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## ABSTRACT

The emerging, multi-disciplinary field of Complex Adaptive Systems (CAS) is an alternative to linear, reductionist thinking. It is based on the observations that realworld systems, regardless of scale, are emergent, complex, adaptive, and evolutionary. In this research the scale of CAS examined range from distances of Planck's constant to Gigaparsecs. CAS has also heavily leveraged the interpretations of several recent Nobel Laureates and assumes too that the world is random, indeterministic, and chaotic. But randomness, chaos, and indeterminism can hardly create the progressive, increasingly harmonious world that we are a part of.

At the heart of this issue lies confusion around what innovation in CAS really is. The essential approach to arriving at a mathematical basis of innovation for CAS here has been to view systems from the outside-in as opposed to from the inside-out and the bottom-up. In this approach innovation is conceptualized as existing in every single space-time point-instant in a system. There is a process of precipitation by which this innovation may express itself through a series of quaternary-based architectural forces that are the prime sources of innovation. These series or arrays of forces may further precipitate by informing organizational signatures. Organizations can be thought of as formations with a unique signature at their center, and can vary in complexity and scale. The unique signature for each organization is usually hidden though by common surface dynamics, and "to innovate" is to work through and change the habitual and common patterns in order to allow the deeper founts of innovation to become active at the surface level. When this happens, it is then that innovation occurs.

Once that is more clearly seen then the erected probabilistic and uncertainty functions assumed to be true of the fundamental layers of nature, will be relegated to their place as interim devices in model building.

The nature of innovation can be progressively elaborated through inductive reasoning to arrive at a mathematical framework for innovation in CAS. Rather than assume a chaotic, random, indeterministic world as a starting point, this framework can be built assuming a purposeful, ordered world characterized by qualified determinism. Equations to provide insight into the inherent innovation bias of our system, the nature of each point in the system, the broad architectural forces behind the development of organizations, the inherent uniqueness of each organization, the way to think about varying cultures or organizations, and the inherent dynamism of our system, form the edifice of this framework. The resulting model can then be used deductively to reinforce observations, and predictively to suggest directions and / or steps to emerging trends.

This research hence, through deriving mathematical equations, and by further applying these to various domains ranging from the quantum, to the atomic, to the cellular, to the astrophysical, has been able to provide mathematical contributions to the theory of CAS and to various CAS application areas.

With respect to the theory of CAS, mathematical contributions have been made to understanding the underlying directional bias of CAS activity, understanding the nature of each point in any CAS, and creating mathematical sets for architectural forces that are posited to be behind the development of any CAS. Further, mathematical contributions have been made to understanding the inherent dynamics in any CAS, the dynamics of stagnation and growth in CAS, and the balance of randomness and determinism of any CAS. Mathematical contributions also extend to framing complexity in CAS, understanding what can drive sustainability of CAS, and arriving at a general set of mathematical operators true of any CAS.

In terms of application areas in the organizational space, mathematical contributions have been made to understanding uniqueness of organizations, the emergence of uniqueness in organizations, and what constitutes varying culture of organizations. Further, existing work done by Nobel Laureate Ilya Prigogine and Alan Turing have been leveraged to further frame organizational transitions, and to frame and model shifts in innovations, respectively.

Further mathematical contributions have been made in a range of CAS areas at different scale and level of complexity. Hence, a series of equations have been derived for the electromagnetic spectrum. Quantum, atomic, and cellular wave equations have been derived building off Schrodinger's existing Wave Equation. Further qualifications have been derived for Heisenberg's Uncertainty Principle and an equation has been derived for the integration of different layers of CAS also using Heisenberg's Uncertainty Principle. Equations for space and time alteration as per Einstein's Theory of Relativity have also been derived.

Additionally, equations for the architectures of quantum particles, periodic table elements, and molecular plans at the cellular level have also been derived. Finally, equations for dark matter and dark energy, non-probabilistic quantum states in quantum computing, and the emergence of CAS in the universe have been derived.

In all over 225 equations in 25 different areas have been derived in this dissertation.
In fact, as suggested by the CAS equation derived for a unified field, everything, from unseen energy fields, to quantum particles, to atoms, to molecules, to cells, and therefore to all animate and even inanimate and even unseen objects, and therefore even any CAS system regardless of scale would have a high-degree of quaternary intelligence embedded in it and exist simultaneously. Quoting Schrodinger: "What we observe as material bodies and forces are nothing but shapes and variations in the structure of space. Particles are just schaumkommen (appearances). The world is given to me only once, not one existing and one perceived. Subject and object are only one. The barrier between them cannot be said to have broken down as a result of recent experience in the physical sciences, for this barrier does not exist." This implicit quaternary-based intelligence likely sheds new light on properties such as
distributed control, uncertainty, paradox, co-evolution, emergence, amongst others, seen as fundamental to CAS.

Thinking about CAS as purposeful, and animated by a mathematically-framed engine of innovation, allows existence to potentially be considered as a unified field. Further, it allows insight and additional solutions to a host of complex problems regardless of scale - at the quantum, cellular, human, organizational, sociotechnical, market, economical, political, and social levels - to be conceptualized, designed, elaborated, and managed differently.

Key words: Mathematics of Innovation, Complex Adaptive Systems, Qualified Determinism, Unified Field Theory, Quantum Particle Architecture, Quantum Level Properties, Periodic Table Architecture, Cellular Architecture, Organizational Architecture, Quantum Computing, EM Spectrum Architecture

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Over the last two years I have had numerous discussions with Tony Hsieh, CEO of Zappos, and Arun Rajan, COO of Zappos, on both the esoteric and practical aspects of Complex Adaptive Systems. Many of these discussions were the result of books on Complex Adaptive Systems, read usually at Tony's recommendation. I have referred to these books through the dissertation and am grateful to both Tony and Arun for these on-going dialogs, in the seemingly serendipitous though now joint journey of uncovering further subtleties of Complex Adaptive Systems. Note though that none of the Complex Adaptive Systems work that Zappos has been engaged in has been examined in this dissertation.

While there are a vast number of luminaries whose work I have researched through the course of this PhD program, I am particularly grateful to the work of Albert Einstein and Erwin Schrodinger, whose integrative approach and catholicity, particularly manifest in the unexpectedness of their thought, has been a source of inspiration in informing my own approach in framing the big-picture with respect to Complex Adaptive Systems.

My own interest in mathematics was awoken through a chance conversation with Jacob Zuniga at a dinner in Berkeley, and subsequently to focused study of some of the works of the contemporary mathematician, Ian Stewart, a little over three years ago. This led to my urge to want to further formulate the fractal-related work I had been engaged in for over a decade, in mathematical terms, and therefore to this PhD
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CHAPTER 1: Introduction \& Background

### 1.1 Introduction

Reductionist thinking and linear modeling has ruled much of our practical problem solving. By contrast, the emerging field of Complex Adaptive Systems (CAS) is a means to understand the spontaneous, self-organizing dynamics of the world. The body of CAS thinking has itself been heavily influenced by the work of Nobel Laureates in multiple fields.

Chemistry Nobel Laureate Ilya Prigogine, in his work on Dissipative Structures (Prigogine, 1997) showed these as open systems that keep their order and may evolve towards a qualitatively different state as a result of exchange of matter and energy with the external environment. Dissipative structures evolve through bifurcations and the alternation of continuity and discontinuity. As he explains, when the thermodynamic force acting on a system reach high levels it can cause the system to become discontinuous and bifurcate in unexpected ways creating an emergent property.

In his work, Nobel Laureate in Physics, Murray Gell-Mann asserts that the fundamental laws are quantum-mechanical. In his book, The Quark and The Jaguar, Murray (Murray, 1994) explores the relationship between the simple and the complex. He asserts that quantum mechanics supplies only probabilities for alternative histories and therefore chance must play a role in the unfolding of the universe. Each alternative history is the result of a large number of accidents. These accidents create regularities that in turn create schema, which results in emergence.

Nobel Laureate in Physics, Feynman, in his book QED: The Strange Theory of Light and Matter (Feynman, 1985), states that the theory he presents will not explain why or how Nature acts the way it does, but will explain with very high accuracy the probability that a photon emitted from a monochromatic light source is detected by a photon detector. Feynman jokingly goes on to suggest that the particles route could be absurd - going around Jupiter, to the local hot-dog stand, before reaching the detector. But this absurdity appears to have been generalized and has become an edifice by which quantum nature is now framed and understood.

Building on the work of such luminaries the world we live in is fundamentally projected as emergent, unexpected, evolving, uncertain, indeterministic, and random. The fundamental characteristics of the emerging field of CAS tend too, to reflect these projections. But if the world were indeterministic and random then chaos would likely be rampant and the world would be characterized by anarchy rather than a general sense of progress. In several of my previous books on fractals in complex systems it is suggested that a pattern of progress emerges regardless of area of consideration (Malik, 2009, 2011, 2015). In his book, 'Does God Play Dice?'

(Stewart, 2002) Stewart leverages Einstein's famous question to pose a new answer to this question by utilizing the field of Chaos. Einstein worried about quantum mechanics, which was generally held to be irreducibly probabilistic. Stewart states "Is it possible that the apparent randomness of the quantum world is actually deterministic chaos?" Einstein himself has stated, "The eternal mystery of the world is its comprehensibility...That the world is comprehensible is a miracle." (Einstein, 2013). In a Brief History of Time, Hawking makes the point that the fact there are dualities in physics, or correspondences between apparently different theories of physics, suggests that there is a unified theory of physics, that further indicates that the universe is governed by a set of rational laws that can be discovered and understood (Hawking, 1988).

The research presented here builds on the interpretation of a fundamentally progressive world. Using reflective reasoning a conceptual analytical framework is constructed. This framework takes the form of a mathematics of innovation active at the heart of CAS to also suggest a resultant refined set of properties true of CAS.

### 1.2 Research Problem

In their book, Complex Adaptive Systems: An Introduction to Computational Models of Social Life (Miller \& Page, 2007), Miller \& Page state that the field of complex systems challenges the notion that by perfectly understanding the behavior of each component part of a system one can then understand the system as a whole. They propose that to really understand 'two' one must not only understand the nature of 'one' as in 'one' and 'one' making 'two', but also understand the nature of 'and'. This approach is also key to the whole body of System Thinking which will be explored in more detail in Section 2.1.1, where the structure of any system is just as important in determining its behavior as the individual components themselves (Sterman, 2002).

This idea is also consistent with a fundamental thought in a Brief History of Time where Hawking suggests that it is very difficult to create a theory of everything. Instead a problem is broken into bits and a number of partial theories are created to model a limited number of observations. But he goes on to say that this approach may be completely wrong. If everything in the universe depends on everything else in a fundamental way, it might be impossible to get a full solution by investigating parts of the problem in isolation. Hence 'and' certainly becomes very important. The question is whether the understanding of dynamics of the visible layer such as 'and', is in itself sufficient to understand the notions of complexity and adaptiveness at a system level in Complex Adaptive Systems (CAS).

While there are several schools of thought relating to CAS, in general there is a core set of attributes considered to describe CAS. In their summary papers on CAS, Chan of MIT (Chan, 2001) and Dodder and Dare of MIT (Dodder \& Dare, 2000) highlight the following representative aspects:

- Distributed control: no centralized single control mechanism that governs system behavior
- Connectivity: Complexity results from inter-relationship, inter-action, and inter-connectivity of elements within a system and between a system and its environment
- Co-evolution: Elements in a system can change based on their interactions with one another and with the environment
- Sensitive dependence on initial conditions: CAS are sensitive due to their dependence on initial conditions
- Emergent order: Complexity in CAS refers to the potential for emergent behavior in complex and unpredictable ways
- Far From Equilibrium: This phenomenon illustrates how systems that are forced to explore their space of possibilities will create different structures and new patterns of relationships
- State of paradox: CAS has dynamics of both chaos and order
- Multiple levels of organization: CAS tends to exist in many levels of organization in the sense that agents at one level are the building blocks for agents at the next level
- Unpredictability: CAS have a future that is hard to predict

As pointed out by Dodder \& Dare the characteristics summarized above have themselves arisen from a study of similarities across CAS. In the natural world, brains, immune systems, ecologies, cells, developing embryos, and ant colonies, all fall under the category of CAS. In the human world, political parties, scientific communities and the economy, are examples of CAS. Human knowledge itself has been suggested to be an example of CAS (Thomas \& Zaytseva, 2016), as has living cities (Gershenson, 2013). In his paper on 'Ecosystems and the Biosphere as Complex Adaptive Systems' (Levin, 1998) Levin points out that it is easy to find discussions of biospheres, ecosystems, economies, brains, organisms as CAS, and summarizes key characteristics as the sustained diversity and individuality of components; localized interactions among those components; an autonomous process that selects from among those components, based on the results of local interactions, a subset for replication or enhancement. In 'Economy as an evolving Complex System' (Arthur, 1997) Arthur states six properties true of all economies and all CAS to include: dispersed interaction, the absence of a global controller, cross-cutting hierarchical organization, continual adaptation, perpetual novelty, and far-from-equilibrium dynamics.

