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INVESTIGATING THE INFLUENCE OF PRE-CALCULUS MATHEMATICS REFRESHMENT MODULE TO FIRST YEAR ENGINEERING STUDENTS IN AN ETHIOPIAN UNIVERSITIES

Student Number: 55781888

Student: Asnake Muluye Bekele

Degree: Doctor of Philosophy in the Subject Mathematics Education in Science and Technology

College of Science, Engineering and Technology South Africa UNISA

Supervisor: Prof Nosisi N Feza

ABSTRACT

The quality of mathematics knowledge attained by students entering university in Science, Technology, Engineering and Mathematics (STEM) fields has been decreasing. There is a need to enhance students' mathematical knowledge in order to maintain the standards of STEM curriculum at university. The rationale of this study was to investigate the influence of Pre-Calculus Mathematics Refreshment module taught using Meta-cognitive skills and Co-operative Learning (MCL), or Co-operative Learning (CL) only, or Traditional lecture (T) intervention method to First Year pre-engineering Students on their Applied Calculus 1 in an Ethiopian university. The study further investigated the influence of Pre-Calculus Mathematics Refreshment module for MCL, or CL, or T intervention method on male and female students' achievement. The refreshment module and Applied Calculus 1 scores were measured through posttest and normal class room score of Applied Calculus 1 result. The dependent variables were student achievement in pre-calculus refreshment Module and Applied Calculus 1. Out of 29 universities in Ethiopia only four were selected to participate in this study. Population of this study was all pre-engineering first year students in those universities in 2016/2017. The sample consisted of 200 pre-engineering university students who studied in four of Ethiopian universities and one class was randomly selected by lottery method from existing pre-engineering classes in each university. Two experimental groups which were taught MCL and the other CL intervention method and two of them were control groups upon whom the control novice with traditional lecture method and control without intervention was applied. In each group 50 students of 25 males and 25 females were purposely selected from sampled class. A pre-calculus mathematics Pre-test was administered first, where the average scores of all students Pre-test result was below 33%. Then, first MCL and CL intervention methods were discussed and exercised for one week before implementing the study. For the study, selected pre-calculus mathematics topics was taught in all classrooms for 32 periods i.e. 50min x32= 26.7hrs at the beginning of the first semester parallel with Applied Calculus 1 for the academic year 2016 / 2017.

The statistical tools used under this procedure include descriptive statistics percentage, mean and standard deviation and inferential statistics, T-test, and one-way analysis of variance (one-way ANOVA). The results show statistically significant differences (Sig 0.00) at the significance level (0.05) between students that learnt pre-calculus refreshment module and control group which did not. Among the students those learned pre-calculus refreshment module through MCL, CL and T method students in the MCL and CL groups' posttest scores significantly different from T group in pre-calculus results both with Sig of 0.00. But there was no significant difference between MCL & CL groups were Sig is 0.97. Additionally, the female students in the MCL group was not significant different from CL and T group, on an impact of refreshment module, in Applied Calculus 1 mathematics where Sig is 0.994 and 0.237 respectively, and CL female group scores significantly different from T group in Applied Calculus 1 results with Sig 0.042. The male students in the MCL and CL groups were significantly different from T group in Applied Calculus 1with Sig of 0.07 and

0.012 respectively. Also, there was a positive correlation between Pre-Calculus refreshment module and Applied Calculus 1 with correlation coefficient of 0.835. Lastly, the result of pre-calculus mathematics posttest scores with the female students in MCL relatively increased than male students, than in CL and T groups, which indicated that MCL benefit more female students than male students. The differences were more in favor of pre-calculus mathematics refreshment with MCL intervention method. To improve success in engineering participation of all students, recommended that a pre-calculus module should be offered by all universities for first year engineering students, structured co-operative learning with purpose has significant gains for effective instruction, and to increase the success rate of female students this study has proven that they are trainable and therefore, meta-cognition skills have to be nurtured for female students.

KEY TERMS:

Refreshment, Pre-engineering, Influence of refreshment, Pre-calculus Mathematics, Applied Calculus 1, Gender difference, Learning theories, Intervention, Meta-cognitive, Co-operative, Lecture centered.

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ACRONYMS

ANOVA - Analysis of Variance

CL - Co-operative Learning Method

CVR - Content Validity Ratio

CLF - co-operative learning with Female

CLM - co-operative learning with Male

MCLF - Meta-cognitive with Co-operative Learning Method with Female

MCL -Meta-cognitive with Co-operative Learning Method

MCL M - Meta-cognitive with Co-operative Learning Method with Male

MoE - Ministry of Education

MT - Male in Traditional lecture method

OVAE - Office of Vocational and Adult Education

STAD - Student-Teams-Achievement Divisions

STEM - Science, Technology, Engineering, and Mathematics.

T -Traditional lecture method

TEAL -The Teaching Excellence in Adult Literacy

TF - Traditional lecture method with Female

TGT -Team Game Tournament

TM - Traditional lecture method with Male

ZPD - Zone of Proximal Development

OPERATIONAL DEFINITIONS

Applied (Calculus) Mathematics I: Mathematics course which enclose the following chapters: vector, matrix, limit and continuity, derivative of function, and integral of functions (Dallas, 2017).

Co-operative learning Method (CL): An intervention method in which female and male students discuss in their small groups that contain three members in each to solve existing problems. The instructor is allowed to assist the groups but the groups and the instructors are not afforded with any meta-cognitive questions sheet (Adams R., 2013).

Meta-cognition: The processes of guiding students to regulate their own learning through planning, monitoring, and evaluation of their current and prior knowledge that helps to activate before, during, and after the existing problem is solved (Vijayakumari & Souza, 2013)

Meta-cognitive with Co-operative Learning (MCL): An intervention method that shows how to deal with meta-cognitive question sheet in co-operative setting. It is the method in which students discuss in their small groups of six members to solve existing problems. In this intervention, the meta-cognitive questions sheet, and the students' dealings offer meta-cognitive strategies to students in the form of planning, monitoring, and evaluation in performing a given task (Ali, 2013).

Pre-calculus Mathematics: for this research it stands for basic algebra, equations and inequalities, function, exponential, logarithmic and trigonometric functions (Stitz & Zeager, 2013) .

Pre-Engineering students: First year first semester university students those had join college of engineering yet did not select their department (Institute of Technology, 2017).

Prior Knowledge: It can be defined as what students acquired and already knew about mathematical content that is potentially relevant for acquiring new applied calculus to be successful (Campbell, 2009), i.e in this study, prior knowledge or background knowledge often used interchangeably.

Quasi-Experimental Design: The quasi-experimental design is an experimental design that does not use random assignment technique to assign samples (e.g. people) to program groups (Thyer, 2011).

Traditional Lecture Intervention Method (T): An intervention method in which the instructor used to explains and manipulate the pre-calculus mathematics and Applied Calculus I concepts and procedures to the whole class (Jochems, 2002).

CHAPTER ONE

INTRODUCTION

1.1.Background

High school graduates who aim to study engineering need to attain strong foundations of mathematics. Hence, Collingwood, Price, and Conroy (2011) propose mathematical readiness for pre-engineering students. Teachers, instructors and mathematics educators in many mathematics departments worldwide want their students to have an in-depth knowledge of basic mathematical concepts and skills, however, students do not have the necessary prerequisite knowledge and skills to be successful at university level work in mathematics (Mulqueen, 2012). According to MacNeal (2015), globally majority of first year pre-engineering students have poor foundational mathematics knowledge and therefore have not attained prerequisite basic mathematical knowledge and skills for applied calculus and related courses. Basic knowledge and skills of mathematics are needed at every step or stage of life; all technologies currently used in different areas and different fields including engineering field based on mathematics (Korn, 2014). As mathematics is a vital tool for the understanding and application of science, technology and engineering, the discipline plays the vital role of a forerunner (Sam William B, 2009). Engineering students should have adequate prior knowledge of pre-calculus mathematics. Prior experience and knowledge of pre-calculus mathematics is a base to build applied calculus concepts for engineering students. Getting students to connect prior knowledge to generate new knowledge is requisite.

Globally there is a massive concern of scholars in every country about the poor achievement of students in mathematics (Siyepu, 2013). According to Lange (2009), it is very difficult for students who have low prior mathematical knowledge, to score “C” and above grade on applied calculus in the learning community. Bumping into the understanding of mathematics challenges their identity and they are potentially pushed to the margin of the learning community (Lange, 2009).

Jennings (2009) indicates that numerous universities are also investigating and trying to improve their students’ background of basic mathematical knowledge and skill. Since Applied Calculus (Applied Mathematics) 1, 2, 3 courses are included in the curriculum of engineering field (Karim, Lelsher, & Liu, 2010), all freshmen pre-engineering university students take Applied Calculus 1, so it is important to prepare students for these courses. Very few universities in other countries for instance Lovric (2009) asserts that at one of the Canadian university and Karim et al. (2010), at the university of Tennessee (Knoxville) implement a programme that diagnoses students’ mathematical knowledge at first year of enrollment to direct their programme in a relevant manner offer pre-calculus mathematics for their students. Korn (2014) states that conceptual understanding needs the student to be active in linking prior knowledge with making adjustments to newly construct and accommodate knowledge. As far as literature is concerned, few studies if any, have investigated the significance level or the influence of this pre-calculus mathematics refreshment for first year pre-engineering university students, so in researchers’ opinion it is important to visualize and fill this gap.

The students’ lack of basic skills in number and algebra, harshly hinders the clarification and development of mathematical ideas, and is one of the critical challenges that mathematics teachers currently face (Karim et al., 2010). The student’s lack of pre-calculus mathematical skills in number and algebra is related to the operation of rational numbers, simplification, rationalization, factorization, approximation, function (exponential, logarithmic, and trigonometric), and absolute value. This problem is observed mostly on the achievement of female students than male students in mathematics class and seems common even in the USA that is one of the developed countries (Campbell, 2009; Gerhand & Philip, 2014). Cunningham, Hoyer and Sparks (2015) report that female students' achievement in STEM (Science, Technology, Engineering, and Mathematics) field was somehow less than male students' achievement. But it is known that prior knowledge is an indispensable material and mental hooks for students to load, build new skill and content knowledge of mathematics (Campbell, 2009).

According to Karim et al. (2010), the main purpose of pre-calculus mathematics for engineering students is to prepare them for applied calculus, but they reported that many students who take applied calculus are not well prepared. As it is indicated in Campbell (2009), considerable research has validated that for academic success of students, connection of prior knowledge to newly acquire knowledge plays an important role. Therefore, assessing and building prior knowledge of students create comfortable situation to grasp and construct new information in their study life. It is not easy for students, who require prior knowledge to activate what they know, to enhance their achievement in a subject. But instructors can identify prior knowledge gaps of their students and activate it by using effective intervention method (Lindblom-Ylänne, Hailikari, & Katajavuori, 2008). According to Lindblom-Ylänne et al. (2008), prior knowledge of pre-calculus mathematics significantly influence pre-engineering students' applied calculus achievement. Hauser (2015) states that, when students engaged and activated their prior knowledge of mathematics to connect across different newly acquire mathematical concepts, they understand and appreciate mathematics as an integrated whole. When students transit from high school to university, there are mathematical problems that they are familiar with and there are mathematical problems that they cannot recall, or have modest experience (Lovric, 2009). For instance, as researcher observed in his classes some first year pre-engineering students cannot operate even elementary addition of integers. Lovric (2005) supports this issue and argues that students come from secondary to tertiary engineering program with prior knowledge gap in mathematics faced a multifaceted observable problematic fact. Lovric (2005), states that gap of prior knowledge is most serious and problematic in mathematical courses, than gaps in other disciplines. Ye Yoon Hong (2009) claims that researchers who wrote on prior knowledge of mathematics show that the students who join university without mathematical preparedness is an issue and that may have its own influence on students' achievement in university mathematics (Ye Yoon Hong, 2009).

According to The American Association of University Women, (2010), one of the specific problems regarding this is related to students' challenges with respect to procedural understanding of algebraic material, particularly related to gender of students. According to Catherine, Christiane, and Andresse (2010), even though boys are considered as outperformed than girls in mathematics, currently girls are minimizing the gaps in average and doing well in mathematics. Catherine et al. (2010), state that in the past few decades the gender gap in mathematics has become narrowed. According to Catherine et al. (2010), currently relative to the past thirty years, female students' mathematics achievement is increasing. On the other hand, Contini, Tommaso, Mendolia, and Dalit (2016) state that in the STEM disciplines, gender differences are widespread in most countries; particularly it was mentioned that female students scored less than their respective male students in mathematics subject.

In addition, Ye Yoon Hong (2009); The American Association of University Women, (2010); Catherine et al. (2010) and Jenning (2009) indicate that, it is very important to identify different genders' prior knowledge difference on pre-calculus mathematics and find an intervention method that minimizes their gaps of prior knowledge and create equal opportunity for both gender separately as well as entirely to learn applied calculus course in engineering class.

Jenning (2009) points out that the transition of prior knowledge of mathematics between high school and university is a base for applied calculus and related course. And Jenning (2009) also indicates that many female students turn away from the way of a field that requires basic understanding of mathematics, like STEM field. Considering the above facts, the researcher believes that it is imperative to examine the influence of reviewing selected topics of pre-calculus mathematics for first year engineering students which requires an intervention strategy that minimizes prior knowledge gaps among genders, and to investigate co-relation of refreshment pre-calculus mathematics and Applied Calculus 1 in Ethiopian universities. Even though, some studies have been conducted to reveal the importance of reviewing pre-calculus mathematics for university students, scholars like Gerhand and Philip (2014) and Hauser (2015), suggest that amending secondary school mathematical content in tertiary courses is very important. There is a gap regarding the influence of pre-calculus mathematics and its co-relation with Applied Calculus 1. Therefore, this study reviewed the influence of pre-calculus mathematics, with focus on basic algebra, equations and inequalities, function, trigonometric functions, exponential and logarithmic functions, with active learning approach as topics mentioned above are the main language and tools for calculus and applied calculus courses.

Creating equal opportunities for both genders to succeed in applied calculus and other related courses are very important. This may happen when equipping and filling the gaps of prior knowledge of those topics for both genders of pre-engineering first year university students by selecting appropriate and effective intervention method (i.e. meta-cognitive with co-operative learning (MCL), co-operative learning alone (CL) or traditional lecture method (T)). It is anticipated to conduct refreshment of pre-university mathematics in pre-engineering classrooms at university.

1.2.Statement of the Problem and Research Questions

Interest in STEM field has been increase and numbers of students' transition from high school to university is increasing in alarming rate in developing countries like Ethiopia. Mathematics is language and tools for STEM field. According to Thomas, De Freitas Druck, Huillet, Nardi, Rasmussen, and Xie (2012), prior knowledge of mathematics influences students' level of competence globally and matters significantly in increasing dropout, especially in engineering field of study. Engineering students' serious challenge has been shortage of vital prior knowledge and skill of basic mathematics, a noticeable problem on tackling engineering related mathematical problems (Thomas et al., 2015). Even though there have been some studies on prior knowledge gaps of transition from high school to university, still in mathematics education it is unpretentious and usually does not point out the achievement of pre-engineering students on pre-calculus mathematics. To this end, this study focuses on investigating the influence of nurturing students' foundational knowledge of basic algebra, equations and inequalities, function, trigonometric, exponential and logarithmic functions.

Passmore (2007) claims that the first reason of university students' failure of mathematics achievement is lack of strong prior knowledge of mathematics. This problem is also common at Ethiopia and currently 70% of students assigned for STEM field by MoE. The researcher in his normal applied calculus teaching class observed that numerous students have been joining college of engineering of Ethiopian universities with weaker mathematical backgrounds. In Ethiopian universities STEM field it is common to see many students' struggle to minimize their gaps of prior knowledge of mathematics, and students' eagerness to deal with the challenge of applied calculus. The investigations held by Thomas, Druck, Huillet, Nardi, Rasmussen, and Xie (2012), support opinion that students are now entering university with weaker mathematical backgrounds as a global issue and changes have to be made to mathematics and engineering programs to accommodate those students.

According to California State Board of Education (2015), improving prior knowledge of science and engineering students of pre-calculus mathematics may influence students' achievement of applied calculus course.

According to Ali (2013), variations of instructional method highly influence students' mathematics achievement. When teaching or nurturing conceptual understanding of mathematics, various educational approaches with gender difference and ability level should be considered (Korn, 2014). Active learning method, that allows students to engage in the lesson (like meta-cognitive with co-operative learning, co-operative learning alone) was found to be better than the traditional lecture instructional approach, especially with respect to achieving higher order cognitive skills (Braun, 2015). According to Laister (2016), students' mathematical achievement can be improved through the provision of meta-cognitive strategies that guide students to regulate and process their own learning by planning, monitoring, and evaluating their prior and newly constructed knowledge that trigger students to think before, during and after tackling a given problem. It is an intervention method that helps students to know what they know, manage and adjust their thinking on existing mathematical problems.

According to Jbeili (2012), co-operative learning is one of the recommended learning strategies that can be appropriate to improve students' achievement in mathematics. Co-operative learning method is a teaching method where pre-engineering first year university students of mixed genders arranged into small groups working co-operatively to enhance their prior knowledge of pre- calculus mathematics. According to Booysen (2018), in co-operative learning environment, students develop creative thinking, feelings of stimulation and enjoyment, by increasing quantity and quality of ideas, and originality of expression in creative ways of solving mathematical problem. Cui-yun (2007) states that co-cooperativeness in solving mathematical problems in group provides opportunity for students' to share, appreciate and comment group members' ideas, computations, and solutions instead of ignoring (personal) or trying to come up with a better computation and solution.

Holy Bible also recommends co-operative work in Ecclesiastes 4:9-12, that in any circumstances or situation two are stronger than one because they help each other (Dake, 2011). According to Vygotsky (1978), learning with understanding takes place when students' in groups and interact to each other to solve an existing problem and receive feedback as a group as well as individually, and be directed that contradicts their current understanding, so then students reconstruct their existing knowledge.

In this co-operative setting, female and male students have opportunity to discuss together to solve problems, complete tasks, get feedback, and celebrate on their achievement. They can share work habits and study skills of more proficient students and build up better understanding and handle pre-calculus mathematics. However, there exists uncertainty as to the intervention method by which improving first year pre-engineering students' prior knowledge of pre-calculus mathematics within co-operative setting group work. Does co-operative group work alone enhance students' prior knowledge of pre-calculus mathematics? Or do they need meta-cognitive strategies with co-operative learning to do so?

In applied mathematics class to create equal opportunities for both male and female genders to enable them to succeed in applied calculus course, it needs to identify an intervention method that benefits more with respect to gender. Do female students benefit more than male students from meta-cognitive with co-operative learning method in university? According to Homayouni, Gharib, Mazini, and Otaghsara (2014), meta-cognitive knowledge helps individuals to solve mathematical problems effectively, but female students perceived as receiving more support and allocated more effort to study.

Moghadama and Mah Khah Fard (2011) observed that mathematical achievement is not determined by biological gender difference. According to Moghadama and Mah Khah Fard (2011), it is the instructors' responsibilities to identify good instructional method that can help both genders, female and male students, promote understanding and skills of mathematical problem solving abilities.

Even though many research have been carried out on the importance of building mathematical background knowledge of first year university students, very few if any studies were found that addresses the influence of refreshment module on selected pre-calculus mathematics in intervention method of meta-cognitive with co-operative learning and co-operative learning alone with respect to gender separately as well as entirely on Applied Calculus 1 achievement.

Specifically, this study was carried out to investigate the influence of refreshment module of pre-calculus mathematics on Applied Calculus 1 and if there was any significant difference in refreshment module of pre-calculus mathematics and Applied Calculus 1 among students taught through meta-cognitive with co-operative learning method, students taught through co-operative learning method, control novice group who were taught through traditional lecture method, and control group who were not taught refreshment module of pre-calculus mathematics. And to examine the effect of intervention method on genders' pre-calculus mathematics and Applied Calculus 1 achievement result.

To explore first year university students' prior knowledge of pre-calculus mathematics and its importance as a refreshment course, the study also examined the students' ability of recalling each item of questions in the selected pre-calculus mathematics in percentage and correlation of posttest result and Applied Calculus 1 achievement.

1.3.Research questions

1. To what extent university first year pre-engineering students recall some basic pre-calculus mathematics?

2. How does the refreshment module of pre-calculus mathematics influence the first year pre-engineering students' achievement in Applied Calculus 1?

3. How does the refreshment module of pre-calculus mathematics through MCL intervention method improve students' achievement in pre-calculus mathematics and Applied Calculus 1, than students those take refreshment module of pre-calculus mathematics through CL intervention method and each achieve better than students those take refreshment module of pre-calculus mathematics through T intervention method?

4. Do male students who take refreshment module of previously acquired skill in pre-calculus through MCL intervention method achieve better than male students who take refreshment through CL intervention method and each achieves better than male students who take refreshment through T intervention method?

5. Do female students who take refreshment module of pre-calculus mathematics through MCL intervention method achieve better than female students those take refreshment module of pre-calculus mathematics through CL intervention method and female students who take refreshment module of pre-calculus mathematics through T intervention method?

6. What is the association between pre-calculus mathematics posttest and Applied Calculus 1 achievement results?

7. Do male students achieve the same as female students who take the refreshment module of pre-calculus mathematics and those who learn through MCL, CL, and T intervention Method?

1.4.General Objective of the Study

The general objective of this study is to investigate the influence of refreshment module of pre-calculus mathematics with meta-cognitive with co-operative learning, co-operative learning, and traditional lecture intervention method to first year pre-engineering students on Applied Calculus 1.

1.5.Specific Objectives of the Study

The specific objectives of this study are:

1. To evaluate the extent that first year pre-engineering students recall some basic pre-calculus mathematics concepts.
2. To evaluate the extent of refreshment module of pre-calculus mathematics influence in improving first year pre-engineering students' achievement in Applied Calculus 1.
3. To identify whether there is any significant difference in refreshment module and Applied Calculus 1 achievement between students taught through Meta-cognitive with Co-operative Learning intervention method (MCL), students taught through the Co-operative Learning alone (CL) and students taught through the Traditional lecture intervention method (T).
4. To identify whether there is any significant difference in refreshment module and Applied Calculus 1 achievement between male students taught through Meta-cognitive with Co-operative Learning intervention method (MCL), male students taught through the Co-operative Learning alone (CL) and male students taught through the Traditional lecture intervention method (T).

5. To identify whether there is any significant difference in refreshment module and Applied Calculus 1 achievement between female students taught through Meta-cognitive with Co-operative Learning intervention method (MCL), female students taught through the Co-operative Learning alone (CL) and female students taught through the Traditional lecture intervention method (T).
6. To explore the association between pre-calculus refreshment module and Applied Calculus 1 achievement.
7. To differentiate the intervention method, which is more effective with respect to gender difference

1.6.Hypotheses

Based on the research questions the following alternative hypothesis formulated:

1. Students who are taught pre-calculus mathematics refreshment will achieve better than students who are not taught pre-calculus mathematics refreshment in Applied Calculus 1 achievement.

2. Students who are taught through MCL intervention method will achieve better than students who are taught through CL intervention method who, in turn will achieve higher than students who are taught through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1.

3. Male students who are taught through MCL intervention method will achieve better than male students who are taught through CL intervention method who, in turn will achieve higher than male students who are taught through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1.

4. Female students who are taught through MCL intervention method will achieve better than female students who are taught through CL intervention method who, in turnwill achieve better than female students who are taught through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1.

5. There is a positive correlation between Pre-Calculus Mathematics refreshment and Applied Calculus 1 achievement.

1.7.Significance of the Study

Producing well educated citizens in any field of study is mandatory for every developed and developing country. These days, education is a baseline of development in developing countries like Ethiopia, to eradicate poverty. In turn with this, as stated in MoE (2010), to fill the needs of countries as it was reflected in educational objectives, where some of general objectives of Ethiopian Education and Training Policy devised are educate citizens to develop their cognitive level and problem solving skills, to make them productive by delivering appropriately related education for students in their levels. In higher education proclamation of Ethiopia stated that the main aim of higher education is to prepare students to be competent professionals, independent thinkers, universal communicators, researchers and scientific societies (FDRE, 2009).

Some of the education objectives listed above are objectives of every country in the world. To achieve those and other objectives of education, effective pedagogical learning theories like under constructivism and cognitive epistemology, meta-cognitive strategies with co-operative learning and co-operative learning alone are required. This in turn requires the use of productive pedagogical approaches to understand, analyze, and synthesis especially mathematical problems that students learned previously and connect it to newly constructed knowledge. In engineering field, when students close up their gaps of prior knowledge of mathematics and construct their new university applied mathematics knowledge and skills to connect to their prior knowledge, they can succeed in all other related engineering courses.

According to Schwartz, Sears, and Chang (2015), students acquire the new knowledge by constructing on the previously acquired knowledge and abilities. Shahinshah (2012) indicates that if students have large gaps on prior knowledge of mathematics, then they are at risk to construct new knowledge of succeeding mathematical courses rather than developing anxiety or phobia. According to Shahinshah (2012), to overcome such kind of problems it is important to design refreshment of educational activities that build and activate students' prior knowledge which enables them to treat existing challenges of lessons meaningfully. The interventions' objectives in this study are to build and activate prior knowledge of pre-engineering students of pre-calculus mathematics that can support learning of Applied Calculus.

According to Fisher, Frey, and Lapp (2012), to achieve this demand it may be helpful to assess students' related prior knowledge of mathematics. Fisher et al. (2012), state that instructors may become more specific in their teaching rather than guessing gaps of students. To be precise instructors need to take quick evaluations of related prior knowledge that awakes students to their gaps of related prior knowledge that make objective of existing content attainable. According to Fisher et al. (2012), assessing prior knowledge of students helps an instructor to get inside of students' mind. Assessing students' background knowledge may help instructors to develop effective instructional strategies that may fill students' gap in respective subject spatially in mathematics. One of the effective ways to build prior knowledge of students is to set strategic plans of revising previous lessons for students. This helps them to study preceding mathematical topics. Setting strategic plans that helps to review previous lessons with effective intervention method for students is instructors' responsibility to facilitate refreshment course. As literature indicates that background knowledge of pre-calculus mathematics has been discussed for several years as it prepares students for any applied calculus course which is essential to study any area of science and engineering (Karim et al., 2010). Looking for refreshment of pre-calculus mathematics with effective intervention method will help to build new knowledge of applied calculus course. Instructors can set their effective instructional intervention method based on educational learning strategies. According to Taber (2001), one of the learning strategies that explained about importance of prior knowledge to build new knowledge is cognitive learning strategies.

The intervention method on meta-cognition with co-operative learning strategies to enhance prior knowledge of pre-calculus mathematics is based on meaningful learning. In co-operative setting, the students' role is constructing their knowledge by diverting their discussion method from knowledge receivers banking method to participatory method in which they build prior knowledge of pre-calculus mathematics for applied calculus courses. Since for all STEM students, especially for engineering college, learning mathematics with understanding is very imperative instructional aspiration to foster meaningful understanding of mathematics with practice look as if it deviates from the norm of constructive active learning (i.e. meta-cognitive with co-operative learning) instruction. According to Stylianides and Gebreil (2007), meta-cognitive strategy helps students to build, activate and master their prior knowledge of pre-calculus mathematics by managing meta-cognitive questions that direct students to ask themselves how to pan to solve a given problem, how to understand, how to monitor, how to evaluate and reason out their solution. This means that meta-cognitive with co-operative learning is a focal point on helping students to be meta-cognitively ready to plan, monitor, evaluate, and solve mathematical problems with understanding and reason their solutions.

Delivering refreshment module of pre-calculus mathematics with co-operative learning and meta-cognitive strategy may help pre-engineering students to build understanding and required skills of pre-calculus mathematics. Individual first year pre-engineering students will be able to acquire not only empirical but also abstract understanding of mathematics and explain the key underlying assumptions behind strategies of handling basic algebra, equations and inequalities, functions, exponential and logarithmic functions, and fills their gap regarding pre-calculus mathematics. This means that it is fundamental for the engineering students.

According to Alemu (2010), many of the Ethiopian university engineering students are not interested to learn applied calculus mathematics due to lack of prior knowledge of basic pre-calculus mathematics. Lovric (2009) also states that new mathematical concepts are built upon cumulative prior knowledge previously acquired. It is impossible to understand accurately and apply an advanced concept of vector, limit and continuity, derivatives and integration without understanding all basic concepts that are used to define it (basic algebra, equations and inequalities, functions, exponential and logarithmic functions) (Lovric, 2009). As a mathematics instructor, the researcher saw that most of the time students lose marks on test and final examination of Applied Calculus 1 due lack of background knowledge of those basic elementary pre-calculus mathematics concepts. For this reason, it is found important to investigate the influence of revising pre-calculus mathematics on Applied Calculus 1 and correlation of revising Pre-Calculus Mathematics and Applied Calculus 1 on first year pre-engineering university students.

The results of this study would present information on alternative instructional intervention method that helps more to close observed gaps of prior knowledge of pre-calculus mathematics in engineering students. The results may be pivotal for closing the gap of first year STEM students in general and first year engineering students' pre-calculus mathematics in particular. Finding of this study will benefit stakeholders that can create conducive atmospheres for study applied mathematics in educational institutions like universities and colleges. Mainly, the study will help mathematics instructors, STEM students, department heads, deans and the Ministry of Education that intend to take measures for addressing the possible problems related to the implementation of revising pre-calculus mathematics through student centered approach in pre-calculus mathematics. In addition, it is hoped that finding of this study contributes in mathematics education to promote the function of meta-cognitive skills together with co-operative learning setting and co-operative learning strategies alone in improving mathematics achievement. If the use of meta-cognitive strategies with co-operative learning and co-operative learning instructional intervention methods prove its' helpfulness in improving mathematics achievement with respect to entire gender or separately, mathematics instructors may select effective intervention methods that can be used to maintain students' mathematical learning with understanding.

1.8.Structure within Conceptual Framework of this Study

According to Musqueeny (2012), it is a common issue that currently majority of first year university students do not have the necessary prior knowledge and skills of pre-calculus mathematics to be successful in mathematics courses at their level. There is a mismatch between prior knowledge and expectations and needs of required achievement of Applied Calculus 1, 2, &3**.**

Building pre-calculus mathematics background knowledge of first year pre-engineering university students by using effective pedagogical intervention method is necessary for improving their background knowledge of pre-calculus mathematics, as well as establishing a good understanding of applied calculus courses. In the context of this study, it is relevant to investigate the extent of first year pre-engineering students' prior knowledge of pre-calculus mathematics which enables them to handle applied calculus courses effectively. It is essential for first year pre-engineering students to include refreshment module of pre-calculus mathematics course in which applied calculus achievement is improved developed to the point where students' ability to do formal induction, deductions and horizontally using applied calculus to other related courses in engineering field. For this reason, this study activates within the structure of an assessment of prior knowledge, input-process, output, and impact of it, which may be presented visually as follows. Structure of conceptual framework of a study is one of the ways that explain the major things of a study graphically (Berman,2013).

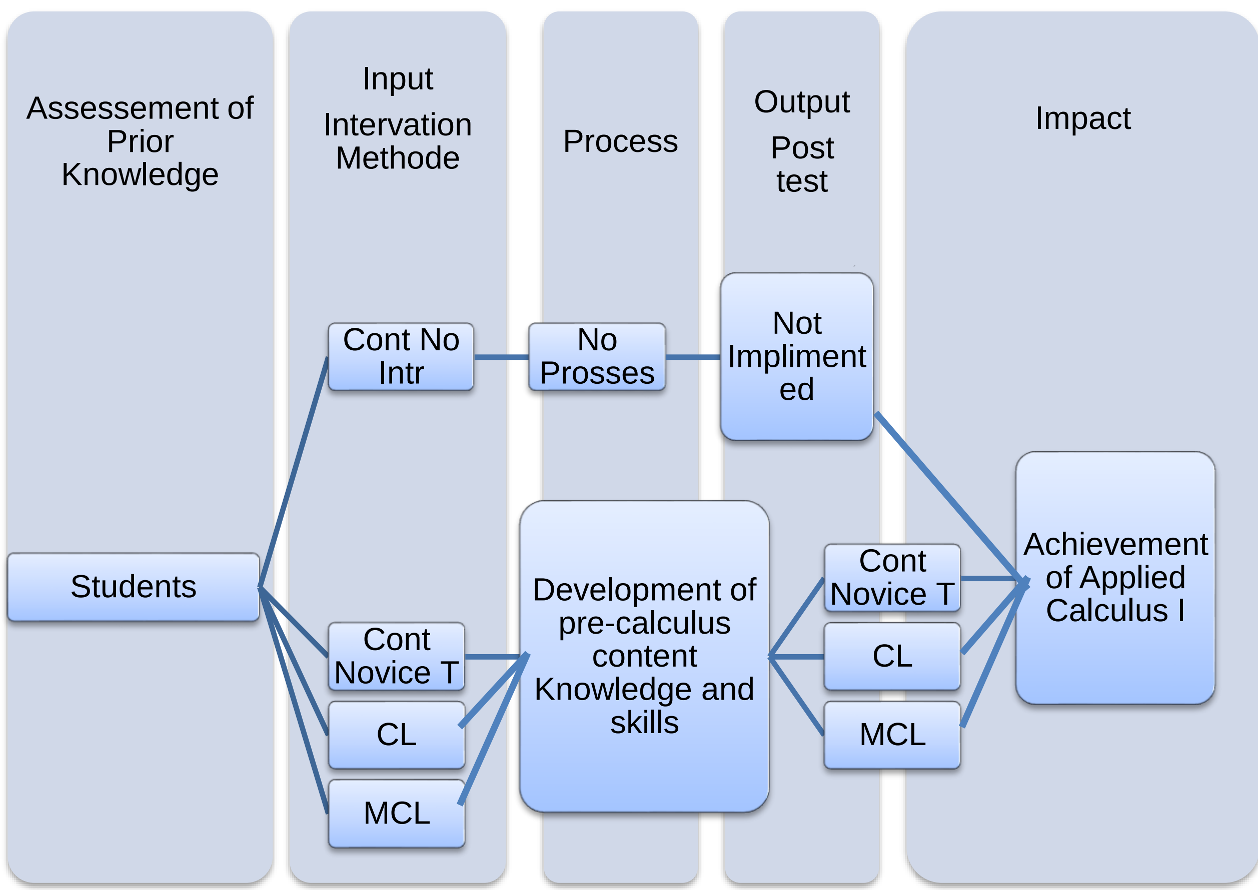


Figure 1: Input – Process – Output – Impact Structure.

The first column of the model describes assessment of prior knowledge of students. It refers to assesses significant different of students with moderator variable, male and female, and its extent to entire sampled students recalled and solved questions on the selected basic pre-calculus mathematics mainly focus on basic algebra, equations and inequalities, functions, exponential and logarithmic functions, and trigonometric functions of Pre-test. First year pre-engineering students had left high school with a certain pre-calculus mathematics conceptual and procedural content knowledge that would have enabled them to apply it in applied calculus and related other course of problem solving situation. This is supported by Tang, Voon, and Hazizah (2010), that prior knowledge of mathematics certainly influences students' mathematical and related course problem solving abilities. Because mathematics is a language and tool for engineering students to describe physical and chemical laws, it is believed that assessing prior knowledge and preparing refreshment module with effective intervention method could be the main motivation to study applied calculus mathematics for engineering field to fill students' knowledge gap properly.

The second column of the model describes the elements of intervention method that is involved in the process that students learn. The second block refers to the sampled first year pre-engineering students in four randomly selected universities as one of the four elements in the process.

The third column depicts the process of control and experimental variables are involved. Specifically, the first Cont. no Intr. group is a control group with no intervention of the refreshment module of pre-calculus. The second group is control novice of traditional lecture method group (Cont. Novice T). This group is a control group to identify effective intervention method for the delivery of refreshment module of pre- calculus mathematics. Students in this group were taught by traditional lecture method as they do in their formal class. The third group is the group that took refreshment module of pre-calculus mathematics with co-operative learning intervention method (CL). In this method, various aspects of selected pre-calculus mathematics problems were discussed based on think-pair- share co-operative learning strategies. In fact, students were taught about the process of think-pair-share method, and then assigned into heterogeneous 6 small groups that contains 3 male and 3 female students. Thus, over a period of two months, during first year first semester academic study as pre-engineering students, development of pre-calculus content knowledge and skills and selection of effective intervention methods is targeted to improve Applied Calculus achievement.

In CL group, after instructor had posed problems, students could jot down their answer to a question then turned to their peers and discussed his or her ideas and then listened to the ideas of his or her partner, and finally they presented their answers to the entire class. Fourth group is the group that took refreshment module of pre-calculus mathematics in meta-cognitive with co-operative intervention method. This method is help students more to be meta-cognitively prepared to plan, monitor, evaluate, and solve mathematical problems with understanding and reason their solutions. In this MCL method, small groups were formed and worked in similar way of CL method, what make different here is, according to Nahil and Eman (2015), students learn how to recreate and analyze thoughts and ideas, and essentially the way to come up with conclusions based on their analysis, which mean that they use meta-cognitive strategies. Instructor introduced the process of meta-cognitive with co-operative learning to students, and distributed meta-cognitive question sheet that was prepared by the researcher (see Appendix 4).

After the problem had been posed in CL setting, the instructor randomly assigned a meta-cognitive questioner, summarizer, and presenter and then he or she described it to make each group member to be aware of his or her role.

The fourth column presents the end product or output of refreshment module of pre-calculus mathematics with different intervention method: students who had acquired significantly necessary knowledge, skills and enhanced their prior knowledge to tackle applied calculus and related engineering course efficiently.

The fifth column presents the desired end product or impact of the process in which students who had taken refreshment pre-calculus mathematics to be performed better or not than the others.

1.9. Structure of Dissertation

In order to position this study consistently within the context of existing research, the first chapter contains the background, statement of the problem and research questions, objectives, significance of the study, and conceptual framework of the study. This is followed by chapter two, reviewed literature of mathematics in engineering, learning mathematics with understanding, and prior academic knowledge of mathematics. This chapter also includes learning theories with special reference to intervention methods that influence to enhance prior knowledge of mathematics such as behaviorism, constructivism, cognitive, co-operative, and meta-cognitive and reviews of pre-calculus concepts. And it is followed by chapter three and in chapter three, the design of the study and research methods used to conduct the investigation are explained. This chapter also includes sampling, intervention method, methods of data analysis, validity and reliability and research ethics. Chapter four presents the result of the study, processing and analysis of data. Finally, chapter five provides the conclusion, recommendations and limitations of the present study.

CHAPTER TWO

LITERATURE REVIEW

2.1.Introduction

Mathematics is a language and tool for all STEM fields such as chemistry, physics, biology, technology, engineering (Shenkut, 2017). Sazhin (1998) states that, even though engineering students cannot expect to perceive mathematics in the same way as professional mathematicians usually do; still now the professional engineers are expected to acquire abstract and empirical understanding of mathematics. STEM fields, especially science and engineering, have close ties with applied mathematics. Kreyszig (2006) asserts that engineering and technology students have been using applied mathematics to solve problems in engineering field of study and it has become a new study area. Teaching mathematics to engineering and technology students is to find the right balance between exhaustively understanding and practical applications of mathematical equations. According to Alfaki and Siddiek (2013), one forceful fact that prior knowledge of mathematical content is one of the strongest indicators of how well students will learn new applied calculus relative to the mathematical content that previously learned which refers to as prior knowledge. In this study, prior knowledge and background knowledge are generally used interchangeably.

Academic background knowledge affects more than the influence of school learning (Marzano, 2004). Currently it is common to see many pre-engineering university students struggling to handle with their mathematical prior knowledge and skills in their first year university course (Cunningham & Rory, 2014), most of pre-engineering university students are not well prepared on pre-calculus mathematics (Karim et al., 2010).

To build meaningful prior knowledge of pre-engineering students for better understanding of applied calculus, it needs the selection of fruitful intervention method like meta-cognitive with co-operative learning, co-operative learning alone strategies considering interest of students toward mathematics with respect to gender difference.

As such the intention of this chapter is to present a review of literature on the mathematics in engineering, understanding mathematics, prior knowledge of mathematics, intervention method of refreshment module, gender difference in mathematics and area of reviewing pre-calculus mathematics for first year university students in engineering field.

2.2.Mathematics in Engineering

Shenkut (2017) and Ali (2013) report that mathematics in engineering is recognized as the base of all engineering fields those found their concepts on mathematics. In addition, Shenkut (2017) asserts that mathematics is an international language that is needed in almost all fields. It is unquestionable that, any individual who is competent in mathematical science in engineering field can compute and achieve other related engineering courses; so, a good achievement in mathematics is important (Ali, 2013).

Important, (2013) assert that for well-educated citizen, knowledge of mathematics is very important. Mathematics is a core subject to science and engineering disciplines (Nahari, 2014). To equip pre-engineering students with required knowledge of pre-calculus mathematics for Applied Calculus 1, intervention method that helps the instructors to create conducive learning environment for students is very crucial. Intervention strategies that have shifted from talk and chalk lecture teaching method of mathematical algorithms and formulas to a constructive view of internalization is mandatory for engineering students (Important, 2013). Engineering students use application of mathematics as a symbol and language of terms that describes, explores, and reasonably interprets to solve mathematical problems. Important (2013) emphasis mathematics is a base for engineering students. Without doubt, majority of engineering courses use mathematics as a tool and language. Therefore, to be successful, engineering students need pre-calculus mathematics refreshment module to activate, understand and connect their prior knowledge of related mathematical concepts and applications that enable them to solve problems and develop skills to use application of mathematics at the time of study. For these reasons, the focus of this study was to build prior knowledge of students using intervention methods such as meta-cognitive with co-operative learning and co-operative learning alone on pre-calculus mathematics, whereas students need basic algebraic operations, procedural competence with numbers and an understanding of fundamental mathematical concepts, which is more than basic skill competition.

According to Alemu (2010), understanding mathematics refers to learning mathematics with procedural fluency and conceptual understanding; it is not only mastering algebra and geometry but also it is a matter of connecting prior knowledge to mathematical ideas those being newly constructed. This understanding of mathematics for engineering students is used as tools to design devices and the language that is used to explain their design and results (Baker, n.d.).

Savoy (2007: 2) states that "Mathematics is an essential tool for engineering students and engineering departments are clearly concerned about the mathematical preparation of many new undergraduates who have taken applied calculus. They feel that many such students are not sufficiently fluent in basic algebra and calculus at the start of their degree courses. As a result, students often experience serious difficulties with the mathematical elements of their degrees and many universities find it necessary to run extra mathematics classes for new engineering undergraduates".

When engineering students develop confidence in using pre-calculus mathematics to understand ideas, concepts and information, they show progress in university engineering applied calculus and related courses. And above all, when quantitative evidence is needed, they need to turn to mathematics (Savoy, 2007). Mathematical preparation for engineering students and building their prior knowledge of pre-engineering mathematical concepts is important for succeeding mathematical courses. Subsequently, it is an urgent need to ensure students' deep understanding and skills like basic algebraic fluency.

2.2.1. Learning mathematics with understanding

Understanding mathematics is essential for full participation in society (Mahajan, 2014). Learning mathematics courses is not only for students' memory of formulas and attains consistent methods for constructing right solutions on paper-and-pencil exercises; rather it is needed to be learned with full understanding (Weber, 2005).

Learning mathematics with understanding is an important intervention goal for engineering students (Stylianides & Gebriel, 2007). This suggests that mathematics practice fosters meaningful learning (Stylianides & Gebriel, 2007). Williams (2011: 96) states that "many students follow rules and execute procedures they do not understand, making it impossible for them to modify or extend their skills to fit new situations or to monitor their performance and catch errors when they occur". Strengthening understanding of basic pre-calculus mathematics is mandatory to overcome challenges in applied calculus mathematics for pre-engineering students. In academic context, understanding is used in relation to intellectual capacity (Gertrude, n.d.).

For instance, in mathematics where instructors frequently ask learners whether they understand mathematical concepts or not (Gertrude, n.d.). Gertrude (n.d.) indicates that in class understanding of mathematics is explore by an achievement having met the goal of teaching. According to Korn (2014), conceptual understanding of mathematics provides a more holistic skill to deal with mathematical equations for pre-engineering students. Liu and Chun-Yi (2011) state that learning mathematics with understanding is not only learning the rules and operations, but also it is about being aware of connections, seeing relationships, and knowledge reconstruction in everything that students do.

Mathematics educators interpreted the word ‘understanding’ in mathematics as follows: Siyepu (2013) states that if mathematics mental representation is a part of an internal network of representation, then mathematical idea is understood. Thus, understanding is determined by connection of prior knowledge to new knowledge being built. Conceptually grounded and well-connected ideas enables student link, remember, make connection and transfer ideas to solve new concepts being built by using previously acquired knowledge. The scholars (Liu & Chun-Yi ,2011; Siyepu, 2013) underline that the virtues of building students background knowledge and improving conceptual understanding stress the significance of the powerful connections established between procedures and concepts in the learning of mathematics. As Applied Calculus 1 is a pre-requisite for engineering field, engineering students are expected to achieve a good grade (i.e A or B) that may let give opportunity for them to pass to the next level. To achieve this goal, the prior knowledge plays the crucial role. So, it is mandatory to assess and build the students’ prior knowledge.

Why it is important to focus on the refreshment module of pre-calculus mathematics is because it is an instrument which facilitates to tackle applied calculus mathematics. In addition, Godino (1996) states that the epistemological and cognitive assumptions in philosophy of mathematics are: (a) solving problematic situation or finding solutions to those internal and external problems that mathematical objects progressively emerge and develop, (b) mathematical objects are socially shared cultural entities, (c) symbolic language that has a communicative function and an instrumental role, and (d) it is logically and conceptually organized system. According to Stylianides and Gebriel (2007), from an epistemological point of view, problems are the source of meaning of mathematical knowledge. Not only in its practical aspects, but also in its theoretical aspects, knowledge emerges from problems to be solved and situations to be mastered (Stylianides & Gebriel, 2007).

Cotttrill (2003) asserts that students' ability to solve mathematical problems mostly rely on four categories. The first is basic background mathematical knowledge, the second is a set of extensive problem-solving techniques, the third is resources, and the last one is the system to bear on the problems' situation. To understand mathematics, students must actively build new knowledge from prior knowledge and experience integrating those four categories mentioned above. Building connections is important activity in engineering field for instructors and students in classrooms where teaching is aimed at building mathematical knowledge.

