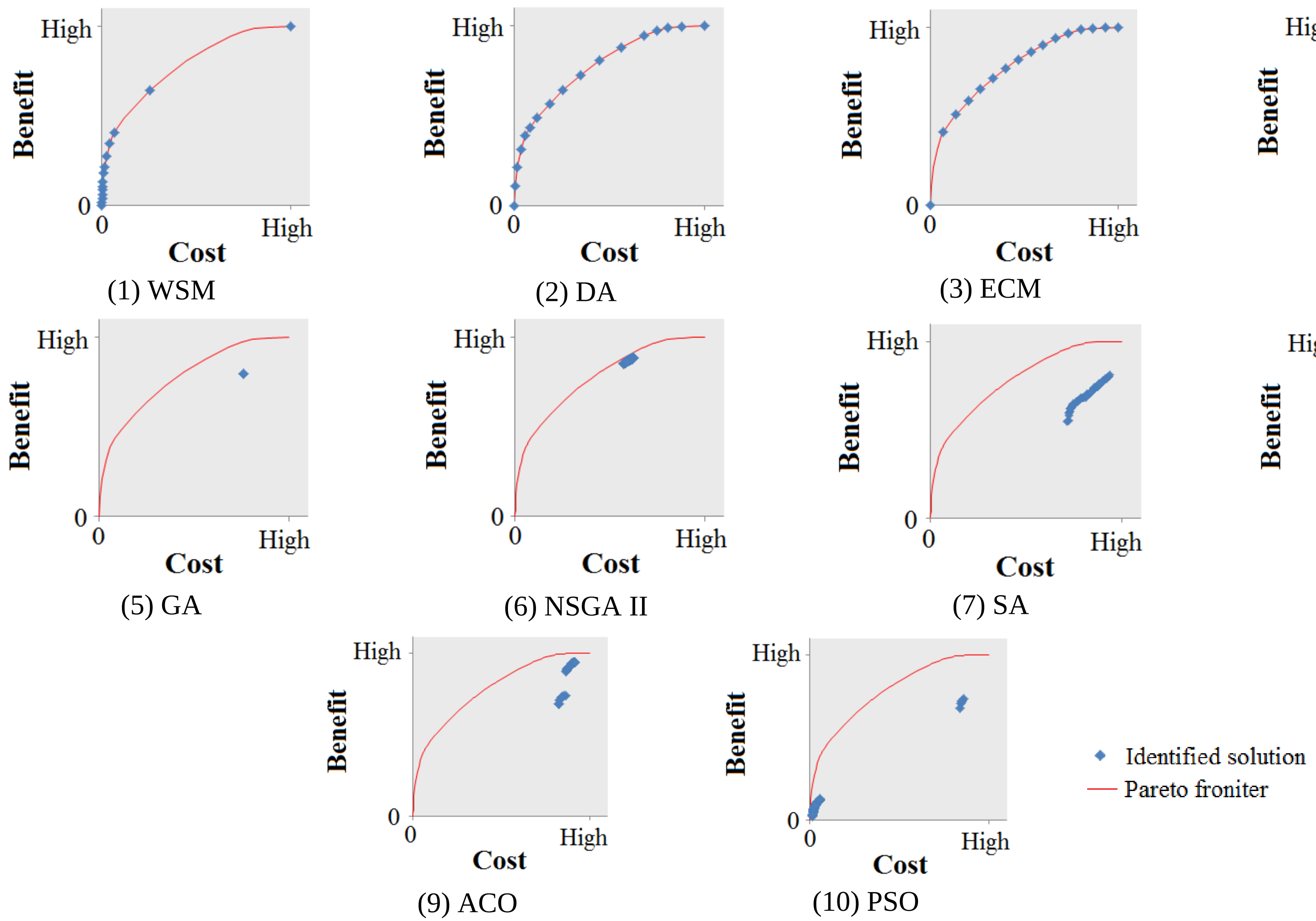


Figure B.3 Identified solutions of Test A3

Table B.6 Criterion values in Test A3

Algorithm	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	14	100	0	0.4114	0.1296	0.2194	127.44	9.10	3.3	Most	1 main parameter	N
DA	17	100	0	0.0596	0.5847	0.0284	139.94	8.23	0	Most	None	N
ECM	16	100	0	0.2055	0.6511	0.0850	174.18	10.89	3.3	Most	1 main parameter	N
RNBI	10	100	0	0.2127	0.0339	0.1279	216.25	21.63	3.3	Most	1 main parameter	N
GA	17.2	0	0.1718	1.1001	0.4745	0.0000	11762.88	11.76	10	Some	2 main parameter	Y
NSGA II	239.1	0	0.0316	1.0315	0.0226	0.0001	15443.02	15.44	10	Some	2 main parameter	Y