These characterizations suggest the nature of the innovative activity at the core of all CAS. The question is can this engine of innovation be succinctly framed and represented? Leveraging a universal metaphor of being able to see the forest from the trees, the supposition in this research is that this engine can be more easily framed and represented by shifting the prevalent point of view. In studying these characteristics, these are all based from a bottom-up point of view. Hence it is perhaps the case that from this point of view it is easier to arrive at characteristics
such as distributed control, unpredictability, emergent order, sensitivity to initial condition, and a state of paradox.

But what if this point of view is changed to look at systems as a whole? This in fact may even be necessary from the point of view of broken symmetries since as Nobel Laureate Anderson suggests in his paper 'More Is Different' (Anderson, 1972) that at higher levels of system complexity existing symmetries are broken and new properties or behaviors that could never have been predicted by just studying the dynamics at a micro-scale come in to being with the broken symmetries. In this case a different set of characteristics for CAS will arise at each subsequent scale. To arrive at a potential engine of innovation valid for all CAS, it will likely be more fruitful to employ an outside-in view that more easily surfaces big picture characteristics (Malik, 2009). Such an outside-in perspective will be most successful if it can in some sense transcend broken symmetries, or in other words, can inform the model to account for the effects or any patterns that emerge due to broken symmetries.

This research hence, explores the development of such a mathematical framework for innovation in CAS, that highlights the essentially innovative quality that exists in all CAS, while also addressing the fundamental attributes of complexity, adaptiveness, co-evolution, amongst other characteristics. As such it will help to create a model of innovation for all CAS regardless of scale. Such a model is needed to address complexities in the socio-ecological and sociotechnical realms that arise when having to deal with different temporal, spatial and social scales, uncertainty, multidimensional interactions, and emergent properties (Rammel et al, 2007).

The research problem hence can be stated as: Can an engine of innovation within CAS be framed such that it will usefully apply across all CAS regardless of level of complexity or scale?

### 1.3 Key Attributes of the Desired Theory

In his paper on a mathematics of life and civilization, 'Scaling: The surprising mathematics of life and civilization' (West, 2014) West points out that the predominance of quarter-power scaling laws across all life forms is surprising because each organism, sub-system, cell type, genome has evolved in its own everchanging environmental niche within a unique history. Using an example, quarterpower refers to the phenomena of requiring only 75\% (multiple of a quarter) increase in metabolic energy when an organism increases in size. The emergence of systematic behavior is what is surprising and suggests that "generic underlying dynamical mechanisms have constrained evolutionary processes, thereby opening a possible window into determining quantifiable emergent laws that capture the essential features and coarse-grained behaviors of living systems."

The question is how deep and far does the systematic behavior go? West has explored an aspect of the systematic behavior as it relates to inputs required by systems as they increase in size, for a range of biological life-forms and development of large cities. In this dissertation, the range of systematic behavior explored has to do with what keeps systems complex, adaptive, and innovative. Further it is supposed that the systematic behavior exists regardless of scale. A systematic mathematics has to be developed to frame this. Key attributes of such a mathematical theory for innovation in CAS should therefore include the following:

- This approach is inherently "outside-in" focused, and will thereby also integrate aspects of the existing "bottom-up" approach to CAS into it. The bottom-up approach is characteristic of key CAS schools of thought such as MIT and Santa Fe Institute, amongst others. In this school the characteristic of emergence is what creates top-down schema. Thus, as suggested by GellMann complex adaptive system interacts with the environment, creates schemata, which are compressed and generalized regularities experienced in those interactions, behaves in ways consistent with these schemata, and incorporates feedback from the environment to modify and adapt its schemata for greater success (Gell-Mann, 2003). The outside-in approach has been the focus of my previous research and is elaborated in a series of books: Connecting Inner Power with Global Change (Malik, 2009), Redesigning the Stock Market (Malik, 2011), and The Fractal Organization (Malik, 2015).
- CAS are characterized by qualified determinism at each level of organizational complexity
- This qualified determinism is orchestrated by a cohesive mathematical framework
- This mathematical framework explains the spectrum of possibility from stagnation to sustainability / progress
- The mathematical framework and the derived equations will hence also provide a basis for innovation
- The mathematical framework advances the field of innovation by creating a series of mathematical equations to better understand innovation
- The mathematical framework for innovation applies to organizations at different levels of complexity from the 'simple' to the more 'complex'. Hence it will provide insight into how innovation happens at the atomic/quantumparticle level, the level of the biological cell, the level of the human being, the level of a team, the level of the corporation, the level of the market, amongst other levels
- The mathematical framework provides insight into further potential development at each level of organizational complexity
- This framework separates the nature of functionality by meta-layers and suggests key dynamics operative at each layer. Under certain conditions the respective meta-level functionality and operations becomes active to bring about complexity and adaptiveness in the visible, surface layer
- The mathematics of innovation suggested in this research constructs a generalized equation of innovation that may exist at multiple-levels of
complexity ranging from the quantum-particle level, through the cellular level, to the larger organizational, market, and system levels. The mathematics constructs functions that highlight key operations for each relevant layer and the interaction between layers to bring about the emergent adaptability and complexity visible in the surface layer.

The following table in Figure 1.3.1 summarizes the key attributes of the desired theory:

| Attribute \# | Key Attributes of Desired Theory |
| :---: | :--- |
| 1 | The theory is derived from a big-picture "outside-in" approach as <br> opposed to a "bottom-up" approach |
| 2 | This approach being inherently "outside-in" focused will thereby <br> also integrate aspects of the existing "bottom-up" approach to CAS <br> into it |
| 3 | CAS are characterized by qualified determinism at each level of <br> organizational complexity |
| 4 | This qualified determinism is orchestrated by a cohesive <br> mathematical framework |
| 5 | This mathematical framework explains the spectrum of possibility <br> from stagnation to sustainability / progress |
| 6 | The mathematical framework and the derived equations will hence <br> also provide a basis for innovation |
| 7 | The mathematical framework advances the field of innovation by <br> creating a series of mathematical equations to better understand <br> innovation |
| 8 | The mathematical framework for innovation applies to <br> organizations at different levels of complexity from the 'simple' to <br> the more 'complex' |
| 9 | The mathematical framework provides insight into further potential <br> development at each level of organizational complexity |
| 10 | This framework separates the nature of functionality by meta-layers <br> and suggests key dynamics operative at each layer |
| 11 | Under certain conditions the respective meta-level functionality and <br> operations becomes active to bring about complexity and <br> adaptiveness in the visible, surface layer |
| 7 |  |

Figure 1.3.1 Key Attributes of Desired Theory
These key attributes will be further affirmed through the surfacing of design principles and working hypotheses generated in the representative literature review in Chapter 2.

### 1.4 Summary of Core Mathematical Framework

This dissertation will present a mathematical model for thinking about innovation at various levels of organizational complexity.

To begin with a series of equations will be derived that will provide deeper insight into the nature of systems. In particular the equations will provide some insight into the inherent innovation bias of our system, the nature of each point in the system, the broad architectural forces behind the development of organizations, the inherent uniqueness of each organization, the way to think about varying cultures of organizations, and the inherent dynamism of our system.

These equations will assist in putting 'science' into the 'art' of innovation. Three basic and ubiquitous states of organization - the physical, the vital the mental - will be built on to suggest six equations that provide insight into the fundamental organizational states of stagnation, stability, entropy, energy, sustainability, and fragmentation. These equations provide insight into the inherent bias of the system to always re-organize based on a more comprehensive possibility.

Equations for the characteristics or properties embedded in a single point will then be derived. The 'point' captures the inherent intelligence that appears to exist in the system, or in any smaller part of it. Hence, equations for the fundamental power, knowledge, presence, and nurturing suggested to exist in each point of our system will be derived. Following this, sets for each of the four characteristics embedded in a point will be derived, that in effect suggest the architectural forces that can create and develop any type of organization.

The hypothesis is that every organization, whether an atom, cell, person, team, corporation, market, or country is unique and that this uniqueness can be specified in terms of elements of the derived sets for power, knowledge, presence, and nurturing. This hypothesis for uniqueness stems from observations at multiple levels.

At the sub-atomic level Nobel Laureate Wolfgang Pauli's 'Pauli Exclusion Principle' states that no two similar fermions, which include fundamental particles with halfinteger spin such as protons, neutrons, and electrons, can occupy the same quantum states simultaneously (Pauli, 1964). Spin has to do with the angle that the particle has to rotate through before being symmetrical with its original state. Half-integer spin particles need to rotate through 720 degrees before being symmetrical with their original state. The implication of the Pauli Exclusion Principle is that fundamental structure and consequently stability comes into being at the atomic level, which as is evident in the periodic table allows the separation of function related to form. This stability related to the underlying structure of atoms implies the basis of uniqueness and diversity. In the absence of the Exclusion Principle matter would just be a dense soup (Hawking, 1988) with particles occupying overlapping space.

At the observable level uniqueness is evident from the immense diversity of distinct species on earth (Mora, 2011) estimated to be over 2 million, and further the uniqueness of every member of each species. This member-level uniqueness is suggested by the difference in non-coding regions of the DNA that may vary in their sequence by about 1 to 4 percent, which in turn result in unique protein binding sequences of each human (Snyder, 2010), as an example, which in turn results in unique observable qualities.

At the astronomical level Einstein's Special Theory of Relativity (Einstein, 1995) suggests that every coordinate system potentially has its own space-time rendering as opposed to there being one absolute space and time. This implies the notion of uniqueness as an implicit property of space.

Hence, an equation for the unique 'signature' of an organization will be derived. Further, the equation for uniqueness will itself be governed by an equation for the emergence of the uniqueness. Having defined an equation for the uniqueness of an organization it now becomes possible to also derive equations for the culture of an organization. Hence, two equations are derived - one for a monoculture and one for a diverse culture. Linking the notion of diversity and uniqueness back to West's observations on scaling may imply that uniqueness and diversity are themselves the result of overarching or systemic "laws". This notion will be explored further in subsequent chapters.

Having expressed specific mathematical models for uniqueness it is then useful to turn full circle and derive equations for the fundamental states of the system begun with: physical, vital, mental, and integral (from the family of nurturing). The derived equations indicate the dynamism in-built into each of these states, and indicate too the conditions for the incarnation or interchange of the meta-levels with the observed states. Subsequently, equations for stagnation and for dynamic growth, based on the dynamism inherent in the four states will be derived.

These equations will provide a framework for thinking about and managing innovation at varying scales of organizational complexity.

### 1.5 Chapter Summary

The study of CAS suggests their inherent innovativeness as evidenced by characteristics such as distributed control, co-evolution, connectedness, and adaptiveness, amongst others. The question is how best to mathematically model this innovativeness. Prevalent thinking tends to view dynamics in CAS as chaotic, random, and probabilistic as would likely be the case if viewed from the bottom-up. But even this view is beginning to shift as the systematic-ness behind instances of CAS is coming into view (refer to Section 1.3). For a comprehensive mathematical model of innovation, it is likely better to view CAS from the outside-in.

The research in this dissertation will proceed along the following lines to structure a possible mathematical modeling of innovation in CAS:

1. Key attributes of the desired theory become overall guiding principles for the research and are suggested by fundamental departures from the existing body of CAS thinking as reviewed in this chapter (Chapter 1).
2. Representative literature review of existing mathematical modeling as it relates to innovation in CAS, and the surfacing of existing design principles and suggested working hypotheses (Chapter 2). The surfacing of design principles and working hypotheses will affirm the key attributes summarized in Figure 1.3. The working hypotheses may be further validated by subsequently exercising the model.
3. Construction of a conceptual analytical framework expressed as a series of equations that will define a mathematical kernel of innovation in CAS that also incorporates several of the hypotheses generated in the literature review (Chapter 4). Note that this dissertation being an exploratory research project suggests that hypotheses are modeled as a first-step, leaving more detailed quantitative validation of hypotheses for further research.
4. Application of the mathematical framework to sample CAS domains to arrive at relevant conclusions about the domains to further our understanding of these domains, while also suggesting the efficacy of the derived model, and initial validation of working hypotheses (Chapter 5). Domains considered include molecular plans at the cellular level, the quantum level, the architecture of quantum particles, the categorization of atoms in the periodic table, the sustainability of CAS, and mathematical operators of CAS. Note that the CAS domains can be considered as the input into the derived mathematical model, with the enhancement of understanding as the output. In several cases the enhancement of understanding also results in predictions in the domains that may be verified in subsequent research.
5. A simulation of the primary mathematical equation for innovation using Vensim Simulator, to descriptively research how innovation in CAS may be altered through manipulating key parameters in the equation (Chapter 6).
6. The application of the derived equations for CAS innovation to a multi-year case study at Stanford University Medical Center to gain further insight into a process of organizational innovation (Chapter 7).
7. The review of some of the existing piece-meal CAS math, leveraging some of Prigogine's and Turing's work amongst other work, in light of the mathematical model developed here to also explore the efficacy of the model in terms of its ability to align with aspects of this existing math (Chapter 8).
8. Exploration of a qualitative proof for the essential mathematical structure of the model through leveraging Einstein's Theory of Relativity (Chapter 8).