2.2.2. Prior academic knowledge of mathematics

Knowledge transfer is impossible without making connection prior academic knowledge with the new knowledge being built (Mils, 2016). One considers that the strongest predictor of academic success is prior academic knowledge which strongly influences students' mathematical ability (Mils, 2016). Murry (2013) indicates that students' prior knowledge at a university is the strongest predictor of their achievement on mathematics. Students' activity and identity depend upon providing task opportunities; where in a sense of success that they achieve by drawing upon prior mathematical knowledge (Grootenboer & Torgensen, 2009). Students' prior knowledge is content knowledge, academic mathematical language and vocabulary necessary for comprehending content information that students have learned both formally in the classroom as well as informally through life experiences (Campbell, 2009).

Background knowledge of basic mathematics is the raw material to accommodate new advanced applied mathematical information of content and skill knowledge (Short, Echevarr, & Vogt, 2013). Short et al. (2013) indicate that some people distinguish the content and skill knowledge as a means to separate experiences students have had in their lives, but the reality is, without prior knowledge of basic mathematics it is very difficult for engineering students to handle the new concept of applied calculus that has been built. Research supports one undeniable fact that, what students have already known about the content is one of the strongest pointers of how well they will learn new information relative to the content (Campbell, 2009; Short et al., 2013; Alfaki & Siddiek, 2013).

Marzano (2004) found that investigating students' background knowledge before lessons begin enhances achievement of students' in newly constructing knowledge. According to Musqueeny's (2012) report, globally instructors in many mathematics departments want their students to have an in-depth knowledge of fundamental skills of mathematics. As reported by Musqueeny (2012), currently most first year university students do not have necessary prerequisite skill to be successful in mathematics course at college or university level. Therefore, it is vital to build background knowledge of students which is the corner block that supports students in mastering new content fruitfully. Marzano (2004) comments that to build or activate students' background knowledge, instructors may use a variety of active learning strategies like, co-operative learning, meta-cognitive with co-operative learning. Even if it is true that students will learn a new content depends upon intervention method that instructors' provide and students' interest, research literature shows that students' background knowledge about the content is the major indicator of how well they will learn new information. Campbell (2009); Short et al. (2013); Alfaki and Siddieki (2013); and Cervetti, Jaynes, & Hiebert (2009) also support that it is important to build background knowledge of pre-engineering university students by using different intervention method. Without identifying and building background knowledge of pre-engineering students, providing Applied Calculus 1 would be privileges to the students those already have prior knowledge (Cervetti, Jaynes, & Hiebert, 2009).

Belina (2012) states that prior knowledge reflects students' engagement for new knowledge that they can make connections from knowledge they have already had to new concepts. Therefore, this procedure refers to meta-cognitive processes, skills, and even self-understanding. Belina (2012) also comments that activating prior knowledge can be done before the lesson or during the lesson by discussing topics and using skills that the students are already familiar with. and by giving Pre-test instructors can determine the level of students' prior knowledge and use this as the foundation to identify students' gap and prepare their intervention. Alfaki and Siddiek (2013) also argue that activating prior knowledge refers to the activities and strategies that are used to bring out what students have already known about a topic. This prior knowledge can be activated by instructors in a number of ways (Fisher et al., 2012).

To minimize gaps on pre-calculus mathematics of pre-engineering university students Instructors determine the intervention methods that can activate students' background knowledge by looking for intervention strategies like, co-operative learning, meta-cognitive learning or other active learning techniques. Pre-engineering university students' prior knowledge of mathematics often interferes with their accurate learning of new concepts of applied calculus due to their misconceptions of basic mathematics and learning strategies. So, reviewing prior knowledge is an important step for mathematics instructors of pre-engineering university students. It helps them to find out misconceptions and to overcome the challenges of prior knowledge gap related to new learning applied calculus. When students develop new ideas of mathematics at any age, they use the old ideas to make sense of the new (Kenney & Kastberg, 2013).

Currently, first year pre-engineering university students have not an adequate amount of prerequisite knowledge of trigonometry, algebra, geometry and logarithm for applied calculus (MacNeal, 2015). MacNeal (2015) explains that students who have been studying applied calculus (advanced calculus) course do not have adequate background knowledge in basic pre-calculus mathematics. Yet, students coming into first year STEM programs fight with prior knowledge of mathematical concepts (Loughlin, Watters, Brown, & Jahnston, 2015). Loughlin et al. (2015) confirme that failure in the STEM field is influenced behave of student's skill of grasping prior knowledge, concepts and application in the context of engineering or other STEM field that often leads to feelings of anxiety, stress and lack of self-confidence, potentially resulting in the students' drop out of university. It has been reported that prior calculus mathematics knowledge serves as one of the strongest predictors of academic achievement of applied calculus (Loughlin et al., 2015).

2.3.Intervention Method

Building students' background knowledge of mathematics is linked to efffective teaching methods, that instructors are required to be conversant with numerous teaching strategies

(Ganyaupfu, 2013). Mahajan (2014) states that most mathematics instructors have been using rote (lecture) approach possibly go for ‘transfer of knowledge’. Aggrawal (1996) states that intervention methods that supports to involve a body of fixed and stereo-typed modes of procedures that are applicable to its appropriate subject as a kind of ritual to be observed by all instructors and in all circumstances. In teaching mathematics, improving a student’s conceptual understanding is important (Korn, 2014).

Mahajan (2014) advocates for pushing students beyond the traditional approach of learning mathematics to broaden their problem-solving abilities and to strengthen their critical thinking skills in further are mandatory. D'Amore (2008) states that instructors explicitly or implicitly use every kind of personal knowledge, method, and belief about ways of finding, learning, or organizing substantial knowledge. The process that students deal with information is an outcome of learning (Cantwell & Scevak, 2013). Such process is the result of higher order representation of knowledge itself. The framework that individuals interpret, accept, or reject information is termed as personal epistemology (Cantwell & Scevak, 2013). Cantwell and Scevak (2013) indicate that epistemological knowledge is seen as significant because it may permit introspection into, and therefore control of, the processes of meta-cognition. This epistemology is essential to construct background knowledge of students empirically for responding to instructive requirements.

Ganyaupfu (2013) states that instructors should create a conducive learning atmosphere to build and improve background knowledge of students engagement in mathematics. Students’ engagement in learning of mathematics significantly failed on instructors’ pedagogical practices and as indicated by Calder and Willacy (2017), engagement in mathematics occurs when students enjoy learning and doing mathematics, and they view the learning and doing of mathematics as valuable, worthwhile task, useful within and beyond the classroom. Students' engagement in mathematics is viewed as instructional strategies, such as behavioral, cognitive, constructive, and co-operative learning.

2.3.1. Behaviorism learning strategies

According to Machisi (2013), the base for the development of behaviorist view of learning is the work of various scholars such as Thorndike, Pavlov, Skinner, Watson, and Hull. As to the assertion of these scholars, learning is considered as observable behavioral change that results from stimulus-response associations made by the learner (Zhou & Brown, 2015). Pacis and Weegar (2012) state that human beings go beyond than just responding to the environment which means that they react to the environment based on their prior experiences. This is clearly observed in mathematical subjects. For instance, memorization of formulas through drilling and attempts to deal with mathematical problems are the base for intervention strategies in the behaviorist theory (Cottrill, 2003).

The learning paradigm of behaviorism represents the original Stimulus-Response (SR) framework of behavioral psychology (Burton, Moore, & Magliaro, 2004): The paradigm for S-R theory is trial and error learning in which certain responses come to dominate others due to rewards (Weegar & Pacis, 2012). The feature of behaviorism is that learning could be adequately explained without referring to any unobservable internal states (Weegar & Pacis, 2012). The behaviorists' earlier studies concentrated on animals before becoming interested in human thinking, states that in any given situation an animal has a number of possible responses, and the action that would be performed depends on the strength of the connection or bond between the situation and the specific action (Burton, Moore, & Magliaro, 2004). The bonds that go together should be taught together. In pedagogical terms, this yields a drill and practice mode of instruction. The purpose of instruction in mathematics is thus seen to be one of drilling into the student the necessary rules and connections until sufficient responses are obtained (Burton, Moore, & Magliaro, 2004).

Weegar & Pacis (2012) further argue that an organism learns mainly by producing changes in response to its environment. In other words, learning is characterized by changes in behavior. This may seem to be a simple truism except for the fact that Skinner argues that a change in behavior is the only characteristic of learning (Weegar & Pacis, 2012).

According to behaviorist principles, all learning processes are fully controlled by the instructor. So the instructor has to understand all of the students' behaviors and sub behaviors involved in the task, as well as the characteristics of the students. Also the instructor has to create an instructional situation that requires students to practice the appropriate behaviors, in proper sequence and with appropriate reinforcement, gradually building more and more behaviors until the target behavior is achieved (Kay & Kibble, 2016). This process requires a great deal of time for complex, intricate tasks such as data classification (Kay & Kibble, 2016). The nature of mathematics as represented by behaviorism portrays mathematics not as a product of human creation but, instead, as existing external to the human minds (Kay & Kibble, 2016).

Burton, Moore, & Magliaro (2004) assert that behaviorism is unable to effectively address the critical issue like how students think, understand, reason, and build knowledge. Students are more than just the sum total of the behaviors that they engage in. Students make plans, remember things, forget things, solve problems, hypothesize, and much more. These aspects of cognition could not be fully understood just by looking at behavior. Moreover, the role of the student in this environment is passive, namely, it is instructor -centered where the instructor selects, explains, demonstrates, and evaluates the instructional activities. According to Cottrill (2003) in behaviorism, students attempt to explain learning with out inferring anything that is going on inside through the observable interactions.

As to the assertion of this study the behaviorist scholars ought to help the students to enhance their achievement in every related field of mathematics, such as operating fractions, simplifying rational algorithm, solving exponential, and logarithmic equations.

2.3.2. Constructivism learning strategies

Muna (2017) states that constructivism has emerged in recent years as a dominant paradigm in education by developing constructivist learning as a substantial approach to teaching and has had a major intellectual impact on the development of pedagogy, rooted in the cognitive development.

The theory of constructivism is generally credited to Jean Piaget, who expressed the mechanisms through which the students internalize knowledge. According to Karimganj (2015), Jean Piaget suggests that individuals construct new knowledge through processes of accommodation and assimilation from their experiences. In the stage of assimilation without changing pre-existing knowledge, students assimilate and incorporate and build the new knowledge into already existing knowledge (Karimganj, 2015). Constructivist learning strategy is incorporated in an epistemological frame work on the movement of prior knowledge to the knowledge that is being newly constructed (Cottrill, 2003). Instructors constantly search for new strategies to help students to understand and connect their past or present experiences to the knowledge being newly constructed (Akpan & Beard, 2016). Akpan and Beard (2016) indicate that strengths of constructivism lie in the construction of knowledge but knowledge cannot be transferred from instructor to student like goods, so instruction must be student-dominated in which instructors serve as facilitators.

According to Karimganj (2015), in constructivist learning students do not easily process or transfer what they passively receive, but they actively construct their own meaning. Constructivism differs from behaviorism by asking questions that inquire the place where knowledge came from. They claim that a person's knowledge is being constructed by individual her /himself in the setting of some environment (Cottrill, 2003). In order to make knowledge useful in a new situation, students must invest a deliberate effort to make sense of the information that comes to them (Karimganj, 2015). Karimganj (2015) states that engineering students to create new knowledge and fit applied calculus courses, they must control, realize, possess their prior knowledge of pre-calculus mathematics. According to Karimganj (2015), educators realize that knowledge cannot be simply discovered in the real world, nor passed from a book or instructor to students as it is expected, but the students construct knowledge from their experience. So, instruction guided by the constructivist learning method enhances students' engagement and facilitate students to construct new knowledge for themselves (Karimganj, 2015). In this view constructivism is an action in which students construct new knowledge based on the prior or existing knowledge that they have brought to newly acquire knowledge learning situations (Akpan & Beard, 2016).

Akpan and Beard (2016) state that students can gain information from different sources, but in constructing their own knowledge, they organize, connect and construct meaning of information to their prior knowledge and experiences. For this reason, it is expected from students to connect the prior knowledge to new information in building. According to Bhattacharjee (2015), during the connections of old information to new information, students may modify their understanding that they accept or reject old information to proceed. What students attain is directly influenced by what students have already known, how they organize input, and how they are able to integrate new constructions to expand their knowledge bases (Akpan & Beard 2016). Constructivism in education represents the shift from behaviorism to cognitive learning strategy that students assert and construct their own knowledge (Korn, 2014; Muna, 2017).

2.3.2.1.Benefits of constructivism

Dada (2015) summarizes the six benefits of constructivism as: (a) when students engage in constructive learning, they actively participate and involve in learning activity and enjoy more rather than passive listeners (b) when students engage in constructive learning, they concentrate on thinking and understanding to work best rather than on rote memorization as constructive learning concentrates on learning how to think and understand (c) when students engage in constructive learning, they create organizing principles of problems at hand with a group or other learning settings (d) when students engage in constructive learning, they are enforced to possess what they know, because constructive strategies focus on the way that students construct their own learning, and design their assessment as well. In constructivist evaluations students engage in articulating knowledge through different ways that enable them to associate new knowledge to their environments and more likely retain problem solving skills (e) in constructive learning strategies students activate, stimulate, and engage in learning activities by linking with their prior knowledge and real-world context (f) in constructive learning strategies, number of heterogeneous students discuss, share and evaluate ideas to each other in their group through communication which help them to enhance social and communicative skills. Students in this constructive learning strategy learn how to express and handle their ideas and accept, respect, and evaluate their peers' ideas clearly in group activities. Students exchange ideas, materials and discuss with others and evaluate their contributions in a socially acceptable manner. This is fundamental to achieve their educational objective as well as to be effective in their interactions in the real world (Dada, 2015).

2.3.3. Cognition and learning strategies

Philosophers, educators and psychologists have forwarded and debated for centuries about theories how children think and learn for years and certain theories like cognitive learning. According to Marsigit (2009), cognitive learning theory have been translated and absorbed as influential and famous into modern terms. The term cognitive development and anything that related to cognitive is founding the work of, Jean Piaget and Lev Vygotsky, where Piaget asserts that a student actively constructs her or his own knowledge of thinking through interaction with surrounding environment, and Vygotsky asserts that a student constructs her or his own knowledge using intellectual inheritance and socio-cultural as the process (Marsigit, 2009).

According to Kilnger (2009.), cognition presents learning as an adaptive process in which knowledge may be transmitted between individuals but is store as internal mental constractions or representations. According to Kilnger (2009), cognitive learning strategy is a strategy that has been shown to yield superior learning outcomes for more experienced students. Students can develop their cognitive learning through one of the active learning strategies like co-operative setting.

Mathematical conceptual instructions involve co-operation with competent students and others as mediators. Thus, the cognitive development in a student is social, which involves the interaction of students to teach each other in their group and the class as whole (Christmas, Kudzai, & Josiah, 2013). In the classroom situations, students' cognitive will be developed by helping each other. According to Vygotsky (1978), the difference between what students can do without help and what students can do with help is called ZPD. Vygotsky (1978) views dealing with peers as an effective way to enhance skills and strategies in co-operative learning exercises where more experienced university students help less experienced peers within the zone of proximal development (ZPD).

The learning activities are required to be designed by instructors to start from what the students can do independently based from prior knowledge to link to existing knowledge with knowledge that they can achieve through peer group discussion (Siyepu, 2013). When activities are designed, instructor may demonstrates how students develop an understanding of mathematical algorithm that is appropriate to study or solve and let the students use prior knowledge to carry out the task by themselves and lead students to develop independency (Siyepu, 2013).

According to Harland (2003), at the stage ZPD occurs, student does not need help from his or her peers. Harland (2003) explains that ZPD occurs when students are able to solve a given problem independently and internalize the problem that has been solved in co-operation with peers and under guidance of others. Siyepu (2013:2) discusses ZPD as "the potential for cognitive development is optimized within ZPD or an area of exploration for which a student is cognitively prepared, but requires help through peer interaction". Therefore, group work activities are essential to enhance problem solving skills and minimize the gap resulted from lack of experience by developing ZPD.

2.3.4. Co-operative learning

Co-operative learning, the pedagogical theory in educational research, has been considered as an important intervention approach (Wolfensberger & Canella, 2015). Co-operative learning has been effective for decades, on the theoretical framework of the socio-cognitive and socio-constructivist views of learning (Wolfensberger & Canella, 2015). Researchers in education believe that co-operative learning is one of the most remarkable and fertile areas of research, and practice in education (Johnson, Johnson, & Stanne, 2000).

Co-operative learning is a learning situation in which two or more students are working together to complete a common task (Adams, 2013). Co-operative learning is working together to solve a problem to arrive at shared goals that each student makes an individual contribution (Kulshrestha & Sonam, 2014; Johnson et al., 2000). In co-operative learning activities, students are formed into heterogeneous small groups of 4-6 students after receiving introduction and direction from the instructor. Then, the group members work the given task together until each individual student effectively understands and completes it where each of them looks for outcomes that are mutually beneficial to them as well as for all other group members (Johnson et al., 2000).

According to Johnson, Johnson, and Roger (1989), in co-operative learning, all small group students share a common destiny gain or loss that swimming together or sinking together, but the effort of co-operative learning strives swimming together rather than sinking together by developing mutual benefit, so that all students in a group gain and give each other's efforts, and know that contributions of each individual group member’s achievement causes feeling proud and equally celebrating together when they show progress successfully.

Content of subject matter affairs a lot co-operative learning to be successful (Terwel, 2011). Almost all content of mathematics permits for specific models of co-operative learning in order to accommodate individual differences among students (Terwel, 2011). Mathematics is more apt for co-operative learning than other domains. So, co-operative learning research is a rapidly growing body of research in higher education regarding mathematics (Johnson el al., 2000).

According to Wolfensberger and Canella (2015), previously defined tasks or learning activities are suitable for co-operative learning. In co-operative learning students actively construct their new knowledge on previously existing knowledge in group discussion which characterized by student-to-student interaction. Co-operative learning is systematic intervention technique in which small groups of 4-6 students work together to achieve a common goal. Co-operative learning is defined as, students' working together with group goals but individual accountability (Vijayakumari & Souza, 2013). In co-operative learning setting students work in groups on mathematical problems that prepared by instructors based on instructor's daily, weekly or semi annually lesson plan (Vijayakumari & Souza, 2013). When students are working together in their small group, they should often inform each other about procedures and meanings, debate over findings, and evaluate how the task is progressing (Johnson et al., 2000).

Co-operative learning is an approach of group work that maximizes the engagement of students while learning and satisfaction that results from working together on a high-achievement team and minimizes the occurrence of those unpleasant situations (Şimşek, 2012). Many scholars confirmed that co-operative learning is effective in university education, than traditional learning strategies, chalk and talk approach instructor-centered lectures and individual assignments (Felder & Rebecca, 2007; Baida, 2010). According to Felder and Rebecca (2007), co-operative learning strategies help students to show signs of improvement of critical thinking with high-level of reasoning skills; better persistence through graduation with higher academic achievement, and higher self-esteem.

2.3.4.1.Use of co-operative learning

In co-operative learning setting, to attain its goals, students may be structured to promote working together to achieve common goals. Traditional lecture method creates a competitive situation which is solo working atmosphere in which students work in opposition to co-operative learning that few or one can manage and achieve a goal (Har, 2013). According to Johnson, Johnson, and Roger (1989), in traditional lecture method, students are negatively interdependent among each others' achievements. Because, in traditional lecture method evaluations is based on norm reference criterion, who scores more can get better grades than other class mates who scores less result and fail to achieve the goal. In traditional lecture method, students work hard individually than their classmates to score good grades or they prefer cheating, because they do not believe they have a chance to win, and to focus on self-interest, personal success and ignore the success and failure of others. It may interfere with students' capacity to solve problem and it also promotes cheating (Oloyede, Adebowale, & Ojo, 2012).

According to Oloyede et al. (2012), competitive learning has many criticisms, including the assertion that says competitive learning promotes high anxiety levels, selfishness, self-doubt, and aggression. Whereas, co-operative learning promotes (a) greater productivity and high achievement, (b) supportive, considerate, task oriented commitment relationships, and (c) develop confidence of social competence and self-esteem, and greater psychological health. In addition, Wolfensberger and Canella (2015) report that the merit of co-operative learning is that it enhances students' academic achievement, intergroup relations, self-esteem, self-confidence, promote academic peer norms, locus of control, time on task and classroom behavior, being sociable with classmates, altruism, and the ability to agree with other's perspective.

2.3.4.2.Strategies for co-operative learning

In co-operative learning, students gain considerable benefit of experience from group work activities which needs planning, monitoring, and evaluation details. Adams (2013) describes that to be successful in co-operative learning there are essential elements or requirements those must be met. These are: (a) specific objective of students' learning outcome should be clearly set: In co-operative learning instructors should come to class with precisely planned learning tasks that students are likely to achieve and be able to do herself or himself after the group task. Instructors should describe Mathematical problems in clear-cut language and abilities in which students are able to acquire academic content, cognitive processing, or skills (Stahl, 1994). (b) All member students in a small group focus on the targeted outcome: Students must identify specified purposes of given problems those belongs to them and be aware that everyone in the classroom is expected to master the common set of ideas or skills. Every student in a group must accept his or her academic outcomes as ones they all must achieve (Stahl, 1994). (c) Instructions and directions must be clearly stated before students engage in their group learning activities. Instructors ought to state instructions or directions precisely and in clear terms those guide the students to perform the task with what materials and when it is an appropriate to perform, what students are to generate as evidence of their mastery of targeted content and skills. (d) Every student must not be distorted academically as a result of being persuaded to be a particular group member without interest rather than feel comfort by thinking that he or she has the critical role to play in contributing their part in a team work or activity that results in achieving academic success equally (Stahl, 1994). (e) In mixed groups instructors are expected to form various groups in which there are four to six students in each according to their race, gender, academic abilities. This helps to organize the group in which there is no chance for students to select the group members based on friendship or clique (Stahl, 1994). (f) Positive interdependence: students are expected to be aware that they benefit when other group members achieve the success or they fail if the other group members fail to achieve which means that they sink or swim together (Johnson et.al., 1998). Students are confronted by their fellow students in the small group with different solution and point of views (Terwel, 2011). This may cause a willingness in students to reconsider their own solutions from a different perspective which helps to develop higher cognitive skills (Terwel, 2011). (g) Individual accountability: Individual students in co-operative learning achieve and compute than solo work.

To encourage students' individual accountability, instructors should facilitate the way to explain what they have discussed and learned to a class mate, and observing and documenting portfolio of each student (Johnson et.al., 1998). The word of God in the Bible, 2 Thessalonians 3:10, says "If you do not work, you do not eat." "If a man he will not work, he shall not eat" (Dake, 2011). (h) Face-to-Face promotive Interaction: Arranging the students into small groups helps to create conducive environment for learners to support, motivate, and praise each other's attempts during group discussion. Students ought to explain how to deal with problems, share experiences, and use prior knowledge to interpret the present learning, as well as deal with challenges through face-to-face interactions in which they are able to reason out, conclude, facilitate learning efforts and provide modeling by academic discussion. In this way, students receive and provide verbal and nonverbal feedback (Terwel, 2011).

Our creator in the Bible, Ecclesiastes 4:9-12, advises us to be together in any activities for instance it is quoted that "Two are better than one, because they have a good reward for their labor. For if they one fall, the one will lift up his fellow: but woe to him that is alone when he falleth; for he hath not another to help him up. Again, if two lie together, then they have heat: but how can one be warm alone? And if one prevail against him, two shall withstand him; and a threefold cord is not quickly broken" (Dake, 2011). This could be a good example of Face-to-Face promotive interaction.

(i) Social skills: when students work together in co-operative setting, they have to engage themselves in interactive activities such as leadership, decision-making, trust-building, communication, and conflict management that they are expected to build these skills, just as academic skills. (j) Group processing: Students should identify what member actions were helpful in ensuring effective working relationships and that all group members achieved learning goals. They also decide which behaviors to keep and which to change. Successes should be celebrated.

2.3.4.3.Types of co-operative learning

For further understanding of co-operative learning, it is imperative to review its types. According to Diamond (2015), employing co-operative learning in the class is not a simple task as assigning students to group and then letting them work. Co-operative learning needs exertion of instructor's that first to determine if co-operative learning is best suited for the lesson at hand to attain the desired objectives of the lesson. There are three main types of co-operative learning (formal, informal and base group co-operative learning) (Kumar, 2014), but for this study, the researcher has focused on formal co-operative learning type. Meanwhile, instructors can choose from the three main types of co-operative learning.

Formal co-operative learning: Involves organized and preplanned co-operative learning efforts and designed to facilitate and monitor by instructors every time to gain the goal in co-operative work (Diamond, 2015). Reference books for the lesson or provided activities can be used to this formal co-operative learning and the students are frequently required to be 2-6 members in each small group to make discussion for few minutes or for the whole period. The role of the instructor in this process is selecting appropriate co-operative learning strategies/techniques, planning and organizing the co-operative learning environment and assignment and then monitoring students' learning to maximize their achievement (Diamond, 2015).

To achieve co-operative learning goals, there are different strategies or mechanisms that can be utilized in formal co-operative learning of two to six students (Schul, 2012). Here are some of those strategies that can be used across subject areas and grade levels which are simple to use. These include Think-Pair-Share, variations of Round Robin, and the Reciprocal Teaching Technique and a well-known co-operative learning technique Jigsaw, Jigsaw II and Reverse Jigsaw.

Think Pair Share: Is a co-operative learning activity that allows students to contemplate posed problems themselves, prior to being instructed to discuss their response with the group. The student may first write down thoughts or simply just brainstorm in his or her head, prompted the student paired with a peer, discusses his or her idea(s), then listens to the ideas of his or her partner, and finally, the groups share out what they discussed with their partner to the entire class and discussion continues (Lightner & Tomaswick, 2017).

Round Robin Brainstorming: Is brainstorming strategy. Like other co-operative learning techniques, the students are assigned to small groups of four to six members in which one student is assigned as the writer of the group. Round Robin discussion also revolves around robin style students generating ideas on a particular content or problem. A prepared question is provided with several answers and the students are required to think and select the appropriate answer for the provided question and then they discuss with the group members regarding the right answer. The students are expected to forward their idea until each individual in the group has got a chance to say something concerning the point of discussion that is about the answer for the provided question in circular setting (round robin style). The mandate of the facilitator of the group at this time is to write what is discussed by the group members and the mandate of the recorder is to record the answer of the group members. The discussion begins from the student who sits next to the recorder and it continues until all group members contribute their part for the discussion up to the allotted time for discussion is stop (Pacis & Weegar, 2012).

Jigsaw I: Students are members of two groups: home group and expert group. In the heterogeneous home group, students are each assigned different questions. In mixed group each student is provided with different questions and then the students leave their group and move to the other group and make discussion on the same topic. In the new group, students learn the material together before returning to their home group. Once back in their home group, each student is accountable for teaching his or her assigned topic (Schul, 2012).

Jigsaw II: First, students are provided with different portions allotted for each student in a small group from the same material to focus on the given task and each member is expected to be a professional concerning the task at hand and teach the other group member students (Schul, 2012).

Reverse Jigsaw: Which is different from Jigsaw I is applied at the time of delivering the portion of the task and in this strategy student in the expert group teach content to the whole class (Alabekee, Amaele, & Osaat, 2015).

Reciprocal Teaching: It is a co-operative learning strategy in which students are expected to be in pairs to make discussion regarding the provided contents or problems. Each group member is required to take turns while reading, asking questions, and receiving the feedback immediately (Alabekee et al., 2015).

STAD (or Student-Teams-Achievement Divisions): In this strategy all of the students are assigned to small groups to work together on presented lesson and then students are consequently tested individually to encourage and improve over all achievement of the group (Alabekee et al.,2015).

Informal co-operative learning: Kumar (2014) states that in this informal cooperative learning students work in temporary small groups to attain their common goal during one class period. The activities of these groups may take few minutes from the provided time to a given period. The students under this strategy are expected to work together behalf of the common goal that had been set (Kumar, 2014). These groups are formed temporarily in the class and changed from lesson to lesson. Although the tasks are provided and the students' progress is monitored by the instructor, the goal of informal co-operative learning is set for a short period of time. Formal co-operative learning, on the other hand, could be applied for projects that can be accomplished for days or weeks. Informal co-operative learning is advantageous as it helps to strengthen specific concepts by making the students engage in a discussion for a period of time and then produce an answer (Kumar, 2014).

Base groups co-operative learning: According to Wong and Teresa (2001), in base group co-operative learning, students are formed and organized into groups for one year or one semester to develop students' academic achievement. In this group setting each student has responsibility to support, assist and encourage their group members who need to progress during the given time of period (Wong & Teresa, 2001). The group members are responsible to help each other to make sure that they all understand intervention concepts, completing their work, and providing moral and social support. In this base group co-operative learning, more committed students become proud of each other's success and the greater influence they have on each other's behavior. If properly disseminated, co-operative base group learning can influence students' achievement and may contribute to enhance quality of education.

2.3.5. Meta-cognition learning strategies

According to Vijayakumari and Souza (2013), meta-cognition learning strategy helps to empower students to possess their knowledge; what one does and does not know and students ability to understand, control, and manipulate their cognitive processes as well as meta-cognition learning strategy is the process of developing self-awareness and evaluation. So, it is the process of thinking about thinking, thought about one's education and learning in past, at present, and in future. It takes account of knowing when, where, how and why to use specific strategies for learning and problem solving (Vijayakumari & Souza, 2013).

Meta-cognition is most commonly divided into two distinct, but interrelated areas (Sajna & Premachandran, 2016). Sajna and Premachanran (2016) state that intention of meta-cognition in education mainly spotlight on two areas; the first is cognitive knowledge, deals about awareness of one's thinking and the second is meta-cognitive regulation, deals about ability to manage one's own thinking processes. Control of cognitive activities is the result of meta-cognition in which students process, manipulate and store information (Cantwell & Scevak, 2013). Conceptualizations of these processes comprise significant recursion of pre knowledge of students, with cognition feeding back to inform meta-cognition which influences the framework of students' personal epistemology (Cantwell & Scevak, 2013). According to Cantwell and Scevak (2013), to improve meta-cognitive ability as well as academic achievement of students, pedagogical strategies and meta-cognitive training with content is needed.

In education meta-cognition plays an important role (Hossen, 2014). Meta-cognition with co-operative learning strategy is one of mathematical learning strategy regarded as high order managerial skills that make use of cognitive processes (Hossen, 2014). Meta-cognitive skill of planning, monitoring and evaluation promote the spirit of skilled professional students (Koorosh, 2008). In the process of learning, inspiring questions are essential for the development of learning abilities of students. Meta-cognitive learning strategy leads students to ask themselves those thought-provoking questions. Meta-cognition is knowledge of 'self-instruction in order to control and organize one's achievement in tasks (Nahil & Eman, 2015). According to Ellis, Denton, and Bond (2014), university instructors can use meta-cognition as thoughtful strategies to enhance students' achievement. Even if pre-engineering students have little prior knowledge of a subject matter, it is possible to raise students' awareness of the personal epistemology that underlies their meta-cognition and self-regulation of applied calculus (Cantwell et al, 2013).

Literacy (2012) states that meta-cognition is an aptitude to apply background knowledge concerning the strategies to monitor the students' planning, following procedures to solve problems, and giving feedback to each other during the team work. And it helps students to choose the right cognitive tool for the required assignment and it plays a vital role in successful learning (Literacy, 2012). Hacker, Dunlosky, and Graesser (2009) indicate that meta-cognitive knowledge develops through children's interactions with peers and adults. When students learn with other students they may exchange hard talk like bite rates language including the ability to reflect on thought and language, through that process they may develops as a function of conflict resolution during social interaction (Hacker et al., 2009). In mathematics perspectives, social constructivists theorize that students construct mathematical knowledge in collaboration with others. From a constructivist point of view, self-awareness and reflection on cognitive processes and states emerge through social interaction with others (Hacker et al., 2009).

2.3.5.1.Meta-cognitive learning strategies Processes

According to Bergey, Deacon, and Parrila (2015), the term meta-cognitive strategies refers to intentional and directed cognitive activities that students can practice to monitor, control, and evaluate their meaning making in mathematical problem solving process in co-operative setting.

According to Nahil and Eman (2015), students are different in thinking, learning, and study based on their application of cognitive strategies (e.g., associating, comparing or contrasting, rehearsing various information from memory, analyzing sounds and images) which helps each of them to achieve a specific purpose, such as identifying, measuring, and solving problems. On the other hand, activities that ensure learning goal attainment are meta-cognitive strategies that contain the strategies which help to get the solution for the given problem. Students' mandate under this strategy is to monitor their own task, to assess, to correct, and to evaluate their progress with respect to the completion of the task and to be aware of distracting stimuli by developing and using such skills and strategies (Nahil & Eman, 2015).

Mathematically meta-cognitive strategies are very important for pre-engineering first year university students to develop capacity of using knowledge to plan, how to deal with a learning task and follow appropriate procedures to solve a problem, reflect on and evaluate results, and modify strategies as needed (Bergey et al., 2015). In addition, meta-cognitive strategies help students to select the appropriate cognitive tool for the activity and it is very important in making the lesson effective (Nahil & Eman 2015). And Nahil and Eman (2015) state that meta-cognitive strategies are the mechanizims in which students know how to recreate and analyse thoughts and ideas, and essentially the way to come up with conclusions based on their analysis.

Therefore, Literacy (2012) recommends that instructors ought to motivate their students by helping them concentrate on the way they construct their knowledge through asking questions themselves, and sharing ideas and experiences with their partner students. This makes them to be successful strategic thinkers of meta-cognitive intervention method. The instructor can also model how to apply questions and lesson plans jointly to create opportunities for students to ask and answer these questions during learning is taking place and the students have been provided with opportunities to ask questions themselves as well as their partners. At the time of preparation, they can ask questions during each phase. During the planning phase, students could ask (e.g. what am I supposed to learn? What prior knowledge will help me with this task? What should I do first? What should I look for in this problem? How much time do I have to complete this? In what direction do I want my thinking to take me?).

During the monitoring phase, students can ask (e.g. how am I doing? Am I on the right track? How should I proceed? What information is important to remember? Should I move in a different direction? Should I adjust the pace because of the difficulty? What can I do if I do not understand?).

During the evaluation phase, students can ask (e.g. How well did I do? What did I learn? Did I get the results as expected? What can I have done differently? Can I apply this way of thinking to other problems or situations? Is there anything I do not understand-any gaps in my knowledge? Do I need to go back through the task to fill in any gaps in understanding? How might I apply this line of thinking to other problems?) (Literacy, 2012).

According to Nahil and Eman (2015), building knowledge needs cognitive as well as meta-cognitive components and by applying cognitive method, students construct their knowledge and they lead, regulate and assess their learning progress. When the students are familiar with meta-cognitive strategies, they will be able to deal with challenges that they face while they are performing tasks by choosing suitable methods and also they are able to make effective decisions concerning the learning task at hand and they gain confidence and become more independent as students (Ali, 2013). The basic meta-cognitive technique includes linking new thoughts to the prior knowledge (Rahimi & Kalal, 2012).

Students usually rehearse what has come to their mind at the time of processing information and they learn from their wrong doings or inaccuracies (Ali, 2013). This processing information strategy inspire students to involve in meta-cognitive interactions, taking place in their mind through which they communicate with themselves and correct their mistakes while learning process is going on.

Students who practice various kinds of meta-cognitive strategies achieve good results (A or B) in examination and they use the appropriate tool to complete the task successfully and they select and identify suitable learning strategies as needed (Nahil & Eman, 2015). In addition, they identify blocks to learning and changing tools or strategies to ensure goal attainment (Nahil & Eman, 2015). Therefore, a meta-cognition plays critical roles in successful learning. It is very important that instructors should help their students to develop meta-cognitive thinking skill.
Vijayakumari and Souza (2013) also reveal that, the instructors are expected to work hard in order to help their students to enhance meta-cognitive skills. They are also expected to create conducive environment in which the students help themselves to learn through meta-cognitive skills.

Developing meta-cognition makes pre-engineering university students aware of the learning process that helps them to be effective students (Rahimi & Kalal, 2012). According to Rahimi and Kalal (2012), meta-cognitive knowledge prepares students to be aware of their own learning as well as thinking, managing, planning, and evaluating their learning process while they are performing learning tasks.

University students' level of meta-cognitive thinking was found to be moderate (Nahil & Eman, 2015). Hence, one way to accelerate the mathematics learning is to teach students how to understand and solve problems more effectively and efficiently (Rahimi & Kalal, 2012).
Nahil and Eman (2015) report that meta-cognitive strategy plays the crucial role when it is compared with other strategies because students identify the effective ways to regulate their own learning through this strategy, and mathematical solution achievement perform at a faster rate.
Cantwell and Scevak (2013: 47) recommend that: "universities are responsible for scaffolding students' epistemological and meta-cognitive abilities (and transitions) as part and parcel of the

tertiary package". Meanwhile, some studies reported that (Erskine, 2009) in university learning meta-cognitive skills and strategies were considered as a waste of time, due to lack of awareness regarding how meta-cognitive skills helped their learning rather than seeking quick result. Jing (2006) states that university students admitted the value of using meta-cognition, but personally they were not interested in using the skills and strategies offered. Instead, students wanted to focus on how to pass the next exam and they would store information to memory, which shows that university students may miss the point of obtaining how to become self-directed learners. Erskine (2009) states that first year university students are ineffective in practicing meta-cognitive strategies because of tend to focus on: 1) how to get good grades than learning, 2) how to memorize than understanding. To create effective meta-cognitive intervention strategies, Erskine (2009) recommends that instructors engage to: 1) discuss importance of meta-cognitive knowledge and regulation; 2) be model and explain strategies and skills in meta-cognition; 3) provide guided practice and help students recognize the tacit processes they use by giving constructive feedback; 4) make meta-cognition as a part of normal class assignment by embedding it with reinforcement.

2.4. Gender Difference in Mathematics

Currently gender equality in terms of participation in, and consciousness of, education has been achieved in most countries (OECD, 2011). On the other hand, in many developing countries like Ethiopia, female students still have poorer educational attainments, especially at the tertiary levels (Korir & Laigong, 2014). Gaps in cognitive skills of males and females at tertiary level are similar across countries (OECD, 2011). According to OECD report, males achieve better than females in mathematics whereas females outperform males in reading in most countries. Cognitive outcomes of gender differences between female and male in STEM field in which female students' achievement is relatively less and more in social science stream in which males achievement is relatively less (Contini et al., 2016). Consequently, females are less likely than males to tackle and to choose Science, Technology, Engineering, or Mathematics (STEM) as field of study at graduate level (OECD, 2011). Palino (2010) states that there was a significant functional relationship between gender, interest towards mathematics, teaching competencies, teaching strategies and techniques and library setting that there is no significant functional relationship between classroom setting and the students' achievement in mathematics. But, the intervention materials and facilities have no significant difference in terms of students' gender, age and year level (Palino, 2010).

According to Kyei and Benjamin (2011) and Kwamina and Adelaide (2015), there is a gender difference in the outcome of mathematics, specifically, their investigation shows that males achieve better than females. Kyei and Benjamin (2011) state that universities should organize refreshement course for students to update their background knowledge for university mathematics. The study which was conducted in Italy, reveals that when comparing equally achieving students, females tend to be less confident in their own abilities and attribute less importance to mathematics than their male peers (Contini et al., 2016). There is a study that was conducted at the level of individual countries (Reilly, Neumann, & Andrews, 2017). According to this study 38 of 45 nations were assessed showed significant gender difference in mathematics. Reilly et al. (2017) state that there were different prototype between OECD and non-OECD nations, where females scored higher than males in mathematics and science achievement across non-OECD nations. Moreover, the achievement of males was more variable than those of females, in most nations (Reilly et al., 2017).

Although the issues that women underachieve in mathematics have changed immensely over the last 30 years, the concepts of female underachieve in mathematics less likely than male are still evident today (Payne, 2015). In most of African countries gender difference in mathematics is significant. Ajai and Imoko (2015) state that in Tanzania, Kenya, Mozambique, Zanzibar, Malawi, males scored significantly higher than females did, but in South Arica, the difference were not significant.

However, Payne (2015) discusses that all studies are biased with already determined conclusion that females do underachieve is not general true. Payne (2015: 87) takes South Africa as a perfect example to falsify that biased idea by stating that "there were no cases of ability differences in university mathematics majors". Reilly et al. (2017:1) state that "While the gender gap in STEM achievement may be closing, there are still large sections of the world where differences remain".

According to Alcocka, Attridgeb, Kennya, and Inglisa (2014), research of mathematics education has described females as a disadvantaged group in mathematics. Alcocka et al. (2014) state that it is vital to identify which resource and intervention method is required to be used and appreciated by which groups to consider additional refreshment module for pre-engineering students.

2.5. Pre-Calculus Concepts

In these pre-calculus concepts five notions are addressed. For each concept, there are discussions of its role in applied calculus, how it can be better integrated in courses previous to calculus, and same examples of each concept are reviewed.

2.5.1. Basic Algebra

According to Star, Caronongan, Foegen, Furgeson, Keating and Larson (2015) algebra is the first concept that requires abstract thinking in mathematics, a challenging skill for many students. One of the major sources of errors students make when working problems in university is an error of not knowing all of the algebraic rules (Collingwood, Price, & Conroy, 2011). Star et al. (2015) state that algebra moves students beyond an emphasis on arithmetic operations to focus on the use of symbols to represent numbers and express mathematical relationships. It is convenient to start the refreshment module for pre-engineering first year students from these basic algebra concepts. The concepts of basic algebra are fundamental to a students' understanding of real numbers, intervals, absolute value, polynomial, radicals and rational expressions, and fractional expressions. Understanding algebra is a key for success in applied calculus courses, including geometry and calculus. It is important that students are confident in handling and manipulating such expressions, so it is critial to refresh pre-engineering students memory regarding some basic algebraic concepts and techniques (Knapp, 2016).

In calculus, we compact on the set of real numbers and its subsets. One important set is the set of **Natural number**: (N) 1, 2, 3, 4… **Integers**: (Z) . . . , -4, -3, -2, -1, 0, 1, 2, 3…

For some time, it was observable that many students struggle with their basic algebraic skills and knowledge as they make the transition to university in applied calculus. Pre-engineering first year university students have problem, specifically when operating negative integers with operation subtraction and addition. Pre-engineering students are familiar with the following operations: addition (+), subtraction (−), multiplication (×) and division (÷) but for the sake of completeness it is important to review some simple rules and conventions. like $-24-45, -23-(-45), -6+(-34)$ and $-20(-32)$. Having an understanding of four operations on Integer may better prepare students for concepts like analyzing vectors and other of mathematical computation, for instance if $a = < -1, 4 >$ and $b = < -6, 3 >$ *are two given vectors*, find $a+b, a-b$ and $2a+3b$. **Rational number:** is a number that expressed in the form of a/b where $a, b \in Z$ and $b \neq 0$, a is numerator and b is denominator . The quotient $\frac{a}{b}$, of two integers a and b (where $b \neq 0$) is called rational number.

According to David, David, and Mathew (2011), the first type of error that engineering students loses points in Applied Calculus 1 is algebra. It is an error of not knowing all of the algebraic rules and mistakes in the selection and use of mathematical symbols. Spatially when they add, subtract and divide fractions. David et al. (2011: 4) stated that "A review of the essential mathematics needed to succeed in calculus". Karim et al. (2010) indicate that adding, subtracting, multiplying and simplifying rational expressions are often difficult concepts for most of pre-engineering students, especially when determining least common denominators and greatest common divisors**.** It is important to review those rules of addition, subtraction, multiplication and division of rational numbers and expressions with variables. Sometimes students are confused with how to use (Least Common Multiple) LCM for adding two or more fractions, a number divided by zero which is undefined and zero divided by numbers is zero. Review of some concepts as follows:

Example: it is important to remember that division by zero is not allowed. However, zero divided by none zero number is zero. Expression such that $\frac{1}{0}, \frac{7}{0}, \frac{0}{0}$ are not defined. But, zero divided by non-zero number is zero. We use the BODMAS principle to determine the order of evaluation: For example, reviewing how to solve the following algebraic expression is useful for pre-engineering students.

Like $a, -\frac{1}{3} + \frac{1}{2} =$ ____ $c, -2\left(-\frac{1}{3} - \left(-\frac{1}{2}\right)\right) =$ ____ $e, \frac{0}{3} + \frac{1}{2} =$ ____

$b, -\frac{3}{4} - \frac{4}{5} =$ ____ $d, \frac{3}{4} + \frac{4}{5} - \frac{3}{10} x \frac{2}{5} \div \frac{3}{2} =$ ____ $f, \frac{1}{5} + \frac{3}{0} =$ ____

Kooij and Godijn (2011) mention that most of the time students in the lower grades of secondary vocational education had practiced algebra based on quantities, rather than skilled and lost many of these skills in the upper grades. The attention of dimensions and units, besides offering a critical perspective on formulas and the meaning of parameters and numbers, also provides some elementary algebraic activity as part of dimensional analysis (Kooij & Godijn, 2011). Consideration for reviewing and working with imprecise numbers appears to be useful, due to their use in engineering. According to Dunn (n.d.), it is clear that algebra helps to solve unknown quantities that represent real things and useful for engineers to realise and revise the numbers, symbols and units which is called dimentional capatibility. In Applied Calculus 1 application of fractional expressions started from unit one vector, for instance helps to solve $a = \frac{1}{3}i + \frac{1}{2}j, and b = \frac{3}{4}i + \frac{4}{5}$ are two given vectors, find $a + b, a - b$ and $2a + 3b$ such kind of questions.

And numbers that cannot be represented as quotients of rational numbers are called irrational numbers

Example: $\sqrt{3}, \sqrt{5}, \prod and \log_{10}^{3}$ 2.131131113…are irrational numbers.