Table B.6 (Continuous)

Algorithm	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
SA	127.1	0	0.2787	0.9016	0.0851	0.0019	33237.61	33.24	10	Some	1 main parameter	Y
TS	297.8	0	0.1335	0.9589	0.0533	0.0004	108711.93	108.71	10	Some	4 main parameter	Y
ACO	101	0	1.988	1.0721	0.0098	0.0021	91775.59	91.78	10	Some	4 main parameter	Y
PSO	58.4	0	0.0522	0.5673	0.0421	0.0044	16567.70	16.57	10	Some	2 main parameter	Y

Note: "Most" means most common perspectives can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

Table B.7 Criterion scores in Test A3

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	8.75	10	10	**9.58**	6.62	1.87	0	**2.83**	10	9.35	**9.67**	3.3	10	7.5	6.01	**6.45**
DA	10	10	10	**10.00**	10	8.96	8.71	**9.23**	8.59	10	**9.30**	0	10	10	7.50	**6.25**
ECM	10	10	10	**10.00**	8.60	10	6.13	**8.24**	4.74	8.02	**6.38**	3.3	10	7.5	6.01	**6.45**
RNBI	6.25	10	10	**8.75**	8.53	0.38	4.17	**4.36**	0	0	**0.00**	3.3	10	7.5	6.01	**6.45**
GA	0	0	3.84	**1.28**	0	7.24	10	**5.75**	10	10	**10.00**	10	5	5	5	**6.25**
NSGA II	7.90	0	8.87	**5.59**	0.66	0.20	10	**3.62**	9.62	9.62	**9.62**	10	5	5	5	**6.25**

Table B.7 (Continuous)

	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
Algorithm	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
SA	3.91	0	0	**1.30**	1.91	1.17	9.92	**4.33**	7.78	7.78	**7.78**	10	5	0.75	5	**5.19**
TS	10	0	5.21	**5.07**	1.36	0.68	9.98	**4.01**	0	0	**0.00**	10	5	0.25	5	**5.06**
ACO	2.98	0	2.87	**1.95**	0.27	0	9.91	**3.39**	1.75	1.75	**1.75**	10	5	0	5	**5.00**
PSO	1.45	0	8.13	**3.19**	5.12	0.50	9.80	**5.14**	9.50	9.50	**9.50**	10	5	0.50	5	**5.13**

APPENDIX C: OPTIMISATION RESULTS OF THE TESTS OF THREE-OBJECTIVE OPTIMISATION (CITY B)

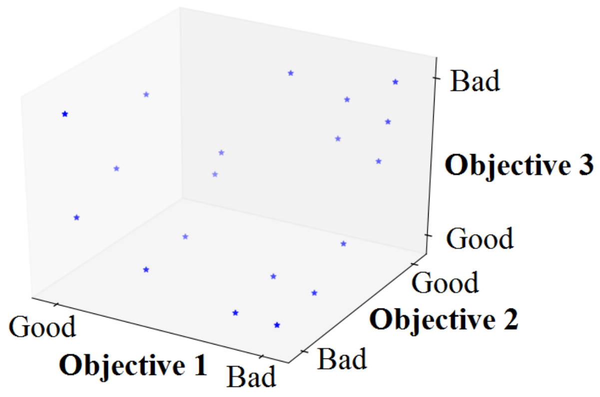
This appendix demonstrates the optimisation results of the tests of the decision making of City B. A summary of these tests is shown in Table C.1; and the optimisation results are shown in the following figures and tables.