This approach is summarized in the table in Figure 1.5.1:

| Step \# | Step/Chapter | Key Output | Notes |
| :---: | :--- | :--- | :--- |
| 1 | Key attributes of the desired <br> theory (Chapter 1) | Overall guiding principles for <br> the research | Suggested by fundamental <br> departures from the existing body |


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| :---: | :---: | :---: | :---: |
|  |  |  | of CAS thinking as reviewed in this chapter |
| 2 | Representative literature review of existing mathematical modeling as it relates to innovation in CAS (Chapter 2) | Generation of design principles and working hypotheses | Design principles are based on existing insight, and working hypotheses are based on possible new insight and will be leveraged to assist in creation of the mathematical model The surfacing of design principles and working hypotheses also affirm the 'key attributes of the desired theory' summarized in Figure 1.3.1 |
| 3 | Construction of a conceptual analytical framework (Chapter 4) | Series of equations that will define a mathematical kernel of innovation in CAS that also incorporates the design principles and working hypotheses generated in the literature review | This dissertation being an exploratory research project suggests that hypotheses are modeled as a first-step, leaving more detailed quantitative validation of hypotheses for further research |
| 4 | Application of the mathematical framework to sample CAS domains (Chapter 5). Domains considered include: molecular plans at the cellular level, properties at the quantum level, the architecture of quantum particles, the categorization of atoms in the periodic table, the sustainability of CAS, and mathematical operators of CAS | Relevant conclusions about the domains to further the understanding of these domains, while also suggesting the efficacy of the derived model, and providing initial validation of working hypotheses | CAS domains can be considered as the input into the derived mathematical model, with the enhancement of understanding as the output. In several cases the enhancement of understanding also results in predictions in the domains that may be verified in subsequent research |
| 5 | Simulation of the primary mathematical equation for innovation using Vensim Simulator (Chapter 6) | Descriptive research on how innovation in CAS may be altered through manipulating key parameters in the equation |  |
| 6 | Application of the derived equations for CAS innovation to a multi-year case study at Stanford University Medical Center (Chapter 7) | Further insight into a process of organizational innovation |  |
| 7 | Review of some of the existing piece-meal CAS math for innovation (Chapter 8) | Further exploration of the efficacy of the model in terms of its ability to align with aspects of this existing math | Existing CAS math include some of Prigogine's and Turing's work amongst other work |
| 8 | Exploration of a qualitative proof for the essential mathematical structure of the model (Chapter 8) | Qualitative proof of mathematical structure of derived model | Leveraging Einstein's Theory of Relativity |

Figure 1.5.1 Proposed Research Approach

The immediate question hence, is what mathematical modeling has been used to structure innovation in CAS so far? This will be explored in Chapter 2.

CHAPTER 2: Representative Literature Review

The theory and research review explores the areas of Systems Dynamics and Complex Adaptive Systems as they relate to the possible construction and elaboration of a mathematical framework of innovation within CAS.

Specifically, Section 2.1.1 explores some relevant notions of system dynamics and system modeling. Section 2.1.2 explores some fundamental properties of CAS that also suggest the inherent variability and mathematical space in CAS. Section 2.1.3 explores rule-based systems and the possible use of genetic algorithms to further tighten the mathematical space that can be generated due to the variability in CAS. Section 2.1.4 on the top-down view of CAS provides suggestions on an implicit direction of CAS, and explores some meaning of probability, chance, and determinism in CAS. Section 2.1.5 explores some mathematical representations of variability and innovation in CAS, and of the measurement of complexity in CAS.

Within these areas the approach has been to highlight key representative pieces. Aspects of existing theories and models are focused on, and design principles and working hypotheses implicitly guided by 'key attributes of the desired theory' summarized in Figure 1.3.1 in Chapter 1, are generated in the following ways: 1. Design principles are generated to highlight existing insight that seems relevant to the development of a mathematical kernel of innovation within CAS.
2. Working hypothesis highlight potential departures from or enhancements to the representative pieces under consideration that may strengthen the overall body of theory in alignment with the development of a mathematical kernel of innovation within CAS. Note that working hypotheses will attempt to be validated through one of the subsequent research steps summarized in Figure 1.5.1 in the previous chapter.

Further, the link between design principles and working hypotheses with the key attributes of the desired theory are elaborated in Figure 2.2.1 in the summary section of this chapter.

This research work is primarily exploratory at this stage, focused on the derivation of a conceptual analytical framework as a possible engine of innovation for CAS. It will therefore leverage the design principles and generated hypotheses that will serve as a means to inductively arrive at an initial conceptual analytical framework expressed through a series of derived mathematical equations.

### 2.1 Theory and Research Review

A review of Systems Dynamics is an ideal starting point since it highlights an

established practical modeling approach. Consideration can then be given to the CAS that needs to be modeled to highlight the essential aspect of innovation at its core. In doing so key views on CAS will be reviewed - the emergent, the rule-based, and the top-down views. Finally the state of mathematics in its attempt to model the kernel of innovation at the heart of CAS will be reviewed.

### 2.1.1 Systems Dynamics

In his paper on System Dynamics (Sterman, 2002), John Sterman of MIT Sloan School of Management suggests that systems thinking is the ability to see the world as a complex system, and to understand how everything is connected to everything else. It is rooted in the scientific method and concepts of nonlinear dynamics, and has grown out of control theory and servomechanisms design, and pioneered at MIT by Jay Forrester in the 1950s (Forrester, 1961).

System dynamics can be thought of as a methodology and mathematical modeling technique for framing, understanding, and discussing complex problems. The basis of the method is the recognition that the structure of any system - the many circular, interlocking, sometimes time-delayed relationships among its components — is often just as important in determining its behavior as the individual components themselves.

Structures of systems generally tend to be illustrated or modeled assuming that all functionality that may cause observable dynamics exists at the same level as the observed dynamics of the system. But it is also possible to incorporate meta-level functions that have an influence on the dynamics of the observable layer. This idea is expressed by MIT's Peter Senge (Senge, 2007), "System dynamics looks for the causality that underlies the longer-term patterns of change in complex systems. We assume there are underlying interrelationships at deeper levels in systems and that once one understands this level, one has unique abilities to influence change. We can't ever understand those levels completely. However, we can reach plateaus of insight."

Senge's work articulates the importance of human values such as vision, purpose, reflectiveness required if systems are to function optimally. The field of system dynamics has been enhanced by his work, key principles of which are encapsulated in his book, The Fifth Discipline: The Art and Practice of the Learning Organization (Senge, 1990).

The five disciplines are:

1. Personal mastery, the discipline of continually clarifying and deepening personal vision, focusing energies, developing patience, and seeing reality objectively.
2. Becoming aware of mental models, that are deeply ingrained assumptions, generalizations, or even pictures of images that influence understanding of and consequent action in the world.
3. Building shared vision, a practice of unearthing shared pictures of the future that foster genuine commitment and enrollment rather than compliance.
4. Team learning, which starts with dialogue, the capacity of members of a team to suspend assumptions and enter into genuine thinking together
5. Systems thinking, The Fifth Discipline that integrates the other four.

The idea of meta-levels is also consistent with some of the most recent developments in theoretical physics to do with strings and branes that may operate in up to 11 dimensions as opposed to the four-dimensions of our known space-time rendering of reality (Greene, 2003).

This notion of meta-levels may lead to the following design principle:
DESIGN PRINCIPLE 2.1.1.1: Complex systems can be better modeled by the explicit inclusion of relevant meta-level functions

As a reminder, as suggested in Figure 1.5.1, Proposed Research Approach, design principles are based on existing insight, and working hypotheses are based on possible new insight and will be leveraged to assist in creation of the mathematical model.

Sterman relates that where as the world is dynamic, evolving, and interconnected, actors in the world tend to make decisions using mental models that are static, narrow, and reductionist. This observation is an illustration of a range of meta-level "mental" functions that influence system dynamics. He suggests that at the very least feedback, delays, and stocks and flows need to be considered in broadening mental models. This may lead to less open-loop, event-oriented world views, that in turn lead to event-oriented, reactionary approaches to problem-solving.

Further, Sterman suggests that to judge causality cues such as temporal and spatial proximity of cause and effect, temporal precedence of causes, covariation, and similarity of cause and effect are leveraged. But in fact in complex systems cause and effect are often distant in time and space. Further, cause and effect, can as in the case of internal mental models influencing external reality, occur in different times and space. If a different time-space reality is considered as being synonymous with a meta-layer, this leads to Design Principle 2.1.1.2 and Working Hypothesis 2.1.1.1 in strengthening the representation of complex systems:

DESIGN PRINCIPLE 2.1.1.2: Systems modeled will be more accurate if meta-layers are included in addition to the surface layer already being considered

WORKING HYPOTHESIS 2.1.1.1: There is a transforming effect on the surface layer that allows new elements to come into being that are not the result of the surface layer only

Further, there is the possibility of using the existing systems modeling insights, as in

Design Principle 2.1.1.3, to shed light on some relationships between layers:
DESIGN PRINCIPLE 2.1.1.3: Feedback, delays, stock and flows can be leveraged to connect meta-levels with surface level for a more complete view of a system

It is also possible to get to relevant system-level meta-level functions by observing how when left to itself systems may change over time. Observable patterns across diverse systems may point to a further set of meta-level trend and possible architectural system patterns. This generates another hypothesis:

WORKING HYPOTHESIS 2.1.1.2: Trends and architectural system patterns, in addition to time-delays, feedback and so on, are a contributing factor to complexity and need to be factored in for a more complete view of a system

Sterman suggests that good systems dynamics models have a broad model boundary. The specification of the equations and the modeling process is key. Specifications of models should not be compromised to achieve analytical tractability, and should include all variables thought to be important whether numerical data to estimate the parameters is available or not. Omitting soft variables is less scientific and less accurate than using best judgment to estimate values.

Further, Peter Checkland has developed a Soft Systems Methodology to be used in the analysis of complex situations where there are divergent views about the definition of a problem. In such situations even the actual problem to be addressed may not be easy to agree upon. To intervene in such situations the soft systems approach uses the notion of a "system" as an interrogative device that will enable debate amongst concerned parties (Checkland, 2006).

General Morphological Analysis was developed by Zwicky as a "totality research" which, "in an unbiased way attempts to derive all the solutions of any given problem". This approach pushes the boundaries of a system by assisting in the discovery of new relationships or configurations, which may not be so evident, or which might have been overlooked by other methods. Importantly, as he reported "it encourages the identification and investigation of boundary conditions, i.e. the limits and extremes of different contexts and actors" (Ritchey, 2013).

System models are usually formulated as systems of high-order, nonlinear, and possibly stochastic differential equations portraying decision rules of agents.

These research suggestions are summarized by the following additional design principles:

DESIGN PRINCIPLE 2.1.1.4: Any mathematics for CAS needs to be based on the whole rather than solely on separately treated parts

DESIGN PRINCIPLE 2.1.1.5: The mathematical model for CAS will be higher-order with multiple meta-levels

The preceding design principles and working hypotheses are summarized in Figure 2.1.1.1:


Figure 2.1.1.1 System Dynamics Design Principles \& Working Hypotheses

### 2.1.2 Bottom-Up Approach to Complex Adaptive Systems

Key schools of thought in the field of CAS are for example MIT and Sante Fe Institute. Both these schools are grounded in the bottom-up emergent reality as a basis.

Leveraging a general definition of Complex Adaptive Systems (CAS), as characterized in an MIT research paper (Chan, 2001), these may be stated as apparently complex behaviors that emerge as a result of often nonlinear spatiotemporal interactions among a large number of component systems at different levels of organization.

Some of the characteristics of CAS as summarized in Section 1.2, summarizing the research problem, are elaborated in the subsequent sub-sections and appear to be a generalization of the surface or visible layer only, without reference to the underlying or deeper-layered system of which the surface phenomena is suggested here to be only an outcome. Hence:

WORKING HYPOTHESIS 2.1.2.1: Insight into CAS and emergent reality will be strengthened by formulating CAS as comprising multiple layers

With reference to a potential mathematics being developed to frame key CAS relationships:

WORKING HYPOTHESIS 2.1.2.2: A CAS mathematics needs to integrate multiple layers together to aim to achieve a clearer relationship between the multiple layers of any complex system

Consideration of the properties of CAS generates further hypotheses.