It is important to review common mistake of operating irrational numbers as follows:

a. $\sqrt{3}+\sqrt{5}=$ ________ e. $\frac{2\sqrt{3}}{3\sqrt{2}}=$ ________

b. $2\sqrt{5}+\sqrt{5}$ ________ $f.\frac{2\sqrt{3}}{3\sqrt{3}}=$ ________

c. $5\sqrt{3}\times 3\sqrt{5}=$ ________ g. $2\sqrt{5}\times\sqrt{5}$ ________

Those example are beneficial to review four operations on irrational number for pre-engineering students because it introduce the concepts of magnitude or norm of vectors before Applied Calculus 1

For instance to find norm and the direction cosines of the vector $a = 2i + 5j + 4k$ is

$\|a\| = \sqrt{2^2+5^2+4^2} = \sqrt{45} = 3\sqrt{5}$, we see that the direction cosine are

$\cos\alpha = \frac{2}{3\sqrt{5}}, \cos\beta = \frac{5}{3\sqrt{5}}, \cos\gamma = \frac{4}{3\sqrt{5}}$

Heinbockel (2012) indicats and includes in his book students that going to take Applied Calculus 1 (Calculus I) be sure that they have had the appropriate background material of algebra and trigonometry or must review algebra that deal real numbers and trigonometry. From literature and my teaching experience of pre-engineering students, it is important to include all sets of number systems in the Refreshment module. Like the following notes

Note: the set of real numbers consists of rational and irrational numbers.

It is usually denoted by ℝ. Rational numbers have a repeated decimal representation;

For example, 1/6 = 0.1666…, 2925/9900 = 0.2954545.

Note: Real numbers are ordered a < b that “a is less than b,” a > b that “a is greater than b”

Number line and intervals: Real numbers can be represented visually as points on a number line, see $\overleftrightarrow{\ldots, -3, -2, -1, 0, 1, 2, e, 3, \pi, \ 4, 5, 6, \ldots}$ and for any real number a and b on the line $\overleftrightarrow{\quad a \quad b \quad}$

When $a \le b$ mean that a is less than or equal to b for example $\sqrt{2} \le 1.42$, $\frac{3}{4} \ge 0.6$.

In all application areas of mathematics, inequalities of numbers play an important role. Lack of attention for inequalities, pre- engineering students may face deficiencies in working with formal definition of limit and derivative concepts. It is important to review inequalities: Once students understand the basic mechanics behind solving an equation, they in theory, will understand how to solve an inequality

> Solving applied problems that involve inequalities requires students understanding to compare $\frac{a}{b}$ *and* $\frac{c}{d}$ they examine ad and bc where if ad $\ge$ bc then $\frac{a}{b} \ge \frac{c}{d}$ and if ad $\le$ bc then $\frac{a}{b} \le \frac{c}{d}$

Here are some examples which seem elementary for pre–engineering students.

> Put the inequalities $<, \ >, \ \le, or \ \ge$ between the following algebraic expressions
>
> a. $\frac{3}{4}$ ___ $\frac{9}{11}$
>
> d. $\frac{6}{5}$ ___ $\frac{7}{5}$
>
> b. $\sqrt{5}$ ___ 2.25
>
> e. If $\varepsilon > \frac{1}{\sqrt{x}}$, then $\frac{1}{x^2}$ ____ ε
>
> c. If, $8 < x < 10$, then $|x - 9| \frac{1}{|\sqrt{x}+3|}$ _______ $|x - 9| \frac{1}{\sqrt{8}+3}$

When dealing with real numbers a, b, x it is customary to use intervals of real numbers (Heinbockel, 2012). The concepts of interval in the set of real number are fundamental to Engineering students, while exercises for solution and domain would require knowledge of interval to choose an interval to keep the solution or domain for given problem. It is expected from pre-engineering students to muster and use intervals having the following understanding of notation of intervals:

Set Notation	Set Definition	Name
$[a,\ b]$	$\{x: a \leq x \leq b\}$	closed interval
$[a,\ b)$	$\{x: a \leq x < b\}$	left-closed, right-open
$(a,\ b]$	$\{x: a < x \leq b\}$	left-open, right-closed
$(a,\ b)$	$\{x: a < x < b\}$	open interval
$[a, \infty)$	$\{x: x \leq a\}$	left-closed, unbounded
$(a,\ \infty)$	$\{x: x < a\}$	left-open, unbounded
$(-\infty,\ b]$	$\{x: x \leq b\}$	unbounded, right-closed
$(-\infty,\ b)$	$\{x: x < b\}$	unbounded, right-open
$(-\infty,\ \infty)$	$\mathbb{R}$	Set of real numbers

For instance, the next two examples illustrate the above points.

a. Find the domain of $f(x) = \sqrt{9 - x^2}$,

where the solution is {x:-3 $< x <$ 3}

b. Find interval of $x\ that\ f(x) = |x|\ is\ differentiable,$

where the solutions are $\{x: x < 0\ and\ x > 0\}\ or$ at $x \neq 0$

In the above example number 'b' we have seen that absolute value notation. There are many mathematical problems that require absolute value concepts in applied calculus.

Recognizing the absolute value in real numbers as well as in function will allow students to apply what knowledge they have of absolute value to the absolute value problem in question.

Absolute values: The absolute value of a `real number a is defined as

$$|a| = \begin{cases} a \; if \; a \geq 0 \\ -a \; if \; a \; < 0 \end{cases}$$

The absolute value of a function f(x) is defined as

$$\text{f(x)} = |x| = \begin{cases} x \; if \; x \geq 0 \\ -x \; if \; x \; < 0 \end{cases}$$

For instance, $|6| = 6,\quad |0| = 0,\quad |-13| = 13$

The distance between two numbers a and b on a number line is given by $|b-a|$. However, better developing this concept may aid pre-engineering students in visualizing absolute values and its properties, as follows;

Properties of absolute value: for any real number a and b then

i. $|ab| = |a||b|$

$ii. \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$iii. |a^n| = |a|^n$

$iv. |a+b| \leq |a| + |b|$

$v. |a-b| \geq |a| - |b|$

According to Musqueeny (2012), typical treatment of simple exponential expressions found in pre-calculus is somewhat the same. The above properties of absolute value number 'iii' indicate pre-engineering university students must retrieve numbers in exponential expressions used as counters to:

Integers as exponents

If a is a real number and n=1, 2, 3, 4 … a positive integer, then

i. $a^n = a.a.a.a \ldots a \; (n \; factors)$ where $a^0 = 1 \; (for \; a \neq 0) and$

ii. $a^{-n} = \frac{1}{a^n}$

Represent repeated multiplication application. Here under note and properties of integral exponents as follows.

Review of all exponent properties

$i,\ a^n a^m = a^{n+m}$	$iv, a^{-m} = \frac{1}{a^m}$	$vii, (a^n)^m = a^{nm}$
$ii, (ab)^n = a^n b^n$	$v, \frac{1}{a^{-m}} = a^m$	$viii,\ \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$iii,\ \frac{a^m}{a^n} = a^{m-n}$	$vi, a^0 = 1$	$xv,\ \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$

For instance, pre-engineering students have prior knowledge that enable to convert $f(x) = \frac{1}{x^2}$ into $f(x) = x^{-2}$ to find the derivative of $f(x)$; where $\frac{d\,f(x)}{dx} = -2x^{-2-1} = -2x^{-3}$ hence, it is expected from pre-engineering students to use and apply the above listed exponential properties in their study time as well as whenever it needs.

It may be worth reviewing radicals and rational exponent properties to be sure pre-engineering students are comfortable with using the properties. The largest advantage of being able to change a radical expression into an exponential expression is being allowed to use all exponent properties to simplify. For example some note and expressions as follows

Radicals and Rational Exponents: review like the expression $b^n = a \geq 0$ (n= 1, 2, 3 . . .) can also be written as $b = \sqrt[n]{a}$ if n = 2, then $b = \sqrt[2]{a}$ is denoted by $\sqrt{a}$.

To avoid ambiguity, we define $\sqrt[n]{a}$ for even n to be the positive n^{th} root of a. Thus $\sqrt{64} = 4, \sqrt[4]{16} = 2, etc.$ Note: that $\sqrt[n]{0} = 0$ for all n=2, 3, 4 …

If n is even and $b < 0$, then $\sqrt[n]{b}$ is not defined.

Rules of Radicals:

a. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ b. $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$ c. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

And it is better, if students practice similar to the following questions

a. $x\sqrt{x}\sqrt[3]{x^2}$ =__ b. (4+ $\sqrt{7}$)(4- $\sqrt{7}$) = __ c. $4.4^{3/2}$ = __ d. $(9/16)^{-3/2}$

Remember that: $\sqrt[n]{a^n} = |a|$ if n is even. And $\sqrt[n]{a^n} = a$ if n is odd

2.5.2. Polynomials expressions

In any science, technology, engineering, economics field a system of polynomial equations is basic for students, normally students start working on polynomial expression from elementary level to tertiary level. Reviewing polynomial terms, factorizations and fractional expressions may fill the gap of pre-engineering students on polynomial expressions. Like polynomial with two terms is called a binomial. If a polynomial contains three terms, it is called a trinomial. Polynomials can be adding/ subtracting the like terms. Polynomials are nice functions that are not only continuous but can be differentiated infinitely many times, and that any continuous function can be approximated by polynomials with small error.

For example:

$5x^2y^3 - x^3y^3) + (x - 2x^2y^3 + x^3y^{3)} = x + 3x^2y^3 \quad and$

$$(5x^2y^3 - x^3y^3) - (x - 2x^2y^3 + x^3y^3) = 7x^2y^3 - 2x^2y^3 - x.$$

To multiply two polynomials, we multiply each term of the first polynomial with each term of the second polynomial. Like $(a + b)(c + d) = ac + ad + bc + bd$.

Factoring: Even though factoring techniques are taught in elementary and high school mathematics course, pre-engineering students still come into applied calculus weak in the area (Karim et al., 2010). Also Karim et al. (2010) state that spending more time and practice on reviewing techniques would be highly advantageous to students as they precede into applied calculus course, where simplifying for limits and derivatives is often a frustrating and difficult portion for them.

For instance

For binomial terms a and b it should be factorized as follows

$a.$ $a^2 - b^2 = (a + b)(a - b)$ b. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

c. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

And by using Pascal triangle, pre- engineering students are expected to factorize as follows

a. $(a+b)^2 = a^2 + 2ab + b^2$ b. $(a-b)^2 = a^2 - 2ab + b^2$

c. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ d. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

e. $(a-b)^{\mathrm{n}} = \cdots$

For instance: To evaluate the following limit question

a)

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} \; then\ the\ solution\ is\ \lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{x^3-2^3}{x^2-2^2} =$$

$$\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = 3$$

b)

$$\lim_{(x,y)\to(0,0)}\frac{x^4-y^2}{x^2-y} \; then\ the\ solution\ is\ \lim_{(x,y)\to(0,0)}\frac{(x^2-y)(x^2+y)}{x^2-y} =$$

$$\lim_{(x,y)\to(0,0)} x^2+y = 0$$

Fractional Expressions: According to Karim et al. (2010), revising fractional expresssions especially partial fractions helps students to prepare themselves for applied calculus as they are instrumental for an integration technique and in Laplace transformations in solving differential and integral equations. Revising this part is beneficial that students could be exposed to converting fractional expressions into partial fractions in pre-calculus, for example as follows;

Note: Any rational expression $\frac{Q(x)}{R(x)}$, factorize the numerator and denominator into linear and quadratic factors and if possible factorize the quadratic factors also. Finally simplify the fraction. For every factor $(ax+b)^n$ papering in the denominator the expressions of the form

$$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\frac{A_3}{(ax+b)^3}+\cdots+\frac{A_n}{(ax+b)^n}$$

Where $A_1,\ A_{2,}\ A_3, \ldots\ A_n$ must be determined.

For every factor of the form$(cx^2+dx+e)^n$, includes expressions of the form

$$\frac{B_1x+C_1}{cx^2+dx+e}+\frac{B_2x+C_2}{(cx^2+dx+e)^2}+\frac{B_3x+C_3}{(cx^2+dx+e)^3}+\cdots+\frac{B_nx+C_n}{(cx^2+dx+e)^n}$$

Where $B_1,\ B_{2,}\ B_3, \ldots\ B_n$ must be determined.

For example

I. $\frac{2x+3}{x^2+2x}=\frac{A}{x}+\frac{B}{x+2}$ $\quad II.\ \frac{2x+3}{x(x+2)^2}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$

II. $\frac{x+1}{2x^2-x-6}=\frac{A}{2x+3}+\frac{B}{x-2}$ $\quad IV.\ \frac{x+1}{(x+2)\,(2x^2+3x+1)}=\frac{A}{x+2}+\frac{Bx+C}{2x^2+3x+1}\ \ etc$

Example: It is expected to convert the following fractions in to partial fraction form

a. $\frac{2x+3}{x^3+2x^2+x}$ $\quad b.\ \frac{x^3-27}{x^2-9}$ $\quad c.\ \frac{x^4-x^2}{x^2+5x-6}$ $\quad d.\ \frac{-x-4}{x^3+x^2+2x}$ $\quad e.\ \frac{1}{x(x+2)^2}$

For instance, student could face in applied Calculus to evaluate the following integrals questions like as follows which requires knowledge of partial fractions.

$i. \int \frac{dx}{x^2-1}$

To solve this, student must know that the partial fraction of $\frac{1}{x^2-1}$, where

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{x^2-1} = \frac{(A+B)x+(-A+B)}{x^2-1}$$

$\begin{cases} A+B=0 \\ -A+B=1 \end{cases} \Rightarrow A = -\frac{1}{2}$ and $B = \frac{1}{2}$, then they can solve easily

$$\int \frac{dx}{x^2-1} = \int \frac{-dx}{2(x-1)} + \int \frac{dx}{2(x+1)} = \frac{1}{2}\left(\int \frac{-dx}{2(x-1)} + \int \frac{dx}{2(x+1)}\right) = \frac{1}{2}(-\ln(x-1) + \ln(x+1))$$

$$= \frac{1}{2} ln\left(\frac{x+1}{x-1}\right) + c$$

One of the most important concepts in fractional expressions is rationalizing radicals of denominators. Exercises for rationalizing radicals of denominators would require students to simplify and solve questions of limit and derivatives in applied calculus.

For example pre-engineering students are expected to rationalize the following denominators

a. $\frac{x-1}{\sqrt{x+3}-2}$ $\quad b.$ $\frac{1}{\sqrt{x}+\sqrt{2}}$ $\quad c.$ $\frac{3-x}{\sqrt{x}-\sqrt{3}}$

Because, without prior knowledge of rationalizing denominators difficult to solve similar to the following questions

$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$

To solve such kind of question one must know rationalizing of denominator, where

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} = \lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{x-1} = 4$$

2.5.3. Equation and inequalities

Equations: To solve an equation or to find all of unknown variables (these values are called solutions) that satisfy a given equation. In other words, a solution is a real number that, when substituted into the equation, gives the identity. Pre-engineering university students have to identify that addition, subtraction, multiplication and division problems involve a number of equations. They must also be able to recognize which number they must solve for in order to achieve the right operation (Williams, 2011). To solve the problem, students would need to recall and review concepts of equation which are bases for Applied Calculus 1 at university, like as follows;

Solving linear equations:

A linear equation $ax + b = 0$ (assume that $a \neq 0$) has only one solution, namely $x = {-b}/{a}$

Pre-Engineering students should able to solve the following equations.

a. $3(x+4) = -4(2-3x)$

b. $\frac{4x-1}{3} + \frac{x}{4} = -2$

c. $\frac{4}{x-3} = \frac{5}{x+2}$

d. $\frac{4x-2}{5} - \frac{3x-11}{4} = 0$

e. $\frac{x}{x+3} - 1 = \frac{8}{x^2-9}$

f. $\frac{2}{x} - \frac{2}{2x} + \frac{4}{3x} = \frac{1}{2}$

g. $\frac{x}{x-3} = \frac{x+1}{x+5}$

Most of pre-engineering students do not have a good understanding of quadratic equations. Instructors who have taught applied calculus for pre-engineering students (without two spatial universities in Ethiopia among currently existing forty-three universities) will attest this widespread problem. In Ethiopia except two special science and technology universities and medical colleges students are randomly distributed for the rest the universities based on their grade and interest. Students tend to remember the quadratic equation formula and discriminate incorrectly and use these miss-remembered formula and rules without making sure they are correct. From my teaching experience this is the major problem of most pre-engineering students in applied calculus course. Williams (2011), states that students who memorize formulas, procedures and rules without understanding cannot extend their knowledge. It is important to review elementary concept of pre-calculus mathematics like quadratic equation as follows:

Solving Quadratic Equations:

A quadratic equation $ax^2 + bx + c = 0$ can be solved by factoring, by completing the square or by using the quadratic formula

The solution of $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $D = b^2 - 4ac$ is called the discriminate of the quadratic equation. If $D > 0$, the equation has two distinct real solutions; if $D = 0$, it has one real solution, and $D < 0$, the equation has no real solutions.

Example: Solve the following and check the answer

a. $x^2 + 5x - 24 = 0$ $b.$ $x^2 + 2x - 2 = 0$ $c.$ $1 + \sqrt{2 - x} = 2x$

For STEM students factorizing and finding roots and having understanding of the above discriminate property of quadratic equations determines their ability to solve and find extreme (maximum or minimum) value of problems of applications of derivatives in Applied Calculus 1.

Inequalities: Background knowledge of students' ability to solve equations and inequalities is crucial to understand the major topics of applied calculus (Smith, 2006). Smith (2006) states that inequalities help students think globally and it may be worthwhile to increase the number of inequalities. According to Lazebnik (2012), inequalities are fundamental notions of modern mathematics. Properties of inequalities provide the main tool for developing applied calculus course. Revising the basic inequalities intended to help first year pre- engineering students to improve their skills in working with inequalities. For example some of them as follows

Note: students have to be careful when working with reciprocals

If $0 < a < b, then\ \frac{1}{a} > \frac{1}{b}$, if $a < b < 0, then\ \frac{1}{a} > \frac{1}{b}$

But the above formulas do not work if one number is positive and the other is negative. For example, $-2 < 4$, but $-1/2$ is not greater than $1/4$

For instance, let us see the solution of $x^2 + 5x + 6 \geq 0$ To solve this kind of inequalities students have to follow the following two steps;

Step one: how to find the solution of $(x + 2)(x + 3) = 0$, which are $x = -2$ and $x = -3$

Step two: how to use a sign chart

	(-∞, -3)	-3	(-3, -2)	-2	(-2,∞)
• x+2	-	-	-	0	+
x+3	-	0	+	+	+
• (x+2)(x+3)	+	0	-	0	+

It follows that the solution consists of the interval (-∞, -3] and [-2, ∞), since the value 0 is allowed. Note that if the inequality is a strict inequality, then the solution would have been (-∞, -3) and (-2, ∞).

According to Kooij and Godijn (2011), numerical values that are used in engineering and other related fields are often values of measured quantities like distance which is expressed in absolute value. First year pre-engineering students start using absolute value notion in Applied Calculus 1 on formal definition of limit, ε *and* δ, method. Recalling the definition of the absolute value and examples are important during refreshment module. For instance, to solve $|4x-3|=2$.

By definition $|4x-3| = \begin{cases} 4x-3 & if\ 4x-3 \geq 0 \\ -(4x-3) & if\ 4x-3 < 0 \end{cases} = \begin{cases} 4x-3=2 & if\ x \geq 3/4 \\ -4x+3=2 & if\ x < 3/4 \end{cases}$

So the solution is $x=5/4$ and $x=1/4$ and to solve the inequalities like $|2x+1| \leq 4$

Solution: $|2x+1| \leq 4 \iff -4 \leq 2x+1 \leq 4 \iff -4-1 \leq 2x \leq 4-1$

$$\frac{-5}{2} \leq x \leq \frac{3}{2} \iff s.s.\,[-\frac{5}{2},\frac{3}{2}]$$

Systems of Equations: Many problems encountered by engineering students are made cumbersome by the necessary solution of large numbers of simultaneous equations (Wilbur, n.d.). There are many occasions that engineering students come across two or more unknown quantities and, two or more equations relating them. These are called simultaneous equations and when asked to solve them they must find values of the unknowns which satisfy all the given equations at the same time. Reviewing some techniques of solving simultaneous equations, like by substitution method and by elimination method, will be helpful in advancing progress in engineering. For instance, it is expected that pre-engineering students solve the following simultaneous equations.

Example: Solve the following Systems of Equations

a. $2x+y=10, 4x-y=2.$ c. 2x-y =13, x+2y = -11

b. $2x-y=-5$, y = x^2+2 $d.$ y = $2x+2, x-y+4=0$

2.5.4. Functions

First year pre- engineering students have to understand the concept of function with its domain and range. Understanding of the concept of function is a person's mental experience assigning some object to the term 'function' (Godino, 1996). This experience should be developed by reviewing it for students. As it is written in Hauser (2015), first year pre-engineering students, should be able to make connections across multiple representations of mathematical function by the time they complete high school. But it is common to see that when students struggle with the concept of functions especially and come up to difficulty using the different representations that are inherent to functions (combination, composition, domain, and range) (Hauser, 2015). In applied calculus the concepts of domain and range of functions are fundamental (Smith, 2006).

Without understanding functions, the learning of other concepts in engineering mathematics may become difficult because understanding basic concept of functions is a prerequisite for learning many other mathematics concepts (Hauser, 2015). Students did not have a depth understanding of the concept function launch his/her self to work hard and cover concept of functions to solve problems at the level of applied calculus (Juddi & Crites, 2012). Operations of calculus (differentiation and integration) are applied to functions, like operations of (addition, subtraction, multiplication, and division) are applied to numbers. That is why understanding of functions is required before study calculus.

Reviewing for instance similar to the following concept of function may strengthen prior knowledge and understanding of pre-engineering students of function.

Note: A function f is a rule that assigns, to each real number x in a set A a unique real number $f(x)$ in a set B. the set A is called the domain of f, and the set of all values $f(x)$ for all x in A is called the range of f.

NB: if domain is not given, it assumed that the domain of f consists of all real numbers for which the formula for f makes sense.

The value of x is called the value of independent variable, and $f(x)$ is called the corresponding value of the function $f(x)$. . The value $f(x)$ is also denoted by y; hence the usual notation $y = f(x)$. In the reviewed module all operations of combinations and compositions are included.

Most applied problems require the limiting of the domain, with an early understanding of the domain and range of functions, students may be better prepared to address applied problems in both pre-calculus and applied calculus mathematics (Smith, 2006).

Example: Every pre-engineering student has to find the domain and the range for the following functions.

a. $f(x) = x^2$ $b.\ g(x) = \frac{1}{x}$ c. $h(x) = \sqrt{x}$ $d.\ f(x) = |x|$ $e.\ f(x) = \frac{1}{x+4}$

$f.$ $f(x) = \frac{x+2}{x^2-2x}$ $g.$ $f(x) = \sqrt{1-x^2}$

Composing function: Attention for reviewing with composition functions, composition of function f and g, is the function gof, defined by $(gof)(x) = g(f(x))$, appears to be useful, due to their use in engineering. Some examples of composition of functions are provided as follows;

Example: Let $g(x) = 1/x$ and $f(x) = x + 4$. Compute

a. gof =______ b. fog =______

Solution a: $gof(x) = g(f(x)) = g(x+4) = \frac{1}{x+4}$

Solution b: $fog(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{x} + 4$

Note: a function that can be obtained from polynomials using elementary algebraic operations and by taking roots is called an algebraic function.

For example, $\sqrt{x}$, $\frac{x^4+\sqrt[3]{x-1}}{x^2+\sqrt{x}}, \frac{x^2-1}{x^6+x-1}, \frac{\sqrt[5]{1-\sqrt{x}}}{\sqrt{x}}$ are algebraic functions.

The functions that are not algebraic are called transcendental. Trigonometric, logarithmic, and exponential functions are examples of transcendental functions.

Exponential and logarithmic functions: Exponential and logarithmic functions are vital mathematical concepts that play central roles in applied Calculus, but students have serious difficulty on these concepts (Weber, 2002). Exponentiation: reviewing exponentiation understanding as a mathematical process is critical for engineering students (Weber , 2002)

One very important special choice for calculus applications is called $e = 2.71828$ ..., (labeled for Swiss mathematician Leonhard Euler, 1707-1783). The function with the exponent e as its base is so important in mathematics, science and engineering that it is referred to as the exponential function (Knapp, 2016). The resulting function $y = f(x) = e^x$, is sometimes considered as "THE" exponential function, denoted exp(x). Other exponential function is $f(x) = b^x$. The inverse of the exponential function $y = b^x$ is, by definition, the logarithm function $y = log_b x$. That is, for a given base b, the "logarithm of x" is equal to the exponent to which the base b must be raised, in order to obtain the value x.

Exponential functions: An exponential function of the form $y = a^r$, where $a > 0$ and x is any real number. Although we can sometimes compute a power of a negative number, such as$(-4)^3$, the exponential function is defined only for positive bases. The domain of $y = a^r$ consists of all real numbers. Since $a^r > 0$ for all x it follows that the range of exponential function $y = a^r$ consists of positive number only.

Exponential and logarithmic functions are used in science and engineering from the beginning. It is important to review briefly their algebraic properties as follows for pre-engineering students.

Working with exponential functions for convenience as follows

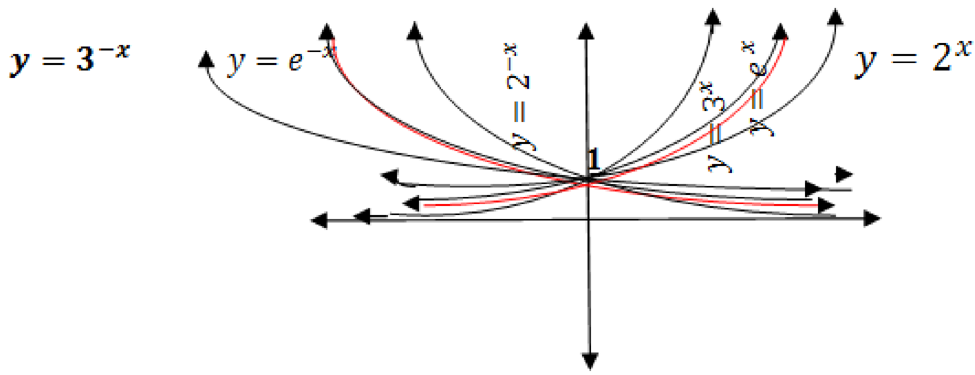
$a^0 = 1,\ a^1 = a \quad a^x a^y = a^{x+y}, \quad (a^x)^y = a^{xy}, \frac{a^x}{a^y} = a^{x-y}, \frac{1}{a^y} = a^{-y},$

For example solving the following equations

a. $3^{x+6} = 9^{2-x}$ b. $\frac{27^{2x-3}}{9^{x-4}}$ $c.$ $(2^x)^3.(4^{2-x})^4$

Outright of it, it is important to visualize graphically as follows. We obtain the so-called special exponential function $y = e^x$. Basically this function is used in a number of applications, from population problems to compound interest and radioactive decay.

The graph of $y = e^x$ and $y = e^{-x}$ are shown below



Logarithms: it appears in all sorts of calculations in engineering and science, business and economics (Mc-TY-Logarithm, 2009). According to Berezoviski (2004) students' understanding of logarithm is very poor, so they are unable to use it as a cognitive tool in their advanced mathematical thinking. University students face difficulties with logarithm while learning applied calculus courses (Kenney & Kastberg, 2013). Kenney and Kastberg (2013: 13), state that: "we share concepts related to logarithms that could help students build an understanding of these functions". According to MacNeal (2015), first year engineering students do not have strong background knowledge of logarithm.

In logarithm the statement $a^m = n$ can be written as $log_a n = m$ where log_a is the logarithm to the base a. Many first year pre-engineering university students seem to have trouble with algebra of logarithms of the following.

$a. log_a^{a^x} = x$ b. $y = a^{log_a^y}$, c. $log_{a^x}^a = \frac{1}{x}$, d. $log_a^a = 1$, e. $log_a^{xy} = log_a^x + log_a^y$

$f. log_a^{\frac{y}{x}} = log_a^y - log_a^x$, g. $log_a^{b^x} = xlog_a^b$, h. $log_{a^x}^b = \frac{1}{x} log_a^b$

And note that $log_e^x = lnx$, where x is the set of domain $(x > 0)$ and its range is the set of all real numbers.

2.5.5. Trigonometric function

Trigonometry is used throughout mathematics, especially in applied calculus (Clark, 2015). Trigonometry is an important course for many engineering fields like architecture, civil, surveying, and it can serve as an important pre-requisite to understand applied calculus (Weber, 2005). Trigonometry is a tool that mathematically forms geometrical relationships. The understanding and application of these relationships are vital for all engineering disciplines.
Trigonometry as an ancient branch of mathematics, it revolutionized hundred years ago by the invention and publication of trigonometric tables and facilitated more recently by the availability of trigonometric tables on slide rules, and scientific calculators (Kissane & Kemp, 2009). Kissane and Kemp (2009), indicate that this topic has been surprisingly neglected in the research work.

Trigonometric functions, identities and properties are used throughout mathematics, especially in applied calculus (Clark, 2015). But according to Rajalingam and Shubashini (n.d.) many university students found that trigonometry as difficult subject because it is not offered in depth in their high schools as much as algebra and geometry were thought hence their knowledge about trigonometry was quite poor. MacNeal (2015), states that first year engineering students do not have strong background knowledge of trigonometric functions. Students struggle with trigonometry at many points during their applied mathematical studies (Fanning, 2016). As cited in Fanning (2016) documents are revealed STEM students' difficulties of trigonometry (e.g Weber, 2005). According to Fanning (2016), most of those documents explained students' difficulties into two categories: that are difficulties of angle and difficulty of pertain sine, cosine, and tangent functions.

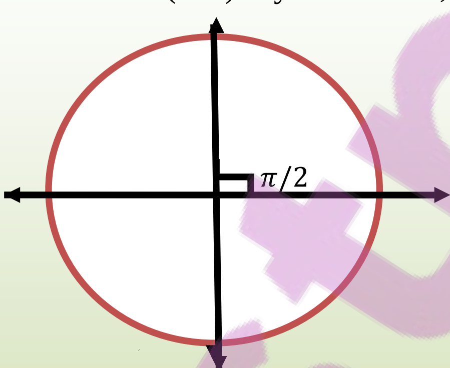
From my observation of five years teaching in university, majority of pre-engineering first year students find that even elementary ideas of trigonometry are difficult.

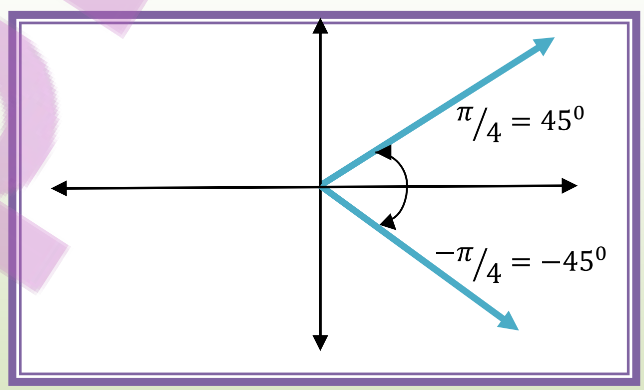
Fanning (2016) states one of the difficulties with angles that incompatibility between unit circle and the ratio approaches associated with radians and degrees respectively, to understand trigonometry. However, students taught degrees and radian interchangeable to measure angle, certain problems are to be done in terms of radian and other problems in terms of degrees, without justification. It is imperative to review the relationship of radian and degree measures of angles for pre- engineering students before reviewing trigonometric functions.

First year pre-engineering students should be able to convert between radians and degrees (Dawson, 2007).

For instance, the relation between radian and degree presented as follows

Angles: Recall that a positive angle is measured counterclockwise from the direction of the positive x-axis. If it is measured clockwise, it is negative. The unit commonly used are degree (^{0}) and radians (rad). By convention, we use radians.

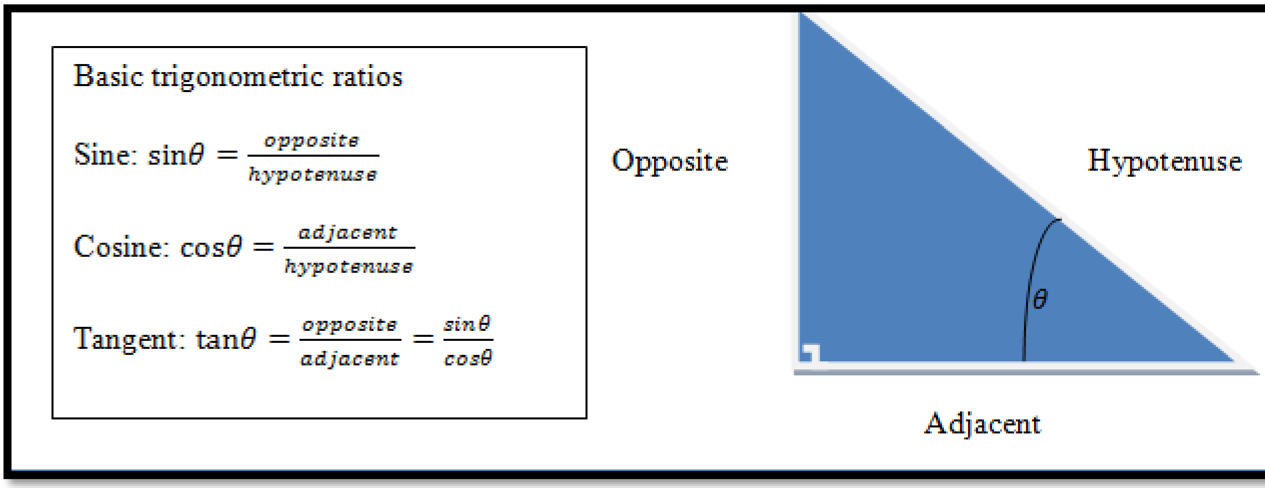




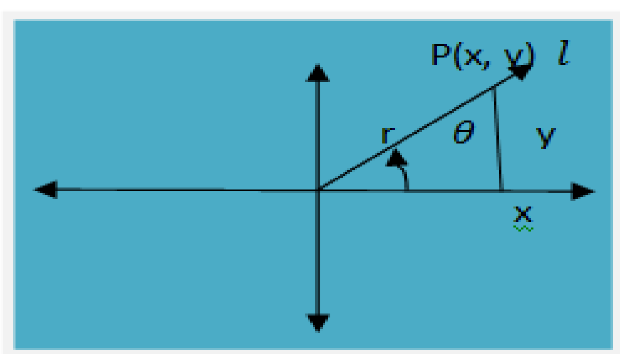
A full revolution equals 360 degree= $2\pi\, radians$. Thus one degree is equal to $\frac{2\Pi}{360} = \frac{\Pi}{180}$ radians. Conversely 1 radian equals $\frac{360}{2\pi} = \frac{180}{\pi}$ degree

Trigonometric Ratios: University pre-engineering students should know the definitions of the trigonometric functions, and be able to use them to find sides and angles of triangles. They should know and be able to use the sine and cosine laws for triangles (Dawson, 2007).

For example: For an acute angle**,** the trigonometric ratios are defined as ratios of lengths of sides in a right triangle.



Let θ be an angle defined by the, $x - axis$, and a line l



Choose a point p anywhere on the line l, and denote p by coordinate (x, y)

Let r be the distance between p and the origin $r = \sqrt{x^2 + y^2}$

Trigonometric ratios for general angles $0 \leq \theta \leq 90^0$, $sin\theta = \frac{y}{r}$, $cos\theta = \frac{x}{r}$, $tan\theta = \frac{y}{x}$, $csc\theta = \frac{1}{sin\theta} = \frac{r}{y}$, $sec\theta = \frac{1}{cos\theta} = \frac{r}{x}$, $cot\theta = \frac{1}{tan\theta} = \frac{x}{y}$, $sin^2\theta + cos^2\theta = 1 = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

Therefore, it is helpful for pre-engineering students reviewing Trigonometric functions starting from elementary concepts like as follows:

Trigonometric functions: pre-engineering university students have to know the trigonometric functions of a few common angles, such as $0^0, 30^0, 45^0, 60^0, 90^0$, and should be able to find the trigonometric functions outside of those common angles in terms of them (Dawson, 2007). For example: $\sin\frac{\pi}{6} = \frac{1}{2}$, $\text{Cos}\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $cot\ \frac{\pi}{6} = \sqrt{3}$, $\csc\frac{\pi}{6} = 2$, $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, $\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$ and $\sin 60^0 = \frac{\sqrt{3}}{2}$, $\text{Cos } 60^0 = \frac{1}{2}$, $cot\ 60^0 = \frac{1}{\sqrt{3}}$, $\csc 60^0 = \frac{2\sqrt{3}}{3}$, $\tan 60^0 = \sqrt{3}$, $\sec 60^0 = 2$.

Let x denote an angle in degrees of radians. Using the general method of defining trigonometric ratios, students should be able to compute the values of the functions $y = sin\, x$ and $y = cos\, x$ for all real numbers x. Since the angles x and $x + 2\pi$ are the same, it follows that periodicity of $sin\, x$ and $cos\, x$; $Sin(x + 2\pi) = sinx$ and $cos(x + 2\pi) = cosx$. These formula state that the values of $sine$ and $cosine$ repeat after 2π.

Developing understanding of trigonometric functions graphs includes being able to classify a relationship as trigonometric functions. For example, understanding part of the graph of $sin\, x\ and\ cosx$ over the interval $[-\pi,\ \pi] or [0,\ 2\pi]$ is called the main period. That part is repeated in both directions to produce the whole graph. $sin\, x = 0\ if\ and\ only\ if\ x = k\pi\ (\ k\ is\ an\ integer)$ and $cos\ x = 0\ if\ and\ only\ if\ x = \frac{\pi}{2} + k\pi\ (\ k\ is\ an\ integer)$. Students should be aware that $-1 \leq sinx \leq 1$ and $-1 \leq cosx \leq 1$. These ideas can be used to build understanding of trigonometric functions and graphs.

Trigonometric functions are characterized by their own unique set of identity properties from other functions. For example: trig identity $sin^2x + cos^2x = 1$ and $tan^2x - sec^2x = 1$, $relation\ between\ x\ and - x \quad sin\left(\frac{\pi}{2} - x\right) = cosx \quad and \quad cos\left(\frac{\pi}{2} - x\right) = sinx, sin(-x) = -sin(x)$, $cos(-x) = cos(x)$, addition and subtraction of trig angles $sin(x \pm y) = sinx\ cos\ y \pm siny\ cosx$, $cos(x \pm y) = cosx\ cos\ y \mp siny\ sinx$, double angles $sin(2x) = 2sinx\ cos\ x$, and Cos2x = $cos^2x + sin^2x = 2cos^2x - 1 = 1 - 2sin^2x$.

2.6. Some Studies about Meta-cognitive with Co-operative Learning Strategies

According to Laister (2016), using meta-cognitive learning strategies improve students' mathematical achievement. The use of meta-cognitive with co-operative learning method has noticeable differential effect and takes part on changing students' mathematics achievement (Dimtsu, 2017). Teaching a mathematical course with meta-cognitive learning strategies (i.e. planning, monitoring and evaluating) can be beneficial to students' promote mathematical achievement (Laister, 2016). Aljaberi and Gheih (2015), state that no statistical significant difference between university male and female students in the level of meta-cognitive thinking skills but according to Laister (2016), using meta-cognitive strategies increases the mathematical achievement of female students. On the other hand, Misu and Masi (2017), state that meta-cognitive awareness of university female students of mathematics department are greater than that of male students. Study of (Vijayakumari & Souza, 2013), conducted in Chinies secondary schools; elicited that meta-cognitive with co-operative learning approach has positive effect on the achievement in mathematics among the students. They suggest that meta-cognitive with a co-operative learning approach could be implemented in the classroom, for improving achievement in mathematics; spatially female students need to be trained in meta-cognitive learning and co-operative learning strategies to enhance their achievement in mathematics (Vijayakumari & Souza, 2013). Whereas, according to Johnson and Johnson (2014), currently in every part of the world co-operative learning is used in schools and universities, in every subject area, and with every age and gender student. Co-operative learning intervention procedure is now accepted and preferred at all level of education (Johnson & Johnson, 2014).

CHAPTER THREE

METHODOLOGY AND RESEARCH DESIGN

3.1.Introduction

The purpose of this study was to investigate the influence of refreshment module of pre-calculus mathematics on Applied Calculus 1 achievement. Refreshment module of pre-calculus mathematics was delivered in intervention method of Meta-cognitive with Co-operative learning (MCL), Co-operative Learning (CL), and Traditional Lecture (L) methods. The study was intended to investigate the effects of Meta-cognitive with Co-operative learning (MCL) and Co-operative learning (CL) methods on pre-calculus mathematics achievement among male and female first year pre-engineering university students in Ethiopia. This chapter describes the methodology that was used in this study. The chapter presents population and sample, the research design, the experimental conditions, procedures and methods of analyses. It also includes discussions that addressed reliability of test instrument with its validity and ethical issue of participant of the study. The study adopted the quasi-experimental quantitative research approach.

3.2.Research Method

3.2.1. Population and sample

This study was conducted in four higher learning institutions of Ethiopia. In Ethiopia students who join a governmental university, are randomly assigned by Minister of Education for each 29 governmental universities except two science and technology universities. Among 29 government universities in 2016/2017, four universities which have equal status of facilities like, library, dormitory, learning classrooms, were randomly selected to participate in the study. Hence, the population of this study included all pre-engineering first year students in those universities in 2016/2017. There were more than ten sections in the selected universities. Simple random sampling, specifically lottery method was used to select one section from each university. As a result, 200 pre-engineering university students who studied pre-engineering courses were taken as the participants of the study. In Ethiopian context, one class accommodates 55 to 65 students. For this study, in each group 50 students of 25 males and 25 females were selected from sampled class by simple random sampling technique.

According to Thyer (2011), quasi-experimental research attempts to determine causal relationships by applying a treatment to one group and comparing experimental group with a control group. Quasi–experimental research is used extensively in education where the subjects are not randomly and it allows the research to occur in its natural setting (Thyer, 2011). Such quasi experimental research in education has to occur in school based research at the beginning of class year (Ross & Morrison, 2003) . Ross and Morrison (2003) state that it is a common application to use Pre-testing or analysis of prior achievement to establish group equivalence and be exposed to two or more similar sections of students to alternative intervention strategies and compare them on designed dependent measures during the year.

As it was a quasi-experimental study, for its sample one section of pre-engineering first year students was randomly selected from each university. Those classes were the classes in which the selected instructors had been assigned.

To implement this study, for selected section that were going to take refreshment module, tutorial class was arranged by the assigned instructors after they had been trained and oriented about intervention. In order to make the tutorial class more effective and attractive, the regular class rooms of Applied Calculus 1 instructors were participated. All of the three participant instructors who taught intervention module of pre-calculus mathematics for pre-engineering students in this study were males, who had the same levels of preparation of education in teaching mathematics (MSc) with more than 5 years of teaching experience in their university.

At the beginning two instructors who would teach experimental groups were exposed to MCL and CL instructional methods and informed and practiced for one-week with researcher. The participating students were informed that the purpose of this study was to achieve good mark (i.e. letter A or B) on Applied Calculus 1 and examine different learning strategies that help in the improvement of prior knowledge of their pre-calculus achievement.

3.2.2. Experimental conditions

One class in each of the four universities was assigned randomly to one of the following four subjects. There were subjects in one of the universities who did not take or participate in the review of pre-calculus mathematics (control group) (n= 50). On the other hand, there were subjects in other university who took part in the intervention of review of pre-calculus mathematics through traditional /lecture/ methods (n=50), and also there were the third group in other university who participated in the intervention of review of pre-calculus mathematics through co-operative learning method (n=50), while the fourth group of subjects in one university who participated in the intervention of review of pre-calculus mathematics through meta-cognitive with co-operative learning.

The three groups that took the refreshment module of pre-calculus mathematics were different from one another in terms of the intervention method and materials used. The MCL group was asked meta-cognitive questions by the instructor and students in this group used meta-cognitive question sheet in co-operative learning setting. The CL group students studied co-operatively without using meta-cognitive question sheet, whereas the T group studied in the traditional lecture method.

3.3.Design of the Study

The method of study used in this research is the quasi–experimental design that identifies whether there is similarity between a comparison group and the treated group with respect to baseline (pre-intervention) characteristics. As stated by White and Sabarwal (2014), quasi–experimental design was conducted to revise an intervention of pre-calculus mathematics in view of understanding teaching-learning process in relation to meta-cognitive with co-operative learning and co-operative learning approach alone on pre-calculus mathematics achievement of pre-engineering first year students.

This research method has been used to assess the influence of revising pre-calculus mathematics, mainly focus on basic algebra (real number, interval, absolute value, polynomials, radicals, and fractional expressions), equations and inequalities (solving linear equations and inequalities in one variable, solving equations and inequalities involving absolute value, simultaneous equations), functions (definition of a function, domain, range). Trigonometry (radian and degree measures, trigonometric functions, identities and equations), exponential and logarithmic functions in the view of understanding teaching-learning process in relation to co-operative learning approach and meta-cognitive with co-operative learning on pre-calculus mathematics achievement of pre-engineering first year students. It has been planned to find out the influence of the independent variable on the dependent one at each of the levels in both cases of moderate variables (male and female).

The independent variable of this study was the intervention method with three categories:

1 Reviewing Pre-Calculus Mathematics through Meta-cognitive with Co-operative Learning intervention method (MCL).

2 Reviewing Pre-Calculus Mathematics through Co-operative Learning intervention method (CL).

3 Reviewing Pre-Calculus Mathematics through Traditional intervention method (T).

The moderator variable was the gender with two categories: 1. Male. 2. Female. The dependent variable was Pre-Calculus Mathematics and Applied Calculus 1 achievement. The study was designed to investigate the influence of pre-calculus mathematics refreshment module on Applied Calculus 1 and to compare three intervention methods: (1) Meta-cognitive with co-operative learning intervention: (2) Co-operative learning intervention, and: (3) Traditional intervention lecture method.

3.4.Materials and Instruments

A major instrument used in this research was the students' pre-calculus mathematics achievement test which had been prepared by researcher and materials that had been used such as manual/module, teacher's action plan as well as a meta-cognitive question sheet.