Table C.1 Summary of the tests of three-objective optimisation (City B)

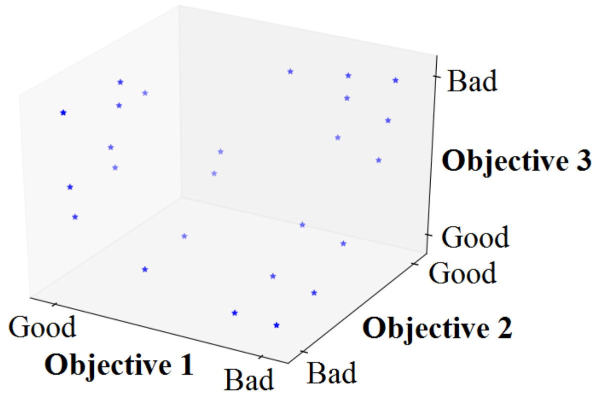
Test Index	Number of Segments (sub-network 1/ sub-network 2/ sub-network 3)	Number of Strategies (sub-network 1/ sub-network 2/ sub-network 3)	Budget (million)
B1	100 (20 / 40 / 40)	4,127 (1,025 / 1,213 / 1,889)	1
B2	600 (200 / 200 / 200)	30,359 (7,232 / 13,419 / 9,708)	4
B3	1,301 (400 / 400 / 501)	72,562 (20,651 / 19,519 / 32,392)	8

None of the heuristics obtains feasible solutions in these test; therefore, heuristics are not discussed. The exact methods are measured based on the adopted implementation and the measurement framework introduced in Section 5.4, where a score of 0 (10) represents the worst (best) criterion value achieved by the tested exact methods. In Test B3, the set of all the Pareto solutions is hard to be obtained. Hence, different criteria are selected within the measurement framework. The implementation is measured based on the author’s viewpoint.