### 2.1.2.1 Distributed Control

Distributed Control is cited as a property of CAS (Chan, 2001; Dodder \& Dare, 2000). In this view there is no single centralized control mechanism that governs system behavior. It is believed that the interrelationships between elements of the system produce coherence, but the overall behavior usually cannot be explained merely as the sum of individual parts.

This view arises when systems are viewed from the surface layer only. If the perception were to change to begin to see the layers that informed the outer layer then a different notion of System Control would emerge. This would then also enhance the set of devices in framing and managing CAS. Hence:

WORKING HYPOTHESIS 2.1.2.1.1: System Control may exist if the system is perceived as being multiple-layered

WORKING HYPOTHESIS 2.1.2.1.2: System Control will enhance the management of CAS

### 2.1.2.2 Inter-Connectivity

Complexity results from the inter-relationship, inter-action and Inter-Connectivity of the elements within a system and between a system and its environment. CAS scientists (Minai, Braha, Bar-Yam, 2006) conclude that a decision or action by one part within a system will influence all other related parts but not in any uniform manner.

However, if one could envision each surface part as the outcome of a deeper edifice and begin to see too the influence and relative strength that one part has in relationship to another, then it would become clearer as to what parts may dominate and why. Hence:

WORKING HYPOTHESIS 2.1.2.2.1: There must be a mathematical function that will put in perspective the relative strength and influence of parts in a system

### 2.1.2.3 Sensitive Dependence on Initial Conditions

CAS scientists claim that there is Sensitive Dependence on Initial Conditions (Chan, 2001). Hence, changes in the input characteristics or rules are not correlated in a linear fashion with outcomes. Small changes can have a surprisingly profound impact on overall behavior. They cite the research of Edward Lorentz, an American physicist, who studied the solutions to equations describing weather patterns (Lorentz, 1993). He concluded that the current state of the weather is no predictor of what it will be in a couple of days time because tiny disturbances can produce exponentially divergent behavior. Because most natural processes are at least as complex as the weather, the world is hence according to them, fundamentally unpredictable. This means the end of scientific certainty, and long-term prediction and control are therefore believed to not be possible in complex systems.

It could be the case though that it is the nature of change that is important in determining its impact. If the change is just a rehashing of existing patterns then it will always have an outcome that has already been experienced, and therefore will not be noticeable. If the change is the result of a new pattern, though, something that is the result of the synthesis of multiple layers of the informing system, then the resulting outcome will likely be quickly noticeable. Hence there is a 'qualified determinism' that is possible. Hence:

WORKING HYPOTHESIS 2.1.2.3.1: There is a qualified determinism that governs outcomes within CAS

### 2.1.2.4 Emergent Behavior

Complexity in complex adaptive systems refers to the potential for Emergent Behavior in complex and unpredictable phenomena. CAS is envisioned as a networked system of many agents acting in parallel. There is constant action and reaction to what other agents are doing, thus nothing in the environment is essentially fixed. Hence from the interaction of the individual agents arises some kind of global property or pattern, something that could not have been predicted from understanding each particular agent.

In his work on Dissipative Structures, Prigogine (Prigogine, 1997) showed these as open systems that keep their order and may evolve towards a qualitatively different state as a result of exchange of matter and energy with the external environment. Dissipative structures evolve through bifurcations and the alternation of continuity and discontinuity. As he explains, when the thermodynamic force acting on a system reaches high levels it can cause the system to become discontinuous and bifurcate in unexpected ways creating an emergent property. Hence physical systems may also have a history associated with them, and in general this history is indeterministic.

It is also possible to say though that the bifurcation is not random and that history is not random, but the result of the habitual patterns that may or may not already have been overcome in that particular system. It is possible that there is some incubation happening that is not visible that will result in a specific bifurcation. While the details of the exact result may not be determined, the direction of the bifurcation may be determined.

This generates a series of working hypotheses:
WORKING HYPOTHESIS 2.1.2.4.1: There is a mathematics of CAS that will determine direction of outcome

WORKING HYPOTHESIS 2.1.2.4.2: Direction of bifurcations in CAS are the result of the play of habitual patterns and forces

WORKING HYPOTHESIS 2.1.2.4.3: There is a meta-level caused incubation that creates the possibility of new patterns emerging at the surface level

## WORKING HYPOTHESIS 2.1.2.4.4: History is the concatenation of expressed forces or patterns

### 2.1.2.5 Far from Equilibrium

In 1989, Nicolis and Prigogine (Nicolas \& Prigogine, 1989) showed that when a physical or chemical system is pushed away from equilibrium, it could survive and thrive. If the system remains at equilibrium, it will die. The "far from equilibrium" phenomenon illustrates how systems that are forced to explore their space of
possibilities will create different structures and new patterns of relationships. Hence:

WORKING HYPOTHESIS 2.1.2.5.1: "Far from equilibrium" only appears so because the deeper layers informing surface phenomenon have not been considered

### 2.1.2.6 State of Paradox

Research in complex adaptive systems (summarized by Toni and Camello, 2010) has indicated dynamics combining both order and chaos. This reinforces the idea of bounded instability or the edge of chaos that is characterized by a state of paradox: stability and instability, competition and cooperation, order and disorder.

Paradox can be unbundled though by thinking of past order as a predominantly 'physical' state (Malik, 2009). Chaos can be thought of as a predominantly ‘vital' state. Order in the making as a predominantly 'mental' state. Hence:

DESIGN PRINCIPLE 2.1.2.6.1: A mathematics of CAS should illustrate the nature of the physical, the vital, and the mental, to therefore unbundle paradox

As an example of a practical level, in their white paper on CAS, Dodder and Dare (Dodder \& Dare, 2000) describe a conference "The Economy as an Evolving Complex System" that took place at the Santa Fe Institute in 1987. The conference was designed to stimulate the cross-fertilization of ideas to deal with what economist Brian Arthur identified as difficult features of real-life economic systems. These include: dispersed interactions, no global controller, cross-cutting hierarchical organization, continual adaptation, perpetual novelty, and out-of-equilibrium dynamics. What emerged was the complexity perspective on economics and economic modeling.

Figure 2.1.2.1 summarizes the design principles and hypotheses generated from the bottom-up consideration:


Design Principle 2.1.2.6.1 A mathematics of CAS should illustrate the nature of the physical, the vital, and the mental to therefore unbundle paradox

Working Hypothesis 2.1.2.1 Insight into CAS and emergent reality will be strengthened by formulating CAS as comprising of multiple layers

Working Hypothesis 2.1.2.2 A CAS mathematics needs to integrate multiple layers together to aim to achieve a clearer relationship between the multiple layers of any complex
system
Working Hypothesis 2.1.2.1.1 System Control may exist if system is perceived as being multilayered

Working Hypothesis 2.1.2.1.2 System Control will enhance management of CAS

Working Hypothesis 2.1.2.2.1 There must be a mathematical function that will put in perspective the relative strength and influence of parts in a system

Working Hypothesis 2.1.2.3.1 There is a qualified determinism that governs outcomes in CAS

Working Hypothesis 2.1.2.4.1 There is a mathematics of CAS that will determine direction of outcome

Working Hypothesis 2.1.2.4.2 Direction of bifurcations in CAS are the result of the play of habitual patterns and forces

Working Hypothesis 2.1.2.4.3 There is a metalevel caused incubation that creates the possibility of new patterns emerging at the surface level

Working Hypothesis 2.1.2.4.4 History is the concatenation of expressed forces or patterns

Working Hypothesis 2.1.2.5.1 Far from Equilibrium only appears so because the deeper layers informing surface phenomena have not been considered

Figure 2.1.2.1 Bottom-Up Design Principles \& Working Hypotheses
Note that the design principles and generated working hypotheses will be used in the formulation of the mathematical model and equations in Chapters 4 and 5, and again referred to in the evaluation in Chapter 8.

### 2.1.3 Rule-Based Systems

In a piece of work on Complex Adaptive Systems, representative of his larger body of work, John Holland (Holland, 1992) of University of Michigan suggests that economies, ecologies, immune systems, amongst other complex systems share similar kernels.

First is their ability to evolve: that is, the ability of parts to adapt and learn. Second is their ability to aggregate behavior, which is not simply derived from action of parts, but is something that emerges from the interaction of the parts. Third is their ability of anticipation: that is the ability to anticipate the consequences of certain responses. These properties, he proposes, are also the prime reason Complex Adaptive Systems are difficult to understand - they are a moving target.

The inherent ability of CAS to adapt, evolve, and create new emergent behavior suggests their basic ability to innovate. In addition to the opinion of the emergent schools of thought, it is possible though that there is a kernel that forms the heart of CAS that oversees or centralizes the innovative function. Hence:

## WORKING HYPOTHESIS 2.1.3.1: The heart of Complex Adaptive System can be represented by a mathematical model for innovation

Holland suggests that standard theories in physics, economics, and elsewhere are of little help in understanding CAS because they concentrate on ideal end-points, whereas CAS are continually changing. They continue to evolve, and exhibit new forms of emergent behavior. History and context also play a critical role. It is the process of becoming that must be studied to gain insight. This gives rise to the following design principle:

DESIGN PRINCIPLE 2.1.3.1: There is a mathematical model that can be developed in which becoming is an integral part. This becoming is exhibited by evolution, by emergent behavior, and has a foundational context and history

Consistent with a diverse body of Complex Adaptive Systems work, Holland's central hypothesis is that there are many distributed, interacting parts with little or nothing in the way of central control. Each part is governed by its own rules. Hence CAS are decentralized, rule-based systems. But it could also be the case there is some centralized control or operating system of which decentralized rule-based systems are instances. This suggestion derives from the object-oriented software

paradigm (Abadi \& Cardelli, 1996) where programming objects can be instances of hierarchical classes with distinct behaviors in addition to some inherited behaviors from a parent class. This suggests that it is possible that there is a single governing equation that influences or even perhaps controls the system.

WORKING HYPOTHESIS 2.1.3.2: There is a centralized equation that governs the adaptation of a system and this same equation governs the adaptation of each independent part

Holland suggests that there are two computational procedures that give systems their evolving structure: credit assignment and rule discovery. He suggests that credit assignment is necessary because one wants the system to evolve towards something. So there is a sense of what "good" performance is and this rewards parts of the system that are pushing the system towards good performance. Reward systems have been suggested to be superior to performance objective-based systems in automating creativity (Nguyen at al., 2016).

The problem though, is that the notion of "good" requires meta-modeling and hence suggests some kinds of central rule-structure.

Similarly the rule discovery process has to generate plausible rules, Holland suggests. But the notion of "plausible" also requires meta-modeling, and hence suggests a centralized rule-structure.

Rules can be thought of as comprising smaller building blocks, which are leveraged in breeding newer rules. Strong rules are selected as parents and new offspring rules are created by crossing the parents. He calls this rule discovery process Genetic Algorithms. Note that genetic algorithms have been increasingly used to optimize systems dynamics (Sholtes, 1994; Onwubolu \& Babu, 2004). Holland suggests this happens because strong rules have "valuable" building blocks inside them. But the notion of "valuable" also implies meta-modeling and hence suggests a centralized structure.

The property of anticipation bases current actions on "expected" outcomes. Expectation too implies meta-modeling and hence some kind of centralized structure.

This possible need for meta-layering reinforces a series of hypotheses in alignment with some of the previous working hypotheses, such as 2.1.1.1, 2.1.2.1, 2.1.2.2, 2.1.2.4.3, and 2.1.2.5.1, and design principles, such as 2.1.1.1, 2.1.1.2, 2.1.1.3, and 2.1.1.5:

WORKING HYPOTHESIS: A Complex Adaptive System is comprised of multiple, tightly bound meta-layers that become progressively active as surface layer conditions are changed

WORKING HYPOTHESIS 2.1.3.4: Stronger rules can be created with elements inspired from meta-layers that inform the surface layer of a system

WORKING HYPOTHESIS 2.1.3.5: When a surface layer building block opens to metalayer influence the resulting process can create building blocks of a different nature than when two surface layer building blocks combine

WORKING HYPOTHESIS 2.1.3.6: A rule will become stronger when a meta-layer building-block is incorporated in new rules

Note that there appears to be a tendency to want to express everything in terms of the surface layer. But there are meta-level dynamics required to successfully explain and even design any system. Adopting a meta-level approach with a different nature of dynamics at each layer would be a useful approach. Interestingly many complex computer-based systems require multiple layers that each exist for a different purpose and yet inform the layers above and below it for a superior overall functioning. For example all computer networks, including the Internet, are built by combining multiple layers with specific purposes. The TCP/IP Reference Model (Tanenbaum, 2002) comprises of a Physical Layer, Data Link Layer, Internet Layer, Transportation Layer, and Application Layer.