3.4.1. Intervention material

The instrumental materials used in this study to identify the influence of students' pre-calculus mathematics achievement on Applied Calculus 1, which was pre-calculus mathematics module, adapted from First Year Survival Guide of Mc Master University (Lovric, 2009) Manual/module. Pre-calculus mathematics' topics chosen for this study were basic algebra, equations and inequalities, functions, exponential and logarithmic functions, and trigonometry; function because these topics of pre-calculus mathematics are the background for the mathematical concepts, problems, issues and techniques that appear in the calculus course. Pre-calculus mathematics has been common language for understanding and describing many aspects of the physical world of science and engineering (Flashman, 2000). Concept of function is, without doubt, one key background tool for the calculus and applied calculus. According to Flashman (2000), being familiar with those pre-calculus concepts and functions which were specified are the crucial base and terms for the calculus, that help applied calculus students to have background knowledge of numbers and variables, equations and functions and applications which are used to relate the quantities included. To investigate the refreshment module of those pre-calculus topics on Applied Calculus 1 achievement, each instructor carried out the intervention for 32 sessions of 50 minutes each, which was about 26.6 hours in the respective universities they were assigned. Explaining the topic was the first procedure of the instructor and next he delivered the allotted exercise for a session (50 minutes) insuring that all of the students in each small group would arrive at the same level of understanding with respect to his objective. A set of meta-cognitive question sheet (See Appendix 4) was set by the researcher based on the meta-cognitive components (planning, monitoring, and evaluation).

3.4.2. Pre-Calculus Mathematics achievement test

3.4.2.1.Pre-test and posttest

Dimitrov and Rumrill (2003) state that most of the time Pre-test is used to identify level of understanding of research participant students before instruction. Pre-tests can be used to identify if there are a knowledge gaps that may not be expected in students' learning and it helps to generate ideas for a future lesson including further instruction of refreshment (Kelly, 2017). Forming an effective pre-test helps to identify areas of students' strengths and weaknesses that can be improved through different intervention method (Kelly, 2017). Posttest measures students' learning. pre-test-posttest is a tool that is used most often to measure changes resulted from experimental approach to compare groups in educational research (Dimitrov & Rumrill, 2003). For this study, one of the reasons of providing the Pre-tests was to compare its results with the outcome of the posttest. For this reason, researcher developed a test (pre and post) which were administered to both experimental and control groups. And the researcher used similar test items for both pre- and post-tests. The Pre-test and posttest were similar and the same in content and procedures to all groups (See Appendix 5).

The Pre-test was administered at the beginning of the program to evaluate prior knowledge extent of recalling those selected concepts of pre-calculus mathematics and identify the students' prior knowledge. Pre-test was scored and analyzed by descriptive statistics such as percentage, mean, St.D, and inferential statistics like ANOVA and independent T-test. It has been thought that identifying the pre-engineering students background knowledge regarding pre-calculus mathematics achievement mainly on topics of pre-calculus mathematics such as basic algebra, equations and inequalities, functions, exponential and logarithmic functions, and trigonometric functions. The above descriptive and inferential statistics helps to see the role of revising pre-calculus mathematics on Applied Calculus 1 achievement result and correlation between the intervention methods and genders with posttest and Applied Calculus 1 achievement result. The posttest was scored and analyzed by descriptive statistics like percentage and mean and inferential statistics such as ANOVA, independent sample T-test. Moreover, the effect size of intervention was computed. The posttest also helps to determine if there were mean scores difference between the MCL, CL, and T groups after treatment and moderator variables (the female with female students of each group, male with male students of each group, male with female students of the whole groups and significance difference). The pre-calculus mathematics achievement test contained sixteen types of test format that demanded 100 short answers was used to minimize probability of cheating. These questions were prepared based on blooms taxonomy proportional to each of selected content.

3.4.3. Implementation of the three intervention methods

To explore effective intervention method that assist to improve achievement of pre-engineering students on basic pre-calculus mathematics the three intervention methods were implemented as follows:

The control groups did not attend pre-calculus mathematics refreshment and T group students learned the course in traditional lecture Method. On the other hand, MCL and CL students were assigned into heterogeneous small groups and each small group was randomly selected to form the group which contained three male and three female (total six) students. The other students in MCL and CL classes formed in small group by applying the same procedure. In this treatment, students were informed about the procedure that they would go through after a week from that moment. Students were informed about the use of intervention; this means that they would be exposed to an intervention method that would help them become more effective managers of their own recalling of pre-calculus mathematics in focused areas and learning activities. That took place for 8 weeks i.e. it was implemented for 32 periods through each intervention method.

Meta-cognitive with Co-operative Learning Method: According to Literacy (2012), in co-operative setting, meta-cognitive strategy is: the strategy that help students to become more strategic thinkers on the existing mathematical problems and process information by asking self-questions and working with other peer students. Dealing with this method, the instructor may motivate students to develop and examine how to focus on existing mathematical problems.

During the first two or three classes instructor can be exemplary to show how to use meta-cognitive question sheet, and expected to encourage and direct students to use self-addressed meta-cognitive questions. And instructor considers four types of meta-cognitive questions into his lesson plan that gives opportunity for students to practice those meta-cognitive questions during their learning tasks. Examples of those four types of meta-cognitive questions as follows: a) Comprehensive questions (e.g. What are the issues raised as a problem?); b) Connective questions (e.g. In what ways the existing problems are similar or different from problems those solved previously?); c) Strategic questions (e.g. What is the simplest and appropriate strategy that helps to solve existing problem?); and lastly, d) Reflective questions (e.g. Does the solution of existing problem is satisfactory? If not, is there any other way to solve it?

Based on the idea mentioned above, the instructor introduced the processes of MCL approach to the students. Then, the discussion was made regarding the importance and the role of this procedure to enhance their achievement in mathematics. The instructor was expected to spend some time in introducing the concepts explicitly that how students become meta-cognitive thinkers within this learning environment. And made them be informed why they would learn meta-cognitive strategies, and how they could use these approaches in solving real-life problems. At the end of discussion on MCL method, students were assigned to groups. Each group was formed by selecting three female and three male students randomly as mentioned in the sections above. After the group arrangement, the students were provided with activities in their groups to solve the problem in a way they were oriented. Each group member was provided with specific activity that he or she could play the role as expected in the group like asking questions, summarizing, recording and presenting. The role was cyclical among the group members and each member was expected to be aware of his or her own role. The role of the group member who was assigned to ask questions would be asking meta-cognitive questions which were listed in the question sheet. The role of the summarizer was to deal with oral questions with respect to the main ideas and key points of the lesson. The role of the recorder was to write-down the steps of solution, the explanations, and the justifications of that solution. The solution was finally presented, explained, and justified to the whole class by the presenter. Each role was displayed several times in the classroom and the procedure was cyclical.

Structures' of MCL in each small group by Think pair share method as follows

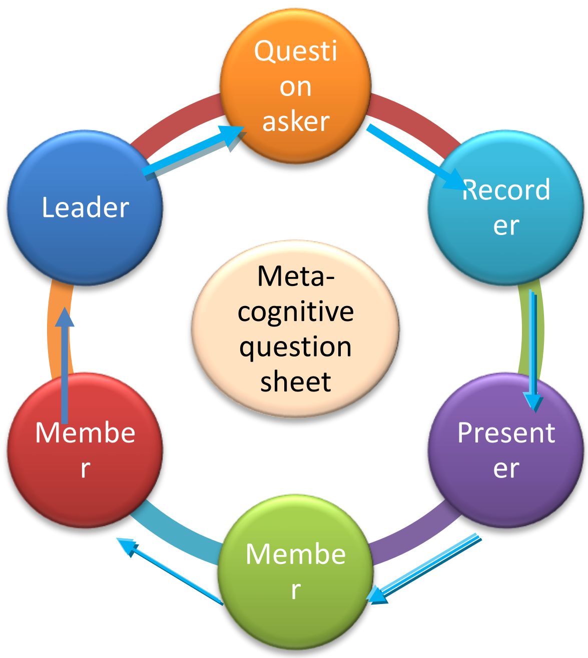


Figure 2: Structures' of MCL in each small group by Think pair share method

In co-operative learning, an instructor introduced the stages of co-operative learning method and made discussion with the students regarding the significance of applying this method in mathematics lesson. To form the small heterogeneous group, the instructor of the CL method followed the same procedure that was applied in the small group formation in MCL method. In this intervention, students got the chance to make discussions with their partners concerning activities provided to them. They also answered the questions, and exchanged their attempts to the neighbor students and made discussions with them to have common understanding with respect to responses that they had provided. This procedure occurred in entire groups in the class. The students able to evaluate their attempts by gathering information through experience sharing among each other at the time of discussion. The instructor also got a chance to assess the students' understanding towards the content of the lesson.

At the end of this procedure, the instructor evaluated students' achievement to ensure whether the students carefully attend the effectiveness of their group members and commemorate their group work achievement together. For the next class, the instructor and students followed the same process and the roles of students would be cyclical as mentioned in the MCL method. The structure of CL in each small group in 'Think pair share method' is presented as follows:

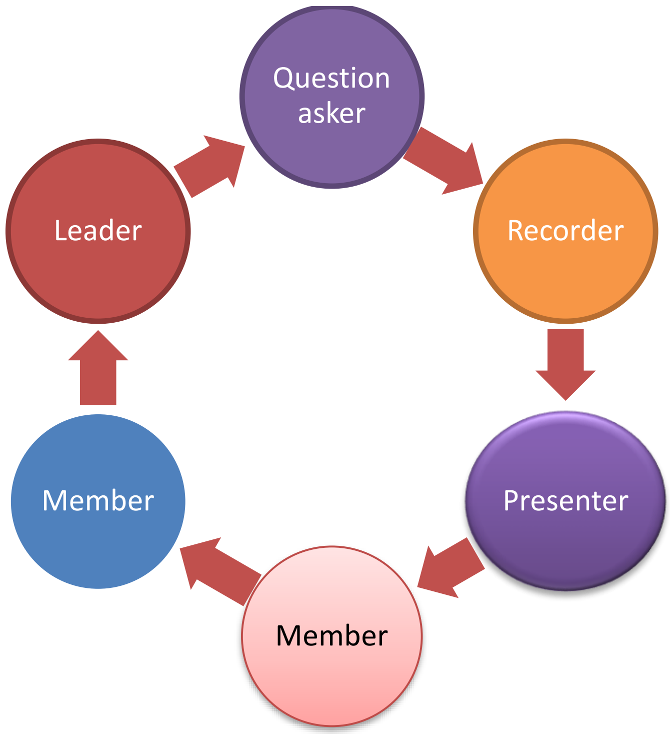


Figure 3: The structure of CL in each small group in 'Think pair share method' is presented

Traditional (T) method: Groups under traditional method attended their tutorial class as they did at the formal class. In other words, the instructor taught the students as usual as he practiced in normal class and the students were not provided with group work activities and meta-cognitive questions.

N.B: There were two control groups from which one was used to compare groups without intervention while the other was used to compare instructional intervention method with traditional lecture method.

After two months delivering intervention of pre-calculus mathematics, at the last session, the students in three groups (MCL, CL, and T) were given posttest of pre-calculus mathematics and at the end of semester, the researcher collected the results of Applied Calculus 1 out of 100% of each group at the four universities.

3.5.Analysis of the Experimental Study Findings

The pre-calculus mathematics achievement test was recorded by the researcher and analysis was made to visualize extents of students' background knowledge of basic pre-calculus mathematics and to decide whether there were any statistically significant differences among the four groups regarding the dependent variables. The statistical tools used under this procedure were descriptive statistics, T-test, and one-way analysis of variance (one-way ANOVA). The tools were used to compare the four mean scores on posttest of pre-calculus mathematics' achievement and on the results of Applied Calculus 1 that they had scored at their normal classroom.

As stated in the manual of Pallant (2010), the one-way ANOVA works for analyzing variance in quantitative data by a single dependent variable. ANOVA is an extension of T-test that is used to identify the significant difference of means. ANOVA itself has two types of tests such as priori contrasts test that take place before the experiment and post hoc test that is applied after the experiment. The researcher applied T-test and ANOVA post hoc tests to analyze the data.

As remarked by Pallant (2010), ANOVA post hoc adjustment with Tukey or Scheffe is used most commonly. ANOVA Hochberg's GT2 can be used if there is difference among group sizes and Games-Howell can be used if a group variance is less than 0.05 (Levenu test gives p value < 0.05). Tukey is used for homogeneous data. When data violet homogeneity, use "Tamhane's T2" because it is the most used test statistics by statisticians (Gupta, 1999).

Based on the above theory, ANOVA was used in this research first to compare posttest pre-calculus mathematics achievement of the three groups and the relationship between pre-calculus mathematics refreshment in respect of Applied Calculus 1 achievement was analyzed using spearman correlation coefficient. Then, ANOVA Post Hoc Test was used with multiple comparison technique to compare male students against male students' Pre-Calculus Mathematics achievement and Applied Calculus 1 across the four groups. With ANOVA, if the significance level is less than 0.05, then there must be significance difference between two groups. But the difference between these groups is specifically unknown in ANOVA. In order to identify the differences, T-Test was applied. In SPSS20, independent sample T Test method was used to compare the mean of pre and posttest one independent variable. For each treatment of independent variables, the differences between values were computed.

The same method was applied to make comparison among students with respect to gender and Pre-Calculus Mathematics achievement and Applied Calculus 1 across the four groups. The statistical analyses in all cases were computed at 0.05 levels of significances.

One technique to judge the efficiency of a given intervention is the effect size that enables us to measure both the enhancement in students' accomplishment for a group of students and the variation of students' achievement expressed on a standardized scale (Coe, 2002). The effect size provides information about which intervention is worth having, specifically valuable to measure the effectiveness of a particular intervention, in relation to some comparison (Coe, 2002).

3.6.Measures to Ensure Validity and Reliability

Before Pre-test was administered, a pilot test was carried out to check the validity of research tools. For pilot test, six instructors who have been teaching Applied Calculus 1 were selected randomly from nearest universities to researcher's university for convenience, those that were 75 km away did not participate to take refreshment module of pre-calculus mathematics to control Pre-test extraneous variable that come from information exchange.

Table 1: Biography of pilot test participant instructors

NO	Age	Qualifications	Experience in years	Training
2	38	MSc in Mathematics	14	Orientation of intervention method was given for all selected Instructors
1	32	MSc in Mathematics	9	
1	29	MSc in Mathematics	7	
2	25	MSc in Mathematics	6	

As shown in Table 1 all participant instructors in conducting pilot test have a similar level of education (MSc degree) and more than six years teaching experience of applied calculus mathematics. There were three purposes to be achieved through the pilot test. First, pilot test was applied to test module material and instrument based on blooms taxonomies. Second, pilot test was used to test content validity and third, pilot test was used to test reliability of the test instruments.

For this purpose, the test instrument was evaluated based on the assertion of Bloom's Taxonomy by the instructors who selected for the pilot study. This assertion was applied to assess students' understanding of topics and their application of higher order thinking skills (DiDonato-Barnes & Fives, 2013). As stated by Abduljabbar and Omar (2015), one should consider the six stages of Bloom's Taxonomy that start with the simplest of knowledge, then comprehension, application, analysis, synthesis and, finally, evaluation. And also an assessment of a given topic of study should be related directly to the amount of class time that allots to cover the objectives and the proportion of summative evaluation (DiDonato-Barnes & Fives, 2013).

In addition, a pilot study was applied to check content validity. Content validity is the mechanism that helps to evaluate the degree to which elements of an assessment instrument is appropriate to and representative for a particular purpose. The researcher applied a pilot test to evaluate the effectiveness and the coverage of content validity which can be used as self-evident measurement that shows the breadth of literature and test instruments (Rahmantya, 2009).

According to Rahmantya (2009), content validity can be evaluated by testing with an eye to decide the establishment of the sampled domain. As mentioned in Rahmantya (2009), content validity ratio (CVR) is a quantitative index for assessing content validity. Therefore, prepared test instrument was given for the instructors who have MSc in mathematics education. It was given to evaluate content validity of the test instrument items by CVR based on quantitative approach to content validity (Rahmantya, 2009). Those instructors were six instructors who have been teaching applied calculus 1 for first year pre-engineering. A purposive sampling technique was used to select the instructors. The instructors were asked independently to judge if the test items reflect the content domain of the study. With N judges, of which $'ne'$ have judged the knowledge required for the item to be essential, $CVR = (ne - N/2)/(N/2)$. where CVR indicates that Content Validity, $'ne'$ indicates number of subject matter evaluator expertise that rates test items need modification or item is essential, N indicates that total number of subject matter evaluator expertise. After instructors' suggestion six (CVR < 0.4) questions were corrected and modified in Pre-test and posttest (see Appendix 1).

The researcher applied test-retest mechanisms to evaluate the reliability of the test instrument. It is believed that test-retest reliability is administered by providing the same test to the same variable on two different occasions within short time interval. Test instrument is thought that there is no considerable difference in the achievement being evaluated between the time intervals of test-retest. The time interval that has been given plays a critical role to measure the validity of pre-and post-test instrument (Muijs, 2004). Whenever we evaluate the same thing twice, the correlation between observations partially depend on the time interval between the two occasions. If the time gap is short, the correlation is high; if the time gap is long, the correlation is low. This is because correlation and given time interval of observations are inversely proportional to each other (Muijs, 2004). This means when the time is closer, there will be mere similar factors that contribute to error (Muijs, 2004). A correlation of test-retest reliability is statistically quantified in the interval of zero and one where 1 being highly correlated in the test and the retest (Muijs, 2004). Perfection is ideal and most researchers accept a lower level, i.e. highly related when all items tend to measure 0.7, 0.8 or 0.9, depending upon the particular field of research.

Based on the principle above, to see the reliability of test instruments of this study, 20 pre-engineering students in 2015/2016 were randomly selected from one of non-participated universities and the test–retest was conducted in 30 minutes' interval and its correlation was 0.998 (see Appendix 2) which indicates that test instrument is reliable.

3.7.Permission to Conduct Research at an Institution (Ethical Issue)

Since the course that was delivered was pre-calculus mathematics, which is not delivering particularly for engineering students in any of Ethiopian universities, it was not depends on universities legislation. For this study to be conducted at the selected universities, consent letter for holding the study was received by the researcher before any research activities (in ethical clearance Appendix 7). In addition, respondents or participants themselves were asked to volunteer and to handle the number of attachments of consent letter of every student, since they have class representative student that they already had been selected, they gave their consent letter through their representative student. Moreover, the participant instructors were classroom teachers that have been teaching Applied Calculus 1 for first year participant students in 2016/2017, and they became the volunteer participants to help the researcher at their part time to conduct the tutorial classes for the study. For this reason, they were asked to give their consent for their willingness to participate in this study (in ethical clearance Appendix 7).

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1.Introduction

In this chapter presentation, analyses, interpretation and discussion of data are presented. The data that used in this study collected through questions of Pre-test, posttest, from pre-calculus mathematics, and class room score results of applied calculus 1 are presented with the help of Tables and figures. The analyses of data took place based on two statistical methods; that are descriptive statistics (percentage, mean, standard deviation (STA.DEV), skewness, and kurtosis) and inferential statistics (T Test, one-way analysis of variance (ANOVA), and correlation of variance). Then the interpretation of hypotheses regarding the influence of pre-calculus mathematics refreshment module to first year engineering students and comparison of the effects of the intervention methods on each group with moderator variable (male and female) in pre-calculus mathematics achievement and interactional effects between each sex and intervention method were identified. Finally, discussion of findings regarding hypothesis followed by summary is presented.

4.2.Results from Pre-test

The Pre-test that had been prepared by the researcher was used to assess not only students' prior knowledge but also it helped the researcher to start the study from the initial ground bases. There were four groups (i.e. MCL CL, Cont T with novice and Cont no int.) with moderator variables (i.e. male and female) and selected pre-calculus mathematics dependent variables. The Pre-test scores were analyzed using descriptive statistics (percentage, mean, Sta.D Skewness, and Kurtosis) and inferential statistics (ANOVA, independent sample T-test). These tools were implemented to check the extent of pre-engineering students recall questions of selected pre-calculus mathematics concepts of each item and to determine scores distribution; if they were different significantly among groups' mean achievements as well as between the same sex across each group, and the whole male students against the whole female students. The data was compiled and analyzed using SPSS computer software package for windows' version 20. The Pre-test results were used to assess and explore groups' equivalence and students' prior knowledge on each item of questions of selected pre-calculus mathematics concepts.

4.2.1. Students' achievement on selected pre-calculus mathematics

In this subsection analysis of Pre-test results of the study is presented. To explore the effect of refreshment of selected pre-calculus mathematics on Applied Calculus 1 for engineering students, first identifying the baseline of students' background knowledge of those selected pre-calculus mathematics is important. For this reason, students were made to take the Pre-test at the first class of the school year. Descriptive statistics was used to examine students' prior knowledge on selected pre-calculus mathematics scores as follows:

Table 2: Descriptive statistics: students' Pre-test results on selected pre-calculus mathematics out of 100%

Descriptive Statistics							
Groups	N	Min	Max	Mean	St.D	Skewness	Kurtosis
- **Cont. no Inter.**	50	0.00	86.00	27.46	20.4	1.07	0.45
- **Cont. T novice**	50	1.00	60.00	27.46	15.95	.32	-0.88
- **CL**	49	6.00	66.00	26.22	17.54	0.97	-0.4
- **MCL**	50	3.00	65.00	25.68	15.47	.81	0.5

Table 2 shows the minimum, maximum, means, standard deviations, skewness and kurtosis for results of Pre-test by class. As table shows there is a slight difference of means across those four groups but, all means of pre-calculus mathematics Pre-test result across four groups (Cont No Int, Cont T, CL and MCL) is less than 28 out of 100.

Before implementing an ANOVA to see the significant difference of groups' achievement scores on Pre-test, normality of groups' were checked and there were no confirmation of lack of normality in the St.D(Cont No Intervention =20.4), skewness (Cont No Intervention = 1.07), and kurtosis (Cont No Intervention = 0.45); St.D (Cont T novice =15.95), skewness (Cont T novice = .32, and kurtosis (Cont T novice = -.88); St.D (CL =17.54) skewness (CL = 0.97, and kurtosis (CL = -0.4); and St.D (MCL =15.47), skewness (MCL = .81, and kurtosis (MCL = 0.5). From the above descriptions, the data distributions skewed to the right side and two groups kurtosis were peaked and the other two groups kurtosis were flat. From definition of Skewness and Kurtosis: - Skewness characterizes the symmetry of collected data distribution. If the distribution of data skewed toward the left, we call it negatively skewed. This happens when Mean < Median < Mode. If distribution of data skewed toward the right, we call it positively skewed. This happens when mean > Median > Mode (Brown, 2011).

According to Brown (2011), kurtosis gives information of group distribution of peakedness or flatness compared to normal distribution. And in number positive kurtosis represents relatively distribution is peaked in contrary negative kurtosis represents relatively flat (Brown, 2011). And Levene test of Pre-test is detected (Sig = 0.094) (See Appendix 3A) which indicates no violation of homogeneity. To run ANOVA first, homogeneity of variance had been identified/ tested to determine the path that would be used. If the p value is significant (less than 0.1 for 90% confidence level), then the variance of the subgroups is not homogeneous estimated using "Tamhane's T_2 (Gupta, 1999), but as it has been observed above the data is homogeneous so ANOVA test was conducted by using the ANOVA for "Post hoc" Tuky path. Having this in mind, the results of pre-calculus mathematics Pre-test is presented as follows:

Table 3: ANOVA of significance difference among totality of all four groups on Pre-test result

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	121.295	3	40.432	.133	.940
Within Groups	59656.300	196	304.369		
Total	59777.595	199			

Source: Survey result. The significant level of mean variation is at p = 0.05

The result represented in Table 3 shows that F= 0.133 and Sig = 0.94 which indicates that the result across four groups on Pre-test of pre-calculus mathematics regarding prior knowledge of pre-engineering students is not significantly different and observed mean difference is very small and calculated effect size (eta square) is 0.002 which is negligible. Since the groups were on the same baseline there was no need to see ANOVA difference indicator post hoc Tuky test among the groups. This means that the results across the four groups were not considerably different.

And to compare the mean score of Pre-test of the male moderator variables of four groups, those are control male students with no intervention (Cont M No Int), control male students with Traditional novice (Cont Novice T M), Experimental male students with co-operative learning (CL M), and Experimental male students with meta-cognitive with co-operative learning (MCL M), descriptive statistics is described as follows:

Table 4: Descriptive statistics of Pre-test result of Moderate Variable (male students)

	N	Min	Max	Mean	St.D	Skewness	Kurtosis
- Cont. no Int M	25	9.00	70.00	33.20	17.35	.47	-0.74
- Cont. Novice T M	25	5.00	56.00	29.72	16.85	0.03	-1.48
- CL M	24	8.00	66.00	31.88	17.26	0.6	-0.68
- MCL M	25	3.00	65.00	26.16	20.33	0.72	-0.72

To explore the effect of refreshment of selected pre-calculus mathematics on Applied Calculus 1 on male pre-engineering students, first let see baseline of male students' background knowledge on those that selected pre-calculus mathematics. Table 4 shows the minimum range from 3 to 9; mean interval from 26 to 33 out of 100; standard deviations interval from 17 to 20 and maximum interval from 56 to 70 for Pre-test scores by selected four classes. As indicated in Table 4, there is slight difference of mean across the four male groups but all means of pre-calculus mathematics Pre-test across the four groups (Cont no Int. M, CL M, Cont. Novice T M and CLM M) is less than 34 out of 100.

Before implementing an ANOVA to see the significant difference of male students groups' achievement scores on pre-test, normality of groups' was checked and there were no confirmation of lack of normality in the St.D (Cont. no Int. M =17.35), skewness (Cont. no Int. M = .47, and kurtosis (Cont. no Int M =-0.74); St.D (Cont. Novice M =16.85), skewness (Cont. Novice M = .03), and kurtosis (Cont. Novice M = -1.48); St.D (CL M =17.26), skewness (CL M = 0.6, and kurtosis (CL M =-0.68); and St.D (CLM M =20.33), skewness (CLM M = 0.72, and kurtosis (CLM M = - 0.72) values where all scores of pre-calculus mathematics result of four groups skewed to the right side and their kurtosis also flat form. And levene's test (pre-test) is detected $p > 0.05$ (Sig = 0.902) not violate the homogeneity of variance (See Appendix 3B). As it is observed above the data is homogeneous so ANOVA test was conducted by using the ANOVA for "Post Hoc" Tuky path and the results of Pre-test is described as follows:

Table 5: ANOVA of male students in each group on Pre-test result

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	715.150	3	238.383	.736	.533
Within Groups	31111.600	96	324.079		
Total	31826.750	99			

Source: Survey result. The significant level of mean variation is at p = 0.05

Table 5 shows that F= 0.736 and Sig = 0.533 the result is not different significantly across four male groups on Pre-test of prior knowledge in pre-calculus mathematics. And observed mean difference is very small and calculated effect size (eta square) is 0.022 which is small effect size. Since the groups were on the same baseline there was no need to see ANOVA difference indicator post hoc Tuky test among the four male student groups. This means that

ANOVA result of Pre-test achievement of selected topics of pre-calculus mathematics was not significantly different among the four male student groups.

Descriptive statistics was applied as follows to compare the mean scores of Pre-test of the female moderator variables of four groups, those were control female students with no intervention (Cont. no Intr. F), control female students with traditional novice (Cont. Novice F), Experimental female students with co-operative learning (CL F), and experimental female students with meta-cognitive with co-operative learning (MCL F).

Table 6: Descriptive statistics of female students in each group on Pre-test result

	N	Min	Max	Mean	St.D	Skewness	Kurtosis
- Cont. no Int. F	25	0	86	21.68	21.88	1.93	3.13
- Cont. Novice F	25	1	76	27.20	18.11	1.01	1.22
- CL F	24	6	58	20.56	16.22	1.73	1.55
- MCL F	25	9	39	25.20	8.64	-0.32	-0.72

To look at the effect of refreshment of selected pre-calculus mathematics on Applied Calculus 1 for female engineering students, first important to determine a baseline of students' background knowledge regarding selected contents of pre-calculus mathematics. Table 6 shows the minimum, maximum, means, standard deviations, skewness, and kurtosis for Pre-test by class. As indicated in Table 6 there was a slight mean difference across the four female groups but, all means of Pre-test across of pre-calculus mathematics result of four groups (Cont. no Int. F, Cont. Novice F, CL F and MCL F) were less than 30 out of 100.

Before implementing an ANOVA to see the significant difference of female students groups' achievement scores on Pre-test, normality of groups' were checked and there were no confirmation of lack of normality in the St.D (Cont. No Int. F =21.88), skewness (Cont. no Int. F = 1.93), and kurtosis (Cont. no Int. F =3.13); St.D (Cont. Novice F =18.11), skewness (Cont. Novice F = 1.01), and kurtosis (Cont. Novice F = 1.22); St.D (CL F =16.22), skewness (CL F = 1.73), and kurtosis (CL F =1.55); and St.D (MCL F =8.64), skewness (MCL F = -0.32), and kurtosis (MCL F = -0.72) values where all scores of pre-calculus mathematics result of three groups skewed to the right side and one groups skewed to the left side, where one on the left is positively skewed and one on the right is negatively skewed. And levene's test of Pre-test is detected (Sig = 0.082) that does not violate the homogeneity of variance (See Appendix 3C). Therefore, the data was homogeneous so ANOVA test was conducted by using the ANOVA for "Post hoc" Tuky path. Results of Pre-test is presented as follows

Table 7: ANOVA of female students in each group on Pre-test result

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	710.840	3	236.947	.828	.482
Within Groups	27477.600	96	286.225		
Total	28188.440	99			

Source: Survey result. The significant level of mean variation is at p = 0.05

The result in Table 7 shows that F= 0.828 and Sig = 0.482 the results were not different significantly across four female groups on Pre-test of prior knowledge in pre-calculus mathematics. Observed mean difference was very small and calculated effect size (eta square) was 0.025, which is small effect size. Since the groups were on the same baseline, there was no need to see ANOVA difference indicator post hoc Tuky test among the four female students' group. This means that the results were not significantly different across the four female students' group on Pre-test achievement of selected topics of pre-calculus mathematics.

To come across the effect of those selected intervention method for refreshment of selected pre-calculus mathematics on gender difference of first year pre-engineering students, first it was important to determine a baseline of students' background knowledge of selected pre-calculus mathematics. Independent sample T-test was used to examine significant difference of male and female students' prior knowledge on selected pre-calculus mathematics scores.

Table 8: Independent Samples T-Test of female and male in each group on Pre-test

Group		**Levene's Test for Equality of Variances**		**t-test for Equality of Means**						
		F	Sig	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Cont. no Int. F	Cont. no Int. M	.058	.810	-2.06	48	.044	-11.56	5.58659	-22.792	-.327
Cont. Int F	Cont. Int M	.375	.543	-.509	48	.613	-2.520	4.94723	-12.467	7.427
CL F	CL M	.191	.664	-2.11	48	.040	-10.16	4.81208	-19.835	-.484
MCL F	MCL M	12.1	.001	-.217	32.	.829	-.9600	4.41802	-9.955	8.035

Source: Survey result. The significant level of mean variation is at p = 0.05

As it is shown in Table 8 an independent-samples T-test was used to see the significance different between male and female students' prior knowledge of pre-calculus mathematics Pre-test scores in each group: For control with no intervention group males and females Levene's Test (Sig = 0.810) which indicate that equal variance was assumed. As it had been seen in Table

4, there was slight difference in scores (M = 33.2, St.D = 17.35) and females (M = 21.68, St.D = 21.88) t (48) = -2.06, p = 0.044, two-tailed. But this group (control with no intervention) had not taken treatment so the difference did not have influence on this study. The magnitude of the variation of means (mean difference = 11.56, 95% CI: –22.792 to -0.327) was moderate effect size (eta squared = 0.08). For control with intervention group males and females Levenes Test (Sig = 0.543) which indicate that equal variance was assumed. The scores were not significantly different as it was seen in Table 4 (M = 29.72, St.D = 16.8) and females M = 27.2, St.D = 18.11) and in Table 8 t (48) = -.509, p = 0.613, two-tailed. The magnitude of the variation of means (mean difference = 2.520, 95% CI: –12.467 to 7.427) was very small effect size (eta squared = 0.005). For CL group males and females Levenes Test (Sig = 0.664) which indicates that equal variance is assumed. The scores were significantly different as it was seen in Table 4 (M = 31.88, St.D = 17.26 and females Table 6 M = 20.56, St.D = 16.22) and in Table 8 t (48) = -2.11, p = 0.040, two-tailed. The magnitude of the variation of means (mean difference = -10.16, 95% CI: –19.835 to -0.484) was moderate effect size (eta squared = 0.08) therefore, researcher considered the difference of Pre-test result when he analyzed the difference on posttest result. For MCL group males and females Levene's Test (Sig = 0. 001) which indicates that equal variance is not assumed. Therefore, the data was non homogeneous so ANOVA test was conducted by using the ANOVA for "Post hoc" Tamhane's T2. The scores were not significantly different as it was seen in Table 4 (M = 26.16, St.D = 20.33 and females Table 6 M = 25.20, St.D = 8.64) and in Table 8 t (32.4) = -.217, p = 0.829, two-tailed. The magnitude of the variation of means (mean difference = -10.16, 95% CI: –9.955 to 8.035) was very small effect size (eta squared = 0.001).

To come across the refreshment effect of selected pre-calculus mathematics on gender difference in entire group of first year pre-engineering students, first, important to determine a baseline of total female and male students' prior knowledge regarding selected pre-calculus mathematics. Descriptive statistics and independent sample T-test were used to examine significant difference of male and female students' prior knowledge on selected pre-calculus mathematics scores as follows

Table 9: Descriptive statistics of Total female (TF) and Total male (TM) students on Pre-test

Between group	**N**	**Min**	**Max**	**Mean**	**St.D**	**Skewness**	**Kurtosis**
		Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
TF	75	0.00	86.00	23.66	16.93	.403	-.91
TM	74	3.00	70.00	30.23	17.93	1.51	2.42

To identify the effect of refreshment of selected pre-calculus mathematics on Applied Calculus 1 for selected entire female and male engineering students, first, important to determine a baseline of students' prior knowledge of pre-calculus mathematics. Table 9 shows the minimum, maximum, means, standard deviations skewness and kurtosis for Pre-test by female and male students. As it is indicated in Table 9, there was mean difference between female and male students, where mean of female students was 23.66 out of 100 and mean of male students was 30.23 out of 100. Before conducting an Independent sample T-Test to compare the achievement scores on the Pre-test by male and female students, the researcher found no evidence of lack of normality in the St.D (TF=16.93), skewness (TF = .403), and kurtosis (TF= -.91), St.D (TM =17.93), skewness (TM = 1.51), and kurtosis (TM =2.42) values where both groups skewed in the right side and kurtosis of female group was flat form and kurtosis of male was peaked form. Evidence of violation of homogeneity of variance from Levene's test (pretest Levene's statistic, p = 0. 111 (See Table 9) equal variances were assumed between male and female students.

Table 10: Independent Samples T-Test between Total of male and female students

Gender		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Female	Male	2.566	.111	-2.5	198	.011	-6.30	2.468	-11.166	-1.434

Source: Survey result. The significant level of mean variation is at p = 0.05

In Table 10 there was statistically significant differences in scores between males and females' t (198) = -2.55, p = 0.011, two-tailed). The magnitude of the variation of means (mean difference = -6.3, 95% CI: –11.166 to -1.434) with a small effect size (eta squared = 0.032).

Summary: The statistical results indicated that male students in Cont. no Intr. and CL groups achieved significantly higher than female students in Cont. no Intr. and CL groups respectively in Pre-test of pre-calculus mathematics; In T and MCL groups, even though mean of male students was slightly higher than female students, there were no significant differences between two genders in Pre-test of pre-calculus mathematics. As total groups of both genders, there was statistically significant difference in scores of Pre-test on pre-calculus mathematics between males and females of pre-engineering first year university students. Therefore, the researcher considered the difference of Pre-test when he analyzed the difference on posttest.

4.3. Research Question Number One

To what extent first year pre-engineering students recall some basic pre-calculus mathematics? Descriptive frequency and percentage of statistical results of pre-engineering students on selected topics of pre-calculus mathematics were presented as follows:

4.3.1. Algebra of Exponents

Simplify the following algebraic exponential expressions? Do not leave negative exponents in your final answer. Leave all answers in fully reduced form.

Table 11: Simplification of algebraic exponential expressions

Item	Male				Female				Total			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	f	%	f	%	F	%	f	%	F	%	f	%
A, y^3 y^4	66	65.2	35	44.9	44	34.1	54	55.1	110	55.3	89	44.7
B, (y^3)^4	57	56.4	43	42.6	57	58.2	41	41.8	114	57.3	84	42.8
C, (3a^4)^2	37	36.6	64	63.4	33	33.7	65	66.3	70	35.2	129	64.8
D, 2^0	60	59.4	40	39.6	45	45.9	49	50	105	52.8	89	44.7
E, (1/4)^(-2)	36	35.6	65	64.4	29	29.6	69	70.4	65	32.7	134	67.3
F, (-2x)^(-4)	10	9.9	91	90.1	8	8.2	90	91.8	18	9.0	181	91.0
G, ((3x^2y^(1))/ (x^(-1) y^2))^(-2)	6	4.9	96	95.1	5	5.1	93	94.9	11	5.0	189	95.0

As it can be seen from Table 11, the first item educes information on the degree of male and female first year pre-engineering students able to simplify different exponents with the same base question. When asked to simplify y^3y^4 item 44.9% of male students and 55.1% of female students i.e. 44.7% of the total students did not correctly recall that it was simplified to $y^{3+4} =$

y^7. Regarding second item $(y^3)^4$, 42.6% of male students and 41.8% of female students i.e. 42.8% of the total students did not correctly recall that this simplified to $y^{3x4} = y^{12}$; the third item educes information on the degree to recall simplification of $(3a^4)^2$. When asked to simplify $(3a^4)^2$, 63.4% of male students and 66.3% female students i.e. 64.8% the total students did not recall that it was simplified to $3^2\ a^{4x2} = 9a^8$; the fourth item educes information on the degree of male and female first year pre-engineering students able to simplify 2^0. To this item 39.6% of male and 52.8% female i.e. 44.7% of the total students did not simplify it; the fifth item educes information on the degree of male and female first year pre-engineering students to convert negative power into positive power of $\left(\frac{1}{4}\right)^{-2}$. On this item 64.4% of male and 70.4% of female students i.e. 67.3% of the total students did not recall that it was convert to 4^2; the sixth item educes information on the degree of male and female first year pre-engineering students able to simplify negative number with variable in negative power $(-2x)^{-4}$. When asked to simplify $(-2x)^{-4}$, 90.1% of male students and 91.8% of female students it i.e.91% of the total students would not correctly recall that it was simplified to $(-2)^{-4}\ x^{-4} = \frac{1}{(-2)^4} x^{-4} = \frac{1}{16x^4}$; the seventh item is the general term of all the above power expression items when students asked to simplify $\left(\frac{3x^2y^{-1}}{x^{-1}y^2}\right)^{-2}$, 95.1% of male students and 94.9% of female students i.e. 95% of the total students did not simplify it correctly.

4.3.2. Polynomials

Table 12: Simplify the following polynomial expressions

Item	Male				Female				Total			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	F	%	F	%	f	%	f	%	f	%	f	%
A, 2x+3y-4x+5y	42	42.4	57	57.6	38	38	62	62	90	45.2	109	54.8
B, (7x^2 y^2+4xy^2-5x)-(4x^2y^2-3xy^2+5)	18	18.2	81	81.8	13	13.4	87	87.6	41	20.6	158	79.4
C, 7a^3 (4a^2-5a)-2a^2 (3a^3-6a^2)	17	17.2	82	82.8	23	23	77	77	50	25.1	149	74.9
D, (2a+b)^2-(2a-b)^2	33	33.3	66	66.7	35	35.7	65	65.3	68	34.2	131	65.8

As it can be seen from Table 12, the first item educes information on the degree of male and female first year pre-engineering students to simplify $2x + 3y - 4x + 5y$; to this item 57.6% of male students and 62% of female students i.e. 54.8% of the total students did not simplify it; the second item educes information on the degree of male and female first year pre-engineering students simplify $7a^3(4a^2 - 5a) - 2a^2(3a^3 - 6a^2)$. For this item 81.8% of male students and 87% of female students i.e. 79.4% of the total students did not simplify it; the third item educes information on the degree of male and female first year pre-engineering students able to simplify$(7x^2y^2 + 4xy^2 - 5x) - (4x^2y^2 - 3xy^2 + 5)$. For this item 82.8% of male and 77% female students i.e. 74.9% of the total students did not simplify it; The fourth item educes information on the degree of male and female first year pre-engineering students able to simplify $(2a + b)^2 - (2a - b)^2$. For this item 66.7% of male and 65.3% of female students i.e. 66% of the total students did not simplify it.

4.3.3. Factorization of basic algebraic expressions

Table 13: Factorize the following algebraic expression in to simplest form

Item	Male				Female				Total			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	F	%	F	%	f	%	F	%	f	%	f	%
A, 10b^3 c^2 - 5 cb^2	24	24.2	75	75.8	25	25	75	75	49	25.6	148	75.3
B, x^2 + 8x + 16	36	36.4	63	63.6	36	36	64	64	74	37.7	124	62.6
C, x^2 - 7x – 18	35	35.4	64	64.6	32	32	68	68	67	33.7	132	66.3
D, x^3 + 1000	4	3	96	97	4	3	97	97	6	2.5	194	97.5
E, 4x^4 -16	1	1	98	99	3	3	97	97	4	2.0	195	98.0
F, 8x^3 – 27	3	3.0	96	97.0	1	1	99	99.	4	2.0	195	98.0

As it can be seen from Table 13, the first item educes information on the degree of male and female first year pre-engineering students regarding how to factorize polynomial questions. To the first item $(10b^3\,c^2 - 5\,cb^2)$, 75.8% of male students and 75% of female students i.e. 75.3% of the total students were not able to factorize it; the second item educes information on the degree of male and female first year pre-engineering students able to factorize $x^2 + 8x + 16$. For this item 63.6% of male students and 64% of female students i.e. 62.6% of the total students were not able to factorize it; the third item educes information on the degree of male and female first year pre-engineering students to factorize $x^2 - 7x - 18$. For this item 64.6% of male and 68% of female i.e. 66.3% total of students were not able to factorize it; the fourth item educes information on the degree of male and female first year pre-engineering students factorize $x^3 + 1000$. For this item 97% of male and 97% of female students i.e. 97.5% of the total students were not able to factorize it; the fifth item educes information on the degree to what extent male and female first year pre-engineering students factorize $4x^4 - 16$. To this item 99%

of male and 97% of female students i.e. 98 % of the total students were not able to factorize it; the sixth item educes information on the degree to what extent male and female first year pre-engineering students factorize $8x^3 - 27$. For this item 97% of male students and 99% of female students' i.e.98% of total students were not able to solve it.

4.3.4. Rational expression and Radicals

Table 14: Solve the following rational expression

Item	Male				Female				Total			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	f	%	F	%	f	%	F	%	f	%	f	%
4. Rational expression: Solve the following												
A. $-2\left(-\frac{1}{2}-\frac{4}{3}+\frac{5}{6}\right)/\frac{2}{3} = \_$	24	38.3	76	61.7	30	31.9	64	68.1	54	27.8	140	72.2
B. $3x + 42 \leq -12$	22	35.8	79	64.2	22	23.4	71	75.5	44	22.7	150	77.3
C. $2x + y = 8$ $and\ x - y = 1$	44	44.7	55	55.3	46	48.9	49	51.1	90	46.4	104	53.6
D. Put the inequalities $<, >, \leq or, \geq$ If $\varepsilon > \frac{1}{\sqrt{x}}$, then $1/x^2$ __ ε	40	40.4	59	59.6	33	35.2	61	64.8	73	37.6	121	62.2
E.. Solve for x $\frac{1}{x^2+5x+6} = \frac{1}{x+3}$, then x =__	27	22	72	72.7	13	13.9	81	86.1	40	20.6	154	79.4
5. Solve the following Rational Exponents and Radicals												

A. $8^{\frac{2}{3}} = \_$	39	36.5	60	60.6	41	44.7	54	55.3	80	41.2	114	58.8
B. $\left(\frac{9}{8}\right)^{\frac{3}{2}} = \_$	5	4.6	94	95.4	1	1.1	94	98.9	6	3	188	97
C. $\frac{2-\sqrt{5}}{2+3\sqrt{5}} = \_$	3	8.1	91	91.9	2	2.1	98	97.9	5	2.5	189	97.5
D. $4^{\frac{-2}{3}} = \_$	1	6.3	93	93.1	4	4.3	95	95.7	5	2.5	189	97.5
E. $\left(\frac{8}{27}\right)^{\frac{-2}{3}} = \_$	21	21.2	78	78.8	14	14.9	80	85.1	35	18.6	158	81.4
F. $\sqrt{\frac{32x^3}{9x}} = \_$	5	8.1	91	91.9	9	5.3	89	94.7	14	7.2	180	92.8

As it is seen under question four in Table 14, the first item educes information on the degree of male and female first year pre-engineering students able to solve rational questions which needs knowledge of BODMAS (Brackets Order Division Multiplication Addition and Subtraction) of mathematical operations. To item $-2\left(-\frac{1}{2}-\frac{4}{3}+\frac{5}{6}\right) \div \frac{2}{3} = \_$ 61.7% of male students and 68.1% of female students i.e. 72.2% of total students were not able to solve it; the second item educes information on the degree of male and female first year pre-engineering students to solve linear inequalities by using transformation rule $3x + 42 \leq -12$. For this item 64.2% of male students and 75.5% of female students i.e. 77.3% of the total students were not able to solve it; the third item educes information on the degree of male and female first year pre-engineering students able to solve simultaneous equation $2x + y = 8\ and\ x - y = 1$. For this item 55.3% of male and 51.1% of female i.e. 53.6% of the total students were not able to solve it; the fourth item educes information on the degree of male and female first year pre-engineering students able to compare rational expression or to put inequality sign($<, >, \leq or, \geq$) based on given information If $\varepsilon > \frac{1}{\sqrt{x}}$, then $1/_{x^2}$ __ ε . For this item 59.6% of male and 64.8% of female students i.e. 62.2% of the total students were not able to put the sign correctly; the fifth item educes information on the degree of male and female first year pre-engineering students to solve rational expression of equations by using LCM or cress-cross method to solve x, $\frac{1}{x^2+5x+6} = \frac{1}{x+3}$. To this item, 72.7% of male and 86.1% of sampled female students i.e. 79.4 % of the total student were not able to answer it; for fifth question the first item educes information on the degree of male and female first year pre-engineering students regarding how to factorize and solve integral numbers with rational exponent $8^{\frac{2}{3}} = \_\_$. For this item 60.6% of sampled male students and 55.3% of sampled female students' i.e.58.8% of the total students were not able to solve it; the second item educes information on the degree of male and female first year pre-engineering students with respect to how to factorize and solve rational expression of numbers with rational exponent $\left(\frac{9}{8}\right)^{\frac{3}{2}} = \_$. For this item 95.4% of male students and 98.9% of female students' i.e.97% of the total students were not able to solve it; the third item educes information on the degree of male and female first year pre-engineering students regarding how to simplify by rationalizing denominator of fractional expressions of real numbers $\frac{2-\sqrt{5}}{2+3\sqrt{5}} = \_$. For this item 91.9% of male students and 97.9% of female students i.e.97.5% of the total students were not able to rationalize the denominator; the fourth item educes information on the degree of male and female first year pre-engineering students able to solve positive integers with negative rational exponent $4^{\frac{-2}{3}} = \_$. For this item 93.1% of male students and 95.7% of female students i.e. 97.5% of the total students were not able to solve it; the fifth item educes information on the degree of male and female first year pre-engineering students able to simplify or solve rational numbers with negative rational exponent $\left(\frac{8}{27}\right)^{\frac{-2}{3}} = \_$. For this item 78.8% of male students and 85.1% of female students' i.e.81.4% of the total students were not able to simplify it; the sixth item educes with information on the degree of male and female first year pre-engineering students able to simplify rational expressions in the radical signs $\sqrt{\frac{32x^3}{9x}}$. For this item 91.9% of male students and 94.7% of female students i.e. 92.8% of the total students were not able to simplify it.