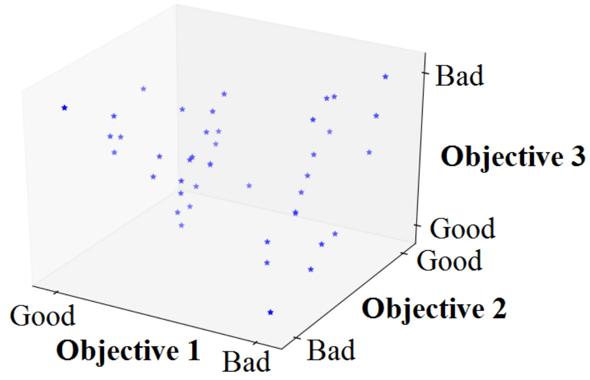
C.1 Optimisation result in Test B1



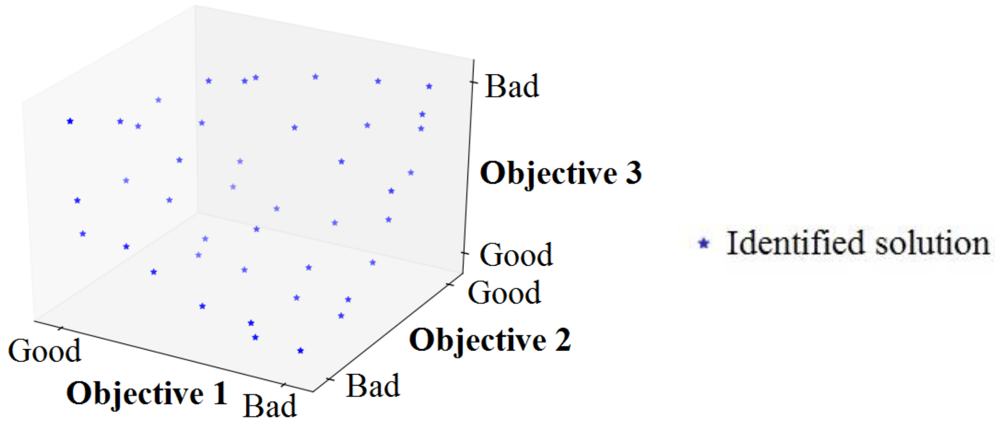
(1) WSM



(2) DA



(3) ECM



(4) RNBI

Figure C.1 Identified solutions in Test B1

Table C.2 Criterion values in Test B1

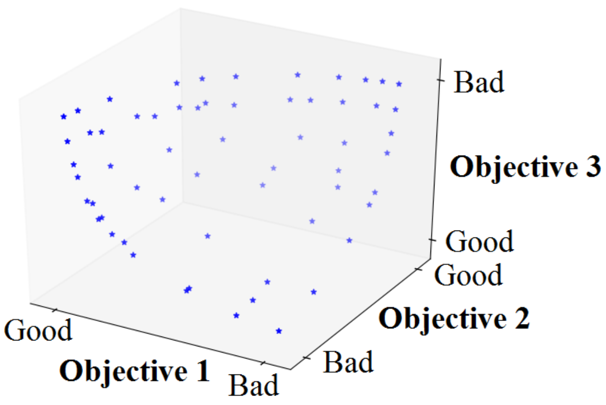
Algorithm	Aspects and Criteria											
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	19	100	0	0.3182	0.4617	0.1096	8.61	0.26	3.33	Some	1 main parameter	N
DA	25	100	0	0.3182	0.6227	0.0701	62.01	5.48	0	Most	None	N
ECM	43	100	0	0.3756	0.0703	0.0843	9.36	0.16	3.33	Most	1 main parameter	N
RNBI	45	100	0	0.1984	0.3588	0.0610	21.53	0.48	3.33	Most	1 main parameter	N

Note: "Most" means most common perspectives can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

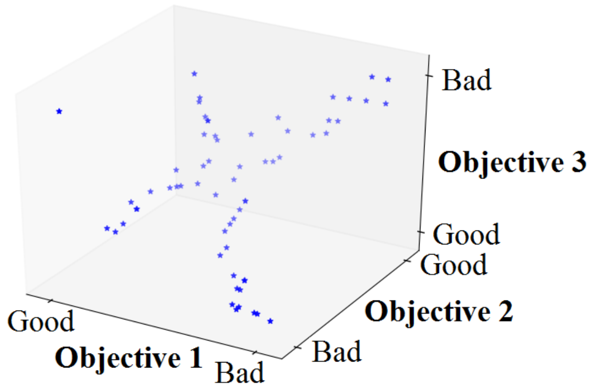
Table C.3 Criterion scores in Test B1

	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
Algorithm	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	3.17	10	10	**7.72**	3.24	7.08	0	**3.44**	10	9.40	**9.70**	3.33	5	5	10	**5.83**
DA	4.17	10	10	**8.06**	3.24	10	8.11	**7.12**	0	0	**0.00**	0	10	10	10	**7.50**
ECM	7.17	10	10	**9.06**	0	0	5.21	**1.74**	9.86	10	**9.93**	3.33	10	5	10	**7.08**
RNBI	7.50	10	10	**9.17**	10	5.2	10	**8.40**	7.58	8.62	**8.10**	3.33	10	5	10	**7.08**