Holland suggests that an appropriate theoretical framework is required to effectively study CAS and that the role of experiment is to inform such theory. He suggests that there are pieces of math that have been developed to better frame CAS. The genetic algorithm approach to rule discovery that assumes a decentralized rule-based structure is one and is further elaborated in Holland's Adaptation in Artificial and Natural Systems (Holland, 1992). There are also formal frameworks that apply to the process of generating internal models through the process of induction. There are pieces from mathematical ecology and mathematical economics that may be applied as well. A piece-meal approach to the development of a mathematics of the whole system appears to be quite an arbitrary approach though. Hence:

DESIGN PRINCIPLE 2.1.3.2: A mathematics of CAS should be derived from the commonality of kernels across all CAS

Figure 2.1.3.1 summarizes the design principles and hypotheses generated from the rule-based view:

## Design Principle 2.1.3.1 There is a

 mathematical model that can be developed in which becoming is an integral part. This becoming is exhibited by evolution, by emergent behavior, and has a foundational context and historyDesign Principle 2.1.3.2 A mathematics should be derived from the commonality of kernels across all CAS

Working Hypothesis 2.1.3.1 The heart of CAS can be represented by a mathematical model of innovation

Working Hypothesis 2.1.3.2 There is a centralized equation that governs the adaptation of a system and this same equation governs the adaptation of each independent part

Working Hypothesis 2.1.3.3 A CAS is comprised of multiple, tightly-bound metalayers that become progressively active as surface layer conditions are changed

Working Hypothesis 2.1.3.4 Stronger rules can be created with elements inspired from metalayers that inform the surface layer of a system

Working Hypothesis 2.1.3.5 When a surface layer building block opens to meta-layer influence the resulting process can create building blocks of a different nature than when two surface layer building blocks combine

Working Hypothesis 2.1.3.6 A rule will become stronger when a meta-layer building block is incorporated in new rule

Figure 2.1.3.1 Rule-Based Systems Design Principles \& Working Hypotheses

### 2.1.4 Top-Down Approach to Complex Adaptive Systems

In the previous sections the bias in modeling CAS is from the bottom-up. By definition this will likely preclude the ability of modeling from a big picture perspective. This section, hence conducts a representative review of the big picture, top-down approach to modeling CAS.

As referred to in Section 1.2, "one and one", that is, the components and operations of a system are instruments to bring about some functional change in the complex adaptive system. In a top-down approach the choice of system-components and the operations between them is suggested to be set in motion by a meta-function that can be thought of as existing in a meta-layer that influences the functioning of the bottom-up "surface" layer.

In Connecting Inner Power with Global Change: The Fractal Ladder (Malik, 2009), an exploration into fractal patterns and biases at levels of increasing system complexity, the case is made that there is a similar pattern that animates progress regardless of the type of organization. An organization, hence, could be an individual, a team, a business unit, a corporation, a market, a country, planet earth, or a system of thought, for example. This pattern is summarized as the movement from the 'physical' to the 'vital' to the 'mental', where the physical can be thought of as reality as characterized by what the eye can see. Necessarily the eye sees what has already been created: hence the physical can also be thought of as an orientation reinforcing the status quo. The vital can be thought of as reality as characterized by the play of energy. Such play often results in the strongest energy winning out, and hence the vital can be thought of as an orientation reinforcing experimentation even bordering on aggression. The mental can be thought of as reality as experienced by thought. Hence this orientation can be thought of as encompassing curiosity, idealism, and the future. In How to Create a Mind (Kurzweil, 2013) Kurzweil maps parts of the brain to similar functions. Hence the mental function is primarily fulfilled by the neocortex, the vital function primarily by the amygdala and hippocampus, and the physical function primarily by the cerebellum.

An organization can center itself in any of the three phases, and will perceive the world, interpret circumstances, and act consistently with the psychology that emanates from that orientation. As an organization progresses though, to thereby increase its degrees of freedom, it is observed that it essentially changes its orientation from the physical to the vital to the mental (Malik, 2009). Further, it is possible to bring about an organization's transformation by causing the shift from the physical, to the vital, to the mental (Malik, 2003). In his book, Exponential Organizations, Singularity University's Ismail also suggests this trend from the 'physical' to an 'information-orientation' (Ismail, 2014) and how the informationorientation allows an organization to be "ten times better, faster, and cheaper than yours". Hence:

DESIGN PRINCIPLE 2.1.4.1: An underlying direction exists in all CAS

Such a design principle may imply a modification or enhancement to the notions of 'chance' and 'probability'. It is possible that things are not random, but rather are the outcome of a working out of a number of forces vying for expression. If so, chance and probability may be thought of as areas where meta-levels also act:

## WORKING HYPOTHESIS 2.1.4.1: Chance and probability are areas where meta-levels

 act. So there is no random chance. Only the working out of influential forces.This change in orientation from the physical, to the vital, to the mental is perceived as being the driver of innovation, and the hypothesis is that the orientations - the physical the vital, the mental - are embedded in the warp and woof of any CAS. Just as the bases of computer-technology is a binary representation and manipulation of zeros and ones, the hypothesis for a broad innovation-technology is a trinary representation and manipulation of the physical, the vital, and the mental. This is so because the physical, the vital, and the mental concisely summarize the orientation of a system and therefore the possibilities available to it.

Many examples of a change in orientation leading to an opening of possibility have been recorded time and time again. Referring to the author's previous work Business Transformation through the Creation of a Complex Adaptive System (Malik, 2003), An Introduction to Fractal Dynamics (Malik, 2004), Connecting Inner Power with Global Change (Malik, 2009), An Integral Perspective on Current Economic Challenges: Making Sense of Market Crises (Malik, 2013), The Flower Chronicles: A Radical Approach to System and Organizational Development (Malik, 2014) and The Fractal Organization (Malik, 2015), as an indication of a global trend now being captured by more and more people in their respective fields. For example, as pointed out earlier Ismail traces the success of new tech-based companies such as Google, Facebook, Uber, Ebay, amongst others and attributes it to a shift from the 'physical' to an 'information' orientation (Ismail, 2014). In University of Pennsylvania's Seligman's observations (Seligman, 2011) there are three tiers that can determine happiness. The first is what he calls 'the pleasant life', the second 'the good or engaged life', the third 'the meaningful life'. "The pleasant life is about using savoriness and mindfulness to enhance the pleasures one experiences" (a physical-orientation). "The good life is about using strengths to increase flow to a point where time stops" (a vital-orientation). "The meaningful life is about using strengths in the pursuit of a goal far vaster than oneself "(a mentalorientation). In thousands of observations Seligman found that increasing pleasures has almost no impact on increasing happiness. On the contrary increasing meaning had the most impact, and increasing flow had a strong impact. It is only when meaning and flow both exist that increasing pleasure may matter. This finding is another instance of what is proposed as a universal, underlying trend in any system, of the shift by its actors from a physical to a vital to a mental orientation in order to increase respective degrees of freedom. In Donella Meadow's observations of most effective places to intervene in a system (Meadows, 2008) she identifies twelve levers and places them in order of their effectiveness. Figure 2.1.4.1 summarizes

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these levers in increasing order of importance, and further identifies the nature of the lever in terms of the physical, vital, or mental orientation:

| Increasing <br> Order of <br> Importance | Lever Name | Lever Description | Orientation |
| :--- | :--- | :--- | :--- |
| 1 | Numbers | Constants and parameters such as <br> subsidies, taxes, standards | Physical |
| 2 | Buffers | The sizes of stabilizing stocks relative <br> to their flows | Physical |
| 3 | Stock-and- <br> Flow <br> Structures | Physical systems and their nodes of <br> intersection | Physical |
| 4 | Delays | The lengths of time relative to the <br> rates of system change | Vital |
| 5 | Balancing <br> Feedback <br> Loops | The strength of the feedbacks relative <br> to the impacts they are trying to <br> correct | Vital |
| 6 | Reinforcing <br> Feedback <br> Loops | The strength of the gain of driving <br> loops | Vital |
| 7 | Information <br> Flows | The structure of who does and does <br> not have access to information | Vital |
| 8 | Rules | Incentives, punishments, constraints <br> Self- <br> Organization | The power to add, change, or evolve <br> system structure |
| 9 | Goals | The purpose of the system | Mental |
| 10 | Paradigms | The mind-set out of which the system <br> - its goals, structure, rules, delays, <br> paradigms - arises | Mental |
| 11 | Transcending <br> Paradigms | Strategically, profoundly, madly, <br> letting go and dancing with the system | Mental |
| 12 |  |  |  |

Figure 2.1.4.1 System Intervention Levers in Order of Importance and Relationship to Orientation [adapted from 'Thinking in Systems' (Meadows, 2008)]

It can be observed that as the effectiveness of a lever increases so does the underlying orientation shift from the essentially physical, to the vital, to the mental. In an article in Harvard Business Review describing how the Chinese internet company Alibaba (Reeves et al, 2015), uses algorithmic thinking to constantly reinvent itself, the crux of the algorithm is also a movement through three sequential phases: discovering what works (physical), adjusting how and how
much experimentation takes place (vital), and influencing customer's preferences (mental).

Hence:

DESIGN PRINCIPLE 2.1.4.2: A trinary representation and manipulation of the physical, the vital, and the mental give rise to a broad CAS-relevant innovation-technology

In "At Home in the Universe" Stuart Kaufmann (Kaufmann, 1995) suggests that a combination of natural selection and self-organization leads to matter organizing itself into complex structures in spite of the forces of entropy. He discusses the second law of thermodynamics that has as a consequence the disappearance of order from equilibrium states. While entropy leads to a sense that an incoherent collapse of order is the natural state of things, he cites abundant evidence of order in our world, from microscopic cells, to plenitude of species, to the postmodern technological era with its exploding rate of innovation.

Coupled with the proposed system-states of the physical, the vital, and the mental, and the observation that it is only in the vital-type conditions that a trend of order can reverse itself, this may lead to another hypothesis:

WORKING HYPOTHESIS 2.1.4.2: Second Law of Thermodynamics gives insight into vital-type conditions

Further, he raises the question as to whether there are any general laws that govern this range of complex activity and further if this would allow us to achieve predictive powers. He cites two commonly accepted characteristics of CAS indeterminism, and sensitivity of CAS to changes in initial conditions - to discard the possibility of achieving predictive powers. However, it is possible that some form or predictability could exist if some form of determinism existed. Hence:

WORKING HYPOTHESIS 2.1.4.3: If some form of determinism were to be established then some ability to predict CAS outcomes would become possible

Kaufmann's hope is to characterize classes of properties of systems that are generic and do not depend on the details. He suggests that it is possible to form theories of system behavior that are insensitive to a full set of descriptive details.

DESIGN PRINCIPLE 2.1.4.3: It is possible to form theories of system behavior that are insensitive to a full set of descriptive details

In stepping back and asking why this similar pattern of progress exists ubiquitously, what this may point to is a reality of an implicit order that occurs in Time and Space:

- Observation \#1: This progress happens regardless of area of the world, and regardless of industry. This points to a characteristic of an implicit-presence, whereby wherever there is the possibility of progress it will happen.
- Observation \#2: This progress happens in spite of tremendous opposition to it, again regardless of field and area. This points to a characteristic of implicit-power.
- Observation \#3: The right instrumentation and the right circumstances seem to be leveraged in order that the progress possible does happen. This points to a characteristic of implicit-knowledge that knows what to leverage and when. Implicit-knowledge implies prerequisite-state that contains a synthesis of contraries, and a consequent-state such that the right component is invoked at the right time. The notion of synthesis of contraries is pressing to the surface in many fields of life. Grand Unified Theories are being worked on in mathematics and physics, for example. In mathematics Frenkel, in his book Love and Math (Frenkel, 2013), refers to the Langlands program as "a Grand Unified Theory of Mathematics because it uncovers and brings into focus mysterious patterns shared by different areas of math and thus points to deep, unexpected connections between them." In his book, Dreams of a Final Theory (Weinberg, 1994), written in the wake of US Congress deliberations and rejection of the Superconducting Super Collider, Nobel Laureate Weinberg suggests progress towards "a theory of everything (ToE)...a hypothetical single, all-encompassing, coherent theoretical framework of physics that fully explains and links together all physical aspects of the universe".
- Observation \#4: The fact that this progress happens so that the degrees-offreedom of the associated actors is continually increased points to an implicit-nurturing.

Hence, one can say that implicit in Space, there are four overarching characteristics - that of implicit-presence, implicit-power, implicit-knowledge, and implicitnurturing (Malik, 2009). If then there is an implicit order in Time and an implicit order in Space, then the notion of organizational causality needs to be re-examined, since the very matrix in which any organizational play can arise is bounded differently than it may have so far been imagined or conceptualized.