4.3.5. Functions

Functions play a major role in the engineering applied Calculus curriculum. However, a student face challenges regarding complex concepts of functions and have difficulty dealing with varies representations that are inherent to functions. Without understanding functions, it is impossible to learn other concepts in undergraduate applied Calculus (Hauser, 2015). The following Table shows the level of pre-engineering first year students' understanding with relation and functions:

Table 15: Solve the following functional equations

Items	**Male**				**Female**				**Total**			
6. Functions	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	F	%	F	%	F	%	f	%	f	%	f	%
A. Let $f(x) = -9 - 3x$ Solve for $x = -1$	60	60.6	39	39.4	49	53.8	42	46.2	109	57	81	43
B. $f(x) = 4\sqrt{x}$ Solve for $x = -1$	43	43.4	56	56.6	42	46.2	49	53.8	85	45	105	55
C. $f(x) = \|2x - 4\|$ Solve for $x = -3$	44	44.4	55	55.6	49	53.8	42	46.2	93	49	97	51
D. let $\|3x + 4\| = 6$	36	36.4	63	63.6	24	26.4	67	73.6	60	32	130	68

$then \quad x = \_$												
E. Let $f(x) = \frac{1}{x^2}$ then $f(0) = \_$	49	49.5	50	50.5	24	26.4	67	73.6	73	38	117	62
7. Domain and Range:												
A $f(x) = 2x + 1,$ $D = \_\_ \quad R = \_\_$	44	44.4	55	55.5	27	29.7	64	70.3	71	37	119	63
B. $f(x) = \sqrt{x^2 - 4}$ $D = \_\_ \quad R = \_\_$	18	18.2	81	81.8	9	9.9	82	90.1	27	14	163	86
C. $f(x) = \sqrt{\frac{x-2}{x-1}},$ $D = \_\_ \quad R = \_\_$	5	5.1	94	94.9	5	5.5	86	94.5	10	5	180	95
D. $f(x) = log_2(x+2)$ $D = \_\_ \quad R = \_\_$	9	9.1	90	90.9	8	8.8	83	91.2	17	9	173	91
8.Operation on Functions: Let $F(x) = x + 2, \quad g(x) = 2x^2, \quad h(x) = \frac{x+1}{x-1}$												
A. $f(2) - g(3) =$	58	58.6	41	41.4	25	27.5	66	72.5	83	43.6	107	56.4
B. $g(x^2)$=	52	52.5	47	47.5	25	27.5	66	72.5	77	40	113	60
C. $\left(g(x)\right)^2 =$	45	45.5	54	54.5	38	41.8	53	58.2	83	44	107	56.3
D. $h(-2) =$	53	53.5	46	46.5	37	40.7	54	59.3	90	47	100	53
9. Composite Function: find the following composition functions, given $f(x) = x^3, g(x) = x - 9, h(x) = \frac{\sqrt{x}-4}{x+4}$												
A. $g\left(h(x)\right) = \_\_\_$	47	42.5	52	52.5	21	23.1	70	76.9	68	36	112	64
B. $f\left(g(-4)\right) = \_\_\_$	27	27.3	72	72.7	35	27.5	66	65.3	62	33	138	69
C.	22	22.2	77	77.8	28	30.8	63	69.2	50	26	140	74

$f(g(h(16)))$ ___												
10. Find the vertical and horizontal asymptotes of the following functions												
A. $f(x) = \frac{2}{x^4-16}$,	16	16.2	83	83.8	21	23.1	70	76.9	37	19	153	81
B. $f(x) = \frac{1-x}{x-2}$,	17	17.2	82	82.8	12	13.2	79	86.8	29	15	161	85
C. $f(x) = \frac{x^3-8}{x+2}$	11	11.1	88	88.9	11	12.1	80	87.9	22	12	168	88
D. $f(x) = \frac{2x^3+2}{x^3+x^2-2x}$	13	13.1	86	86.9	7	7.7	84	92.3	20	11	170	89

Question number 6 in Table 15 focuses on numerical solution of functions. Regarding this, one can observe that the first item of question number 6 educes information on the degree of male and female first year pre-engineering students to find numerical solution of $f(x) = -9 - 3x$ at $x = -1$. For this item 39.4% of male students and 46.2% of female students i.e. 43% of the total students were not able to solve numerical $f(-1)$; the second item educes information which helps to assess students' skill of working out square root on the set of real numbers $f(x) = 4\sqrt{x}$ at $x = -1$. To this item 56.6% of male students and 53.8% of female students i.e. 55% of the total students did not determine its solution; the third item educes information on the degree of students' achievement on the concept of numerical solution of absolute value functions $f(x) = |2x - 4|$ at $x = -3$. For this item 55.6% of male students and 46.2% of female students i.e. 51% the total students were not able to solve it; the fourth item educes information on the extent of finding the solution set of absolute value equation $|3x + 4| = 6\ then\quad x = \_$. For this item 63.6% of male and 73.6% of female i.e. 68% of the total students were not able to find the solution set; the fifth item educes information to what extent the students understand numerical value of function with denominator zero $f(x) = \frac{1}{x^2}$

then $f(0) = \_$. For this item 50.5% of male 73.6% of female students i.e. 62 % of the total students were not able to decide it. Question number 7 in Table 15 was focuses on domain and range of functions. The basic idea that differentiates functions from relation is domain and range.

Relations have no restriction on pairing domain and range, while function restricted on one element of domain paired only with one element of range. The set of all values included in a function and have real value of numbers is the domain of a function. For this reason, the domain excludes the division of any number by zero, negative square roots, as well as zeros and negative numbers of logarithms. So, domain of functions matters like division by zero, negative square roots, logarithms of zero and negative numbers are values that operation cannot take. Students' understanding of domain and range of functions are fundamental for any Calculus course especially for applied calculus. Basically definition of a function needs domain and range. Regarding this, in Table 15 one can observe the extent of pre-knowledge of first year pre-engineering students on domain and range of different functions. Under question number 7 the first item educes information on the degree of male and female first year pre-engineering students to find domain and range of $f(x) = 2x + 1$ which is very elementary and linear function. For this item 55.5% of male students and 70.3% of female students i.e. 63% of the total students were not able to find domain and range; the second item educes information which helps to assess students' ability to find out domain and range of radical function $f(x) = \sqrt{x^2 - 4}$. Pre-engineering students ensured that domain and range of radical function would be a real number that radicand was not-negative. For this item 81.8% of male students and 90.1% of female students i.e. 86% of the total students did not answer it correctly; the third item educed information of students' achievement on how to find the domain of radicals and rational functions in composition. Pre-engineering students inquired that domain of composition of radical and rational function is the intersection of domain of radical and rational function $f(x) = \sqrt{x-2/_{x-1}}$. For this item 94.9% of male and 94.5% of female students i.e. 95% of the total students were not able to find out the answer; the fourth item educed information on the extent of finding the domain and range of logarithmic function like $f(x) = log_2(x+2)$. For this item 90.9% of male and 91.2% of female students i.e. 91% of the total students were not able to find domain and range.

Question number 8 focused on operation of two or more function ($F(x) = x + 2$, $g(x) = 2x^2$, $h(x) = \frac{x+1}{x-1}$) with numbers and variables. The first item educed information on the extent of male and female first year pre-engineering students regarding how to find numerical value of $f(2) - g(3) =$__ . For this item 41.1% of male students and 72.5% of female students i.e. 56.4% of the total students were not able to solve the equation; the second item educed information which helped to identify students' ability to substitute any variable into a given function $like\ g(x^2)$ =_. For this item 47.5% of male students and 72.5% of female students i.e. 60% of the total students were not able to answer correctly; the third item educed information concerning students' awareness about how to simplify functions in power. Pre-engineering students ensured that the difference between substituting power of variable and simplification of power of function like $\left(g(x)\right)^2 =$. For this item 54.5% of male and 58.2% of female students i.e. 56.3% of the total students were not able to simplify it; the fourth item educed information about how to find the numerical solution of negative number over negative function like $h(-2) = -$. For this item 46.5% of male and 59.3% female i.e. 53% of the total students were not able to find the solution. Question number 9 in Table 15 focuses on numerical solution and simplification algorithm of composite function ($like\ f(x) = x^3, g(x) = x - 9,\ h(x) = \frac{\sqrt{x}-4}{x+4}$.

The first item educed information on the degree of male and female first year pre-engineering students to simplify algorithm of composite function of $g\left(h(x)\right) = \_\_$. For this item 52.5% of male students and 76.9% of female students i.e. 64% of the total students were not able to put correct algorithm; the second item educes information concerning the students' ability to use numerical solution for composition function $like\ f\left(g(-4)\right) = \_\_\_$. For this item 72.7% of male students and 65.3% of female students i.e. 69% of the total students were not able to answer it correctly; the third item educes information of students' skill on how to find numerical solution of three composed functions like $f(g\left(h(16)\right)) = \_\_$. For this item 77.8% of male and 69.2% of female students i.e. 74% of the total students were not able to find numerical solution. Question number 10 in Table 15 focused on vertical and horizontal asymptote of functions. The concepts of vertical and horizontal asymptote are fundamental to a student's understanding of Applied Calculus 1, 2, 3 in the field of engineering. Normally, asymptote is related to the graph of a function and point on the graph, namely it is mandatory for engineering students to identify, vertical, horizontal and oblique asymptote. The graph of a function is said to be near to the line asymptotically and the line is an asymptote of the graph of a function if the distance of the graph with respect to its fixed line approaches toward zero. While polynomial functions do not have vertical and horizontal asymptote, they often occur in rational functions. Especially vertical asymptote is basic for innovation of the limit concept of calculus like $\lim_{x\to 0^+} {}^{1}/_{x} = \infty$ rather than saying undefined at $x = 0\ which\ is\ a\ vertical\ asymptote$. Concerning this in Table 15 one can observe that the first item educed information on the degree of male and female first year pre-engineering students to find vertical and horizontal asymptote of $f(x) = \frac{2}{x^4-16}$, where it needs the concept of factorizing the denominator $x^4 - 16$ by using the concept of $(a + b)(a - b) = a^2 - b^2$ to $(x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 +$

$4)$ $and\ finding\ the\ zeros\ of\ equation.$ For this item 83.8% of male students and 76.9% of female students i.e. 81% of the total students were not able to find vertical and horizontal asymptote; the second item educes information about students' ability of computing and finding quotient of coefficient of equal leading degrees for horizontal asymptote and the zeros of denominator for vertical asymptote of $f(x) = \frac{1-x}{x-2}$. For this item 82.8% of male students and 86.8% of female students i.e. 85% of the total students were not able to find vertical and horizontal asymptote; the third item educed information on the extent of students' ability on the concept of factorizing third degree polynomial and simplify $f(x) = \frac{x^3-8}{x+2}$. For this item 88.9% of male and 87.9% of female i.e. 88% of the total students were not able to find vertical and horizontal asymptote; the fourth item educed information on how to use more than two or more techniques together at the same time of $f(x) = \frac{2x^3+2}{x^3+x^2-2x}$. For this item 86.9% of male and 92.3% female i.e. 89% of the total students were not able to find vertical and horizontal asymptote.

4.3.6. Exponential and Logarithmic functions

Exponential and logarithmic functions are essential mathematical concepts that play vital roles in advanced mathematics. Researchers and educators alike have recognized central roles of exponential and logarithmic functions in applied Calculus (Weber, 2002.).

As stated in Musqueeny (2012), that "Logarithm now seen as the inverse of the exponential function or as a meaningful application in mathematical sciences yet students' understanding of this mathematical concept is restricted. Students regularly report "seeing" the material in earlier coursework but report they have forgotten the "rules." However, once presented the right rule they are able to achieve the needed calculations". For this reason, it is significant to revise the concepts of exponential and logarithmic functions for the pre-engineering students.

The researcher reports an empirical study in which he investigated students' understanding of exponential and logarithmic functions as follows

Table 16: Converting exponential function into logarithmic and vice versa

Item	**Male**				**Female**				**Total**			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	f	%	f	%	F	%	f	%	f	%	f	%
11. Exponential and Logarithmic Functions												
A. $3^x = 243, then\ x = \_$	48	49.5	49	50.5	47	49.5	50	51.5	95	48	97	51
B. $8^x = 4, then\ x = \_$	47	48.5	50	51.5	35	36.1	62	63.9	82	41	112	59
C. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$ then x = _	39	40.2	58	59.8	42	43.3	55	56.7	81	41	113	58
D. $7^x = \frac{1}{49}$, then x = __	42	43.3	55	56.7	36	37.1	61	62.9	78	39	116	61
E. $\frac{4^x}{4^{2x}} = 64\ then\ x =$	29	29.9	68	70.1	24	24.7	73	75.3	53	27	141	73
F $\left(\frac{1}{16}\right)^{x-3} = 8^{2x-1}, then\ x =$	15	7.7	82	84.5	8	8.3	89	91.7	23	14	171	86
G. $25^{\sqrt{x}} = 625^x$ then =	7	7.2	90	92.8	2	2.1	95	97.9	9	5	185	95
H. $\left(\frac{1}{2}\right)^x = 32$	36	37.1	61	62.9	27	27.8	70	72.1	63	32	131	68

12. Exponential to Logarithmic												
A. $a^x = b$, then $log\_\_ = -$	28	28.9	69	71.1	17	17.5	80	82.5	45	23	149	77
B. $10^3 = 1000$ then $log\_\_ =$	25	25.8	72	74.2	19	19.6	78	80.4	44	22	150	78
C. $9^0 = 1$, then $log\_\_ -$	22	22.7	75	77.3	12	12.4	85	87.6	34	17	160	83
D. $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$, then $log\_\_$	23	23.7	74	76.3	10	10.3	87	89.7	33	17	161	83
13. Logarithm to Exponential												
$A. log_2 64 =$, then exp formis	19	19.6	78	80.4	14	14.4	83	85.6	33	17	161	83
B. $log_8 1 =$ then exp formis	18	18.4	79	81.4	9	9.3	88	90.7	27	14	167	86
C. $log_{\frac{1}{3}}\left(\frac{1}{9}\right) =$, then exp formis	20	20.6	77	79.4	10	10.3	87	89.7	30	15	164	85
$D. log\ 0.01 =$, then exp formis	20	20.6	77	81.4	12	12.4	85	87.6	32	16	162	84
E. $ln\, x = a$, then exp formis	11	11.3	86	88.7	5	5.2	92	94.8	17	9	177	91
14. Properties of Logarithm:												
A., $log_4 2x + log_4 4x =$	11	11.3	86	88.7	2	2.1	95	97.9	13	7	181	93
B. $log_2 4x - log_2 8x =$	20	20.6	77	79.4	8	8.2	89	91.7	28	14	166	86
C, $log \frac{4x}{3y} =$	16	16.5	81	83.5	12	12.4	85	87.6	28	14	166	86
$D. log_3 x + log_3 y =$	8	8.2	8	91.8	3	3.1	94	96.9	11	6	183	94

			9									
E. $3(log_2 x + 2log_2 y - log_2 z)$	4	4.6	93	95.4	1	1.1	96	98.9	5	3	189	97
15 Solve the following equation												
A. $log_5(x+2) = 1$	34	35.1	63	64.9	26	26.8	71	73.2	60	30	134	70
B. $3\ln 2 + \ln(x-1) = \ln 24$	10	10.3	87	89.7	3	3.1	94	96.9	13	7	181	93
C. $log_3 x = log_3 2 + log_3(x-3)$	4	4.1	93	95.9	2	2.1	95	97.9	6	3	188	97
D. $\ln 5 - \ln x = -1$	2	2.1	95	97.9	0	0	97	100	2	1	192	99

Exponents and logarithms continue to play an important role in mathematics (most significantly in calculus), science, and engineering (Kastberg & Rechael, Links in learning logarithms, 2017). Therefore it is important for students to understand exponents and logarithms as real numbers as well as the characteristics of functions. Table 16 shows the extent of recalling exponential and logarithmic functions.

Question number 11 focuses on finding the solution of exponential functions, regarding this in Table 16 one can observe that the first item induces information on the degree of male and female first year pre-engineering students to find the solution of $3^x = 243, then\ x =$ __, Which is necessary for understanding law of exponential of 3^x of is the product of x factors 3 where x is positive integer and 243 represents mathematically product of three. For this item 50.5% of male students and 51.5% of female students i.e. 51% of the total students were not able to find exact value of x; the second item educed extent to find the solution of $8^x = 4, then\ x =$ __.

Which is necessary for understanding law of exponential of 8^x is the product of x factors 2 and 4 represent mathematically that factors of two, where x is a fraction. For this item 51.5% of male student and 63.9% of female students i.e. 59% of the total students were not able to find exact solution; the third item educed information on pre-engineering students' extent to find the solution of $\left(\frac{3}{4}\right)^x = \frac{27}{64}, then\ x = \_$ which is necessary for understanding law of exponential of $\left(\frac{3}{4}\right)^x$ is the product of x factors $\frac{3}{4}$ and $\frac{27}{64}$ represent mathematically that factors of $\frac{3}{4}$. For this item 59.8% of male student and 56.7% of female students i.e. 58% of the total students were not able to find exact solution; fourth item educed information on the degree of male and female first year pre-engineering students to find the solution of $7^x = {}^{1}/_{49}, then\ x = \_$. It is necessary for understanding law of exponential function $\mathbf{7^x}$ is the product of x factors 7 where x is negative integer and ${}^{1}/_{49}$ represent mathematically that factors of 7. To this item 56.7% of male students and 62.9% of female students i.e. 61 % of the total students were not able to find the solution; fourth item educed information on the degree of male and female first year pre-engineering students to find the solution of $\frac{4^x}{4^{2x}} = 64\ \ then\ x =$. Which is necessary for understanding law of exponential with the same base of numerator and denominator of $\frac{4^x}{4^{2x}}$ is the product of x factors 4 where x is negative integer and 64 represent mathematically product of 4. For this item 70.1% of male students and 75.3% of female student i.e. 73% of the total students were not able to find the solution; fourth item educes to find the solution of $\left(\frac{1}{16}\right)^{x-3} = 8^{2x-1}, then\ x =$. This is necessary for understanding law of exponentials with transformation of exponents x factors 4 , where x is an integer. For this item 84.5% of male students and 91.7% of female students i.e. 86% of the total student were not able to find exact solution; sixth item elicited extents to find the solution of $25^{\sqrt{x}} = 625^{x}$ then x =__. This is the product of x factors 5 where x is positive integer. For this item 92.8% of male students and 97.9% of female students i.e. 95% of the total students were not able to find the solution of x; seventh item educed information on the degree of male and female first year pre-engineering students to find the solution of $\left(\frac{1}{2}\right)^{x} = 32$, which is necessary for understanding law of exponent which needs conversion of positive exponent to negative exponent of $\left(\frac{1}{2}\right)^{x}$ the product of x factors 2 where x is a negative integer and 32 represents mathematically the factors of two. For this item 62.9% of male students and 72.1% of female students i.e. 68% of the total students were not able to find solution of x.

The inverse of the exponential function is the logarithmic function and vice versa. Typically, logarithmic function is an inverse function for raising a number to a power where the exponent is the output of the function. Recalling the relationship that results from the composition of inverse functions directs to understand the logarithmic function as a key tool for solving exponential equation (Kastberg & Rechael, 2017). Understanding of the exponential function and inverse functions could serve as prerequisite for the students' comprehension of the logarithmic function (Kastberg, 2002).

Question number 12 and 13 in the above Table 16 were elicit first year pre-engineering students' understanding of exponential function and inverse function to handle logarithmic functions. In the above Table 16 the first item focused on converting exponentials function with variable into logarithmic functions. When asked to convert $a^{x} = b \; into \;\; log_{\_\_}\,\_ = -$, 71.1% of male students and 82.5% of female students i.e. 77% of the total students did not correctly convert it as they were expected to converts it $log_{a}\, b = x$; the second item assessed conversions of numerical expression of exponentials into logarithmic. For this item 74.2% of male students and 80.4% of female students i.e. 78% of the total students did not correctly convert it; the third item asked to convert $9^0 = 1$ $to\ log\_\_\ \_ = -$ by recalling the inverse relationship of $9^0 = 1$, students could understand and solved why $log_9 1 = 0$ and 77.3% of male students and 87.6% of female students i.e.83% of the total students did not correctly convert it; the fourth item asked to convert $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $to\ log\_\_\ \_ = -$ where 76.3% of male students and 89.7% of female students i.e.83% of the total students did not correctly convert it.

In similar manner question number 13 in Table 16 elicited extent of pre-engineering students' understanding to find solution and conversions of logarithm function to exponential function. Under this question item one $log_2 64 = -to\ exp,$ 80.4% of male students and 85.6% of female students i.e. 83% of the total students did not able to find solution; item two $log_8 1 = -\ to\ exp,$ 81.4% of male students and 90.7% of female students i.e. 86% of the total students did not able to find solution; item three $log_{\frac{1}{3}}\left(\frac{1}{9}\right) = -\ to\ exp$ 79.4% of male students and 89.7% of female students i.e. 85% of the total students were not able to convert.

Under question number 14, item four and item five focused on two bases. Which are used much more commonly than any other bases and deserve special mention (Mc-TY-Logarithm, 2009). The first base is 10. Logarithms to base 10, $log\, 10$, are often written simply as log without explicitly writing a base down. The expression like $log\, x$ indicate that the base is 10. Based on this assumption when asked to convert $log\, 0.01 = -to\ exp$ 81.4% of male students and 87.6% female students i.e. 84% of the total students did not able to convert, where the answer was to 10^{-2}. The second common base is e. The symbol e is called the exponential constant and has a value approximately equal to 2.718. This is a number like π in the sense that it has an infinite decimal expansion. Base e is used because this constant occurs frequently in the mathematical modeling of many physical, biological, economic and engineering applications

(Mc-TY-Logarithm, 2009) Logarithms to base e, $log_e x$, are often written simply as lnx. If you see an expression like lnx you can assume the base is e. Such logarithms are also called Naperian or natural logarithms. When asked to convert $ln\, x = a\ into\ exp,$ 88.7% of male students and 94.8 of female students i.e. 91% of the total students did not able to convert, where the answer was $e^a = x$.

In Table 16 under question number 15 four items were described that needs variety of ideas and methods of solving logarithmic equations. When pre-engineering university students asked to find the solution of $log_5(x + 2) = 1$, 64.9% of male students and 73.2% of female students i.e. 70% of the total students were not able to find solution, where the solution was $5^1 = x + 2$ which implies $x = 3$; for item number two $3\, ln\, 2 + ln(x - 1) = ln\, 24$, 89.7% of male students and 96.9% of female students i.e. 93% of the total students did not able to find solution, where the solution was $ln\, 8(x - 1) = ln24 \iff 8(x - 1) = 24 \Longrightarrow x = 4$;

For item number three $log_3 x = log_3 2 + log_3(x - 3),$ 95.9% of male students and 97.9% of female students i.e.97% of the total students did not able to find solution, where the solution was $x = 2(x - 3) \iff x = 6$. In Ethiopian mathematics curriculum exponential and logarithmic functions are introduced in grade ten and brief discussions with common and natural logarithmic functions including applications are given in grade eleven. Based on countries curriculum it is expected, pre-engineering university students to solve like item four ($ln\, 5 - lnx = -1$), 97.9% of male students and 100% of female students i.e. 99% of the total students did not able to find solution, where the solution was $ln \frac{5}{x} = -1 \iff e^{-1} = \frac{5}{x} \iff x = 5e$.

4.3.7. Trigonometric functions

According to Rajalingam and Shubashini (n.d.), in tertiary education trigonometry has a strong relation with all disciplines. Trigonometry is an important subject for all scientific fields that make use of trigonometry area in the field of STEM and some social sciences, like economics, music theory (Rajalingam & Shubashini, n.d.). Rajalingam and Shubashini (n.d.) state that engineering students design different types of design in their study time and on their day-to-day work activities in their field by using trigonometric ideas, like design of building, machinery cars, planes, ships. One can observe that how important trigonometry is in tertiary education as they produce engineers. Trigonometry is originated from triangle. It studies about triangle based on the given length of the sides and angles, for instance when the length of two sides and including angle are given, one can find the remaining side and measurement of angles of a given triangle. Once engineering students know and practice all measurements of the triangle (structure) they can begin building and defining the relative scope of the engineering and applied calculus course that she or he is undertaking. For this reason, it is important to revise the concept of trigonometry for pre-engineering students while empirical study of Pre-test which explore students' skill and understanding of basic trigonometric expressions and functions as follows:

Table 17: Solve the following trigonometric expressions and functions

Item	Male				Female				Total			
No	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
16	f	%	F	%	F	%	F	%	f	%	f	%
i, Convert the following angles in to radian $\boldsymbol{30^0 =}$ _, $\boldsymbol{45^0}$ = _, $\boldsymbol{120^0}$ =	37	37.4	62	62.6	30	30.3	69	69.7	67	34	131	66
ii, Convert the following radian in to angles $\boldsymbol{\frac{5\pi}{4}}$ = _ $\boldsymbol{\frac{7\pi}{4}}$ = _, $\boldsymbol{\frac{-3\pi}{4}}$ = _	38	38.4	61	61.6	28	28.3	71	71.7	66	33	132	67
iii, Find the exact values of the trig function $\boldsymbol{cos\ 150^0}$ = _ $\boldsymbol{sin\frac{5\pi}{4}}$=_	33	33.3	65	66.3	20	20.2	79	79.8	53	27	145	73
iv, Find the angle in degree $\boldsymbol{sin^{-1}\left(\frac{1}{2}\right)}$ = _ $\boldsymbol{tan^{-1}(-\sqrt{3})}$ = _	37	37.4	62	62.6	25	25.3	74	74.7	62	31	132	69
Simplify the following trigonometric												
V, $\boldsymbol{cos^2 x\ (1 + tan^2 x)}$ =	40	40.4	59	59.6	26	26.3	73	73.7	66	33	132	67
Vi, $\boldsymbol{\frac{1}{1+tan^2 x}}$ =	31	31.3	68	68.7	23	23.2	76	76.8	54	27	144	73

Vii, $\frac{1}{1+\cos x}+\frac{1}{1-\cos x}=$	14	14.3	84	85.7	14	14.1	85	85.9	28	14	169	86
Viii, $\frac{\cot x}{\csc x}=$	16	16.2	83	83.8	9	9.1	90	90.9	25	13	173	87
ix, $\frac{1-\cos^2 x}{\csc x}=$	30	30.3	69	69.7	13	13.1	86	86.9	43	23	155	77
Show the following												
X, $\tan x \cot x = 1$	13	13.3	86	69.9	19	19.2	80	80.8	32	16	166	84
Xi, $\frac{secx-cosx}{\tan x}=sinx$	15	15.2	84	84.8	14	14.1	85	85.9	29	15	169	85
Xii, $\frac{secx}{\cos x}-\frac{tanx}{\cot x}=1$	21	21.2	78	78.8	8	8.1	91	91.9	29	15	169	85

Question number 16 in Table 17 focused on conversion of angle in degrees into angle in radians and vice versa, numerical solution of sine and cosine functions and simplification trigonometric function. In Table 17 one can observe that the first item educed information on the degree of male and female first year pre-engineering students to convert angles in degrees into angles radians ($30^0 = \_,\ 45^0 = \_,\ 120^0 = -$). For this item 62.6% of male students and 69.7% of female students i.e. 66% of the total students were not able to convert degrees into angles, where the conversion are $30^0 = \frac{\pi}{6}, 45^0 = \frac{\pi}{4}$ and $120^0 = \frac{2\pi}{3}$; the second item were converse of the above ($\frac{5\pi}{4} = \_\ \frac{7\pi}{4} = \_, \frac{-3\pi}{4} = \_$) where 61.6% of male students and 71.7 % of female students i.e. 67% of the total students were not able to convert, where the conversion were $\frac{5\pi}{4} = 225^0$, $\frac{7\pi}{4} = -315^0$, $\frac{-3\pi}{4} = -135^0$; the third item ($\cos 150^0 = \_$, $\sin\frac{5\pi}{4}$=_) were asked to find numerical solution 66.3% of male students and 79.8% of female students i.e. 73% of the total students were not able to solve, where the solution were $\cos 150^0 = \frac{\sqrt{3}}{2}$ and $\sin\frac{5\pi}{4} = \frac{\sqrt{2}}{2}$; and for fourth item ($\sin^{-1}\left(\frac{1}{2}\right) = \_\_, \tan^{-1}(-\sqrt{3}) = \_\_$), 62.6% of male students and 74.7% of female students i.e. 69% of the total students were not able to find numerical solutions that the solution were $sin^{-1}\left(\frac{1}{2}\right) = 30^0$, $tan^{-1}(-\sqrt{3}) = 120^0$); and the following items were focused on simplifications of trigonometric functions $cos^2 x\,(1 + tan^2 x) = -$, 59.6% of male students and 73.7% of female students i.e. 67% of the total students were not able to simplify, where $cos^2 x\,(1 + tan^2 x) = cos^2 x + \frac{cos^2 sin^2 x)}{cos^2 x} = 1$; for item $\frac{1}{1+tan^2 x} = -$, 68.7% of male students and 76.8% of female students i.e. 73% of the total students were not able to simplify, where $\frac{1}{1+tan^2 x} = cos^2 x$, for item $\frac{1}{1+cos\,x} + \frac{1}{1-cos\,x} = -$, 85.7% of male students and 85.9 of female students i.e. 86% of the total students were not able to simplify, where $\frac{1}{1+cos\,x} + \frac{1}{1-cos\,x} = \frac{1}{sin^2 x} = csc^2 x$; for item $\frac{cot\,x}{csc\,x} =$ 83.8% of male students and 90.9% of female students i.e. 87% of the total students were not able to simplify, where $\frac{cot\,x}{csc\,x} = cosx$; for item $\frac{1-cos^2 x}{csc\,x} =$ 69.7% of male students and 86.9% of female students i.e. 77% of the total students were not able simplify, where $\frac{1-cos^2 x}{csc\,x} = sin^3 x$. the following items number x, xi, and xii focused on proof of trigonometric identities. For item ($tan\,x\,cot\,x = 1$), 69.9% of male students and 80.8% of female students i.e. 84% of the total students were not able to proof, where $tan\,x\,cot\,x = \frac{sin\,x}{cos\,x}\frac{cos\,x}{sin\,x} = 1$; for item ($\frac{secx-cosx}{tan\,x} = sinx$), 84.8% of male students and 85.9% of female students i.e. 85% of the total students were not able to proof, where $\frac{secx-cosx}{tan\,x} = \frac{\frac{1-cos^2 x}{cosx}}{tan\,x} = sinx$.

4.3.8. Summary of research question number one

Descriptive statistics analyses were conducted to answer research question number one. The study shows that over all group mean (30.24 out of 100), St.D (17.95), Skewness (0.45), and

Kurtosis (-0.90). Frequency and percentage of correct and incorrect answered on pre-calculus mathematics by two genders (Male and Female pre-engineering university students) were examined for each item of questions in each selected topics of pre-calculus mathematics separately. Generally, percentage of all correctly answered pre-calculus mathematics Pre-test item of questions was below 35%. Then the mean score and standard deviation of each group with moderator variable were analyzed. The results indicated that all groups mean of Pre-test were in the interval of 20 to 33 out of 100.

4.4. The Experimental Study Results

In order to measure the influence of pre-calculus mathematics refreshment module on Applied Calculus 1 it is useful to look at different representations (i.e. descriptive and inferential statistics) of the same data. Pre-calculus mathematics module intervention with different instructional methods began for three groups after the Pre-test of pre-calculus mathematics conducted. The treatment group received 32 periods of selected pre-calculus mathematics with different intervention methods (i.e. traditional lecture (T), co-operative learning (CL) and meta-cognitive with co-operative learning (MCL) intervention method. The control group received no intervention of pre-calculus mathematics instruction. The treatment group received the same amount delivery time to improve their background knowledge of selected pre-calculus mathematics. The posttest was administered to those three groups after completed their intervention module.

4.4.1. Testing hypothesis one

Hypothesis one stated that, students who are taught pre-calculus mathematics refreshment would achieve better than students who are not taught pre-calculus mathematics refreshment in Applied Calculus 1 achievement.

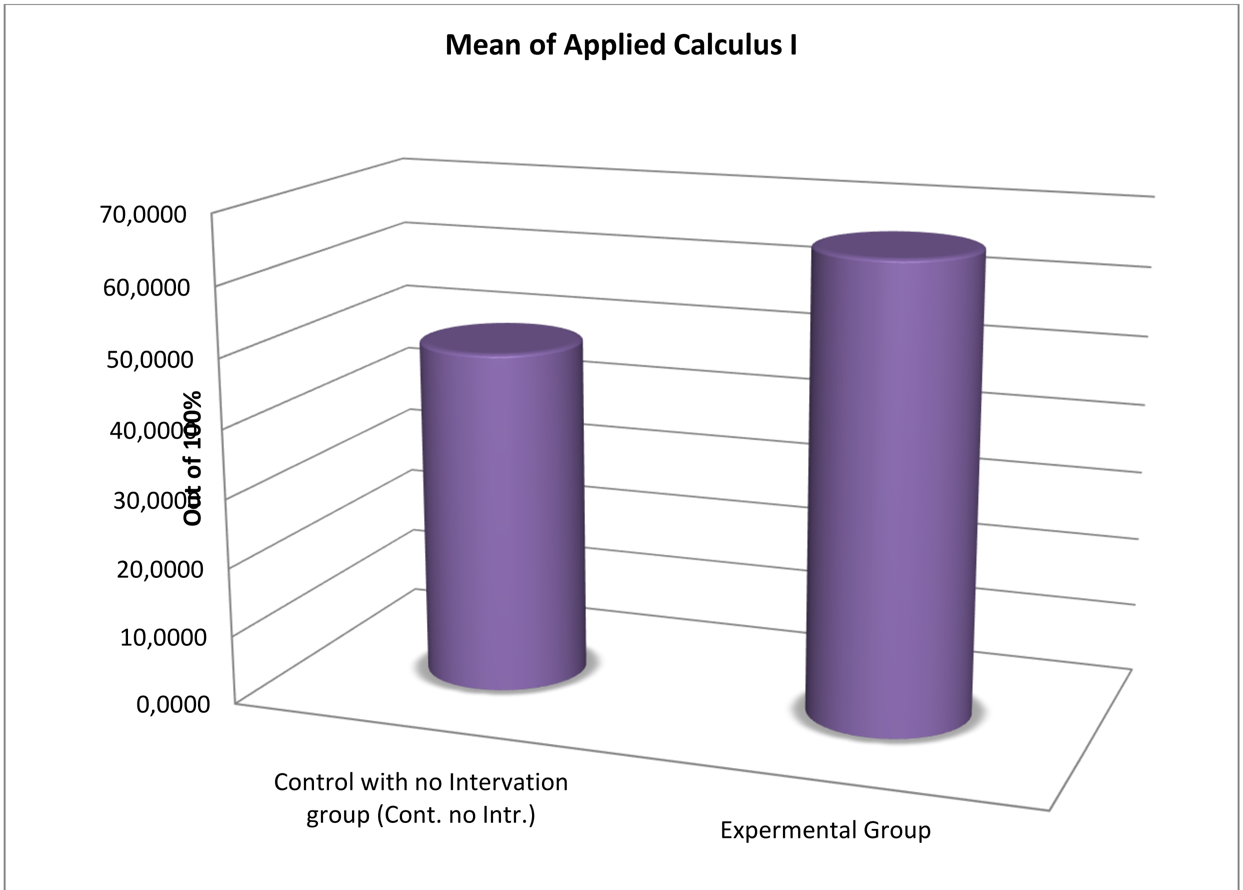
To find out the effect of the experimental training program on the independent variables, the analysis of descriptive statistics with bar graph and independent sample T-test was done for the experimental and the control groups that did not take intervention course. The impact results of the descriptive statistics of Applied Calculus 1 variables are presented in Table 18.

Table 18: Descriptive statistics of Applied Calculus 1 scores of control and experimental groups

Group	N	Mean	St.D	Std. Error Mean
Control with no intervention (Cont. no Intr.)	52	49.37	18.94	2.62716
Experimental groups (T, CL & MCL)	141	66.26	12.09	1.01794

Table 18 summarizes the descriptive statistics for the dependent variables and Applied Calculus 1 achievement by the experimental and control without intervention groups. The scores of experimental group on Applied Calculus 1 Mean = 66.26, St.D = 12.09 and the scores of control without intervention group on Applied Calculus 1 Mean 49.37, St. D 18.94.

For further understanding, mean of Applied Calculus 1 of the experimental and control groups are graphically shown in Figure 4 as follows.



	Control with no Intervation group (Cont. no Intr.)	Expermental Group
Mean of Applied Calculus I	49,3654	66,2553

Figure 4: Bar graph of Mean of Applied Calculus 1 of Experimental and Control without Intervention Groups.

To see significant difference of experimental and control without intervention group on Applied Calculus 1 independent sample T-test was done to find out the significant differences between the paired means of the experimental and control groups in Table 19.

Table 19: Independent Samples T-Test of Applied Calculus 1 between Control and Experimental Groups

Groups		**Levene's Test for Equality of Variances**		**t-test for Equality of Means**						
		F	Sig.	t	df	Sig. (2tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Control with no Intr	All experimental	19.34	0.00	-5.99	66.91	*.000	-16.9	2.820	-22.514	-11.266

* Source: Survey result. The significant level of mean variation is at p = 0.05

As it is shown in Table 19 independent-samples t-test was conducted to see the extent of refreshment module of pre-calculus mathematics on Applied Calculus 1 scores: for control with no intervention group and experimental group Levene's Test (Sig = 0.00) which indicated that equal variance is not assumed. From Tables 18 descriptive statistics and Table 19 there was significant difference in scores for experimental group (M = 49.37, St.D = 18.94) and control with no intervention (M = 66.26, St.D = 12.09; t (193) = -5.99, p = 0.00, two-tailed). The magnitude of the variation of means (mean difference = 16.89, 95% CI: –22.514 to -11.266), this means that the intervention of pre-calculus, had a main influence on Applied Calculus 1. Their significant effect accounted for 16% of the variance was large effect size (eta squared = 0.158).

Concerning result of Applied Calculus 1, it was directly taken from class room Applied Calculus 1instructors of Applied Calculus 1 rosters that he assessed out of 100% by formative and summative assessment technique.

4.4.1.1.Summary of hypothesis one

The statistical results confirmed the hypothesis, showing that students that were taught pre-calculus mathematics refreshment module would achieve better than students that were not taught pre-calculus mathematics refreshment module in Applied Calculus 1 achievement result with large effect size.

4.4.2. Testing hypothesis two

Hypothesis two stated that, students who are taught through MCL intervention method achieve better than students who are taught through CL intervention method who, in turn would achieve better than students that are taught through T intervention method in pre-calculus students' achievement. To test this hypothesis, first let show the descriptive statistics in tables and bar graph, then explored the influence of intervention methods CL, MCL and T, Traditional lecture method (T) used as a control group.

Table 20: Descriptive Statistics of Posttest Result and Applied Calculus 1. Achievement

	Refreshment Module			Applied Calculus 1		
	Control (Novice) T	CL	MCL	Cont (Novice) T	CL	MCL
N	49	50	49	47	47	47
Mean	49.90	71.08	72.41	58.36	70.55	69.85
St.D	24.76	22.79	9.57	10.23	11.5	10.60
Skewness	.30	-.23	-.16	1.32	.267	-.062
Kurtosis	-.75	-1.57	1.187	-.749	-.985	-.77

Table 20 shows the means, standard deviations, Skewness and Kurtosis for posttest of pre-calculus mathematics and achievement of Applied Calculus 1. On posttest mean (Cont. Novice T = 49.9), St.D (Cont. Novice T = 24.76), skewness (Cont. Novice T = 0.30, and kurtosis (Cont. Novice T = - 0.75); Mean (CL = 71.08), St.D (CL = 22.79), skewness (CL = - 0.23), and kurtosis (CL = -1.57); and mean (MCL = 72.41), St.D (MCL = 9.57), skewness (MCL = - 0.16), and kurtosis (MCL = 1.187). And on Applied Calculus 1 mean (Cont. Novice T = 58.36), St.D (Cont. Novice T = 10.23), skewness (Cont. Novice T = 1.32, and kurtosis (Cont. Novice T = - 0.749); Mean (CL = 70.55), St.D (CL = 11.5), skewness (CL = 0.267), and kurtosis (CL = - 0.985); and mean (MCL = 69.85), St.D (MCL = 10.60), skewness (MCL = - 0.062), and kurtosis (Exp MCL = - 0.77).

For further understanding means of posttest and Applied Calculus 1 of the experimental groups (CL & MCL) and control Novice groups (T) are graphically shown in Figure 5.

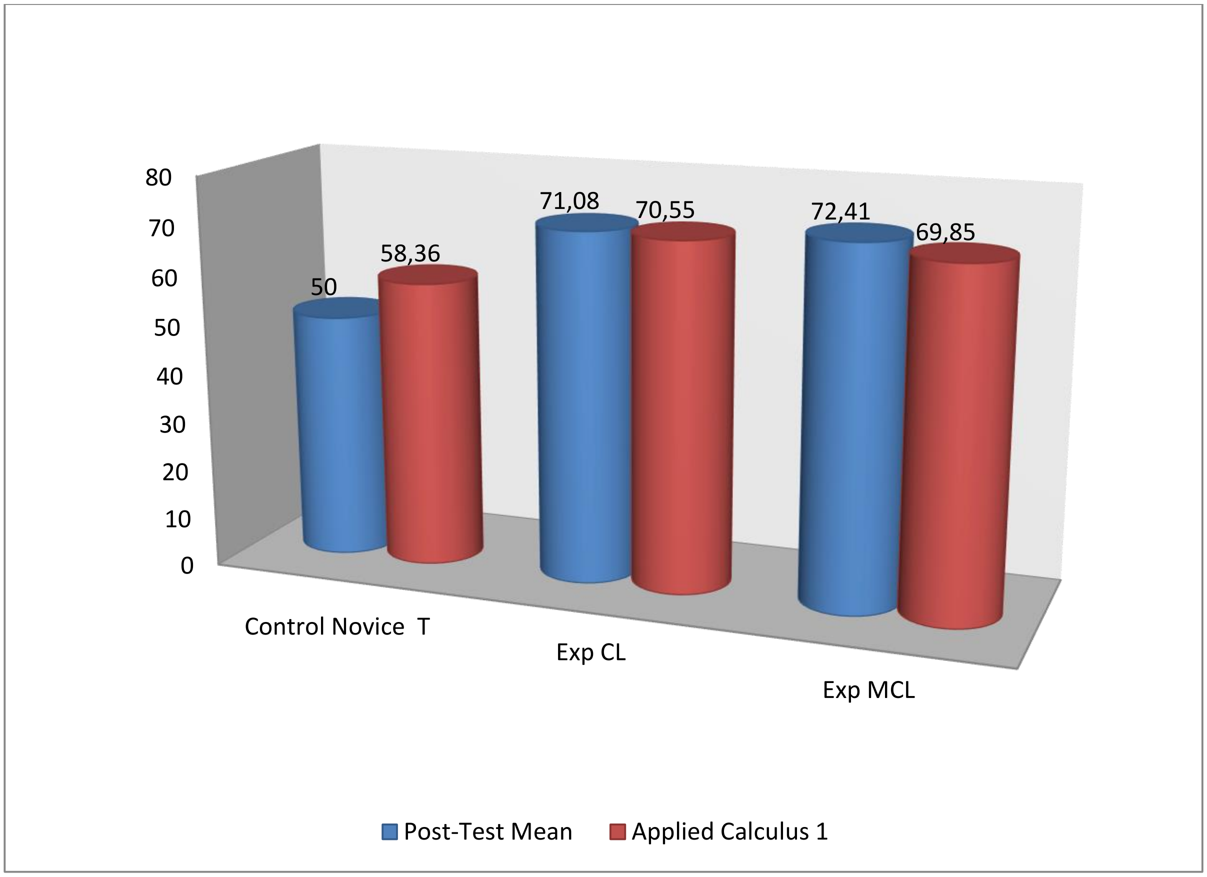


Figure 5: Bar graph of Mean of Posttest and Applied Calculus 1 of Cont. Novice T and Total Exp groups

Testing hypothesis two if there were statistically significant differences across students who have got lesson through MCL intervention method, CL intervention method and T intervention method in pre-calculus students' achievement SPSS20 software package to run ANOVA was conducted. To run ANOVA first, it is important to determine which path to use by testing homogeneity of variance. And homogeneity of variation from Levene's test was p=0.00 among groups on refreshment module (See Appendix 3D). So, the data violets homogeneity, it is better to use "Tamhane's T2" because it is the most used test statistics by statisticians (Gupta, 1999). ANOVA test was conducted by using the ANOVA for "Post Hoc" "Tamhane's T2" path and the results of posttest as follows in Table 21.