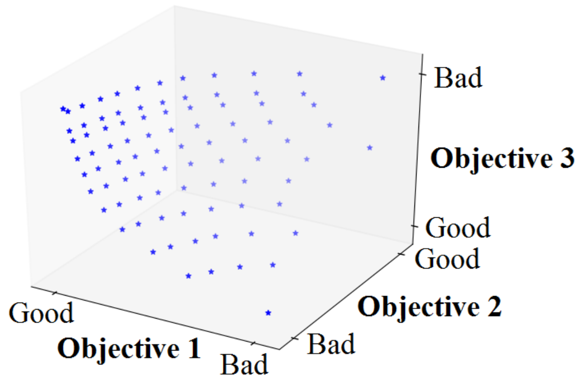
C.2 Optimisation result in Test B2



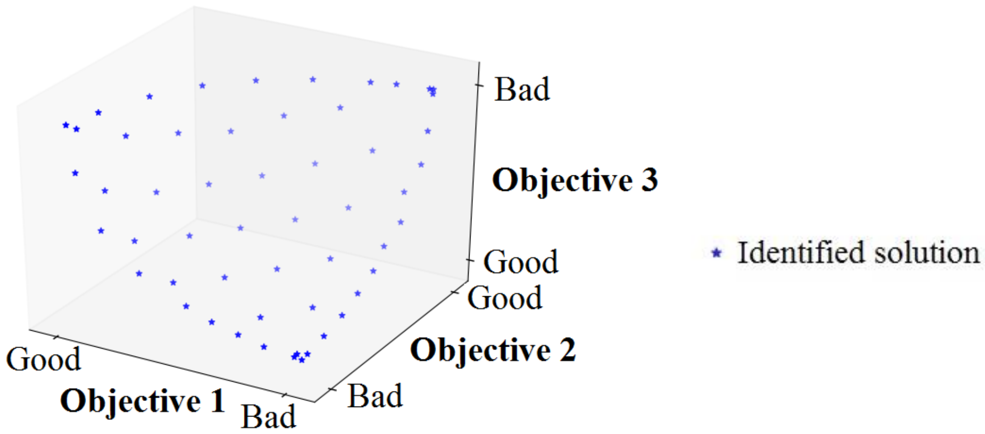
(1) WSM



(2) DA



(3) ECM



(4) RNBI

Figure C.2 Identified solutions in Test B2

Table C.4 Criterion values in Test B2

Algorithm	Aspects and Criteria												
	Solution Quality			Solution Distribution			Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Coverage error	Uniformity level ratio	Spacing	Running time (second)	Time per solution (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	60	100	0	0.2075	0.1912	0.0548	63.66	1.06	3.33	Some	1 main parameter	N
DA	62	100	0	0.4459	0.0058	0.0848	226.43	3.65	0	Most	None	N
ECM	87	100	0	0.2662	0.3084	0.0497	114.38	1.31	3.33	Most	1 main parameter	N
RNBI	54	100	0	0.1515	0.1027	0.0636	181.49	3.36	3.33	Most	1 main parameter	N

Note: "Most" means most common perspectives can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

Table C.5 Criterion scores in Test B2

Algorithm	Aspects and Criteria																
	Solution Quality				Solution Distribution				Computation Time Measures			Implementation Considerations					
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Coverage error	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	4.41	10	10	**8.14**	8.10	6.13	8.55	**7.59**	10	10	**10**	3.33	5	5	10	**5.83**
DA	4.56	10	10	**8.19**	0	0	0	**0**	0	0	**0**	0	10	10	10	**7.50**
ECM	6.40	10	10	**8.80**	6.10	10	10	**8.7**	6.88	9.02	**7.95**	3.33	10	5	10	**7.08**
RNBI	3.97	10	10	**7.99**	10	3.20	6.05	**6.42**	2.76	1.12	**1.94**	3.33	10	5	10	**7.08**