It is also interesting to note that mainstream CAS thinkers are beginning to leverage space and time in more comprehensive ways in their interpretation of CAS. For example in her article, Life's Information Hierarchy (Flack, 2014), Flack suggests that the explanation for complex, multi-scale structure of biological and social systems lies in their manipulation of space and time to reduce uncertainty about the future. She states: "To many, hierarchical organizations suggests the nesting of components or individuals into groups, with these groups aggregating into yet larger groups. But this view - at least superficially - privileges space and matter over time and information. Many types of neural coding, for example, require averaging or summing over neural firing rates. The neurons' spatial location - that they are in proximity - is, of course, important, but at least as important to the encoding is their behavior in time."

Figure 2.1.4.2 summarizes the design principles and hypotheses that have arisen in this section:


Figure 2.1.4.2 Top Down Design Principles \& Working Hypotheses

### 2.1.5 Mathematics in CAS

The question is how has the innate innovative ability in CAS been expressed mathematically? In section 2.1.1 on system dynamics the notion of modeling a system, with its implication of running simulations is perhaps one way in which the mathematics of CAS can be expressed and explored. In Section 2.1.2 the fundamental properties of CAS were explored when CAS are viewed from the bottom up. These properties consisting of distributed control, inter-connectivity, far
from equilibrium, amongst others suggest the immense variability inherent in CAS. This variability may be framed by differential equations and in the subsequent subsection, 2.1.5.1, Nobel Laureate Ilya Prigogine's use of differential equations in exploring dissipative structures is explored. The possible mathematical space generated in CAS calls for optimization and in Section 2.1.3 on rule-based systems the notion of representing systems as rules, and further of optimizing systems through the use of genetic algorithms further tightens the mathematical space that can be generated in CAS. Section 2.1.4 on the top-down view of CAS provides suggestions on an implicit direction of CAS, a possible meaning of probability and chance, and the further use of determinism that can all figure in a comprehensive mathematical model of innovation in CAS. While a detailed modeling will be explored in Chapter 4 on Model Development, an existing approach to modeling innovation in CAS, leveraging Turing's activator-inhibitor equations (Turing, 1952) is explored in Section 2.1.5.2. These equations that deal with pattern formation can provide insight into the subsiding of old and the growth of new patterns. Sections 2.1.5.3 and 2.1.5.4 explore the notion of measuring the complexity of CAS.

### 2.1.5.1 Prigogine's Dissipative Structures

Dissipative Structures exist as stabilities far from equilibrium. They are characterized by formation of complex structures that tend to exhibit long-range correlations between interacting particles. Mathematically, dissipative structures can be thought of as consisting of states $x(t)$, inputs $u(t)$, and outputs $y(t)$ such that the energy stored in the emerging structure $V(x)$, can never exceed the energy supplied to the system, uy (Prigogine, 1967).

This can be represented by the inequality (Prigogine-Dissipative Inequality):

$$
\frac{d V(x(t))}{d t} \leq u(t) \cdot y(t)
$$

It has been suggested that $\mathrm{V}(\mathrm{x})$ may play the role of Lyapunov functions that function to stabilize systems (Willems, 1998). [Note that in this dissertation only the derived equations will be numbered. The handful of pre-existing equations, such as the preceding one, will remain unnumbered, and will be referred to by the Deriver-Subject nomenclature. Such equations are also summarized in Appendix 7.]

These dissipative structures can be thought of as the frame within which the many unique organizations that surface or manifest exist. Hence:

DESIGN PRINCIPLE 2.1.5.1.1: Prigogine's Dissipative Structures are the observable frames within which organizational transitions express themselves

Further, it may be suggested that there are different kinds of dissipative structures depending on what kind of organizational transition is seeking to express itself. Hence:

WORKING HYPOTHESIS 2.1.5.1.1: The general framing of Prigogine's Dissipative Structures can be further detailed depending on the nature of the transition taking place

### 2.1.5.2 Turing's Activator-Inhibitor Equations

In his paper on Complex Adaptive Systems: Exploring the Known, the Unknown, and the Unknowable (Levin, 2002), Simon Levin relates how pattern formation deals with the dynamics of collectives and with understanding how patterns manifest at the level of those collectives are mediated at the level of the individual entities. A common framework for investigating pattern formation involves the use of systems of diffusion and reaction. Levin relates how Alan Turing, the mathematician and computer scientist, applied such a framework to understand how an undifferentiated egg develops into a highly differentiated organism.

Turing suggested (Turing, 1952) the use of an activator ( u ) and an inhibitor (v). The activator stimulates production of both entities, whereas the inhibitor inhibits it. The relevant system of equations, where the diffusion rates for $u$ and $v$ are $D_{u}$ and $D_{v}$ is (Turing Activator-Inhibitor Equations):

$$
\begin{aligned}
& \frac{\partial u}{\partial t}=f(u, v)+D_{u} \nabla^{2} u \\
& \frac{\partial v}{\partial t}=g(u, v)+D_{v} \nabla^{2} v
\end{aligned}
$$

Levin points out that these equations are the preferred model for many forms of pattern formation. If innovation is assumed as generally being the breaking of certain existing patterns and the formation of new ones, then it is possible that this set of equations may also be leveraged in any mathematics of innovation. This may occur by thinking of $u$ as being the 'new innovative pattern' and $v$ as being the 'existing pattern(s)'. Hence:

DESIGN PRINCIPLE 2.1.5.2.1: Turing activator-inhibitor equations provide some insight into mechanics of how old patterns are replaced by newer, more innovative ones

But further, if the presence of meta-layers is assumed, it may also be possible that there is a meta-logic that is the initiating and determining force behind the emergence of new patterns, of which the Turing activator-inhibitor equations are only an expression of what is happening on the surface. The hypothesis thus is:

WORKING HYPOTHESIS 2.1.5.2.1: Turing activator-inhibitor equations need to be put in context by meta-layer logic

### 2.1.5.3 Complicated vs. Complex

There has been much written about the difference between complicated and complex systems (Aral, 2016; Efatmaneshnik et al., 2012). Leveraging off a representative piece, Learning to Live with Complexity that appeared in Harvard Business Review (Sargut \& McGrath, 2011), here are some relevant points:

- Simple systems contain few interactions and are predictable
- Complicated systems have many parts but operate in patterned ways
- Complex systems are imbued with features that may operate in patterned ways but whose interactions are continually changing
- Complex systems interact in unexpected ways
- It is harder to make sense of things because the degree of complexity may exceed our cognitive limits
- Analytical tools have not kept up with complexity thinking
- Three properties determine the complexity of an environment. The first, multiplicity, refers to the number of potentially interacting elements. The second, interdependence, relates to how connected those elements are. The third, diversity, has to do with the degree of their heterogeneity. The greater the multiplicity, interdependence, and diversity, the greater the complexity.
- In Complicated systems one can predict the outcome given starting conditions. In complex systems the same initial conditions can create different outcomes depending on interaction of elements in the system and it is difficult to model complex systems because elements interact continuously and unpredictably

The differentiations point to some useful aspects to be considered in a mathematics for CAS. Hence:

DESIGN PRINCIPLE 2.1.5.3.1: A CAS mathematics will become more robust by considering multiplicity, interdependence, and diversity of a system

Further, the notion that system complexity may be a function of multiplicity, interdependence, and diversity suggests too that the simple, complicated, and complex systems may be related by multiplicity, interdependence, and diversity. That is, they are all different expressions of CAS, perhaps at different levels of maturity:

WORKING HYPOTHESIS 2.1.5.3.1: There is a relationship between simple, complicated, and complex systems that is made sensible through the consideration of multiplicity, interdependence, and diversity

### 2.1.5.4 Measuring Complexity

Having considered aspects of a CAS math for the creation of dynamic structures within CAS by leveraging insights from Prigogine's Dissipative Structure Inequality and Turing's Activator-Inhibitor Equation the next question is how is complexity in CAS measured?

In their paper 'Complex Adaptive Systems and Complexity Theory: Inter-related Knowledge Domains' Dodder and Dare (Dodder \& Dare, 2000), summarize that the measurement of complexity has tended to couple two complementary aspects: knowledge and ignorance of the system. Along the dimension of ignorance, entropy or ignorance provides a measure of complexity by determining the disorder of the system, which in turn establishes a measure of ignorance about a system. Along the dimension of knowledge, a measure of complexity is the information processed or exchanged by the system. Another measure of complexity is the Algorithmic Information Content, which relates complexity to the minimum amount of information needed to describe the system, as measured by the shortest computer program that can generate that system.

In his note on measures of complexity, Professor Seth Lloyd of MIT's d'Arbeloff Laboratory for Information Systems and Technology (Lloyd, 2001) suggests that complexity of a system is related to how hard is it to describe, how hard is it to create, and what is its degree of organization. The difficulty of description is typically measured in bits. Measures may include information, entropy, algorithmic information content, or minimum description length, amongst others. Difficulty of creation is typically measured in time, energy, dollars, etc. Measures may include computational complexity; information-based complexity, logical Depth, cost, amongst others. Degree of organization is itself divided into two quantities: a) difficulty of describing organizational structure; b) amount of information shared between the parts of a system as the result of this organizational structure.

These measures though appear to view CAS as static. Thinking of CAS as dynamic generates additional hypotheses:

## WORKING HYPOTHESIS 2.1.5.4.1: Complexity in a system will increase

WORKING HYPOTHESIS 2.1.5.4.2: The higher the degree of meta-level dynamics active, the more complex the system is going to be

WORKING HYPOTHESIS 2.1.5.4.3: Complexity of system is proportional to emergence
WOKRING HYPOTHESIS 2.1.5.4.4: Complexity of system proportional to level of innovation in system

WORKING HYPOTHESIS 2.1.5.4.5: The more complete the model of a CAS is, the more accurate will the measure of complexity be

The following figure, 2.1.5.1 summarizes the design principles and hypotheses generated in Section 2.1.5:

```
    Design Principle 2.1.5.1.1 Prigogine's
    Dissipative Structures are the observable
frames within which organizational transitions
            express themselves
```

Design Principle 2.1.5.2.1 Turing activatorinhibitor equations provide some insight into mechanics of how old patterns are replaced by newer, more innovative ones

```
Design Principle 2.1.5.3.1 A CAS mathematics
    will become more robust by considering
multiplicity, interdependence, and diversity of a
            system
```

Working Hypothesis 2.1.5.1.1 The general framing of Prigogine's Dissipative Structures can be further detailed depending on the nature of the transition taking place

Working Hypothesis 2.1.5.2.1 Turing activatorinhibitor equations need to be put in context by meta-layer logic


Figure 2.1.5.1 Mathematics in CAS Design Principles and Hypotheses

### 2.2 Literature Conclusion \& Need for a New Theory

Fueled by the bias of emergence and distributed control there is a whole body of CAS thought that builds upon that initial foundation. The next model or theory therefore builds only upon the last and the notion of centrality or meta-level functioning that would assume a top-down bias in CAS, seems to be missing.

System modeling and dynamics (reviewed in Section 2.1.1) and any resulting optimization due to rule-based systems and genetic algorithms (reviewed in Section 2.1.3) will only be as good as accepted properties of CAS. But if the properties are bottom-up (reviewed in Section 2.1.2) by definition they will likely not contain any overarching or central kernel. It is necessary to understand CAS from the top-down (reviewed in Section 2.1.4) in order to surface any initiatives focused on the development of a possible centralized kernel of innovation. Mathematics of innovation in CAS, currently built on an emergent and piece-meal approach (reviewed in Section 2.1.5) may be fruitfully accelerated if it proceeded from the top-down. Existence of CAS meta-level logic would perhaps even offer a different interpretation to emergent phenomena while not negating any of it. In the absence of such centrality it is difficult to construct a cohesive model of innovation for the system as a whole. It is also difficult to begin to frame any mathematics and by default an existing piece-meal approach to mathematics is then easily adopted.

To arrive at an understanding of wholeness of systems it appears that a centralized, meta-level logic, focused on emergent and other innovativeness, and framed using mathematical equations would be a fruitful approach.