Table 21: ANOVA on intervention of refreshment module posttest results

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	15676.987	2	7838.493	19.174	.000
Within Groups	59278.007	145	408.814		
Total	74954.993	147			

*. The significant level of mean variation is at0.05 level.

As it is shown in Table 21 the result of one-way among-groups analysis of variance was conducted to explore the influence of intervention method, as measured by pre-calculus mathematics posttest. Participants were divided into three groups according to their intervention method (control Novice T; CL; MCL). There was a statistically significant difference among the group at the $p < .05$ level in pre-calculus mathematics test scores for the three intervention groups: $F (2, 145) = 19.174$, $p = 0.00$. The significance difference accounted for 21% of the variance was large effect size, calculated using eta squared (eta squared = 0.21).

To identify the significance difference between each groups, ANOVA Multiple Comparison test was conducted by using the ANOVA for “Post Hoc” "Tamhane's T2" path and the results of posttest as follows in Table 22

Table 22: ANOVA Multiple Comparisons of Intervention Method in each Group of Posttest

Dependent Variable: Post V Tamhane						
Group (I)	Group (J)	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Control Novice T	CL	-21.18204*	4.78545	.000	-32.81	-9.55
	MCL	-22.51020*	3.79235	.000	-31.82	-13.20
CL	MCL	-1.32816	3.50118	.974	-9.91	7.25

*. The significant level of mean variation is at0.05 level.

As it shown in Table 20 descriptive statistics and Table 22 Post-hoc comparisons using Tamhane's test indicated that the mean score for CL (M = 71, St.D = 22.79) was not significantly differ from MCL (M = 72.41, St.D = 9.57); the magnitude of the variation of means (mean difference = -1.32816, 95% CI: -9.91to -7.25) and p = 0.974 which was no significant difference between CL and MCL intervention methods on pre-calculus mathematics. Cont. Novice T (M = 49.90, St.D = 24.76) which was significantly different from both CL and MCL: the magnitude of the differences in the mean T & CL and T & MCL (mean difference = -21.18204, 95% CI: -32.81 to -9.55) and p = 0.000, and (mean difference = -22.51020, 95% CI: -31.82 to -13.20) and p = 0.000 respectively, this means that there was significant difference between T & CL and T & MCL intervention method on pre-calculus mathematics.

To see the influence of intervention methods of refreshment module of pre-calculus mathematics on Applied Calculus 1, ANOVA on Applied Calculus (Mathematics) 1 result of the three groups as follows in Table 23

Table 23: ANOVA of MCL, CL, T and Cont. No Intr. Groups on Applied Calculus 1

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	15241.683	3	5080.561	27.951	.000
Within Groups	34354.483	189	181.770		
Total	49596.166	192			

*. The significant level of mean variation is at0.05 level.

As it is shown in Table 23 the result of one-way among-groups analysis of variance was conducted to explore the impact of intervention method on Applied Calculus 1 achievement. Participants were divided into four groups according to their intervention methods control Novice T; CL; MCL and Control without intervention. There was a statistically significant difference among the group at the $p < .05$ level in Applied Calculus 1 achievement scores for the three intervention groups and Control without intervention group: $F (3, 189) = 27.951$, $p = 0.00$. The significant difference accounted for 69% of the variance was large effect size, calculated using eta squared (eta squared = 0.69).

To identify which group was significantly different from the other, first determined which path to use, homogeneity of variance was tested. It violated the homogeneity of variation from Levene’s test ($p = 0.00$) among groups on Applied Mathematics 1 (See Appendix 3E). ANOVA Multiple Comparisons test was conducted by using “Post Hoc” Tahamane’s T2 path and the results of Applied Calculus 1 test as follows in Table 24.

Table 24: ANOVA Multiple Comparisons test of Applied Calculus 1 scores

(I) Groups	(J) Groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Cont. no Intr.	Cont. Novice T	-8.996*	3.021	.023	-17.147	-.845
	CL	-21.188*	3.116	.000	-29.579	-12.79
	MCL	-20.486*	3.049	.000	-28.705	-12.26
Cont. Novice T	CL	-12.191*	2.243	.000	-18.224	-6.158
	MCL	-11.489*	2.149	.000	-17.268	-5.710
CL	MCL	.702	2.279	1.00	-5.428	6.832

*. The significant level of mean variation is at0.05 level.

As it shown in Table 20 descriptive statistics and Table 24 ANOVA Post-hoc comparisons using the Tamhane's T2 test, showing overall differences for the pre-calculus mathematics refreshment module intervention method on dependent variable 'Applied Calculus 1 achievement scores', indicated that the mean score for CL (M = 70.5, St.D = 11.48) was not significantly differ from MCL (M = 69.70, St.D = 10.07); the magnitude of the variation of means (mean difference = 0.702, 95% CI: -5.43 to 6.83) and p = 1.00 this means that there was not significant differ between CL and MCL groups on Applied Calculus 1 achievement.

Cont. Novice T (M = 58.36, St.D = 10.23) was significantly different from both CL and MCL; the magnitude of the differences in the mean T & CL and T & MCL (mean difference = -12.191, 95% CI: -18.224 to -6.158) and p = 0.000, and (mean difference = -11.489, 95% CI: -17.268 to -5.710) and p = 0.000 respectively, this means that there was significant difference between T & CL and T & MCL groups on Applied Calculus 1 achievement. Cont no Int (M = 49.36, St.D = 18.94) was significantly different from all that toke refreshment of pre-calculus mathematics module i.e. T, CL and MCL groups; the magnitude of the differences in the mean Cont. no Intr. &T, Cont. no Intr. & CL and Cont. no Intr. & MCL (mean difference = -8.996, 95% CI: -17.147 to -0.845) and p = 0.023, (mean difference = -21.18781, 95% CI: -29.579 to -12.796) and p = 0.000 and (mean difference = -20.486, 95% CI: -28.705 to -.845) and p = 0.023, (mean difference = -21.188, 95% CI: -29.579 to -12.265) and p = 0.000 respectively, this means that there was significant difference between Cont. no Intr. & T, Cont. no Intr. & CL and Cont. no Intr. & MCL groups on Applied Calculus 1 achievement respectively.

4.4.2.1.Summary of hypotheses two

This hypothesis is partly supported by the statistical outcome of the study. As the study showed the students that took a lesson through MCL & CL intervention method achieved significantly better than the students that took a lesson through T intervention method in pre-calculus Mathematics and Applied Calculus 1, but students that took a lesson through MCL did not achieve significantly better than students that took a lesson through CL intervention method in Pre-Calculus Mathematics and Applied Calculus 1.

For further understanding the following figure 3 shows that the influence of refreshment module of pre-calculus mathematics on pre-engineering first year university students that taught pre-calculus mathematics through MCL, CL and T intervention method in Applied Calculus 1.

Pretest, outcome of eight weeks' intervention posttest and normal class of Applied Calculus 1 mean of the experimental and control groups are graphically shown in Figure 6.

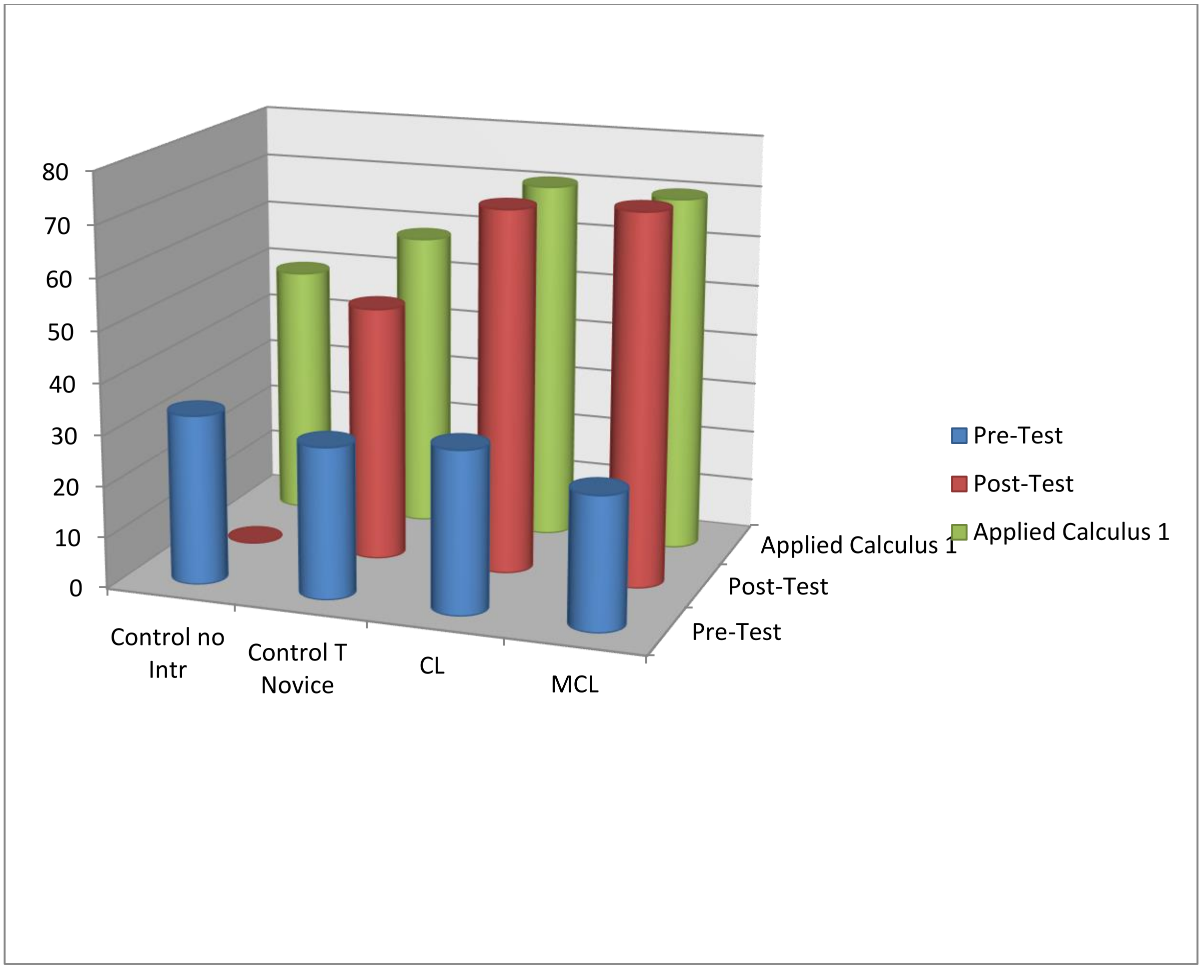


Figure 6: Means of Pre, Post and Applied Calculus 1 of the Experimental and Control Groups

4.4.3. Testing hypothesis three

Hypothesis three stated that, Male students who are taught through MCL intervention method would achieve better than male students who are taught through CL intervention method who, in turn would achieve better than male students who are taught through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1 in student achievement.

Posttest and Applied Calculus 1 scores of descriptive statistics of the experimental and control groups of male students are presented in the table and figure below

Table 25: Descriptive statistics of Posttest and Applied Calculus 1 of male students in each group

	Refreshment Module				Applied Calculus 1		
	Male Cont. Novice T	Male CL	Male MCL	Cont. no Intr. M	Male Cont Novice T	Male CL	Male MCL
N	24	25	24	25	25	24	25
Mean	53.917	71.280	74.375	49.46	61.120	72.458	73.760
St.D	26.831	24.239	9.406	21.82	9.40773	.929	-.444
Skewness	-.018	-.262	-.390	.344	.929	-.444	-.058
Kurtosis	-.845	-1.741	-.262	-1.066	-.361	-.435	-1.144

The above Table 25 of the posttest and Applied Calculus 1 means of the experimental and control groups on male students indicates that in case of MCL experimental group, the posttest and Applied Calculus 1 were mean (74.375), St.D (9.406), skewness (-0.390) and Kurtosis (-0.262) and mean (73.760) , St.D (-0.444), skewness (-0.058) and Kurtosis (-1.144) respectively. In case of CL experimental group, the posttest and Applied Calculus 1 were mean (71.280), St.D (24.239), skewness (-0.262) and Kurtosis (-1.741) and mean (72.458), St.D (0.929), skewness (-0.444) and Kurtosis (-0.435) respectively. In case of T control group, the posttest and Applied Calculus 1 were mean (53.9), STA.DEV (26.8), skewness (-0.018) and Kurtosis (-0.845) and mean (61.12), St.D (9.4), skewness (0.929) and Kurtosis (-0.361) respectively.

Posttest and Applied Calculus 1 mean scores of the Experimental and Control Groups on male students are graphically shown in Figure 7.

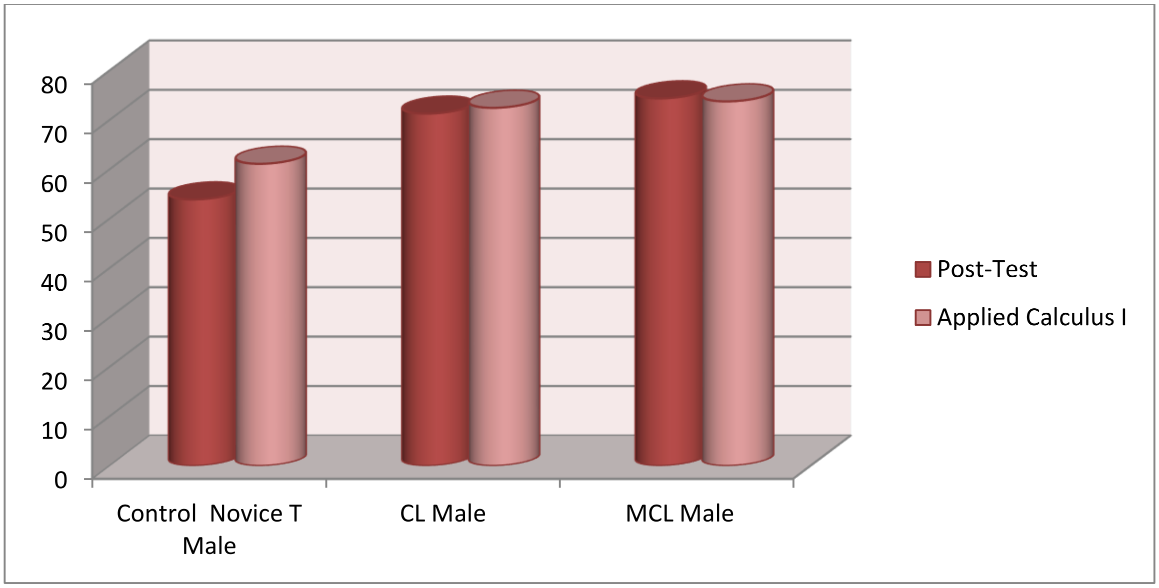


Figure 7: Bar graph of Mean of Posttest and Applied Calculus 1 of males control and Total Exp groups

To see the influence of intervention method of refreshment module of pre- calculus mathematics for pre-engineering first year university male students, ANOVA on pre-calculus mathematics result of the three male groups as follows in Table 26

Table 26: ANOVA on posttest result of male students in each group

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	5859.173	2	2929.586	6.272	.003
Within Groups	32694.498	70	467.064		
Total	38553.671	72			

*. The significant level of mean variation is at0.05 level.

As Table 26 shows there was a significant difference, $p < 0.05$, among the three male groups on pre-calculus mathematics posttest result $F(2, 70) = 6.272$, $p = 0.003$. The large effect size 15%, calculated using eta squared, was 0.15.

To identify which group is significantly different from other, first determine which path to use, homogeneity of variance was tested. It violated the homogeneity of variation from Levene's test (p = 0.00) among groups on pre-calculus mathematics (See Appendix 3F). ANOVA test was conducted by using "Post Hoc" Tahamane's T2 path and the results of pre-calculus mathematics posttest as follows.

Table 27: ANOVA multiple comparisons of posttest result of male students

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Cont. Novice T	CL	-17.363	7.314	.064	-35.485	.758
	MCL	-20.458*	5.803	.004	-35.176	-5.740
CL	MCL	-3.095	5.214	.913	-16.246	10.056

*. The significant level of mean variation is at0.05 level.

As it is shown in Table 25 descriptive statistics and Table 27 Post-hoc comparisons using the Tamhane's T2 test indicated that the mean scores of male students for CL (M = 71.28, St.D = 24.23) was not significantly different from male students for MCL (M = 74.38, St.D = 24.23); the magnitude of the differences in the mean CL & MCL (mean difference = -3.095, 95% CI: -16.246 to 10.056) and p = 0 .913, this means that there was not significantly different from male students in CL & male students in MCL on selected pre-calculus mathematics posttest scores.

Even though there is a great mean deference between Cont. Novice T and CL male groups, Cont. Novice T (M = 53.92, St.D = 26.83) was not different significantly from CL; the magnitude of the differences in the mean Cont. Novice T & CL male groups (mean difference = -17.36, 95% CI: -35.485 to 0.758) and p = 0.064, this means that there was not significantly different from male students in Cont. Novice T & CL male students on selected pre-calculus mathematics posttest scores. Cont. Novice T (M = 53.92, STA.DEV = 26.83) was different significantly from male students for MCL where the magnitude of the differences in the mean Cont. Novice T & MCL male groups (mean difference = -20.458, 95% CI: -35.176 to -5.740) and p = 0.004.

To see the influence of intervention method of refreshment module of pre-calculus mathematics on Applied Calculus 1 for pre-engineering first year university male students, ANOVA on Applied Calculus 1 results of the four male groups as follows in Table 28

Table 28: ANOVA on Applied Calculus 1 result of male students in each group

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	9589.796	3	3196.599	13.31	.000
Within Groups	22816.118	95	240.170		
Total	32405.914	98			

*. The significant level of mean variation is at0.05 level.

Table 28 shows that there was a significant difference, $p < 0.05$, among the four male groups on Applied Calculus 1, $F (3, 95) = 13.31$, $p = 0.00$. The small effect size 29%, calculated using eta squared, was 0.29.

To identify which group is significantly different from other; first determine which path to use, homogeneity of variance was tested. It violated the homogeneity of variation from Levene's test (p = 0.001) among groups on Applied Calculus 1 (See Appendix 3G). ANOVA test was conducted by using "Post Hoc" Tahamane's T2 path and the results of Applied Calculus 1 achievement scores as follows.

Table 29: ANOVA multiple comparisons test on Applied Calculus 1 result of male in each group

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Cont no Int	Cont. Novice T	-11.66000	5.01624	.144	-25.5898	2.2698
	CL	-22.998*	5.026	.000	-36.956	-9.040
	MCL	-24.300*	5.149	.000	-38.552	-10.047
Cont. Novice T	CL	-11.338*	3.463	.012	-20.850	-1.826
	MCL	-12.640*	3.638	.007	-22.629	-2.650
CL	MCL	-1.301	3.653	1.00	-11.338	8.734

The significant level of mean variation is at 0.05 levels.

In Table 25 descriptive statistics and Table 29 Post-hoc comparisons using Tamhane's T2 test indicated that the mean score of Applied Calculus 1 of male students for Cont. no Intr. (M = 49.46, St.D = 21.82) was not significantly different from male students for Cont. Novice T (M = 61.12, St.D = 12.19); the magnitude of the differences in the mean Cont. no Intr. & Cont. Novice T (mean difference = -11.66, 95% CI: -25.589 to 2.269) and p = 0.144, this means that even though, their mean difference was high, there was not significantly different from male students in Cont. no Intr. & Cont. Novice T on Applied Calculus 1 achievement scores. Cont. no Intr. was significantly different from both CL (M = 72.45, St.D = 12.05) and MCL (M = 73.76, St.D = 13.71); the magnitude of the differences in the mean of male students in Cont. no Intr. & CL and Cont. no Intr. & MCL (mean difference = -22.998, 95% CI: -36.956 to -9.040) and p = 0.000 and (mean difference = -24.300, 95% CI: -38.552 to -10.047) and p = 0.000 respectively, this means there was significant difference between male students in Cont. no Intr. & CL and Cont. no Intr. & MCL on Applied Calculus 1 achievement scores. Cont. Novice T was significantly different from both CL and MCL; the magnitude of the differences in the mean of male students in Cont. Novice T & CL and Cont. Novice T & MCL (mean difference = -11.338, 95% CI: -20.850 to -1.826) and p = 0.012 and (mean difference = -12.640, 95% CI: -22.629 to -2.650) and p = 0.007 respectively, this means that there was significant difference between male students in Cont. Novice T & CL and Cont. Novice T & MCL on Applied Calculus 1 achievement scores.

4.4.3.1.Summary of hypothesis three

This hypothesis is partly supported by the statistical outcome of the study. As the study showed the male students that took a lesson through MCL intervention method achieved significantly better results than male students that took a lesson through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1 achievement. Male students that took a lesson through CL intervention method achieved better results than male students that took a lesson through T intervention method in Applied Calculus 1, but they did not achieve significantly better in pre-calculus mathematics. And male students that took a lesson through MCL intervention method did not achieve significantly better than male students that took a lesson through CL intervention method in Pre-Calculus Mathematics and Applied Calculus 1 achievement.

For further understanding the following Figure 8 shows that the influence of refreshment module of pre-calculus mathematics on pre-engineering first year university male students taught through MCL, CL and T intervention method on Applied Calculus 1.

Pretest, eight weeks' posttest and normal class of Applied Calculus 1 mean of the experimental and control groups of males are graphically shown in Figure 8.

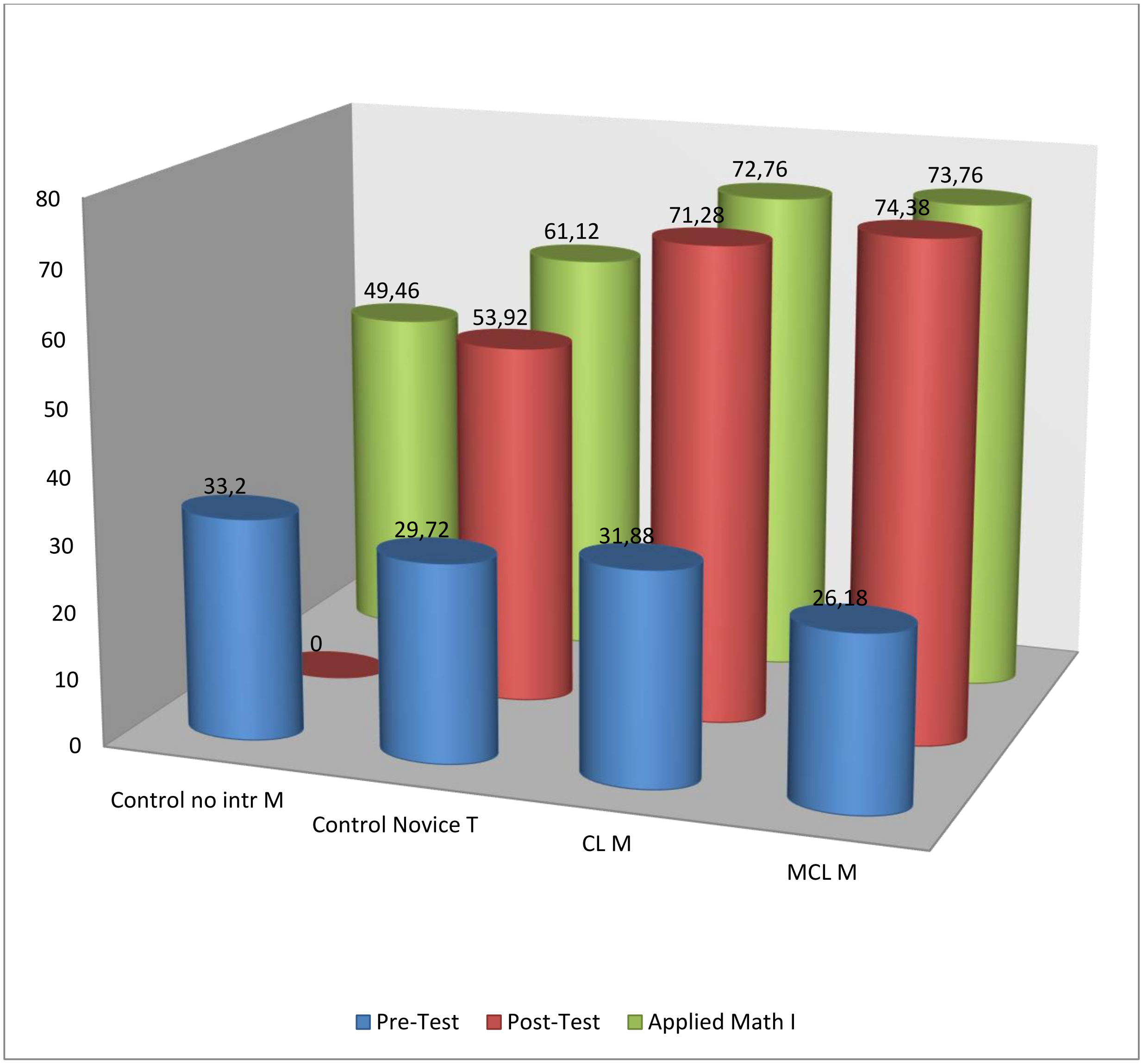


Figure 8: Mean of male students on pre, posttest and Applied Calculus 1 of the experimental and control groups

4.4.4. Testing hypothesis four

Hypothesis four stated that, female students who are taught through MCL intervention method would achieve better than female students who are taught through CL intervention method that, would achieve better than female students who are taught through T intervention method in pre-calculus and Applied Calculus 1 student achievement.

Posttest and Applied Calculus 1 scores of the experimental and control groups of female students are presented in the following table and figure.

Table 30: Descriptive statistics of Posttest and Applied Calculus 1 of female student in each group

Groups		Mean	N	St. D	Min	Max	Kurtosis	Skewness
Posttest	Cont NoviceTF	46.040	25	22.46	13.00	95.00	-.260	.629
	CL F	70.880	25	21.74	34.00	99.00	-1.439	-.208
	MCL F	73.200	25	10.57	50.00	90.00	-.480	-.293
	Total	63.373	75	22.505	13.00	99.00	-.776	-.351
Applied Calculus Result	Cont. no Intr. F	46.590	22	17.03	30.00	75.50	.837	.837
	Cont. Novice T F	56.44	25	10.35	40.00	81.00	.768	1.025
	CL F	65.304	23	11.36	47.00	91.00	.440	.640
	MCL F	63.318	22	12.12	39.00	84.00	-.814	-.085
	Total	57.945	92	14.58	30.00	91.00	-.575	-.023

The above Table 30 of the posttest and Applied Calculus 1 means of the experimental and control groups on female students shows in case of MCL experimental group, the posttest and Applied Calculus 1 were mean (73.2) , **St. D** (10.57), skewness (-.293) and Kurtosis (-.480) and mean (63.318), **St. D** (12.12), skewness (-0.085) and Kurtosis (-0.814) respectively. In case of CL experimental group, the posttest and Applied Calculus 1 were mean (70.88), **St. D** (21.74), skewness (-0.208) and Kurtosis (-1.439) and mean (65.3), **St. D** (11.36), skewness (0.640) and Kurtosis (0.44) respectively. In case of Cont. Novice T group, the posttest and Applied Calculus 1 were mean (46.04), **St. D** (22.46), skewness (.629) and Kurtosis (-0.26) and mean (56.59), **St. D** (10.35), skewness (1.025) and Kurtosis (.768) respectively. In case of Cont. no Intr. group, they had not posttest and Applied Calculus 1 were mean (46.59), **St. D** (17.03), skewness (0.837) and Kurtosis (0.837).

The bar graph of achievement posttest and Applied Calculus 1 of female students as follows

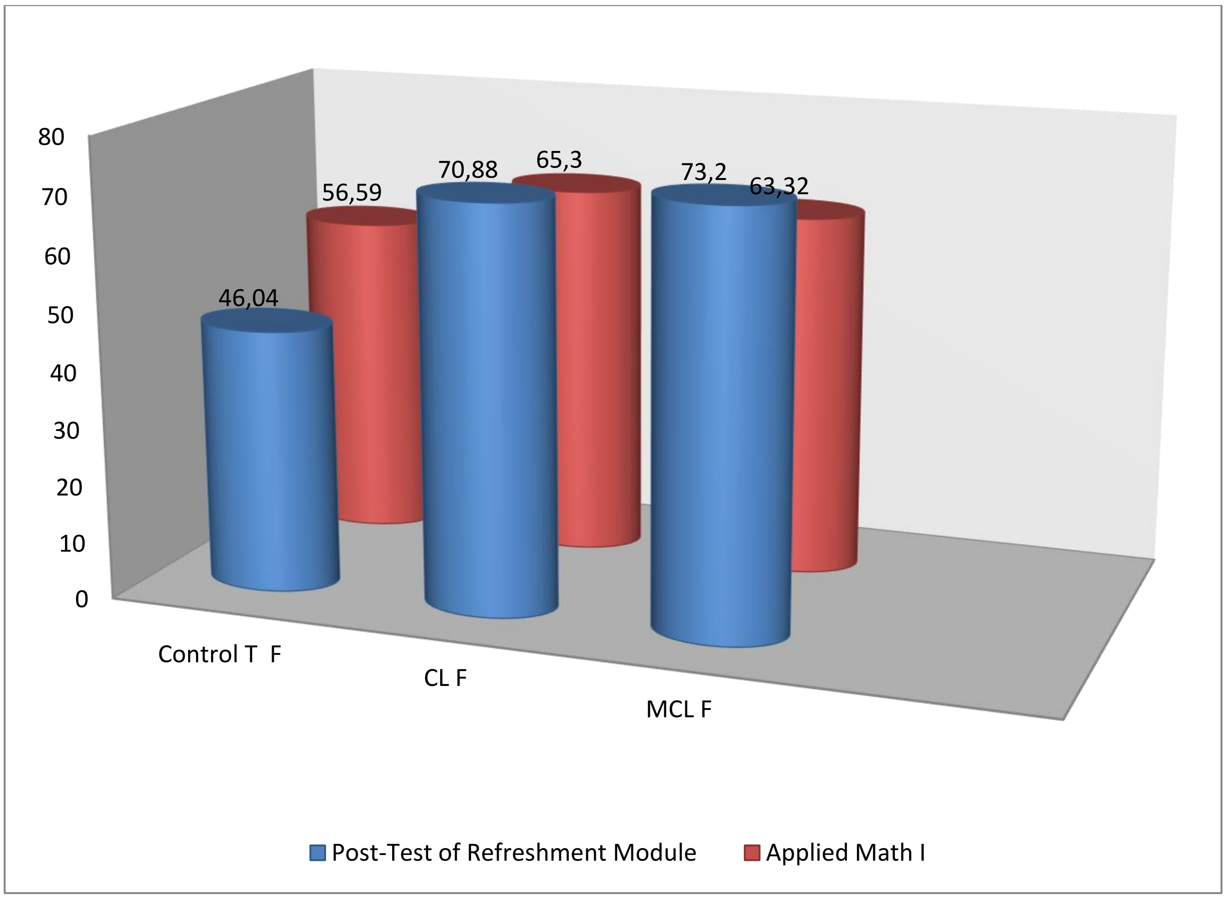


Figure 9: Bar graph of Mean Posttest and Applied Calculus 1 of Females control and Total Exp groups

To see the influence of intervention method of refreshment module of pre-calculus mathematics for pre-engineering first year university female students, ANOVA on pre-calculus mathematics posttest results of the three female groups presented in the following Table 31

Table 31: ANOVA on pre-calculus mathematics posttest result of the three female students' groups

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	11333.947	2	5666.973	15.606	.000
Within Groups	26145.600	72	363.133		
Total	37479.547	74			

*. The significant level of mean variation is at0.05 level.

As Table 31 shows there was a significant difference, $p < 0.05$, among the three female groups on pre-calculus posttest result $F (2, 72) = 15.606$, $p = 0.000$. The large effect size 30%, calculated using eta squared, was 0.30.

To identify which group is significantly different from other; first determine which path to use, homogeneity of variance was tested. Homogeneity of variation from Levene's test ($p = 0.00$) violated homogeneity among groups on pre-calculus mathematics (See Appendix 3H). ANOVA test was conducted by using "Post Hoc" Tahamane's T2 path and the results of pre-calculus mathematics posttest of female students presented in the following Table 32:

Table 32: ANOVA multiple comparisons test of post- test of female students in each group

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Cont. Novice T F	CL F	-24.840*	6.252	.001	-40.309	-9.370
	MCL F	-27.160*	4.966	.000	-39.627	-14.692
CL	MCL	-2.320	4.836	.951	-14.451	9.811

*. The significant level of mean variation is at0.05 level.

As it is shown in Table 30 descriptive statistics and Table 32 Post-hoc comparisons using Tamhane's test indicated that the mean score of female students for CLF (M = 70.88, St.D = 21.74) was not significantly different from female students for MCLF (M = 73.2, St.D = 10.75); the magnitude of the differences in the mean CLF & MCLF (mean difference = -2.320, 95% CI: -14.451 to 9.811) and p = 0 .951, this means that there was not significant different between female students in CL & female students in MCL on selected pre-calculus mathematics posttest scores. Cont. Novice TF (M = 46.04, St.D = 22.46) was significantly different from both CLF and MCLF; the magnitude of the differences in the mean of Cont. Novice TF & CLF and Cont. Novice TF & MCLF (mean difference = -24.840, 95% CI: -40.309 to -9.370) and p = 0.001 and (mean difference = -27.160, 95% CI: -39.627 to -14.692) and p = 0.000, this means that there was significant difference between female students in Cont. Novice TF & CLF and Cont. Novice TF & MCLF on selected pre-calculus mathematics posttest scores.

To see the influence of intervention method of refreshment module of pre- calculus mathematics on Applied Calculus 1 for pre- engineering first year university female students, ANOVA on Applied Calculus 1 results of the four female groups presented in the following Table 33

Table 33: ANOVA on Applied Calculus 1 result of the female students in each group

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	4773.608	3	1591.203	9.595	.000
Within Groups	14593.620	88	165.837		
Total	19367.228	91			

*. The significant level of mean variation is at0.05 level.

Table 33 shows that there was a significant difference, $p < 0.05$, among the four female groups on Applied Calculus 1 result F (3, 88) = 9.595, p = 0.000. The large effect size 25%, calculated using eta squared, was 0.25.

To identify which group is significantly different from other; first determine which path to use, homogeneity of variance was tested. It violated the homogeneity of variation from Levene's test (p = 0.017) among groups on Applied Calculus 1 (See Appendix 3I). ANOVA test was conducted by using "Post Hoc" Tahamane's T2 path and the results of Applied Calculus 1 results of female students presented in the following Table. 34:

Table 34: ANOVA Multiple Comparisons test of Applied Calculus 1 result of female students in each group

(I) Groups	**(J) Groups**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
					Lower Bound	Upper Bound
Cont. no Intr.	Cont. Novice TF	-9.849	4.180	.138	-21.530	1.832
	CLF	-18.713*	4.336	.001	-30.776	-6.650
	MCLF	-16.727*	4.457	.004	-29.099	-4.355
Cont. Novice TF	CLF	-8.864*	3.146	.042	-17.526	-.201
	MCLF	-6.878	3.311	.237	-16.025	2.269
CLF	MCLF	1.98617	3.50652	.994	-7.6892	11.6615

*. The significant level of mean variation is at 0.05 levels

As it is shown in Table 30 descriptive statistics and Table 34 presents the results of Post-hoc comparisons using Tamhane's T2 test showed that the Applied Calculus 1 mean scores of female students for Cont. no Intr. (M=46.59, St.D = 17.03) was significantly different from female students for CLF (M = 65.3, St.D = 11.36) and MCLF (M = 63.32, St.D = 12.12); the magnitude of the differences in the mean Cont. no Intr. F & CLF and Cont. no Intr. F & MCLF (mean difference = -18.713, 95% CI: -30.7767 to -6.650) and p = 0 .001 and (mean difference = -16.727, 95% CI: -29.099 to -4.355) and p = 0 .004 respectively, this means that there was a large significant difference between female students in Cont. no Intr. F& CLF and Cont. no Intr. F & MCLF on Applied Calculus 1. Cont. no Intr. (M=46.59, St.D = 17.03) was not significantly differ from female students for Cont. Novice TF (M = 56.44, St.D = 10.35); the magnitude of the differences in the mean Cont. no Intr. F& Cont. Novice TF (mean difference = -9.849, 95% CI: -21.530 to 1.832) and p = 0 .138, this means that there was not significant differ between female students in Cont. no Intr. F & Cont. Novice TF on Applied Calculus 1. Cont. Novice T F was significantly different from CLF; the magnitude of the differences in the mean Cont. Novice TF & CL (mean difference = -8.864, 95% CI: -17.526 to -.201) and p = 0 .042, this means there was a significant differ between female students in Cont. Novice TF & CL on Applied Calculus 1. Cont. Novice TF and CLF was not significantly different from MCLF; the magnitude of the differences in the mean Cont. Novice TF & MCLF and CLF & MCLF (mean difference = -6.878, 95% CI: -16.025 to 2.269) and p = 0.237 and (mean difference = 1.986, 95% CI: -7.689 to 11.661) and p = 0 .994 respectively, this means that there was not significant different between female students in Cont. Novice TF & MCLF and CLF & MCLF on achievement of Applied Calculus 1.

4.4.4.1.Summary of hypothesis four

This hypothesis was partly supported by the statistical outcomes of the study. As the study showed the female students that took lessons through MCL intervention method achieved significantly better than female students that took a lesson through T intervention method in pre-calculus mathematics. Female students that took a lesson through CL intervention method achieved significantly higher than female students that took lessons through T intervention method in Pre-Calculus Mathematics and Applied Calculus 1. And female students that took lessons through MCL intervention method was not achieved better than female students that took lessons through CL intervention method in Pre-Calculus Mathematics and Applied Calculus 1 achievement.

For further understanding the following Figure 10 shows that the influence of refreshment module of pre-calculus mathematics on pre-engineering first year university female students that took lessons through MCL, CL and T intervention method in Applied Calculus 1 Pretest, outcome of eight weeks' intervention posttest and normal class of Applied Calculus 1 mean of the experimental and control groups of female students are graphically shown in Figure 10.

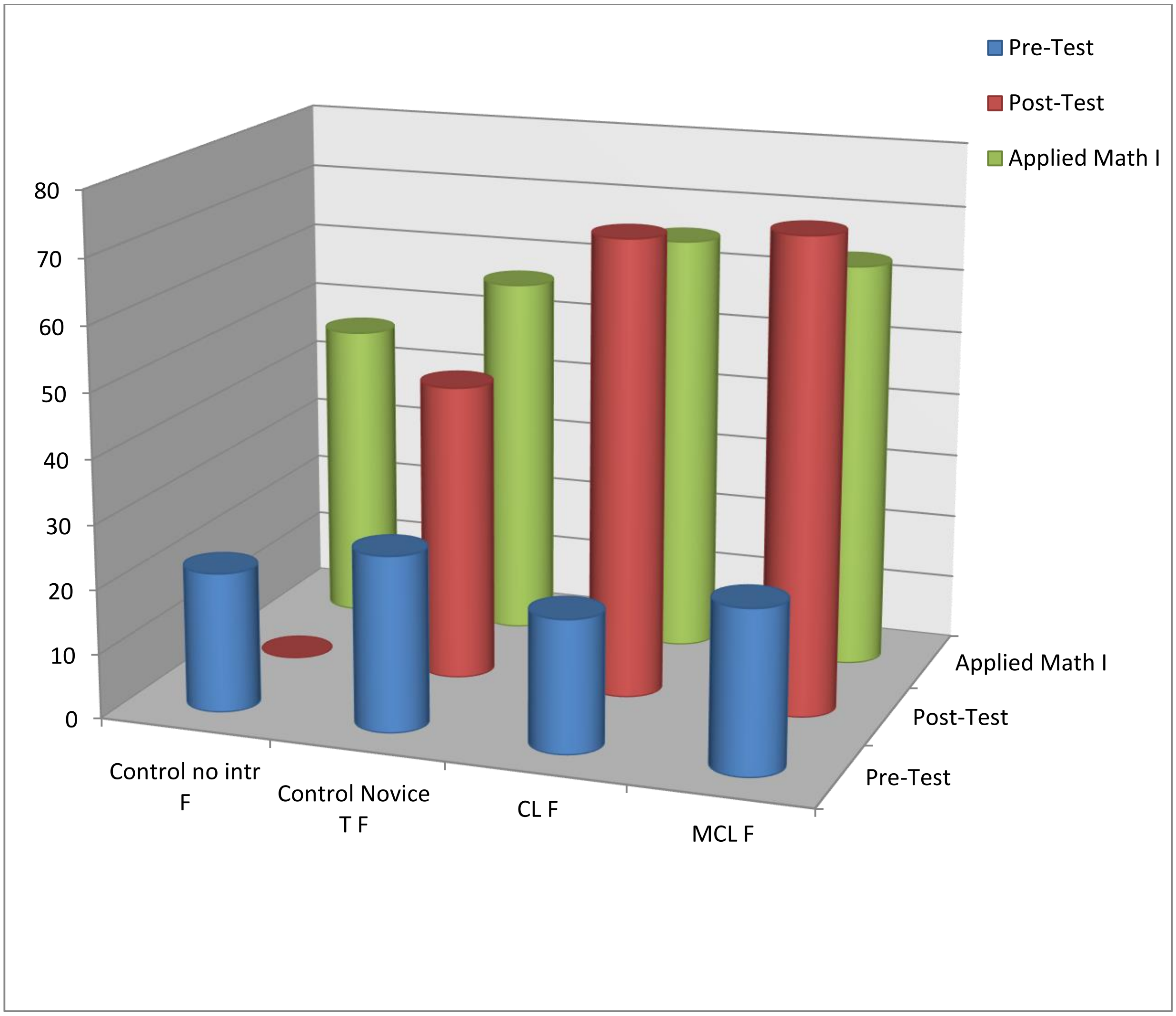


Figure 10: Bar graph of Mean of males Pre, Post and Applied Calculus 1 of the Experimental and Control Groups

4.4.5. Testing hypothesis five

Hypothesis five stated that, there is a positive correlation between pre-calculus mathematics refreshment module and Applied Calculus 1 achievement.

Table 35: Pearson correlation of selected pre-calculus mathematics refreshment and Applied Calculus 1 achievement

Refreshment Module & Applied Calculus 1	Female students		Male students		Entire students	
	Correlation coefficient	Sign (2-tailed)	Correlation coefficient	Sign (2-tailed)	Correlation coefficient	Sign (2-tailed)
	0.787	0.00	0.835	0.00	0.834	0.00

Correlation is significant at the 0.01 level (2-tailed)

To show correlation between pre-calculus mathematics refreshment and Applied Calculus 1 achievement researcher used Pearson correlation coefficient. As shown in Table 35 the results also indicated that female students' pre-calculus mathematics achievement was significantly correlated with Applied Calculus 1 achievement, with a correlation coefficient of 0.787 and $p < 0.05$. The results also indicated that male students' pre-calculus mathematics achievement was significantly correlated with Applied Calculus 1 achievement, with a correlation coefficient of 0.835 and $p < 0.05$. Refreshment module of pre-calculus mathematics test scores had a higher correlation with Applied Calculus 1 scores for first year pre-engineering students, with a correlation coefficient of 0.834 and $p < 0.05$ for total sampled students.

Linear correlation of selected content pre-calculus mathematics refreshment module and Applied Calculus 1 of first year pre-Engineering students are graphically shown in Figure 11.