C.3 Optimisation result in Test B3

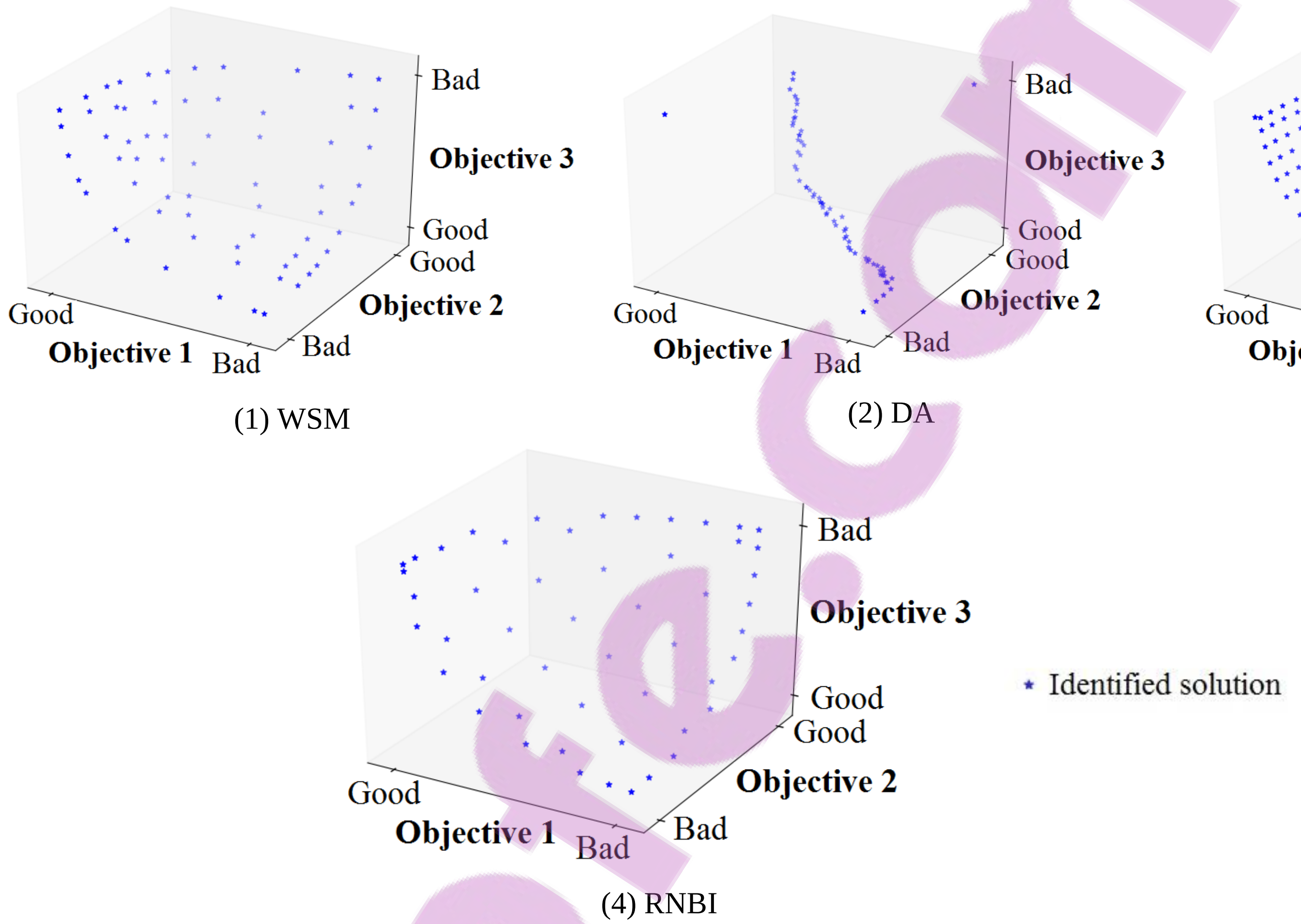


Figure C.3 Identified solutions in Test B3

Table C.6 Criterion values in Test B3

Algorithm	Aspects and Criteria										
	Solution Quality			Solution Distribution		Computation Time Measures		Implementation Considerations			
	Number of solutions	Percentage of Pareto solutions	Distance	Uniformity level ratio	Spacing	Running time (second)	Time per solution/iteration (second)	Ease of implementation	Expect result	Parameter calibration	Stopping criteria
WSM	66	100	0	0.4170	0.0374	195.20	2.95	3.33	Some	1 main parameter	N
DA	67	100	0	0.0027	0.1317	629.28	9.39	0	Most	None	N
ECM	94	100	0	0.3939	0.0567	433.99	4.62	3.33	Most	1 main parameter	N
RNBI	51	100	0	0.2365	0.0561	650.19	12.75	3.33	Most	1 main parameter	N

Note: "Most" means most common perspectives can be easily achieved;
"Some" means only some common perspectives can be easily achieved;
"N" means this algorithm does not need pre-defined stopping criterion; and
"Y" means this algorithm needs at least one pre-defined stopping criterion.

Table C.7 Criterion scores in Test B3

Algorithm	**Aspects and Criteria**														
	Solution Quality				Solution Distribution			Computation Time Measures			Implementation Considerations				
	Number of solutions	Percentage of Pareto solutions	Distance	**Score of solution quality**	Uniformity level ratio	Spacing	**Score of solution distribution**	Running time	Time per solution/iteration	**Score of computation time**	Ease of implementation	Expect result	Parameter calibration	Stopping criteria	**Score of implementation**
WSM	4.85	10	10	**8.28**	10	10	**10**	10	10	**10**	3.33	5	5	10	**5.83**
DA	4.93	10	10	**8.31**	0	0	**0**	0.46	3.43	**1.95**	0	10	10	10	**7.50**
ECM	6.91	10	10	**8.97**	9.44	7.96	**8.7**	4.75	8.30	**6.53**	3.33	10	5	10	**7.08**
RNBI	3.75	10	10	**7.92**	5.64	8.02	**6.83**	0	0	**0**	3.33	10	5	10	**7.08**

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