A review of existing literature surfaces a number of design principles and working hypotheses in alignment with the overall 'key attributes of the desired theory' proposed in Section 1.3. Figure 2.2.1 (modification of Figure 1.3.1) elaborates the mapping between each of the design principles and working hypotheses and the underlying key attributes. As a reminder a design principle is an insight that has already been surfaced by existing research. A working hypothesis is proposed as a further insight not found in the representative literature review conducted to date. Both design principles and working hypotheses will be leveraged in constructing the conceptual analytical framework in Chapter 4. Note that only working hypotheses though, will be further validated to the relevant extent in subsequent chapters in this dissertation.

| Attribute \# | Key Attributes of Desired <br> Theory | Design <br> Principle | Working <br> Hypotheses |
| :---: | :--- | :--- | :--- |
| 1 | The theory is derived from a big- | 2.1 .1 .4, | 2.1 .1 .2 |
|  | picture "outside-in" approach as | 2.1 .3 .2, |  |
| opposed to a "bottom-up" approach | 2.1 .4 .1 |  |  |

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| 2 | This approach being inherently "outside-in" focused will thereby also integrate aspects of the existing "bottom-up" approach to CAS into it |  | 2.1.2.1, 2.1.2.1.1 |
| :---: | :---: | :---: | :---: |
| 3 | CAS are characterized by qualified determinism at each level of organizational complexity |  | $\begin{aligned} & \text { 2.1.2.2.1, } \\ & \text { 2.1.2.3.1, 2.1.4.3 } \end{aligned}$ |
| 4 | This qualified determinism is orchestrated by a cohesive mathematical framework |  | $\begin{aligned} & \hline \text { 2.1.2.4.1, } \\ & \text { 2.1.2.4.2, } \\ & \text { 2.1.2.4.3, 2.1.2.4.4 } \end{aligned}$ |
| 5 | This mathematical framework explains the spectrum of possibility from stagnation to sustainability / progress | 2.1.4.3 | 2.1.2.1.2, 2.1.4.2 |
| 6 | The mathematical framework and the derived equations will hence also provide a basis for innovation | 2.1.4.2 | 2.1.3.1 |
| 7 | The mathematical framework advances the field of innovation by creating a series of mathematical equations to better understand innovation | $\begin{aligned} & \text { 2.1.2.6.1, } \\ & \text { 2.1.5.1.1, } \\ & \text { 2.1.5.2.1, } \\ & 2.1 .5 .3 .1 \end{aligned}$ | $\begin{aligned} & \text { 2.1.5.1.1, } \\ & \text { 2.1.5.2.1, 2.1.5.3.1 } \end{aligned}$ |
| 8 | The mathematical framework for innovation applies to organizations at different levels of complexity from the 'simple' to the more 'complex' |  | $\begin{aligned} & \hline 2.1 .3 .2,2.1 .5 .4 .1 \\ & 2.1 .5 .4 .2, \\ & 2.1 .5 .4 .3, \\ & 2.1 .5 .4 .4,2.1 .5 .4 .5 \end{aligned}$ |
| 9 | The mathematical framework provides insight into further potential development at each level of organizational complexity | 2.1.3.1 |  |
| 10 | This framework separates the nature of functionality by metalayers and suggests key dynamics operative at each layer | $\begin{aligned} & \text { 2.1.1.1, } \\ & \text { 2.1.1.2, } \\ & \text { 2.1.1.3, } \\ & \text { 2.1.1.5 } \end{aligned}$ | 2.1.2.2 |
| 11 | Under certain conditions the respective meta-level functionality and operations becomes active to bring about complexity and adaptiveness in the visible, surface layer |  | $\begin{aligned} & \text { 2.1.1.1, 2.1.2.5.1, } \\ & \text { 2.1.3.3, 2.1.3.4, } \\ & \text { 2.1.3.5, 2.1.3.6, } \\ & \text { 2.1.4.1 } \end{aligned}$ |

Figure 2.2.1 Link between 'Key Attributes of Desired Theory' \& Design Principles and Working Hypotheses

Figure 2.2.2 summarizes the design principles and hypotheses tree:


Figure 2.2.2 Summary Design Principles and Hypotheses Tree
Keeping the underlying research question posed in Section 1.2 in mind - can an engine of innovation in CAS be framed such that it will usefully apply across all CAS regardless of level of complexity or scale - Chapter 3 will suggest a research strategy, research methodology, and research instruments to achieve this.

CHAPTER 3: Research Design and Methodology
The literature review in Chapter 2 suggested a bias in the conceptualization of CAS such that they are perceived as being emergent, distributed, and unpredictable. These generalizations have themselves been built on the work of several Nobel Laureates who have all interpreted fundamental quantum-level phenomena in the same way. The resultant perception of the lack of centralization in CAS does not allow for a meaningful kernel of innovation to be constructed. The mathematics constructed to-date has perhaps therefore also tended to be piece-meal and as a result there does not appear to be a single centralized mathematics that can frame CAS as a cohesive whole animated by the essential property of innovation.

In a way of looking at this it is apparent that the lack of a single mathematical framework for innovation for CAS may be the result of a bottom-up look at CAS. If however, CAS were to be perceived from the outside-in then its holistic nature and the nature of the innovation inherent in it might surface more easily.

The research problem, building on the gap identified in theory in Chapter 2 and as summarized in Section 2.2, is what would a mathematical framework for innovation in CAS look like?

The following research strategy, research methodology and research instruments illustrate the investigation of the fundamental research problem.

### 3.1 Research Strategy

In his book 'Supersymmetry and Beyond: From the Higgs Boson to the New Physics' Kane (Kane, 2013) states that only now scientists are beginning to ask why the world works the way it does, as opposed to the normal research focus of how the world works the way it does. What was considered to be in the realm of philosophy has now become normal in the realm of particle physics research. Einstein asked that question decades ago, but it is only now that the question can be answered with the birth of theoretical, as opposed to experimentally-based frameworks, such as String Theory. Development in String Theory has proceeded through studying the theory itself, as opposed to the interaction of experiment with theory. In perhaps a similar vein the mathematical model developed here is also primarily grounded in theoretical as opposed to experimental ponderings. Subsequently the theoretically constructed mathematical model is then applied and 'tested' in several ways as elaborated by the following components of the research strategy. The research strategy follows from the nature of the research question, to frame a holistic mathematical engine for innovation in CAS, and can be thought of as the essential approach to be employed to further explore the research problem under consideration (Creswell, 2013). The framing of such a mathematical engine that addresses the key attributes, design principles, and working hypotheses therefore employs a qualitative approach and is elaborated in the research strategy outlined below:

1. Construction of a conceptual analytical framework: Construct a conceptual analytical framework that incorporates the design principles and hypotheses generated in the literature review. This analytical framework will be expressed as series of equations that will define a mathematical kernel of innovation in CAS.
2. Application of the conceptual framework: The mathematical framework will then be applied to sample CAS domains to arrive at relevant conclusions about the domains to further the understanding of these domains, using a process of deductive logic. The difference between deductive and inductive logic is illustrated by Vickers in the Stanford Encyclopedia of Philosophy (Vickers, 2016): "Deductive logic, at least as concerns first-order logic, is demonstrably complete. The premises of an argument constructed according to the rules of this logic imply the argument's conclusion. Not so for induction: There is no comprehensive theory of sound induction, no set of agreed upon rules that license good or sound inductive inference, nor is there a serious prospect of such a theory. Further, induction differs from deductive proof or demonstration (in first-order logic, at least) not only in induction's failure to preserve truth (true premises may lead inductively to false conclusions) but also in failing of monotonicity: adding true premises to a sound induction may make it unsound."
3. Simulation of the key equation in the analytical framework: The key mathematical equation that frames a possible process for innovation in CAS will then be simulated to descriptively research how innovation in CAS may be altered through manipulating key parameters contained in the equation.
4. Case study: The derived equations will then be applied to a case study to gain further insight into a process of organizational innovation.
5. Reinterpretation of piece-meal CAS math: The piece-meal CAS math will be reviewed in light of the mathematical model developed here.
6. Hypotheses review: The initial set of hypotheses will be reviewed in light of the results of the previous components of the overall research strategy.
7. Qualitative proof of structure of mathematical model: A qualitative proof or need for the essential mathematical structure of the model, comprising of several layers each focused on some essential dynamic, will be explored.

### 3.2 Research Methodology

The research methodology elaborating how each component (Buys, 2014) of research strategy will be fulfilled is the following:


1. Construction of a conceptual analytical framework: The methodology is to use the process of induction in arriving at the mathematical framework. Note that while a deductive argument "is intended by the arguer to be (deductively) valid, that is, to provide a guarantee of the truth of the conclusion provided that the argument's premises (assumptions) are true", "an inductive argument is an argument that is intended by the arguer merely to establish or increase the probability of its conclusion" (IEP Staff, 2016). Existing mathematical functions will be leveraged where appropriate, and new mathematical functions will be defined and used where none currently exist. Starting from the outside-in, certain pervasive and relevant patterns will be mathematized. Then using induction, several equations will be derived to reflect observations and hypotheses about the outside-in view of CAS. Equations will in general build on the previous one and the simplest initial observations will then be converted into summary equations using previous equations developed.
2. Application of the conceptual framework: The resultant framework will then be applied to several CAS domains. Since the process of induction was used in constructing the conceptual analytical framework, the process of deduction is used in the application of the framework to each of the CAS domains to thereby increase the validity of the framework. The first CAS domain is at the level of the cell, where a substantial body of knowledge about cell-function has already been developed. The mathematical framework will re-interpret certain observations about molecular plans being used by all cells to suggest an equation of innovation at the cellular level. Second, it will be applied to the domain of quantum properties to reinterpret some fundamental conclusions that several Nobel Laureates have arrived at, and that stand at the bases of many perceptions about CAS itself. Third, it will be applied at the level of quantum-particle classification. Fourth, it will be applied to the domain of atoms by studying a four-fold classification of the Periodic Table. Fifth, it will be applied to the domain of CAS itself to deduce conditions for sustainability of CAS. Sixth, it will be applied to reinterpret fundamental properties of CAS itself. Note that the application to the corporate level of complexity will take place by means of a case study as elaborated in point 4 below.
3. Simulation of the key equation for innovation in the analytical framework: Using Vensim Simulator, the key equation for CAS innovation will be constructed. A graphical user-interface will allow key parameters that cause a jump in the level of system-innovation to be modeled. Interactive sessions will allow the level of system-innovation to be graphed and studied as parametric inputs change.
4. Case study: An approximately three-year case study conducted by the researcher of this dissertation at Stanford University Medical Center will be discussed to illustrate the practical action of several key equations for innovation in CAS. Hence a descriptive illustration of how the equations may work practically will be highlighted.
5. Piece-meal CAS math review: The question is to what degree are the piece-meal areas that have been considered able to integrate into the general mathematical model as being developed here. If this integration is noticeable the degree of confidence may be higher that the developed model may be considered deep and wide enough to be a basis of thinking about a more general mathematics for CAS.
6. Hypotheses review: The hypotheses initially generated will be reviewed in light of the research conducted in the preceding steps. The associated body of knowledge for System Dynamics and CAS as laid out in the literature review will be revisited.
7. Qualitative proof of structure of mathematical model: Implication of Einstein's special and general theory of relativity will be leveraged to gain insight into essential structure that must exist in any coordinate system.

### 3.3 Research Instruments

Given that the research problem is the construction of a mathematical model of innovation in CAS, the instruments used are qualitative in nature and are essentially reduced to 'codes' (Cooper \& Schindler, 2014) such as 'cohesiveness', 'simplification', 'relationship', 'ability', amongst others, as summarized in Figure 3.3.1. The choice of research instrument by which each component of the research strategy is probed into is also elaborated by Neuman (Neuman, 2013) who draws a distinction between data used in quantitative versus more qualitative research. Being that the research question is qualitative in nature data would be more in the form of trends, generalizations, taxonomies, as opposed to precisely measurable variables:

1. Construction of a conceptual analytical framework: The cohesiveness of the derived mathematical equations becomes the instrument of research. It will need to be observed if the generation of a new equation is consistent with the equations already generated and the understanding of innovation in CAS as a whole.
2. Application of the conceptual framework: The mathematical equations will have to be applied to CAS domains to deduce findings. Further insight and simplifications to the properties in the target domains are the instrument of research.
3. Simulation of the key equation in the analytical framework: As different input parameters are selected in the simulation, the overall level of system innovation will change. Detailed graphs depicting relationships will be highlighted to illustrate some of the relationships in the derived equations. The relationship of adjustable parameters to system innovation is the instrument of research.
4. Case study: The ability of equations to frame organizational innovation and change is the instrument of research.
5. Piece-meal CAS math review: Cohesiveness of integration of piece-meal math equations into the main model is the instrument of research. Data about how well they integrate into the derived math model is the focus of analysis.
6. Hypotheses review: Addressing generated hypotheses in the derived mathematical model is the research instrument.
7. Proof of structure of mathematical model: Ease with which existing established theory suggests structure of mathematical model derived here.