Regression→ Curve Estimation

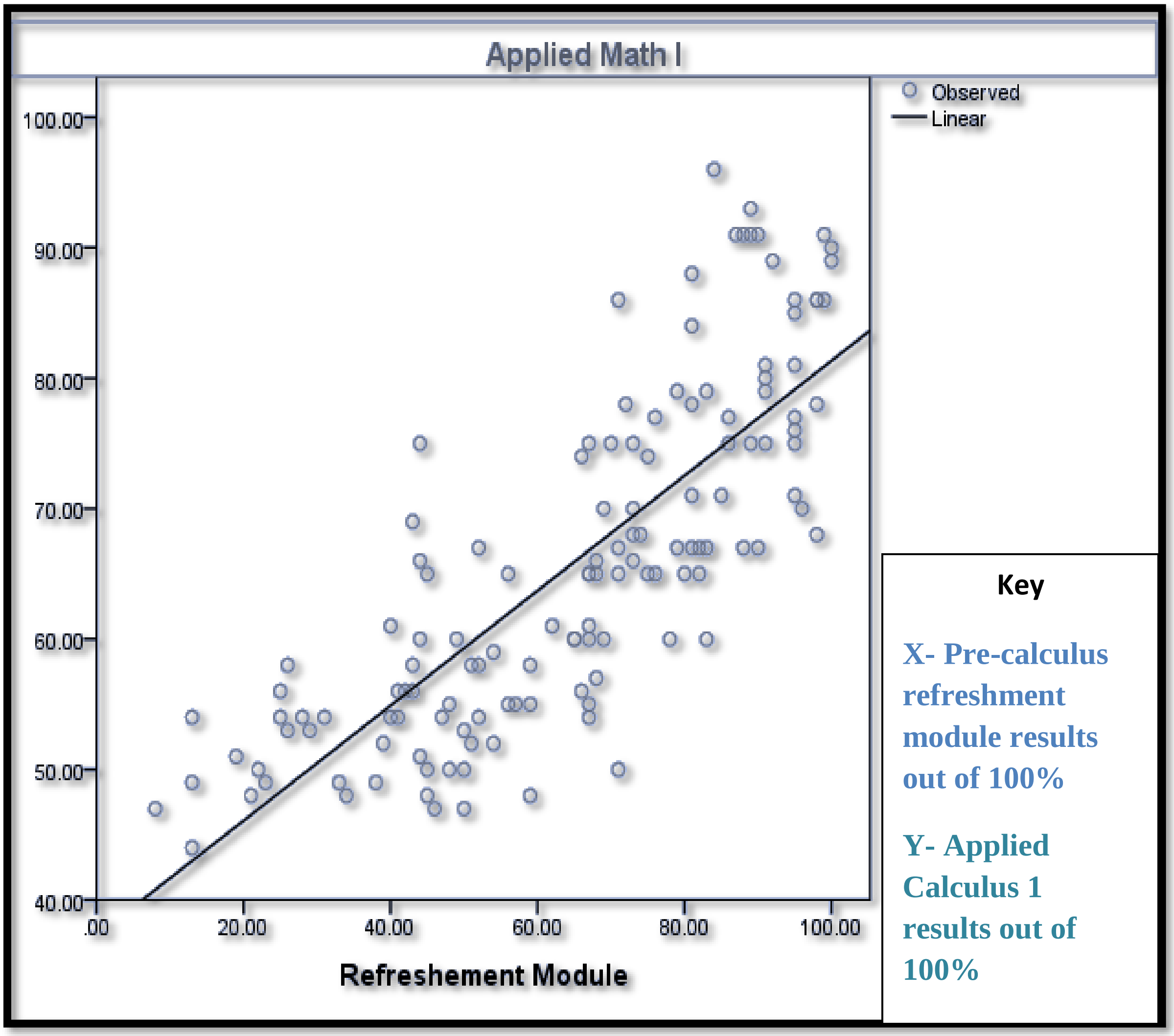


Figure 11: Linear correlation co-efficient of pre-calculus mathematics refreshment module and Applied Calculus 1

4.4.5.1.Summary of hypothesis five

The statistical results support the hypothesis, that is, there is a positive correlation between Pre-Calculus Mathematics refreshment module and Applied Calculus 1 achievement.

4.4.6. To answer research question number seven

Research question number seven states that: Do the achievement of male students the same as female students who would token refreshment module of previously acquired skill in pre-calculus through MCL, CL and T Intervention Method?

To answer the above research question number seven Pre-test, Posttest and Applied Calculus 1 scores of descriptive statistics of the experimental and control groups of male and female students are presented side by side in the following Table 36 and Figure 12.

Table 36: Descriptive statistics of overall tests of gender difference in each group

Group	Mean of pre- test	St.D of pre - test	Mean of post –test	St. D of post -test	Mean of App Math I	St.D of App Math I
Cont. no Intr. F	21.68	21.88	.	.	46.59	17.03
Cont. no Intr. M	33.24	17.35	.	.	49.46	21.83
Cont. Novice T F	27.2	18.11	46.04	22.46	56.44	10.35
Cont. Novice T M	29.72	16.85	53.92	26.83	61.12	12.19
CL F	20.56	16.22	70.88	21.75	65.30	11.36
CL M	31.88	17.26	71.28	24.24	72.46	12.05
MCL F	25.2	8.64	73.2	10.58	63.31	12.12
MCL M	26.16	20.33	74.35	9.41	73.76	13.51

For further understanding the following Figure 12 shows the gender difference in each experimental and control groups on Pre-Calculus Mathematics and Applied Calculus 1 achievement of pre-engineering first year university female and male students.

Pretest, outcome of eight weeks' posttest and normal class of Applied Calculus 1, means of the experimental and control groups of female and male students are graphically shown in the following Figure 12.

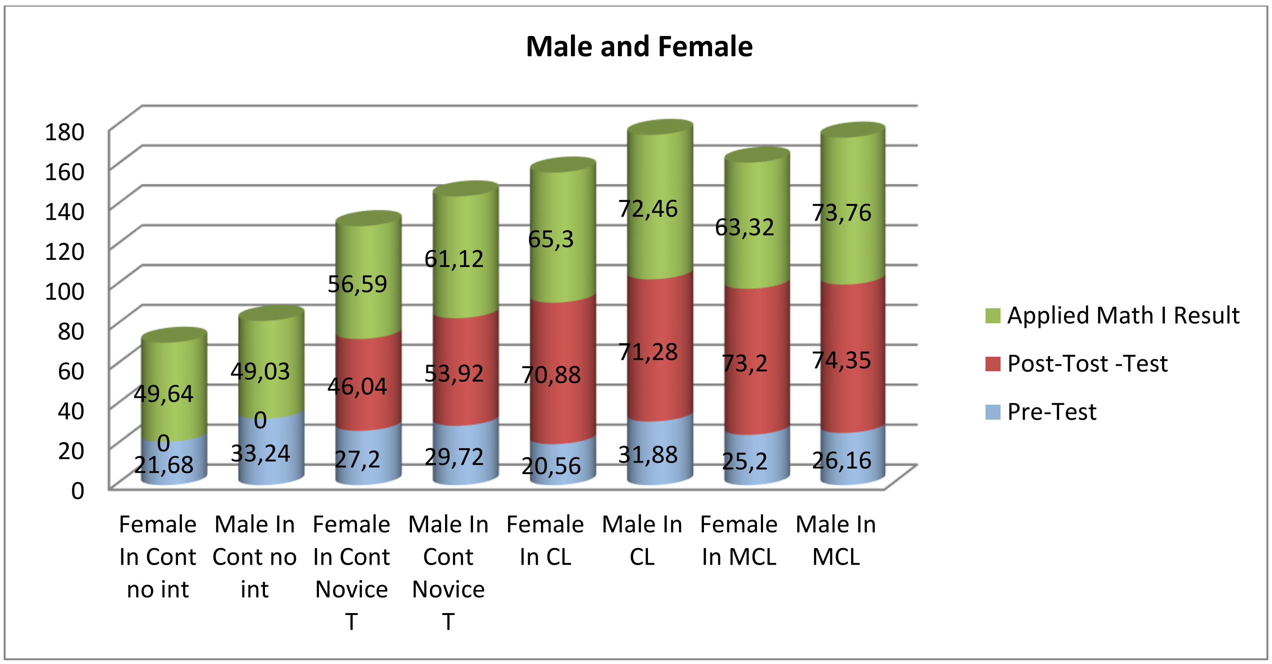


Figure 12: Bar graph of Means of male and female students on Pre, Posttest and Applied Calculus 1

To see gender difference on mathematics achievement of first year pre-engineering university students, Independent sampling T-test on pre-calculus mathematics posttest results and Applied Calculus 1 of the groups presented in the following Table 37:

Table 37: Independent Samples T–Test of gender difference in each group on posttest result

Groups		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Cont. Novice T Male	Female	1.23	.274	-1.12	47	.270	7.88	7.05794	-22.07	6.32208
CL Male	Female	1.08	.303	-.061	48	.951	0.40	6.51277	-13.49	12.6948
MCL Male	Female	.474	.494	-.410	47	.684	1.18	2.86430	-6.937	4.58724

*. The significant level of mean variation is at 0.05 levels

In Table 36 and Table 37 differential statistics (M= mean and St.D = standard deviation) and for inferential statistics, an independent-samples t-test respectively conducted to compare the pre-calculus mathematics scores for male and female students in each group. There was not significantly differ from Cont. Novice T male students (M = 53.92, St.D = 26.83) and female students (M = 46.04, St.D = 22.46); t (47) = -1.12, p = 0.27, two-tailed) and their mean variation (mean difference = 7.88, 95% CI: –22.07 to 6.32) with very small size effect 0.2% (eta squared = .002). There was not significantly differ from CL males (M = 71.28, St.D = 24.24) and CL females (M = 70.88, St.D = 21.75); t (48) = -0.061, p = 0.951, two-tailed) and their mean variation (mean difference = 0.4, 95% CI: –13.495 to 12.69) with very small size effect 1.9%

(eta squared = .019). And there was not significantly differ from MCL males (M = 74.37, St.D = 9.41) and MCL females (M = 73.20, St.D = 10.58); t (47) = -0.41, p = 0.684, two-tailed). Their mean variation (mean difference = 1.18, 95% CI: –6.94 to 4.59) was very small size effect 0.5% (eta squared = .005). Table 37 shows there was not significant differ between the two genders in each intervention method (MCL, CL and T) on posttest of pre-calculus mathematics results.

Table 38: Independent Samples T–Test of gender difference in each group on Applied Calculus 1 achievement result

Between Gender In		**Levene's Test for Equality of Variances**		**t-test for Equality of Means**						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Cont. no Intr. F	Cont. no Intr. M	1.76	.192	-.504	44.36	.617	-2.87	5.69	-14.34	8.60
Cont. Novice T F	Cont. Novice T M	2.44	.125	-1.46	46.8	.150	-4.68	3.20	-11.12	1.76
CL F	CL M	.313	.578	-2.09	44.99	.042	-7.154	3.42	-14.03	-.27
MCL F	MCL M	.417	.522	-2.79	44.97	.008	-10.44	3.74	-17.97	-2.91

*. The significant level of mean variation is at 0.05 levels

In Table 36 differential statistics and Table 38 inferential statistics an independent-samples t-test respectively was conducted to compare the Applied Calculus 1 scores for male and female students in each group. There was not significantly different from Cont. no Intr. Females (M = 46.59, St.D = 17.03) and Males (M = 49.46, St.D = 21.92); t (44.36) = -.504, p = 0.617, two-tailed). Their variation of means (mean difference = -2.87, 95% CI: -14.34 to 8.60) was very small size effect 0.6% (eta squared = .006). There was not significantly different from Cont. Novice T Females (M = 56.44, St.D = 10.35) and Males (M = 61.12, St.D = 12.19); t (46.77) = -1.46, p = 0.150, two-tailed). Their variation of means (mean difference = -4.68, 95% CI: -11.12 to 1.76) was very small size effect 4.5% (eta squared = 0.045). There was a significant difference in scores for CL Females students (M = 65.36, St.D = 11.36) and CL Males students (M = 72.46, St.D = 12.05); t (44.99) = -2.09, p = 0.042, two-tailed). Their variation of means (mean difference = -7.154, 95% CI: -14.03 to -.27) was moderate size effect 9.2% (eta squared = .092). And there was a significant difference in scores for MCL Females students (M = 63.31, St.D = 12.12) and MCL Males (M = 73.76, St.D = 13.51); t (44.97) = -2.79, p = 0.008, two-tailed). Their variation of means (mean difference = -10.44, 95% CI: -17.97 to -2.91) was large size effect 15% (eta squared = 0.153). Table 38 shows there was not significant different between the two genders in Cont. Novice T intervention method but there was a significant difference between two genders in MCL & CL intervention method on Applied Calculus 1 mathematics result.

Generally, the following Table 39 and Figure 12 show the totality gender difference on mathematics achievement of pre-engineering first year university students.

Table 40: Descriptive statistics of total gender difference on Pre-test, Post –Test and Applied Calculus 1 achievement

Gender	**Pre- Test**		**Post –Test**		**Applied Calculus 1**	
	Mean	STA.DEV	Mean	STA.DEV	Mean	STA.DEV
Female	23.95	16.955	63.373	22.505	57.945	14.588
Male	30.25	17.929	66.589	23.140	64.116	18.184

Pretest, outcome of eight weeks' intervention posttest and normal class of Applied Calculus 1 mean of experimental and control groups of male and female students are graphically shown in the following Figure 13.

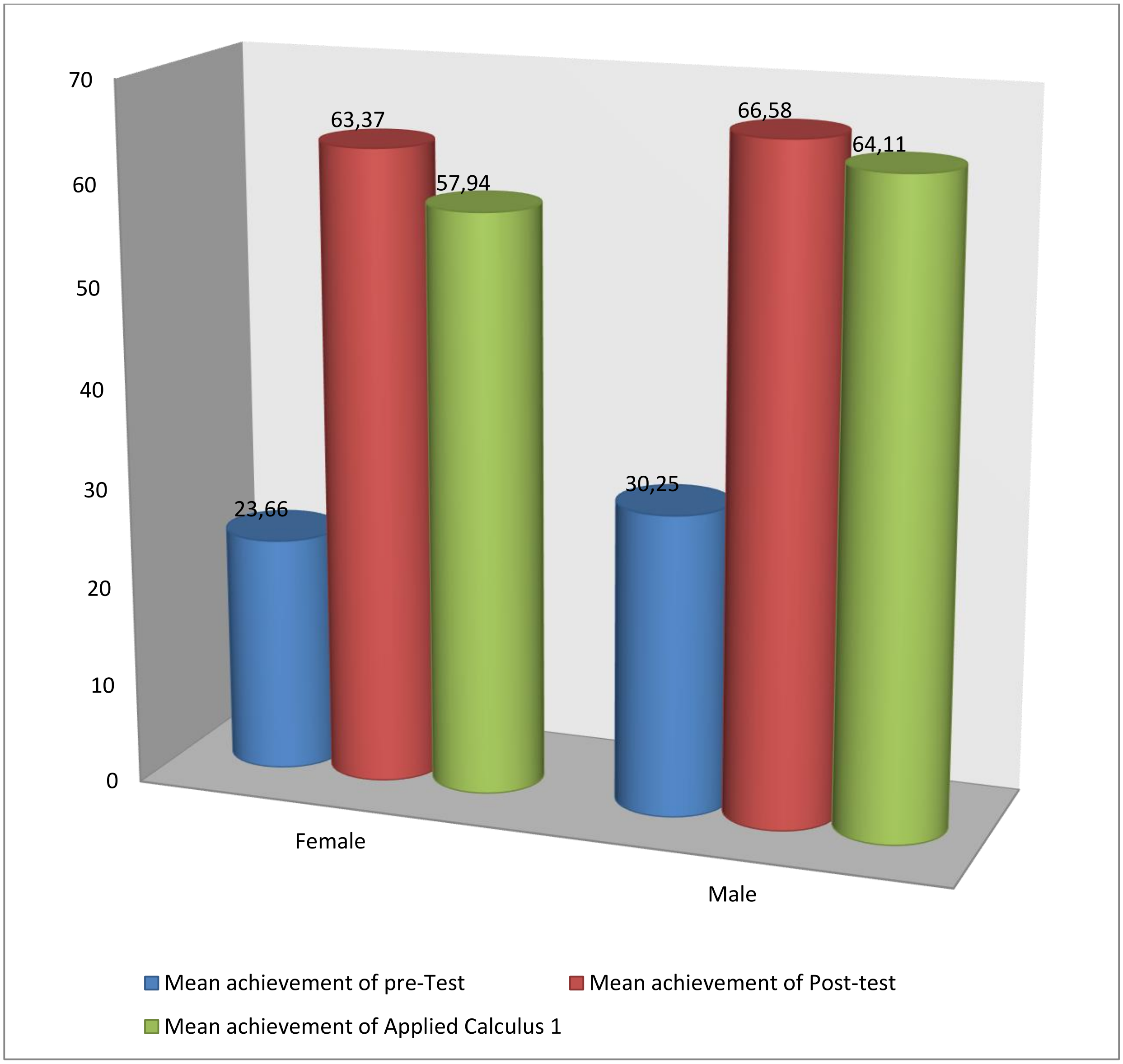


Figure 13: Gender difference on mathematics achievement

Table 41: ANOVA of total gender significant difference on Pre-test, Post –Test and Applied Calculus 1 achievement

			Sum of Squares	df	Mean Square	F	Sig.
Gender onPre-test * (Male & female) Eta Square.032	Between Groups	(Combined)	1984.50	1	1984.5	6.52	.011
	Within Groups		60287.50	198	304.48		
	Total		62272.00	199			
Gender on Post - Test * Eta Squared .230	Between Groups	(Combined)	17575.66	5	3515.1	8.48	.000
	Within Groups		58840.098	142	414.37		
	Total		76415.757	147			
Gender Applied Calculus 1 * Eta Squared .034	Between Groups	(Combined)	1815.648	1	1815.6	6.63	.011
	Within Groups		51773.142	189	273.93		
	Total		53588.791	190			

Table 40 shows that overall, there was significant differences between genders (male & female students) on Pre-test, Posttest and Applied Calculus 1 achievement, $p < 0.05$; on Pre-test Results $F(1, 198) = 6.52$, $p = 0.011$. The small effect size 3.2%, calculated using eta squared, was 0.032. On Posttest Results $F(5, 142) = 8.48$, $p = 0.000$. The large effect size 23%, calculated using eta squared, was 0.23; on Applied Calculus 1 results $F(1, 189) = 6.63$, $p = 0.011$. The small effect size 3.4%, calculated using eta squared, was 0.034. Table 40 presents there was a significant difference between two genders in Pre-test, Posttest and Applied Calculus 1 mathematics results.

4.4.6.1.Summary of research question number seven

Research question number seven asks about the achievement of male and female students that took a refreshment module of pre-calculus mathematics through MCL, CL and T Intervention Method is the same or not. The statistical results showed there was a significant difference between two genders in posttest and Applied Calculus 1, and separately there was a significant difference between two genders in MCL & CL intervention method on Applied Calculus 1. But there was no significant difference between two genders in each intervention method (MCL, CL and T) on the score of posttest of pre-calculus mathematics.

CHAPTER FIVE

DISCUSSION AND CONCLUSION

5.1.Introduction

The major objective of this study was to find out the influence of pre-calculus mathematics refreshment module with different intervention methods to first year pre-engineering students in Ethiopian government universities. The study further investigated the effect of intervention methods, as general as well as with respect to gender difference, in activating prior background knowledge of pre-engineering first year university students on selected pre-calculus mathematics topics i.e. basic algebra, equations and inequalities, function, exponential and logarithmic functions, and trigonometric functions with different intervention method (meta-cognitive with co-operative learning, co-operative learning and traditional lecture method).

In this chapter interpretation of Pre-test and post-test results and discussion of the findings, summary and conclusions, and implications derived from the findings are reported. The limitations of the study and recommendations with suggestions for further research are also presented.

5.2.Interpretation of Pre-test

5.2.1. **Pre-test differences between groups**

The analysis of the Pre-test results was carried out before the intervention. The result showed that there were no statistically significant mean difference among the four groups (control and experimental). As the Pre-test showed the score of the whole four groups, the male students in four groups as well as the female students in four groups were not significantly different. Whereas the significant difference was seen between genders (female and male) in the Cont. no Intr. and CL groups. And also the score of the total of two genders in the four groups showed statistically significant mean difference. This analysis indicated that male students' mean score results were found to be higher than female students' mean scores on the pre-calculus mathematics Pre-test scores.

According to literature review in most countries male and female students differ in their mathematics achievement. Even if some of the findings are contradictory, the results regarding background knowledge of pre-calculus mathematics has generally shown that males tend to do better than females (Korir & Laigong, 2014; OECD, 2011; & Payne, 2015). So the researcher included appropriate intervention method to this study in order to minimize the gap of prior knowledge of pre-calculus mathematics resulted from gender differences on pre-engineering students.

5.2.2. Research question number one: Extent of first year pre-engineering students recalls some basic pre-calculus mathematics concepts.

For question number one descriptive statistics analyses were applied to answer; what extent university first year pre-engineering students recalls some basic pre-calculus mathematics? The study showed that over all groups mean was 30.24 out of 100, St.D was 17.95, Skewness was 0.45, and Kurtosis was -0.90. Frequency and percentage of correct and incorrect items answered by two genders were examined separately for each item of questions in each selected topics of pre-calculus mathematics. In most of the items, percentage of correctly answered items was below 35%. Then mean score and standard deviation of each group with moderator variable were analyzed and the results indicated that all means of Pre-test of groups were in the interval of 20 to 33 out of 100. These findings are similar to the findings of the studies held by various scholars that many students who are going to take Applied Calculus 1 are not well prepared on pre-calculus mathematics and as a result it is common to see that they have been facing challenges regarding their skills and knowledge of mathematics in their first year university course (Karim et al., 2010; Cunningham & Rory, 2014).

5.3. Discussion

5.3.1. Research hypothesis one

The first research hypothesis asserted that pre-engineering first year university students who received refreshment of selected pre-calculus mathematics contents with different intervention methods (MCL, CL and Cont. Novice T) would achieve better applied calculus mean scores than pre-engineering first year university students who did not receive such instruction (Cont. no Intr.).

In this study the effectiveness of refreshment of selected pre-calculus mathematics module, which builds conceptual understanding and procedural fluency of background knowledge, on Applied Calculus 1 achievement was examined. According to Baker (n.d.), learning mathematics with understanding involves competency in basic skills where mathematics is a tool used by engineers. And Savoy (2007), also states that students become fluent in using mathematics to communicate concepts, ideas and information when they have prior knowledge of pre-calculus mathematics. Applied calculus 1 is pre-request for pre-engineering students, to overcome their achievement of Applied calculus 1 conceptual understanding of mathematics provides a more holistic equation for them (Korn, 2014). Learning with understanding refers to connection and knowledge construction in everything that students do (Liu & Chun-Yi, 2011).

These findings therefore indicate the importance of social construction of knowledge and co-operative learning gains towards understanding of mathematical concepts. According to Stylianides (2007), epistemological meaning of mathematical knowledge starts from problems. To tackle problems students' experiences and knowledge are needed (Short et al., 2013). What students already know (Background knowledge) about the content is one of the strongest pointers of how well they will learn new information relative to the content (Campbell, 2009; Short et al., 2013; Alfaki & Siddiek , 2013). The fact that students that took a refreshment module of pre-calculus mathematics (improved their back ground Knowledge) in MCL, CL & T appeared to achieve higher on the Applied Calculus 1 than students who did not take refreshment module in Cont. no Intr. is in turn with the findings of Marzano (2004); Belina, (2012); MacNeal (2015); and Loughlin et al. (2015). Currently first year pre-engineering students have not enough background prerequisite knowledge of basic pre-calculus mathematics. This causes failure on the engineering students' because of lack of grasp prior knowledge concepts and the way to apply them in the context of their discipline of choice. This shortage of prior knowledge by itself often leads to feelings of anxiety, stress and lack of self-confidence and potentially results in the students dropping out of University. So to overcome these problems as the findings also indicated building background knowledge of pre-calculus mathematics before beginning a lesson that enhances students' achievement is the crucial issue for pre-engineering university students.

5.3.2. Research hypothesis two

This hypothesis was partly supported by the statistical outcome of the study. As the study showed the students that took lessons through MCL and CL intervention methods achieved significantly better result than the students that took lessons through T intervention method on Pre-Calculus Mathematics and Applied Calculus 1. Whereas, students that took lessons through MCL intervention method did not achieved better than that took lessons through CL intervention method on Pre-Calculus Mathematics and Applied Calculus 1 achievement.

The outcome of this study is align with Marzano (2004) activating students' background knowledge and creating long term retention of students' understanding by applying intervention method play a major role on their new knowledge being constructed. However, still now most of the mathematics instructors have been using traditional (T) lecture approach. This idea is also agreed by the scholars like Mahajan (2014). According to Cottrill (2003), Traditional (T) lecture approach is behaviorist learning strategy that students attempt to explain learning, without inferring anything that is going on inside, through the observable interactions. It confirmed that control group that took a refreshment of pre-calculus mathematics module through traditional (T) intervention strategies both in Pre-Calculus Mathematics posttest and Applied calculus 1 achievement mean was much less than those two groups who took a refreshment of pre-calculus mathematics module through CL and MCL intervention strategies.

Hein (1991) states that pushing students towards constructive learning approach to learn mathematics with understanding strengths problem solving abilities and mathematical thinking skills. Karimganj (2015) states that knowledge is internalized when students actively use their prior knowledge and engaged to construct their new knowledge. In addition Korn (2014), states that knowledge which involves carrying out actions or operations cannot be instilled ready-made into students but must actively built up by them. Thus, sharing experiences with others is essential to success since they will always be exposed to a variety of experiences in which they will have to cooperate and navigate among the ideas of others (Dada, 2015).

Based on the constructivist theory that students who have dealt with the CL and MCL strategies where expected to be set into small groups. For this reason, the whole class was divided into small groups that contain 6 students (3 males & 3 females) respectively. These small groups were provided with activities in order to work together to complete the common task co-operatively. According to Jonson et al. (2000) CL provides the opportunities for individuals search for remarkable outcomes that are beneficial to themselves as well as beneficial to all other group members. The outcomes of this research align with the findings of other scholars like Felder and Rebecca (2007). Felder and Rebecca (2007), state that co-operative learning in higher education is effective. Therefore, the CL and MCL method provided the students with the opportunities to stretch and extend their understanding of Pre-Calculus Mathematics and Applied Calculus 1 more than the students that took refreshment module of pre-calculus mathematics through the T method who worked individually. Students who took refreshment module of pre-calculus mathematics through MCL intervention strategies were also provided meta-cognitive question sheet, beside co-operative setting, which directs students to ask themselves meta-cognition managerial skills of planning like, What am I supposed to learn?, monitoring like, Am I on the right track?, and evaluation like, What did I learn? during learning task. Even though, a growing body of research (Vijoyakumari, 2013) strongly supports a function on meta-cognition of students that involve the student standing outside their process as they focus on thinking about their thinking and understanding the kind of information that she/he used in solving a problem.

According to Rahimi and Kalal (2012) meta-cognition with co-operative learning is one way to accelerate mathematics learning more efficiently and effectively. But the outcome of this study align with the findings of Erskine (2009) in university learning meta-cognitive skills and strategies were considered as wastage of time as to the thought of male students, since they are not able to see how meta-cognitive skills help their learning rather than seeking quick result. Pre-engineering first year university students that took lessons through MCL did not achieve significantly higher than students that took lessons through CL intervention method in (1) Pre-Calculus Mathematics, and (2) Applied Calculus 1.

5.3.3. Research hypothesis three

This hypothesis was partly supported by the statistical outcome of the study. As the study showed the male students that took lessons through MCL and CL intervention methods achieved significantly better result than the male students that took lessons through T intervention method on Pre-Calculus and Applied Calculus 1. Whereas, male students that took lessons through CL have achieved significantly better than male students that took lessons through T intervention method in Applied Calculu1. However, they did not achieve significantly better in pre-calculus mathematics. And male students that took lessons through MCL intervention method did not achieve significantly better than male students that took lessons through CL intervention method in Pre-Calculus Mathematics and Applied calculus 1. Working by using meta-cognitive questions co-operatively further gave the opportunity to male students in MCL method actively engaged in lesson by asking questions each other focusing on the meta-cognitive questions that help them to cope with challenges that direct to think, plan, monitor, explain, elaborate and evaluate, and justify to tackle the given problem. MCL strategy provided male students with opportunity to make justifications, to reason out and focus on the relationship between the previous and new tasks that enhanced their understanding. But the outcome of this study indicated that the mean of male students that took lessons through MCL method was score greater than who took lessons through CL method though there was no statistically significant difference in Pre-Calculus Mathematics and Applied calculus 1.

5.3.4. Research hypothesis four

This hypothesis was partly supported by the statistical outcome of the study. As the study showed the female students that took a lesson through MCL and CL intervention methods achieved significantly better result than the female students that took lessons through T intervention method on Pre-Calculus and Applied Calculus 1. Female students in MCL achieved slightly higher in selected pre-calculus mathematics mean scores than female students that took lessons through CL method.

The assertion that female students that took lessons through MCL method enhanced their achievement in pre-calculus mathematics mean score of posttest than students that took lessons through CL Method and this assertion supported by Nahil & Eman (2015). As these scholars stated that the students who were familiar with meta-cognitive strategies achieve better and perform their tasks effectively because of meta-cognition that plays the significant role in rehearsing and analyzing ability of their background knowledge. Since MCL method creates opportunity for the students to manage, plan and examine the tasks delivered to them, it helps to enhance their achievement and that may be the reason why the female students that took lessons through MCL method achieved better results than female students that took lessons through CL method.

However, female students in MCL who received the meta-cognitive with co-operative learning instruction in pre-calculus mathematics did not score higher mean in the Applied Calculus 1, than the students that took lessons through their usual classroom T, and students in CL.

This hypothesis was partly supported by the statistical outcome of the study. As the study showed the female students that took lessons through MCL and CL intervention methods achieved significantly better results than the female students that took lessons through T intervention method on pre-calculus. Female students that took lessons through CL intervention method achieved significantly better than female students that took lessons through T intervention method in Applied Calculus 1, whereas, female students that took lessons through MCL did not achieve significantly better than female students that took lessons through T intervention method in Applied Calculus1. And female students that took lessons through MCL intervention method did not achieve significantly better than female students that took lessons through CL intervention method in Pre-Calculus Mathematics and Applied Calculus 1.

5.3.5. Research hypothesis five

The fifth hypothesis states that, there is a positive correlation between Pre-Calculus Mathematics refreshment module and Applied Calculus 1 achievement. The finding of this study showed that there was correlation between Pre-Calculus Mathematics refreshment and Applied Calculus 1 achievement. Regarding gender, female students' pre-calculus mathematics achievement was significantly correlated with Applied Calculus 1 achievement, with a correlation coefficient of 0.787 and $p < 0.05$ and male students' pre-calculus mathematics achievement was significantly correlated with Applied Calculus 1 achievement, with a correlation coefficient of 0.835 and $p < 0.05$. Refreshment module of pre-calculus mathematics examination scores has a higher correlation with Applied Calculus 1 scores for first year pre-engineering students, with a correlation coefficient of 0.834 and $p < 0.05$ for the sampled students of the total class. The statistical results support the fifth hypothesis, which asserts that there was a positive correlation between pre-calculus mathematics refreshment module and Applied Calculus 1 achievement on pre-engineering university students.

5.3.6. Research question number seven

Is the achievement of male students the same as female students who take refreshment module of previously acquired skill in pre-calculus through MCL, CL and T intervention Method?

The statistical results indicated that; male students in Cont. No Intr. and CL groups achieved significantly higher than female students in Cont. No Intr. and CL groups respectively inPre-test of pre-calculus mathematics. In T and MCL groups, even though mean of male students is slightly higher than female students, there were no significant differences between two genders in Pre-test of pre-calculus mathematics. In general, there was statistically significant difference in scores of Pre-test on pre-calculus mathematics between males and females of pre-engineering first year university students. After intervention there was no significant difference between two genders in each intervention method (MCL, CL and T) on posttest of pre-calculus mathematics result. Female students taught through the CL and MCL method worked in their small groups with the male students together to complete given tasks and solve problems.

According to Dada (2015), in co-operative setting students construct their knowledge based on constructivism which makes the students take risk of what they learn because CL and MCL methods are considered as base for the students' ability to explain and ask questions to each other and often the students have the skill of how to deal with evaluating their works. Through CL and MCL method, female students were supplied with varies strategies that supported them to justify and evaluate their solution. Consequently, female students minimized their gaps on mathematics achievement with their counterparts' to male students. This is important especially for female students to be successful in mathematical courses, as they got opportunity to share ideas and experiences of others that helped them to be co-operative.

As it is seen from this study, the mean of female students in CL and MCL group was slightly higher, though the difference observed between the female students that took lessons through CL and MCL method was not statistically significant, i.e. mean scores on posttest of MCL was slightly higher than CL group.

MCL strategies that the students use to think, to study and learn and motivated to recall information from memory that helps them to associate or to compare and contrast variety of information that the students use to resolve the existing problems are the critical role played by MCL method. On the other hand, Bergey et al (2015), asserts that meta-cognitive strategies challenged particularly female students the skill of how to apply the knowledge of planning to approach activities in learning processes. However, students in MCL group were equipped with knowledge of meta-cognitive strategies that helped then to manage and monitor their own performance regarding how to proceed with learning tasks, how to solve problems and evaluating the solution of the solved problems.

Therefore, the meta-cognitive questions sheet, that supported to activate prior knowledge related to the new concepts, facilitated to arouse female students' schema and as a result it enabled them recalling information, elaborating and representing in solving the problems. This finding coincides with the argument of Payne (2015), which concludes that female students do underachieve in solving the problems mathematics, were not general true. As this finding shows bringing meta-cognition questions sheet with co-operative setting to the immediate attention of university students, who are expected to fill the gaps of mathematical computation of problem solving, assists to fully utilize their potential to solve including complex problems.

As it has been seen from Pre-test result findings female students was different significantly from their partner male students in pre-calculus achievement and after intervention the gap become not significant. Particularly, those female students that took lessons through MCL and CL method with their partner male students narrowed the gap that had been seen in Pre-test and the post test result indicated that there was no significant difference in pre-calculus posttest result. But there was a significant difference between two genders in Applied Calculus 1 results. The higher increase in mean scores gained by male first year university students was found to be significant on all the three tests, Pre-test, and Applied Calculus 1 achievement scores. This finding further support the argument of Leonard et.al (2011), there was a gender difference in the outcome of mathematics, that male achieve better than female students. And this finding support Palinos' (2011) finding that there was a significant relationship between gender and teaching strategies and techniques; that value in bringing co-operative learning and meta-cognition with co-operative learning to the immediate attention of university students in closing the gap of two genders in mathematics achievement. Meta-cognition with co-operative learning for university students help them to be active user of the methods in order to reflect whether they are effective or not while using them in different topics.

5.4.Summary and Conclusions

It has been investigated in this study about the extent of first year pre-engineering university students' recalled or retrieved some selected basic pre-calculus mathematics contents and the influence of Pre-Calculus Mathematics refreshment module on Applied Calculus 1 result on first year pre-engineering university students achievement. Overall (Control and experimental) groups' mean achievement of Pre-test on selected pre-calculus mathematics was under average whereas after intervention of those selected pre-calculus mathematics topics overall Control with Novice T and experimental groups that took lessons of refreshment course outperformed than the control no Int. group that did not take lessons of refreshment course. MCL group scored slightly higher than CL which was not significantly different in both Pre-test and Applied Calculus 1 measures. However, meta-cognitive with co-operative learning is more fruitful than co-operative learning alone for first year university students. The female students that took lessons through the MCL and C methods achieved more than female students that took lessons through the T methods in pre-calculus mathematics achievement and Applied Calculus 1. The female students that took lessons through the MCL, CL & T methods achieved more than female students who did not take refreshment module in Applied Calculus 1. This study also showed that refreshment of pre-calculus mathematics with different intervention method, especially meta-cognitive with co-operative learning method is an effective method to achieve the goal of building background knowledge of female students in helping them to learn Applied Calculus 1 with more understanding.

The male students that took lessons through the MCL and CL method did not achieve more than the male students that took lessons through the T method in pre-calculus mathematics, but achieved more than the male students that took lessons through the T method in Applied Calculus 1. The male students who attended lessons through the MCL method did not achieve more than male students that took lessons through the CL method in Applied Calculus 1. MCL and CL methods were highly effective for both male and female students. The finding particularly indicated that the students who attended lessons of refreshment module of pre-calculus mathematics through CL method score better than the MCL groups in Applied Calculus 1 that they learned in their usual classroom and it can be concluded that refreshment of pre-calculus mathematics highly influenced students' achievement of Applied Calculus 1 and helped the students to fully benefit from refreshment Module and co-operative learning intervention method.

Therefore the key contribution of these findings are (1) to improve performance of mathematics and develop positive attitude towards engineering field, a pre-calculus module should be offered by all universities for first year engineering students,(2) structured co-operative learning with purpose has significant gains for effective instruction, and (3) to increase the success rate of female students. This study has proven that they are trainable and therefore, meta-cognition skills have to be nurtured for female students.

5.4.1. Implications for future research

This finding strongly supports the assertion of Leonard et.al. (2011) and Palino (2011) which recommended that universities should organize mathematical refreshment course with effective learning strategies, that foster teaching and learning of applied mathematics and related course in engineering field of study, for first year university students. The findings of this study also raise several questions for further research: First, there are no formal interviews or/ and observations conducted in this study. Therefore, it would be interesting to conduct a qualitative research in the future to investigate attitudes and perceptions of students toward refreshment of pre-calculus mathematics module. It is especially necessary to assess how male and female students make exchanges while they are applying MCL and CL methods in the classroom. Second, geometric part of pre-calculus mathematics is an interesting area for future research. Therefore, the influence of geometry with intervention of meta-cognitive with co-operative learning and co-operative learning is worth further investigation. The result of the present study that focused on the importance of refreshment of pre-calculus mathematics with MCL or CL method was found to be effective for pre-engineering students. However, it needs further investigation regarding its success for university students on mathematics and natural science on different mathematical topics.

Third, the findings of this study call for the design of additional survey regarding prior knowledge of pre-calculus mathematics. Furthermore, it is important to assess the extent of first year pre-engineering or other related field students prior knowledge of mathematics for calculus course.

5.5. Limitations of the Study

This study sought to investigate the influence of pre-calculus mathematics refreshment module to first year engineering students of two genders in Ethiopia. This study was conducted in the tutorial class that was arranged by natural class room instructors. The following short comings may restrict generalizing the findings to all similar areas of the study. First, as it is guise-experimental study, the samples of the study limited to first year pre-engineering students of four universities out of thirty one universities in Ethiopia. The results found in this study may not be generalized to university students in other field of study area. Second, this study which was conducted on refreshment of pre-calculus mathematics for university students was limited to the review of "basic algebra, equations and inequalities, absolute value, function, exponential, logarithmic and trigonometric functions" and it may not be possible to generalize the outcome of the study to the other pre-calculus mathematics concepts like geometry, matrix and vectors.

6. Reference

Abduljabbar, D., & Omar, N. (2015). Exam questions classification based on bloom's taxonomy cognitive level using classifiers combination. *Journal of Theoretical and Applied Information Technology, 78(3),* 147-155.

Adams, R. (2013). *Cooperative learning effects on the classroom. Master Thesis* , Submitted to Michigan University.

Aggrawal, J. (1996). *Principle, methods and techniques of teaching* . New Delh: Vikas Publishing house Pvt Lid.

Ajai, J. T., & Imoko, B. (2015). Gender differences in mathematics achievement and retentions scores: A case of problem-based learning method. *International Journal ofResearch in Education and Science (IJRES),1(1),* , 44-50.

Akpan, J., & Beard, L. A. (2016). Using constructivist teaching strategies to enhance academic outcomes of students with special needs. *Universal Journal of Educational Research, 4(2)*, 392-398.

Alabekee, E. C., Amaele, S., & Osaat, S. (2015). Effect of cooperative learning strategy on students learning experienceand achievements in Mathematics. *International Journal of Education Learning and Development 3(4)* , 67-75.

Alcocka, L., Attridgeb, N., Kennya, S., & Inglisa, M. (2014). Achievement and behaviour in undergraduate mathematics: personality is a better predictor than gender. *Research in Mathematics Education*, 16(1), 1-17.

Alemu, B. M. (2010). Active learning approaches in mathematics education at universities In Oromia, Ethiopia. *DE Thesis, Submitted to UNISA* .

Alfaki, I. M., & Siddiek, A. G. (2013). The role of background knowledge in enhancing reading comprhension. *world Journal of English Language, 3(4)* , 42-66.

Ali, H. O. (2013). Factors Affecting students' academic performance in mathematical science departement in tertiary institutions in Nigeria. *US-China Education Review A, 3(12), 905-913*

Aljaberi, N., & Gheith, E. (2015). University students' level of metacognitive thinking and their ability to solve problems. *American International Journal of Contemporary Research* , 5(3), 121-134.

Baida, M. (2010). *Cooperative learning techniques as a form of group work in higher education*. Retrieved from https://search.yahoo.com/yhs/search?hspart=adk&hsimp=yhs-adk_sbyhp&p.

Baker, U. (n.d.). *Mission of mathematics*. Retrieved from https://www.bakeru.edu/mathematics/

Belina, R. G. (2012). Prior or background knowledge. *Mid - continent comprehensive center (MC 3) regional ELL/CCSS Task Force* (pp. 1-12). Oklahoma: University of Oklahoma.

Berezoviski, T. (2004). *An inquiry into high school students understanding of Logarithms.* Simon Fraser University Master thesis .

Bergey, B., Deacon, S., & Parrila, R. K. (2015). Metacognitive reading and study strategies and acadamic achievement of university students with and without a history of reading difficalties. *Journal of Learning Sisabilities* , 50(1), 1-14.

Berman, J. (2013). Utility of a conceptual framework within doctoral study: A researcher's reflections. *Issues in Educational Research, 23(1)* , 1-18.

Bhattacharjee, J. (2015, July). Constructivist approach to learning–an effective approach ofteaching learning. *International Research Journal of Interdisciplinary & Multidisciplinary Studies (IRJIMS), 1(6)* , 65-74.

Booysen, R. (2018). *The role of cooperative learning in the development of critical thinking* . Retrieved from http://www.iacesa.co.za

Braun, B. (2015). *Active learning in mathematics, Part III: Teaching techniques and environments*. Retrieved from https://blogs.ams.org/matheducation/2015/10/01/active-learning-in-mathematics-part-iii-teaching-techniques-and-environments/

Brown, S. (2011). *Measure of shape: Skewness and kurtosis*. Retrieved from https://www.scribd.com/document/180694167/SkewStatSignif-pdf

Burton, J., Moore, D., & Magliaro, S. (2004). *Behaviorism and instructional technology: Handbook of research for educational communications and technology*. Virginia: Virginia Polytechnic Institute and State University.

Calder, H & Willacy, N. (2017). Making mathematics learning more engaging for students in health schools through the use of Apps. *Education science MDPI* , pp. 1-20.

California State Boord of Education. (2015). *precalculus chapter of the mathematics framework for california public school.* Sacramento CA, U.S.A: California Departement of Education.

Campbell, L. C. (2009). *101 Proven strategies forstudents and teachets success. California, U.S.A.*

Cantwell, R., & Scevak, J. (2013). Personal epistemological and metacognitive awareness in first year preservice education student. *Australian Journal of Educational & Developmental Psychology* , 13(2013), 44-57.

Catherine, H., Christianne, C., & Andresse, S. R. (2010). *Why so few? Women in science, technology, engineering, and mathematics.* Washington, DC: AAUW.

Cervetti, G., Jaynes, C., & Hiebert, E. (2009). *Inceasing opportunitis to acquir knowledg through reading.* Berkeley: University of Colorado.

Christmas, D., Kudzai, C., & Josiah, M. (2013). Vygotsky's zone of proximal development theory: what are its implications for mathematical teaching? *Greener journal of social sciences, 3(7)* , 371-377.

Clark, U. (2015). *Trigonometric limits*. Retrieved from http://mathclarku.edu/~ma120/

Coe, R. (2002). What effect size is and why it is important. *Education Line*. England: British Educational Research Association, University of Exeter.

Collingwood, D. H., Price, K. D, & Conroy, M. M. (2011). *Precalculus.* Washington: University of Washington.

Contini, D., Tommaso, M., Mendolia, S. S., & Dalit, C. (2016). *The gender gap in mathematics achievement: Evidence from Italian Data.* Bonn, Germany: IZA Discussion Paper No. 10053.

Cottrill, J. (2003). An overview of theories of learning in mathematics education research: Retrieved fromhttp://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.668.7545 pdf.

Cui-yun, C. (2007). IEDC method: A new approach to promote students' communicative competence. *US-China Education Review,4(9)* , 30-34.

Cunningham, A., & Rory, W. (2014). *Maths for Chemistrs.* Birmingham Great Bretain: University of Birmingham mathematics support center.

Cunningham, B., Hoyer, K. M., & Sparks, D. (2015). Gender differences in science, technology, engineering, and mathematics (STEM). Retrieved from https://ia800400.us.archive.org/9/items/ERIC_ED554303/ERIC_ED554303.pdf

Dada, S. (2015). Constracrive leaaarning theory: A paradign for teaching and learning. *Journal of Research and Method in Education* , 5(6), 66-70.

Dake, F. (2011). *Annotated reference Bible: King James vorsion text eight priting.* Lawrenceville, Georgia: Dake publishing, Inc.

Dallas, U. (2017). MATH1325 - Applied calculus I. Retrieved from https://catalog.utdallas.edu/2017/undergraduate/courses/math1325

D'Amore, B. (2008). Epistemology didactics of mathematics and teaching practices. *Mediterranean Journal for Research in Mathematics Education 7(1)* , 1-22.

David, P., David, H., & Mathew, M. (2011). *Precalculus.* Washington: University of Washington.

Dawson, R. (2007). *Preparing for university calculus.* Halifax, Nova Scotia canada: Saint Mary's Universities.

Diamond, A. (2015). *What is cooperative learning? - Strategies, Benefits & Definition.* Retrieved from https://study.com/./what-is-cooperative-learning-definition-theory.ht.

DiDonato-Barnes, N., & Fives, H. (2013). Classroom test construction: The power of a table of specifications. *Practical assessment, Research & Evaluation* , 18(3), 1-7.

Dimitrov, D., & Rumrill, P. (2003).Pre-test-posttest designs and measurement of change. *Speaking of Research, 20(2003)* , 159–165.

Dimtsu, A. (2017). Differential effect of meta-cognitive strategy use on mathematics achievement among college juniors: Mediating effects of mathematics andenglish language ability self beliefs . *Psychology and Behavioral Science International Journal* , 5(4), 1-18.

Dunn, D. (n.d.). *Mathematics for engineering*. Retrieved from http://www.freestudy.co.uk

Ellis, A., Denton, D., & Bond, J. (2014). An analysis of research on metacognitive teaching strategies. *Proceda 116(2014)* , 4015-4024.

Erskine, D. (2009). Effect of prompted reflection and meta-cognitive skill instruction on University freshmen’s use of meta-cognition. *PhD Dessertation; Brigham Young University*.

Fanning, J. (2016). *Student responses to instruction in rational trigonometry*. Retrieved from http://web.sigmaa.maa.org/rume/crume2016/Papers/RUME_19_paper_6.pdf

FDRE, (2009). *Higher education proclamation.* Addis Ababa: Federal Negarit Gazeta of the Federal Democratic Republic of Ethiopia.

Felder, R., & Rebecca, B. R. (2007). Cooperative Learning. *P.A. Mabrouk,(Ed.), Active Learning: Models from the Analytical Sciences, ACS Symposium Series 970, Chapter 4,. Washington, DC: American Chemical Society,2007* , 34–53.

Fisher, D., Frey, N., & Lapp, D. (2012). Building and activating students' background knowledge: It's what they already know that counts. *Middle School Journal, 43(3),* 22-34.

Flashman, M. (2000). *Sensible precalculus: Algebra and the elementary functions*. Retrieved from http://www.users.humboldt.edu/flashman/book/$Precalc/p_ch1/ch1A.html.

Ganyaupfu, E. (2013). Teaching methods and students' academic performance. *International Journal of Humanities and Social Science Invention* , 2(9), 29-35.

Gerhand, S., & Philip, M. (2014). The impact of taking a college pre-calculus course on students' college calculus performance. Retrieved from *http://dx.doi.org/10.1080/0020739X.*

Gertrude, K. (n.d.). *A quest for understading understanding in mathematics learning.* Botswana: University of Botswana.

Godino, J. D. (1996). *Mathematical concepts, their meaning, and understanding.* Spain: University of Granada .

Grootenboer, P., & Torgensen, R. (2009). Towords a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of mathematics Education, 5(3),* 255-266.

Gupta, V. (1999). *SPSS for beginners.* Congress: VJBooks.

Hacker, D., Dunlosky, J., & Graesser, A. C. (2009). *Metacognition in educational theory and practice.* New York & London: Routledge.

Har, L. B. (2013). *What is cooperative learning?* The Hong Kong institute of education: Retrieved from https://www.eduhk.hk/aclass/Theories/cooperativelearning.pdf.

Harland, T. (2003). Vygotsky's zone of proximal development and problem-based learning: Linking a theoretical concept with practice through action research. *Teaching in Higher Education*, 8(2), 263-272.

Hauser, L. (2015). *Precalculus students' achievement when learning functions: influences of opportunity to learn and technology from a university of Chicago school mathematics project study.* florida: Scoholar Commons; Graduate Theses and Dissertations.

Homayouni, A., Gharib, K., Mazini, F., & Otaghsara, A. (2014). Comparative investigation of mathematics anxiety and learning mathematics in male and female students of distance educationsystem. *International Journal of Teaching and Education* , 2(2), 34-38.

Hossen, T. (2014). The effectiveness of metacognitive strategy awareness in reading comprehension: The case of Iranian university EFL students.

Important, M. (2013). *Mathematics is important*. Retrieved March 9, 2018, from https://mathsreports.wordpress.com.

Institute of Technology, A. A. U. (2017). *Pre-Engineering School*. Retrieved 7 12, 2017, from http://www.aait.edu.et/pre-engineering-school.

Jbeili, I. (2012). The effect of cooperative learning with metacognitive scaffolding on mathematics conceptual understandingand procedural fluency. *International Journal for Research in Education (IJRE),32*, 1-58.

Jennings, M. (2009). Issues in bridging between senior secondary and first year university mathematics, . *Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia The University of Queensland , 1*,273-274.

Jing, H. (2006). Learner resistance in metacognition training? An exploration of mismatches between learner and teacher agendas. *Language Teaching Research, 10(1),* 95-117.

Jochems, L. V. (2002). Changing a traditional lecturing approach into an interactive approach:Effects of interrupting the monologue inlectures. *Int. J. Engng Ed,18 (3*), 275-284.

Johnson, D., Johnson, W., & Roger, T. (1989). Cooperative and Competition. In R. T. David W. Johnson, *Theory and Research* (pp. 65-84). Edina: MN.

Johnson, D., Johnson, W., & Roger, T. (1998). *Introduction to cooperative learning*. Retrieved from http://wordpress.org/

Johnson, D., & Johnson, R. (2014). *Using technolog yto revolutionize cooperative learning:an opinion*. Retrieved from https://www.frontiersin.org/journals/psychology.

Johnson, D., Johnson, R. T., & Stanne, M. (2000). *Cooperative learning method: A meta-analysis.* Minneapolis: MN: University of Minnesota Press. U.S.A.

Juddi, A. B., & Crites, T. (2012). *Preparing students for calculus.* Flagstaf, AZ, U.S.A: Mathematics and Statistics, Northern Arizona University.

Karim, M., Lelsher, D., & Liu, C. (2010). Pre-calculus algebra: Original purpose-its relevancy to students' needs- and some suggestions. *KBM Journal of Science Education 1(2)* , 33-39

Karimganj, S. P. (2015). Constractivist approah to learing an efective approach of teaching learning jayeeta bhattacherjee. *International research Journal of Interdisciplinary, Multidisciplinary Studies* , 1(6) 65-74.

Kastberg, S. (2002). Understanding mathematical concepts: the case of the logarithmic function. (Doctoral dissertation, University of Georgia).

Kastberg, S., & Rechael, K. (2017). Links in learning logarithms. *Australian Senior Mathematical Journal 27 (1)*, 1-20.

Kay, D., & Kibble, J. (2016). *Learning theories 101: application to everyday teaching and scholarship*. Retrieved from https://www.physiology.org/doi/full/10.1152/advan.00132.2015

Kelly, M. (2017). *Key Advantage ofPre-test*. Retrieved from https://www.thoughtco.com/importance-and-uses-of-pretests.

Kenney, R., & Kastberg, S. (2013). Links in learning logarithms. *Australian Senior Mathematics Journal, 27(1)*, 12-20.

Kilnger, C. (2009). *Behaviourism, cognitivism, constructivism, or connectivism? Tackling mathematics anxiety with 'isms' for a digital age*. Retrieved from http://www.behaviorism. pdf

Kissane, B., & Kemp, M. (2009). *Teaching and learning trigonometry with technology*. Retrieved from http://researchrepository.murdoch.edu.au/id/eprint/6245/

Knapp, A. (2016). *Basic Algebra*. Retrieved 10 20, 2017, from https://www.math.stonybrook.edu/~aknapp/download/b2-alg-inside.pdf

Kooij, V., & Godijn, A. (2011). Algebra in science and engineering. In A. G. Van Der Kooij. Colorado: Community College in Colorado.

Koorosh, P. (2008). Analyzing rate of cognitive and metacognitive strategies in learners with respect to interactive effect of learning environments (University and guidance school) and location (urban and rural). *British Educational Research Association Annual Conference,* (pp. 1-8). Edinburgh: Heriot-Watt University.

Korir, D., & Laigong, B. C. (2014). An assessment of the differences in mathematics and chemistry between boys and girls in secondary schools in Bomet District, Kenya. *International journal of Advanced research, 2(2),* 1348-1355.

Korn, J. (2014). *Teaching conceptual understanding of mathematics via a hands-on approach.* Thesis. (Senior Honors Thesis, University of Liberty).

Kreyszig, E. (2006). *Advanced engineering mathematics / Erwin Kreyszig.-9th ed.* Singapore: Wiley & Son.

Kulshrestha, A., & Sonam, M. (2014). Implementation of cooperative learning in science: A developmental-cum-experimental study. *Educational research International: Http:// dx.doi.org/10.1155/2014* , 1-7.

Kumar, K. (2014). *Cooperative learning in special education and its Advantages and disadvantages*. Retrieved from http://integratelearning108.com/events.php?id=289

Kwamina, A., & Adelaide, K. (2015). Gender Differences and Mathematics Achievement of Senior High School Students: A Case of Ghana National College. *Journal of Education and Practice ,6 (33)* 67-74.

Kyei, B., & Benjamin, A. (2011). Some gender differences in performance in senior high mathematics examinations in mixed high schools. *AmericanJournal of Social and ManeggementSciences, 2(4)*, 348-355.

Laister, N. (2016). *Metacognition and student achievement in mathematics. Education and Human Development Master's Thesis 611*. Retrieved from http://digital.com

Lange, T. (2009). Difficalties, meaning and marginalisation in mathematics learning as seen Through Children's Eyes. www.learning.aau.dk.

Lazebnik, F. (2012). *Note on mathematical inequalities*. Retrieved from http://www.lazebnik/papers/inequalities245pdf

Lightner, J., & Tomaswick, L. (2017). *Active learning – Think, Pair, Share. Kent State University Center for Teaching and Learning*. Retrieved from http://www.kent.edu/ctl/educational-resources/active-learning-think-pair-share/.

Lindblom-Ylänne, S., Hailikari, T., & Katajavuori, N. (2008). The relevance of prior knowledge in learning and instructional design. *American Journal of Pharmaceutical Education 113* , 1-7.

Literacy, T. E. (2012). *TEAL Center Fact Sheet No. 4: Metacognitive Processes | Adult ...* Retrieved from https://lincs.ed.gov/state-resources/federal-initiatives/teal/guide/metacognitive

Liu, H., & Chun-Yi, S. (2011). Skills development: A web-based approach in higher education. *The Turkish online Journal of Education Technology, 10(2)*, 140-143.

Loughlin, W., Watters, D., Brown, C., & Jahnston, P. (2015). Snapshot of mathematical background demographics of a Broad Cohort of first year chemistry science students. *International Journal of Innovation in Science and Mathematics Education, 23(1),* 21-36.

Lovric, A. K. (2005). Transition from secondary to tertiary mathematics: McMaster University experience, *International Journal of Mathematical Education in Science and Technology,36(1), 149-160*.

Lovric, M. (2009). *Mathematics review manual*. Retrieved from http://ms.mcmaster.ca/lovric/rm/MathReviewManual.pdf.

Machisi, E. (2013). Exploring solution strategies that can enhance the achievement of low performing grade 12 learners in some mathematical aspects. (Master *Thesis*, University of South Africa)

MacNeal, S. (2015). *Students preraredness in college Mathematics.* Worceste: Worcester polytechnic Institute: Retrieved from https://web.wpi.edu/Pubs/E-project/Available/E-project-012915-152427/unrestricted/MQP.MacNeal.v2.pdf.

Mahajan, S. (2014). *Mathematics for 21st Century:What should students learn?* Boston, M.A, U.S.A: Center for Curriculum Redesign.

Marsigit, M. (2009). *The role of cognitive development theory for mathematics education*. Retrieved from http://marsigitpsiko.blogspot.com/2009/01/role-of-cognitive-development-theory.html

Marzano, R. J. (2004). *Building background knowledge for academic achievement. Alexandria VA*:A

Mc-TY-Logarithm. (2009). *Mc-TY-Logarithms*. Retrieved from http://www.mathcentre.ac.uk.

Mils, S. (2016). Conceptual understanding: A concept analysis. *The qualitetive report* ,21(3), 547-557.

Misu, L., & Masi, L. (2017). Comparison of metacognition awareness of male and female students based on mathematics ability in department of mathematicsEducation of Halu Oleo university. *International Journal of Education and Research*, 5(6), 43-50.

MoE, E. (2010). *Education sector development program IV.* Addis Ababa/ Ethiopia: Federal Ministry of Education.

Moghadama, A., & Mah Khah Fard, M. (2011). Surveying the effect of metacognitive education on the on the mathematics achievement of 1stgrade highjunior school female students in educational district 5. *International Conference on Education and Educational Psychology*, 29(2011), 1531-1540.

Muijs, D. (2004). *Doing quantitative research in SPSS.* Thousand Oaks, CA, U.S.A: Sage Publications.

Muna, A. (2017). Principles of "constructivism" in foreign language teaching. *Journal of Literature and Art Studies* , 7(1), 97-107.

Murry, J. (2013). The factors that influence mathematics achievement at the Berbice campus. *International Journal of Business and Social Science* , 4(10), 150-160.

Mulqueen, E. (2012). *How do students acquire an understanding of logarthmic concepts?* (PhD Dissertation, Kent State University)

Nahari, N. (2014). *Mathematical skills and attitudes of first year engineering students.* (MA Thesis, University of Dublin City).

Nahil, M., & Eman, G. (2015). University students' level of metacognitive thinking and their ability to solve problems. *American International Journal of Contemporary Research, 5(3)* , 121-133.

OECD. (2011). Report on the gender initiative: gender equality in education,employment and entrepreneurship. *Meeting of the OECD Council at Ministerial Level* (p. 2). Paris.

Oloyede, E. O., Adebowale, O. F., & Ojo, A. A. (2012). The Effects of competitive, cooperative, and individualistic classroom interaction models on learning outcomes in mathematics in Nigerian senior secondary schools. *ISRN Education, doi:10.5402/2012/263891* , 1-8.

Pacis, D., & Weegar, M. (2012). *A comparison of two theories of learning behaviorism and constructivism as applied to Face-to-Face and Online Learning National University. San Diego*. Retrieved from https://docplayer.net/37764570-A.html

Palino, C. R. (2010). *"Effectiveness of teaching mathematics as perceived by the students of Balian national high school A.Y. 2008-2009.".* Balian: Balian National High School.

Pallant, J. (2010). *SPSS survival manual. A step by step guide to data analysis using SPSS 4th edition.* Australia: Allen & Unwin Book Publish.

Passmore, T. (2007). Secondary-tertiary transition: What mathematics skills can and should we expect This Decade. *Mathematics: Essential Research, Essential Practice, 2* , 464-472.

Payne, L. (2015). Gender differences in mathematics ability at an undergraduate level. *journal of the British Education Studies Association* ,1(1), 86-102.

Rahimi, M., & & Kalal, M. (2012). Metacognitive strategies awareness and success in learning English as a foreign language: An overview. *Procedia - Social and Behavioral Sciences, 3(2012),*, 73 – 81.

Rahmantya, K. (2009). *Content validity ratio*. Retrieved from http://www.blackwellreference.com/public/tocnode?id=g9780631233176_chunk.

Rajalingam, S., & Shubashini, A. (n.d.). *The teaching and learning of trigonometry in tertiary education.* Miri, Sarawak, Malaysia: School of Engineering and Science, Curtin University.

Reilly, D., Neumann, D. L., & Andrews, G. (2017). Investigating gender differences in mathematics and Science. *Research in Science Education.* , doi: 10.1007/s11165-017-9630-6), 1-20.

Ross, S. M., & Morrison, G. R. (2003). *Experimental research methods*. Retrieved from https://www.researchgate.net/publication/201382131

Sajna, J., & Premachandran, P. (2016). A study on the metacognitive awareness of secondary school students. *Universal Journal of Educational Research, 4(1)*, 165-172.

Savoy, P. (2007). *Attracting the best students of mathematics into engineering.* London: Mathematics in Education and Industry.

Sazhin, S. (1998). Teaching mathematics to engineering school. *International Journal of Engineering, 14*, 145-152.

Schul, J. (2012). Revisiting and old friend: The practice and promise of cooperative learning for the twenty-first century. *The Social Studies* , 102 (2), 88-93.

Schwartz, D., Sears, D., & Chang, J. (2015). *Reconsidering prior knowledge Palo Alto, CA, U.S.A* stanford.University.

Shahinshah, B. (2012). Teaching of mathematics at elementary level through making connection between previous knowledge in mathematics and its use in existing situation. *Journal of Educational and Instructional Studies in the World* ,2(1), 2146-7463.

Shenkut, H. (2017). Factors affecting learning mathematics in the case assosa university college of natural science. *International Journal of Education, Culture and Society, 2(1),* 6-12.

Short, E., Echevarr, M., & Vogt, J. (2013). *Making content comprehensible for english Learners.* Boston: SIOP.

Şimşek, U. (2012). The effects of reading-writing-presentation and group investigation methods on students' academic achievementsin citizenship lessons. *Eğitim Bilimleri Araştırmaları Dergisi – Journal of Educational Sciences Research* ,2(2), 190-203.

Siyepu, S. (2013). The zone of proximal development in the learning of mathematics. *south African Journal of education* , 33(2), 1-13.

Smith, M. (2006). *Pre-calculus concepts fundamental to calculus.* (MSc Thesis, University of Akron).

Stahl, R. (1994). *The essential elements of cooperative learning in the classroom.* Retrieved from www.ericdigest.org.

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R. (2015). *Teaching strategies for improving algebra knowledge in middleand high school students (NCEE 2014-4333)*: Washington, DC: National Center for Education Evalua-tion and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education.

Stitz, S., & Zeager, J. (2013). *Pre-calculus.* Lakeland: Lakeland Community College.

Stylianides, A., & Gebriel, J. (2007). Learning mathematics with understanding of the learning principles and standards for school mathematics. *TMME 4 (1)* , 101-126.

Taber, K. (2001). The mismach between assumed priorknowledge and the learner's conceptions' a typology of learning impediments. *Educational Studies* , 27(2), 159-171.

Tang, H., Voon, L., & Hazizah, J. (2010). The relationships between students' underachievement in mathematics courses and influencing factors. *Procedia Social and Behavioral Sciences* ,8(2010), 134-141.

Terwel, J. (2011). Cooperative learning and mathematics education: A happy marriage? *Paper presented at the OECD / France workshop*, (pp. 23-24). Paris : VU University *France*.

The American Association of University Women (2010). *Why So Few? Women in Science, Technology, Engineering, and Mathematics*. Retrieved from https://www.aauw.org/files/2013/02/Why-So-Few-Women-in-Science-Technology-Engineering-and-Mathemathics.

Thomas, M. O., Druck, I., Huillet, D., Nardi, E., Rasmussen, C., & Xie, J. (2012). *Key mathematical concepts in the transition from secondary to university*. Retrieved from http://faculty.math.tsinghua.edu.cn/~jxie/papers/icme2012_ST4.pdf orhttp://www.math.auckland.ac.nz/~thomas/ST4.pdf

Thomas, M., Klymchuk, S., Hong, Y., & Kerr, S. (2015). *The transition from secondary to tertiary mathematics education.* Wellington, New Zealand: Teaching and Learning Initiative Website: www.tlri.org.nz pg 1-10

Thyer, B. A. (2011). *Pocket guides to social work research method: Quasi-experimental research designs.* Oxford : Oxford University Press.

Vijayakumari, S., & Souza, M. (2013). Metacognitive cooperative learning approach to enhance mathematics. *International Journal of Education and Psychological Research, 2(2), 111-119.*

Vygotsky, L. (1978). Intraction between learning and development. In *Reading on the development of children* (pp. 79-91). New York: Harvard univeersity press.

Weber, K. (2002). *Students' understanding of exponetial and logarithmic functions.* Murray: Murray State University KY 42071. U. S. A.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal* , 17(3), 91–112.

Weegar, A., & Pacis, D. (2012). *A comparison of two theories of learning-behaviorism and constructivism as applied to face-to-face and online learning.* San Diego, CA, USA: E-Leader Manila.

White, H., & Sabarwal, S. (2014). Quasi-expermental design. *Impact Evaluation 8, UNICEF Office of Research, Florence.* , 1-16.

Wilbur, J. (n.d). *The mathematical solution of simultaneous equations.* Retrieved from http://www.wilbur36pdf

William, S., Joshua, B., & Alice, E. (2009). Gender differences and mathematics achievement of rural. *Proceedings of Epistem. New Dehli*: McMillian.

Williams, H. (2011). *A conceptual framework for student understanding of logarithm*. Retrieved from http://scholarsarchive.byu.edu/etd

Wolfensberger, B., & Canella, C. (2015). Cooperativelearning about nature of science with a case from the history of science. *International Journal of Environmental & Science Education,10(6),* 865-889.

Wong, S., & Teresa, T. (2001). Group work in science learning.- international scenarios and implications for teaching and learning in Hong Kong. Retrieved from https://www.eduhk.hk/apfslt/v2_issue2/wongts/wongts7.htm

Ye Yoon Hong, S. K. (2009). A comparison of teacher and lecturer perspectives on the transition from secondary to tertiary mathematics education. *International Journal of Mathematical Education in Science and Technology,* 40(7), 877–889.

Zhou, M., & Brown, D. (2015). *Educational learning theories*. Retrieved from http://oer.galileo.usg.edu/education-textbooks/1

Appendix 1: Content Validity Ratio of pilot test panelists

Item	It need correction	It is essential	CVR
Q12. b	6	0	-1
Q13.d	4	2	-0.33
Q13. e	3	3	0.0
Q14.a	2	4	0.33
Q14. b	2	4	0.33
Q15.c	5	1	-0.66
	All the rest their CVR > 0.7	Retained in the Pre-test and posttest	

Appendix 2: Test-retest Correlation for testing Reliability of the Instruments

		pilotT1	PilotT2
pilotT1	Pearson Correlation	1	.998**
	Sig. (2-tailed)		.000
	N	20	20
PilotT2	Pearson Correlation	.998**	1
	Sig. (2-tailed)	.000	
	N	20	20

**. Correlation is significant at the 0.01 level (2-tailed).

Appendix 3: Test of Homogeneity of Variances of Pre-test and Posttest

No,	Levene Statistic	df1	df2	Sig.
A. Pre-test Value among total students	2.163	3	196	0.094
B. Pre-test Value among male students	0.192	3	96	0.902
C. Pre-test Value among female students	2.299	3	96	0.082
D. Posttest Value among total students	23.666	2	145	0.00
E. Applied Calculus 1 Achievement Value of total students	11.121	3	189	0.00
F. Post test Value among male students	14.828	2	70	0.00
G. Applied Calculus 1 Achievement Value of male students	5.769	3	95	0.001
H. Posttest Value among female students	7.415	2	72	0.001
I. Applied Calculus 1 Achievement Value of female students	3.565	3	88	.017

Appendix 3: Meta-Cognitive Question Sheet

META-COGNITIVE QUESTION SHEET

A. During the planning phase: students can ask:
   1. What am I supposed to learn?
   2. What prior knowledge will help me with this task?
   3. What should I do first?
   4. How much time do I have to complete this?
   5. In what direction do I want my thinking to take me?

B. During the monitoring phase: students can ask:
   1. How am I doing?
   2. Am I on the right track?
   3. How should I proceed?
   4. What information is important to remember?
   5. Should I move in a different direction?
   6. Should I adjust the pace because of the difficulty?
   7. What can I do if I do not understand?

C. During the evaluation phase: students can ask:
   1. How well did I do?
   2. What did I learn?
   3. Did I get the results I expected?
   4. What could I have done differently?
   5. Can I apply this way of thinking to other problems?
   6. Is there anything I do not understand or any gaps in my knowledge?
   7. Do I need to go back through the task to fill in any gaps in understanding?
   8. How might I apply this line of thinking to other problems?

Appendix 4:Pre-test and Posttest Instrument

THIS IS TO INVESTIGATE THE INFLUENCE OF PRE-CALCULUS MATHEMATICS RE-FRESHMENT MODULE TO FIRST YEAR ENGINEERING STUDENTS

Pre/post Test

Code____________________ ____________

Time allowed 3: hr

1. Exponents: simplify the following expressions. Do not leave negative exponents in your final answer. Leave all answers in fully reduced form.

a. y^3y^4 =___________
b. $(y^3)^4$ =___________
c. $(3a^4)^2$ =__________
d. 2^0 = __________
e. $\left(\frac{1}{4}\right)^{-2}$ =__________
f. $(-2x)^{-4} =$ _______
g. $\left(\frac{3x^2y^{-1}}{x^{-1}y^2}\right)^{-2} =$ _____

2. Polynomials: simplify the following polynomials

a. $2x + 3y - 4x + 5y =$ _____________
b. $3a^3(4a^2 - 5a) - 2a^2(3a^3 - 6a^2) =$ _______________
c. $2(10y^2 + 4xy^2 - 5x) - 5(4x^2y^2 - 3xy^2 + 5) =$ ________________
d. $(2a + b)^2 - (2a - b)^2 =$ _____________________

3. Factoring: Factorize the following

a. $10b^3c^2 - 5b^2c =$ _________
b. $x^2 + 8x + 16 =$ __________
c. $x^2 - 7x - 18 =$ __________
d. $x^3 + 1000 =$ _____________
e. $4x^4 - 16 =$ ______________
f. $8x^3 - 27 =$ _____________

4. Rational expression: Solve the following

a. $-2\left(-\frac{1}{2} - \frac{4}{3} + \frac{5}{6}\right) \div \frac{2}{3} =$ _______
b. $3x + 42 \leq -12$ find the solution set of x _______________
c. Solve the following system of equation
   $2x + y = 8$ *and* $x - y = 1$, then $x =$ _____, and $x =$ ______
d. Put the inequalities $<, >, \leq or, . \geq$

   If $\varepsilon > \frac{1}{\sqrt{x}}$, then $\frac{1}{x^2}$ ____ ε

e. Solve for x

$$\frac{1}{x^2+5x+6} = \frac{1}{x+3}$$, then x = ______

f. Simplify $3 + \frac{2}{3y+6} + \frac{y-3}{y^2-4} =$

5. Solve the following Rational Exponents and Radicals

a. $8^{\frac{2}{3}} =$ ___________

d. $4^{\frac{-2}{3}} =$ ___________

b. $\left(\frac{9}{8}\right)^{\frac{3}{2}} =$ __________

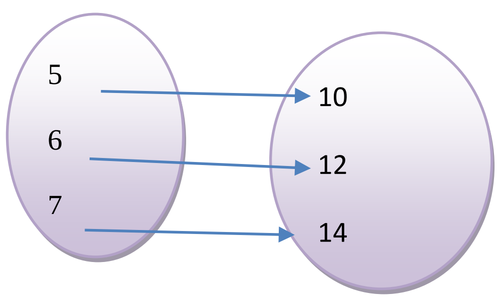
e. $\left(\frac{8}{27}\right)^{\frac{-2}{3}} =$ __________

c. $\frac{2-\sqrt{5}}{2+3\sqrt{5}} =$ _____________

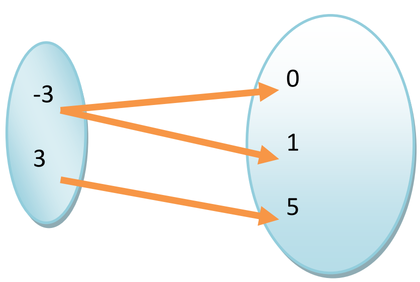
f. $\sqrt{\frac{32x^3}{9x}} =$ ____________

6. Relation and functions: I decide whether the given are a function/relation

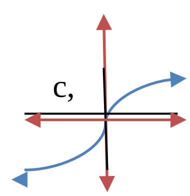
a.



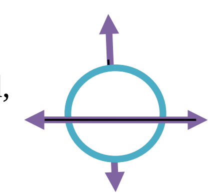
b.



c,



d,



ii. Solve the following

a. let $f(x) = -9 - 3x\ find$ $\quad i, f(1) =$ _______ $\quad ii, f(-1) =$ _______

b. let $f(x) = 4\sqrt{x}\quad find$ $\quad i, f(1) =$ ________ $\quad ii, f(-1) =$ _______

c. let $f(x) = |2x-4|\ find$ $\quad i, f(-3) =$ ________ $\quad ii, f(7) =$ _______

d. let $|3x+4| = 6\ then\quad x =$ ________

e. let $f(x) = \frac{1}{x^2}$ then $f(0) =$ ________

7. Domain and Range: find domain and range of the following functions

a. $f(x) = 2x + 1$, $D =$ ____ $R =$ ____

b. $f(x) = \sqrt{x^2 - 4}$, $D =$ ____ $R =$ ____

c. $f(x) = \sqrt{\frac{x-2}{x-1}}$, $D =$ ____ $R =$ ____

d. $f(x) = log_2(x + 2)$, $D =$ ____ $R =$ ____

8. Operation on Functions: Given $f(x) = x + 2, g(x) = 2x^2, h(x) = \frac{x+1}{x-1}$

Find

a. $f(2) - g(3) =$ __________

b. $g(x^2)=$ ____________

c. $\left(g(x)\right)^2 =$ _____________

d. $h(-2) =$ _____________

9. Composite Function: find the following composition functions, given

$f(x) = x^3$, $g(x) = x - 9$, $h(x) = \frac{\sqrt{x}-4}{x+4}$

a. $g\left(h(x)\right) =$ _____________

b. $f\left(g(-4)\right) =$ _____________

c. $f(g\left(h(16)\right)) =$ _____________

10. Asymptotes: Find the vertical and horizontal asymptotes of the following functions

a. $f(x) = \frac{2}{x^4-16}$, $VA =$ ______________ $HA =$ ________________

b. $f(x) = \frac{1-x}{x-2}$, $VA =$ ______________ $HA =$ ________________

c. $f(x) = \frac{x^3-8}{x+2}$, $VA =$ ______________ $HA =$ ________________

d. $f(x) = \frac{2x^3+2}{x^3+x^2-2x}$, $VA =$ ______________ $HA =$ ________________

11. Exponential and Logarithmic Functions

Solve the following exponential equations

a. $3^x = 243$, $x =$ ____________ ,

b. $8^x = 4$, $x =$ _____________

c. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$, $x =$ ______________

d. $7^x = \frac{1}{49}$, $x =$ ______________

e. $\frac{4^x}{4^{2x}} = 64$ $x =$ _____________

f. $\left(\frac{1}{16}\right)^{x-3} = 8^{2x-1}$, $x =$ ______________

g. $25^{\sqrt{x}} = 10^{x}$, $x =$ _____________

h. $\left(\frac{1}{2}\right)^{x} = 32$, $x =$ _____________

12. Exponential & Logarithm: write exponential equation in logarithmic form

a. $a^x = b$, *then* log___ ____

b. $10^3 = 1000$, *then* log___ ____

c. $9^0 = 1$, *then* log___ ____

d. $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$, *then* log___ ____

13. Logarithm to Exponential: write logarithmic equation in exponential form

a. *The exponential form of* $log_2 64$ *is* ____________

b. *The exponential form of* $log_8 1$ *is* ____________

c. $log_{\frac{1}{3}}\left(\frac{1}{9}\right) = 2$ *then exponential form is* ____________

d. $log\, 0.01 = -2$ *then exponential form is* ____________

e. $ln\, x$ *then exponential form is* ____________

14. Properties of Logarithm:

a. $log_4 5x + log_4 6x =$ _____________

b. $log_2 4x - log_2 6y =$ _____________

c. $log\frac{4x}{3y} =$ _______________________

d. $2\, log_3 x + log_3 y =$ _____________________

e. $3(log_2 x + 2log_2 y - log_2 z) =$ _________________

15. Solve the following equation

a. $log_5(x+2) = 1$ then x = ________

b. $3\ln 2 + \ln(x-1) = \ln 24$ then x = ________

c. $\log_3 x = \log_3 2 + \log_3(x^2 - 3)$ then x = ________

d. $\ln 5 - \ln x = -1$ then x = _______

16. Trigonometry

i. Conversion of angles

a. Convert the following angles in to radian $30^0 =$ ___ $45^0 =$ _____, $120^0 =$ _______

b. Convert the following radian in to angles $\frac{5\pi}{4} =$ ___, $\frac{7\pi}{4} =$ ____, $\frac{-3\pi}{4} =$ _______

ii. Find the exact values of the trig function of the following angles:

a. $\cos 150^0 =$ _____ b, $\sin\frac{5\pi}{4}=$_______ c, $\tan 300^0 =$ _____

iii. Find the angle in degree form and in radian form for the following problems

a. $\sin^{-1}\left(\frac{1}{2}\right) =$ ____ b, $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$ ____ c, $\tan^{-1} -\sqrt{3} =$ _____ d,$\tan^{-1} 0 =$ _____

iv. Simplify the following trig expressions as much as possible

a. $\cos^2 x\,(1 + \tan^2 x) =$

b. $\frac{1}{1+\tan^2 x} =$

c. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} =$

d. $\frac{\text{Cot}\, x}{\csc x} =$

e. $\frac{1-\cos^2 x}{\csc x} =$

v. Show the following

a. $\tan x \cot x = 1$

b. $\frac{\sec x - \cos x}{\tan x} = \sin x$

c. $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$

Thank you

Appendix 5: UNISA Ethical Clearance

UNISA | college of science, engineering and technology

Dear Asnake Muluye Bekele (5578888)

Date: 2016-01-26

Application number:
2015 CGS/ISTE_022

REQUEST FOR ETHICAL CLEARANCE: (The role of Pre-Calculus Mathematics refreshment in University of Ethiopia: with reference to first-year Engineering students)

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your research study as set out in your proposal and application for ethical clearance.

Therefore, involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CRIC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:
http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Appendix 6: Ethical Clearance of CSET UNISA

Please note that the ethical clearance is granted for the duration of this project and if you subsequently do a follow-up study that requires the use of a different research instrument, you will have to submit an addendum to this application, explaining the purpose of the follow-up study and attach the new instrument along with a comprehensive information document and consent form.

Yours sincerely

Prof Ernest Mnkandla
Chair: College of Science, Engineering and Technology Ethics Sub-Committee

Prof IOG Moche
Executive Dean: College of Science, Engineering and Technology

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College of Science, Engineering and Technology

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University of South Africa
College of Science, Engineering and Technology
The Science Campus
C/o Christiaan de Wet Road and Pioneer Avenue,
Florida Park, Roodepoort
Private Bag X6, Florida 1710
www.unisa.ac.za/cset

UNISA | college of science, engineering and technology

Appendix 6: Consent letter of Departement of Mathematics of each Universities

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Dilla University
College of Natural & Computational Sciences

Ref. No DU/CNCS/09/08
Date 27/01/08

LETTER TO PERMISSION TO CONDUCT RESEARCH

TO:- WHOM IT CONCERN

COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCE

Tameru Demsess (PHD)

College Dean

Subject:-Request for Benefication

A M BEKELE (ASNAKE MULUYE) is a PhD student of Mathematics education in science and technology at UNISA. He is by now working on a research project; hence looking for information from your university. This is therefore to request your good office to help him in providing necessary information.

We thank you in advance

With Best Regard

Tameru Demsis (PhD)
Dean, College of Natural and Computational Sciences

Dilla University

TEL 0468310251 | P.O Box 419 | Fax 0463312674 | E-mai Dilla @telecom.net.et

ዲላ ዩኒቨርሲቲ
የተፈጥሮና ቀመር ሳይንስ ኮሌጅ

Dilla University

College of Natural & Computational Sciences

Ref. No DU/CNCS/15/08

Date Oct, 15, 2015

To: Institute for Science & Technology Education, University of South Africa (UNISA)

Subject: Giving Letter of Consent

Mr. A M Bekele requested our university to get letter of consent from the participant in his research work. The department of Mathematics in behalf of **DILLA – UNIVERSITY** communicated all mathematics instructors in the department to give their permission to participate in the study by explain his research project and shown positive response.

Therefore, we would like to inform your institution that MR A M Bekele is allowed to conduct his PHD project in our University as well as has got permission from all mathematics instructors (participant of the research project) of our university to participate in the study by giving all the necessary data he need in the study.

With best regards

Temam (Dr.)

Dean, College of Natural and Computational Sciences

TEL 0468310251
P.O Box 419
Fax 0463312674
E-mai Dilla @telecom.net.et

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Wachemo University

Faculty of Natural and Computational Science

Department of mathematics

Date 06/10/2015

Ref. No./:WCU/FNCS/MATH/003/08

To: Institute for Science & Technology Education, University of South Africa (UNISA)

Subject: Giving Letter of Consent

Mr. A M Bekele requested our university to get letter of consent from the participant in his research work. The department of Mathematics in behalf of **WACHEMO UNIVERSITY** communicated all mathematics instructors in the department to give their permission to participate in the study by explain his research project and shown positive response.

Therefore, we would like to inform your institution that MR A M Bekele is allowed to conduct his PHD project in our University as well as has got permission from all mathematics instructors (participant of the research project) of our university to participate in the study by giving all the necessary data he need in the study.

With best regards

Bude W.

Head, Department of Mathematics

Phone No: +251-046-555-1910 Fax: +251-046-555-1930 P. O. Box 667

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ሳይንስ ት/ቤት

Tel. (+) 251 (0) 462 206105
Fax (+) 251 (0) 462 204975

Hawassa University
School of Mathematical &
Statistical Sciences

P.O.Box 5, Hawassa, Ethiopia
E-mail: deanfns@ethionet.et

ቁጥር SMSS/104/15
Ref.No
ቀን 08/10/15
Date

To: Institute for Science & Technology Education, University of South Africa (UNISA)

From: School of Mathematical and Statistical Sciences

ሀዋሳ ዩኒቨርሲቲ
ማቲማቲካል እና ስታትስቲካል ሳይንስ ት/ቤት
School Of Mathematical & Statistical Science
Hawasaa University

Subject: **Giving Letter of Consent**

Mr. A M Bekele requested our university to get letter of consent from the participant in his research work. The department of Mathematics in behalf of HAWASSA – UNIVERSITY communicated all mathematics instructors in the department to give their permission to participate in the study by explaining his research project and shown positive response.

Therefore, we would like to inform your institution that Mr. A M Bekele is allowed to conduct his PHD project in our University as well as has got permission from all mathematics instructors (participant of the research project) of our university to participate in the study by giving all the necessary data he need in the study.

With best regards

Appendix 7: Consent form for Representative Teachers Participants in Research

Request by: ASNAKE MULUYE BEKELE

Cell phone: 251911547262

Email: snkmuluye14@yahoo.com

Dear Rep Fiseha Wasu

You are one of the teachers selected out of all first year Engineering applied Calculus teachers in this university to participate in a study with title Investigating the influence of Pre- Calculus Mathematics Re-Freshment module to First Year Engineering Students in an Ethiopian university: with special reference to first year engineering students. The information obtained will help improve future students' achievement in applied Calculus I.

If you agree to participate you will be asked to implement a re-freshment module to enhance students' conceptual understanding in pre-calculus mathematics. Your students will also be required to take pre-calculus mathematics concept test. Confidentiality will be ensured at every step of the study. During and after data are collected you will be identified by a code to guarantee anonymity.

Your cooperation will contribute more for the completion of the study. If you have any reservation, your decision to carry on or quit is respected.

I Fiseha Wasu consent to participating in the research with title: Investigating the influence of Pre- Calculus Mathematics Re-Freshment module to First Year Engineering Students in an Ethiopian university: with special reference to first year engineering students. I also give my permission to the researcher to use the data collected for the solely purpose of this study, I understand that the researcher will ensure confidentiality as it is explained above.

Name Fiseha Wasu Signed [signature] Date **13/11/2015**

Witness Masreshaw Walle Signed [signature] Date 13/11/2015

Appendix 8: Consent form for Representative Students Participated in Research

Request by: ASNAKE MULUYE BEKELE

Cell phone: 251911547262

Email: snkmuluye14@yahoo.com

Dear Rep Fedasa Adisu

You are enrolled in one of the sections selected to participate in a study with title: Investigating the influence of Pre- Calculus Mathematics Re-Freshment module to First Year Engineering Students in an Ethiopian university: with special reference to first year engineering students. The information obtained will help improve future students' achievement in applied Calculus I.

If you agree to participate you will be required to take pre-calculus mathematics concept test. You will also be asked to permit the researcher to release the scores obtained in this class. Confidentiality will be ensured at every step of the study. During and after data are collected you will be identified by a code to guarantee anonymity.

Your cooperation will contribute more for the completion of the study. If you have any reservation, your decision to carry on or quit is respected.

I Fedasa Adisu consent to participating in the research with title: Investigating the influence of Pre- Calculus Mathematics Re-Freshment module to First Year Engineering Students in an Ethiopian university: with special reference to first year engineering students. I also give my permission to the researcher to use the data collected for the solely purpose of this study, I understand that the researcher will ensure confidentiality as it is explained above.

Name Fedasa Adisu Signature ______ Date **13/11/2015**

Witness Bulo Abu Signature ______ Date __________

Appendix 9:Pre-test result of male and female students

Cont No Male	T Male	CL Male	MCL Male	Cont No Female	T Female	CL Female	MCL Female
56	52	18	28	86	29	12	36
43	56	9	64	75	24	12	25
47	47	28	24	59	55	26	32
62	53	47	65	10	31	20	39
54	44	22	8	6	76	58	28
70	51	27	54	12	30	58	9
28	23	30	55	14	20	10	28
24	18	43	3	10	20	13	15
24	31	51	53	0	32	8	22
21	5	22	10	12	21	12	30
57	36	27	28	12	63	11	32
13	8	10	6	45	34	15	35
43	14	22	18	18	44	18	30
35	37	66	20	11	21	14	39
33	46	16	32	16	17	6	17
18	17	15	22	13	28	10	22
32	7	50	23	18	26	17	26
24	20	64	26	2	26	10	12
10	47	30	4	10	41	18	10
42	36	25	57	5	1	53	16
22	8	8	24	13	5	14	17
36	12	26	15	23	10	14	30
15	27	51	9	32	4	54	27
13	36	60	3	22	15	17	28
9	12	30	3	18	7	14	25

Appendix 10: Posttest result of male and female students

Cont No	T	CL	MCL	Cont No	T	CL	MCL
Male	Male	Male	Male	Female	Female	Female	Female
	67	67	89		50	80	75
	73	95	81		45	88	50
	13	90	89		29	83	65
	8	91	76		26	48	67
	51	95	73		28	76	62
	52	44	72		19	45	56
	86	43	87		73	49	71
	41	91	71		50	67	81
	54	91	81		41	98	78
	23	95	56		21	52	79
	75	100	73		44	98	71
	31	95	68		95	65	90
	67	69	84		52	40	88
	26	38	79		89	57	67
	100	33	81		68	69	81
	91	45	68		43	34	83
	66	44	71		83	96	86
	74	44	67		25	95	59
	13	95	55		13	82	72
	98	48	72		59	99	78
	59	42	70		25	85	64
	40	92	60		51	98	66
	39	99	81		22	43	75
	47	66	81		54	82	77
		70			46	43	89

Appendix 11: Applied Calculus 1 Achievement result of male and female students

Cont. No Male	T Male	CL Male	MCL Male	Cont. No Female	T Female	CL Female	MCL Female
19	52	55	67	30	40	58	60
22	61	91	78	50	45	86	78
38.5	78	70	70	40	81	65	84
46	74	85	56	34	50	65	70
47.5	67	75	88	75	79	60	47
37.5	54	69	84	40	53	47	55
65	66	66	65	40	48	86	52
76	52	89	65	35	54	65	73
70	90	76	50	75.5	56	75	80
35	49	49	55	61	51	67	67
60	54	71	75	75	53	60	58
81.5	55	75	96	45	56	61	73
90.5	53	75	77	31	75	50	79
75	47	49	65	45	51	67	65
62	49	77	63	30	48	91	73
45	79	60	93	45	58	67	60
28	54	81	56	70	68	71	49
27	68	65	87	32.5	66	60	67
45	52	75	79	34	44	55	53
31	77	80	91	75	58	48	39
43	56	89	73	30	58	70	59

Appendix 12: Turnitin Originality Report

Document Viewer

Turnitin Originality Report

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Final Thesis By Asnake Bekele

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Appendix 13A: Language Edited Certificate

From: John and Marilyn Barton [bartonj@vodamail.co.za]
Sent: Wednesday, June 20, 2018 3:35 PM
To: 'ASNAKE MULUVE BEKELE'
Subject: Certificate

This is to certify that I, John James Barton proofread and edited A. M. Bekele's Thesis.

I found that the initial language as well as the contents therein were in line with the prescribed parameters of what is required for a Phd Thesis.

Furthermore, the subject matter is most needed to all Pre- Calculus students.

Yours faithfully,

J. Barton D.D

20-06-2018

Appendix 14B: Language Edited Certificate

From	Belilew Molla (PhD in Teaching English as a Foreign Language, TEFL)
Sent	14.03.2019
To:	Asnake Muluye Bekele
Subject:	Certificate

This is to certify that I **Belilew Molla (PhD)** edited and proof read the dissertation entitled: Investigating the Influence of Pre-calculus Mathematics Re-Freshment Module to First Year Engineering Students in Ethiopian Universities: With Reference to First year Engineering Students. I found the language and flow of ideas well organized and meet the standard required for a PhD thesis.

I got the topic on which the research focused interesting and of great contribution for pre-calculus students. This work will contribute a lot to the field if it is published and disseminated to the stake holders.

With regards,

Belilew Molla (PhD)

College of Social Sciences and Humanities
Department of English Language and Literature

Appendix 15: Curriculum Vitae

Curriculum Vitae -ASNAKE MULUYE BEKELE

P.O.Box 419 Dilla University/ Ethiopia Email snkmuluye14@yahoo.com

Personal details

ID number (Passport): EP4447237
Date of birth: 05 Jan 1979
Nationality: Ethiopian
Languages: English, Afrikaans

Educational Background

Tertiary education:

Haramaya University	MSc in Mathematics	2011 - 2012
Haramaya University	MEd in Mathematics	2008 - 2010
Dilla University	BEd in Mathematics	2000 - 2004
Bahir Dar Teachers College	Diploma in Mathematics	1996 - 1997

Secondary education:

Harar Medaniyalem secondary School	9^{th} - 12^{th} grade	1992 - 1994

Relevant experience

Work experience

Job Title	**Month/Year – Month/Year**
Mathematics Instructor in Dilla university/Ethiopia	Oct 2012/13 .- 2018
Mathematics Teacher in Dilla secondary and preparatory school	Sept 2004 - 2008
Mathematics Teacher in three different Junior school of Gede'o Zone	Jul 1998 - 2003

Community involvement

- Prepared one week workshop training with partners for Gede'o Zone high school mathematics teachers in 2013 in title: Strengthen high school mathematics teachers' mathematics lesson delivery system and conceptual understanding.
- Prepared and delivered training on cooperative learning strategies and its' implementation for both students and instructors' in computational and natural science college of Dilla University 2015
- Delivered refreshment of basic pre-modeling mathematics for MSc GSI staff member students in 2016

Publications

1. Asnake Muluye (2016): Assessment of teachers' perception and understanding of continuous assessment: The case of secondary school mathematics teachers in Gede'o Zone. *IJESC Vol. 6, Issue No 7. pp 1965-1980*
2. Asnake Muluye (2016): Extent of under graduate class understanding on basic exponent and polynomial questions of pre-calculus mathematics: A case of an Ethiopian university BSc Mathematics graduating students: BEST: International Journal of Humanities, Arts, Medicine and Sciences *(BEST: IJHAMS) ISSN (P): 2348-0521, ISSN (E): 2454-4728 Vol. 4, Issue No 9, pp 1-8.*
3. Asnake Muluye, Kassahun Nigatu and Halgeyo Jiloo (2017): Mathematics teachers' practice of continuous professional development (CDP) in Gede'o Zone Grade 5-8: *BEST: International Journal of Humanities: ISSN(P): 2348-0521, ISSN(E):2454-4728, Vol. 5, Issue No 2, pp 145-156.*

Interests

• Enjoy reading Bible and traveling magazines, doing physical exercise.
• Enjoy visiting historical places