Figure 3.3.1 summarizes the research instrument and the corresponding 'code':

| $\#$ | Research Instrument | Code |
| :---: | :--- | :--- |
| RI1 | Cohesiveness of derived mathematical equations | "Cohesiveness" |
| RI2 | Further insight and simplification of properties in <br> target domains | "Insight and <br> Simplification" |
| RI3 | Relationship of adjustable parameters to system <br> innovation | "Parameter <br> Relationship" |
| RI4 | Ability of derived mathematical equations to frame <br> organizational change and innovation | "Frame Change" |
| RI5 | Cohesiveness of integration of piece-meal <br> mathematics into main model | "Integration" |
| RI6 | Addressing of generated hypotheses in the <br> literature review | "Addressing <br> Hypotheses" |
| RI7 | Ease with which established theory suggests <br> structure of mathematical model | "Established Theory" |

Figure 3.3.1 Research Instruments(RI)

### 3.4 Summary

In their book, Business Research Methods, Cooper \& Schindler, contrast qualitative and quantitative research methods. Figure 3.4.1 illustrates some of the key differences highlighted.

|  | Qualitative | Quantitative |
| :---: | :---: | :---: |
| Focus of Research | - Understand and interpret | - Describe, explain, and predict |
| Researcher Involvement | - High-researcher is participant or catalyst | - Limited; controlled to prevent bias |
| Research Purpose | - In-depth understanding; theory building | - Describe or predict; build and test theory |
| Research Design | - May evolve or adjust during the course of the project <br> - Often uses multiple methods simultaneously or sequentially <br> - Consistency is not expected <br> - Involves longitudinal approach | - Determined before commencing the project <br> - Uses single method or mixed methods <br> - Consistency is critical <br> - Involves either a cross-sectional or a longitudinal approach |
| Data Type and Preparation | - Verbal or pictorial descriptions <br> - Reduced to verbal codes (sometimes with computer assistance) | - Verbal descriptions <br> - Reduced to numerical codes for computerized analysis |
| Data Analysis | - Human analysis following computer or human coding; primarily nonquantitative <br> - Forces researcher to see the contextual framework of the phenomenon being measured-distinction between facts and judgments less clear <br> - Always ongoing during the project | - Computerized analysis-statistical and mathematical methods dominate <br> - Analysis may be ongoing during the project <br> - Maintains clear distinction between facts and judgments |
| Insights and Meaning | - Deeper level of understanding is the norm; determined by type and quantity of free-response questions <br> - Researcher participation in data collection allows insights to form and be tested during the process | - Limited by the opportunity to probe respondents and the quality of the original data collection instrument <br> - Insights follow data collection and data entry, with limited ability to reinterview participants |

Figure 3.4.1 Contrasting Qualitative and Quantitative Research (Cooper \& Schindler, 2014)

Since the problem being researched in this dissertation is qualitative in nature, the research strategy, methodology, and instruments used to validate the problem identified therefore leverages several elements as summarized in Figure 3.4.1 identified by Cooper \& Schindler as elaborated below.

Hence, since the 'Focus of Research' is to interpret existing observations from the outside-in to construct a mathematical model for innovation in CAS, 'Researcher Involvement' has been high. The 'Research Purpose' is the building of a theory and mathematical model for innovativeness in CAS. The 'Research Design' has used multiple methods sequentially, starting with the construction of a conceptual analytical framework detailed by mathematical equations derived inductively, to the application of these equations deductively in several CAS domains to thereby gain further insight into the respective dynamics of these CAS domains. Further a simulation to research descriptively the parameters of the key equation of innovation is followed by a case study to illustrate the framing of organizational innovation leveraging the model of innovation. Subsequently the cohesiveness of the existing piece-meal mathematics is tested with respect to the mathematically derived model. A hypothesis review is conducted, and finally a qualitative proof for the essential structure of the mathematical model is suggested leveraging Einstein's special and general theories of relativity. The 'Data Type' is illustrated by the
summary 'codes' in Figure 3.3.1, and the 'Data Analysis' essentially leveraging the codes, and the 'Insights and Meaning' offered by the deeper understanding of the CAS domains, validates the derived mathematical model.

The lynch-pin or backbone of this research effort is the development of the mathematical model for innovation in CAS, which is the focus of Chapter 4.

CHAPTER 4: Model Development

This chapter proceeds to develop a mathematical model to frame innovation in CAS. This mathematical model is the lynch-pin of this dissertation and all research instruments identified in Chapter 3 are dependent on the model derived here. Figure 4.1 adapts the similar Figure 3.3.1 to highlight the relationship of all the research instruments to the derived mathematical model:

| $\#$ | Research Instrument | Code | Relationship |
| :---: | :--- | :--- | :--- |
| RI1 | Cohesiveness of derived <br> mathematical equations | "Cohesiveness" | Model derived <br> (this chapter) |
| RI2 | Further insight and simplification of <br> properties in target domains | "Insight and <br> Simplification" | Dependent <br> (Chapter 5) |
| RI3 | Relationship of adjustable parameters <br> to system innovation | "Parameter <br> Relationship" | Dependent <br> (Chapter 6) |
| RI4 | Ability of derived mathematical <br> equations to frame organizational <br> change and innovation | "Frame <br> Change" | Dependent <br> (Chapter 7) |
| RI5 | Cohesiveness of integration of piece- <br> meal mathematics into main model | "Integration" | Dependent <br> (Chapter 8) |
| RI6 | Addressing of generated hypotheses <br> in the literature review | "Addressing <br> Hypotheses" | Dependent <br> (Chapter 8) |
| RI7 | Ease with which established theory <br> suggests structure of mathematical <br> model | "Established | Dependent <br> Theory" |
| (Chapter 8) |  |  |  |

Figure 4.1 Relationship of Research Instruments to Mathematical Model

### 4.1 Inherent System Bias

At a very basic level, keeping in mind the three ubiquitous states discussed in Section 2.1.4.1 - physical, vital, mental - six equations that provide approximations to situations such as Stagnation, Stability, Entropy, Energy, Sustainability, and Fragmentation are first suggested. These situations are particularly relevant to the management of technology and even sociotechnical systems that often comprises multiple directions and trends, such as stagnation, or stability, or fragmentation of technologies, for example, in any given segment. These equations are derived from the possibility and the obstacle provided by the three ubiquitous states.

Hence, if the physical orientation exists in its capacity as an obstacle, then it may be suggested that while the physical is leading (represented by the subscript 'L' for leading), it is doing so as an obstacle (represented by the superscript, '-') and hence the status quo will always remain as it is, and this will result in stagnation. This can be represented by the function in Equation 4.1.1:

Stagnation $=f n\left(P_{L}^{-}\right)$

## Eq 4.1.1: Stagnation

On the flip-side, if the physical is leading in a progressive capacity this results in stability of established structure, and is a great foundation for other things to be built. Stability can be represented as in Equation 4.1.2:

Stability $=f n\left(P_{L}^{+}\right)$
Eq 4.1.2: Stability
If the vital is leading in its rapacious capacity, this will result in entropy as reflected in Equation 4.1.3:

Entropy $=f n\left(V_{L}^{-}\right)$
Eq 4.1.3: Entropy
In other words, with the random play of energy as captured by the Second law of Thermodynamics, the sum of the entropies of the participating thermodynamic systems increases (Van Wylen \& Sonntag, 1985). Rather than existing as a universal state, the Second Law of Thermodynamics is true where the negative-vital is leading.

If the vital is leading in a balanced capacity, this will result in energy where and when it is needed as represented by Equation 4.1.4:

Energy $=f n\left(V_{L}^{+}\right)$
Eq 4.1.4: Energy
If the mental is leading so that an idea is pitted against another idea, then fragmentation will result. This is reflected in Equation 4.1.5:

Fragmentation $=f n\left(M_{L}^{-}\right)$

## Eq 4.1.5: Fragmentation

If the mental is leading so that ideas are combined around the greatest idea, this will result in sustainability, as reflected in Equation 4.1.6:

Sustainability $=f n\left(M_{L}^{+}\right)$
Eq 4.1.6: Sustainability

Given these it can be suggested that the outcome of any circumstance is going to be a function of which one of these six states is the strongest, as represented by Equation 4.1.7:

Outcome $_{\text {circumstance }}=\operatorname{Strongest}\left(P_{L}^{-}, P_{L}^{+}, V_{L}^{-}, V_{L}^{+}, M_{L}^{-}, M_{L}^{+}\right)$
Eq 4.1.7: Outcome of Circumstance
In general it is conditions of stability, energy, and sustainability as opposed to stagnation, entropy, and fragmentation, that are observed and hence it can be suggested that any system prefers these states and reinforces them.

### 4.2 Nature of a Point in a System

The 'point' captures the inherent intelligence that appears to exist in the system. This inherent intelligence is approximated by the four observations previously arrived at that appear true of systems, as suggested in Section 2.1.4 on a review of the top-down approach to CAS.

Starting with Observation \#1, introduced in Section 2.1.4: 'Progress happens regardless of area of the world, and regardless of industry. This points to a characteristic of an implicit-presence, whereby wherever there is the possibility of progress it will happen.'

This implicit-presence can be referred to as system-presence. Translating this into an equation, the notation System $_{P r}$ is given to system-presence. This systempresence is true across any considered Time-Space continuum starting from a timespace boundary ' 0 ' to a time-space boundary ' N '. This notion is characterized by the notation $T S_{0 \rightarrow N}$. Within that boundary from 0 to ' N ', the 'presence' is such that it will always seize an opportunity to cause a shift from the physical-leading to the vital-leading, and from the vital-leading to the mental-leading. Research in evolutionary economics also suggests that a firm's openness to its external environment can improve its ability to innovate (Nelson and Winter, 1982). Evolutionary economists highlight the role of search in helping organizations to find sources of variety, allowing them to create new combinations of technologies and knowledge. Openness of an organization surely allows it to be more receptive to System $_{\text {Pr }}$.

The notion that the presence seizes on opportunity as characterized by the notation:
Presence
Opportunity

The shift from physical-leading to vital-leading and vital-leading to mental-leading is characterized by:

$$
\begin{array}{lll}
P_{L} & \rightarrow & V_{L} \\
V_{L} & \rightarrow & M_{L}
\end{array}
$$

Hence in this approach it is suggested that:
System $_{P r} \equiv T S_{0 \rightarrow N}\left[\begin{array}{c}\text { Presence } \\ \downarrow \\ \text { Opportunity }\end{array}\left[\begin{array}{ccc}P_{L} & \rightarrow & V_{L} \\ V_{L} & \rightarrow & M_{L}\end{array}\right]\right]$

But there is something else about this presence as well. All other developments take place in it. That is, it provides a container of sorts in which the plays of implicitpower or system-power, implicit-knowledge or system-knowledge, and implicitnurturing or system-nurturing can take place. This notion is summarized by the notation:

## Container $_{\left[\begin{array}{l}\text { System }_{P} \\ \text { System }_{K} \\ \text { System }_{N}\end{array}\right]}$

Hence, combining these various components, an equation for 'system-presence', Equation 4.2.1, arises:

Eq 4.2.1: System Presence

Moving to Observation \#2, reviewed in Section 2.1.4: ‘This progress happens in spite of tremendous opposition to it ('it' refers to 'progress'), again regardless of field and area. This points to a characteristic of implicit-power.'

This implicit-power can be referred to as system-power. Constructing an equation for system-power, the notation System $_{P}$ is used to represent system-power. Any endeavor will always be met with resistances of various kinds. The resistances that arise along the physical dimension are referred to as $P_{R}$. The resistances that arise along the vital dimension are referred to as $V_{R}$. The resistances that arise along the mental dimension are referred to as $M_{R}$. In the fruition of any endeavor one or all of these types of resistances may arise. Further, resistance of one kind often feeds on
resistance of another kind, and to generalize the resistances encountered in an endeavor may be characterized as the product of the three types of resistance
$P_{R} * V_{R} * M_{R}$
These resistances arise across any considered Time-Space boundary from 0 to ' N ', and therefore it may be said that the power of the system is such that:
power $>\sum_{T S=0}^{N} P_{R} * V_{R} * M_{R}$

An equation for 'system-power', Equation 4.2.2, hence, is the following:
System $_{P} \equiv$ power $>\sum_{T S=0}^{N} P_{R} * V_{R} * M_{R}$

Eq 4.2.2: System Power
Moving to the Observation \#3, reviewed in Section 2.1.4: 'The right instrumentation and the right circumstances seem to be leveraged in order that the progress possible does happen. This points to a characteristic of implicit-knowledge that knows what to leverage and when.' This idea is consistent with the work of Cohen and Levinthal, who argue that the ability to exploit external knowledge is a critical component of innovative performance (Cohen and Levinthal, 1990). It is further reinforced by the work of Laursen and Salter, who investigate the influence of search strategies for external knowledge (Laursen and Salter, 2006). They have proposed the concepts of breadth and depth as two components of the openness of individual firms' external search strategies as critical in increasing innovation.

This implicit-knowledge can be referred to as system-knowledge. Translating this into an equation, the notation, System $_{K}$, is used for system-knowledge. This System $_{K}$ is such that it leverages the right instrumentation and circumstance to bring about the progress that is possible. This concept of 'instrumentation' is denoted by the subscript ' $I$ '. The concept of 'circumstance' is denoted by the subscript ' $C$ '. Both instrumentation and circumstance can be of a physical, vital, or mental type and this possibility is denoted by:

$$
\left[\begin{array}{l}
P_{I, C} \\
V_{I, C} \\
M_{I, C}
\end{array}\right]
$$



Further, the notion that the 'knowledge' is such that it 'leverages' the right instrumentation and circumstance is depicted by:

## Knowledge <br> $\downarrow$ <br> Leverage

This act of leveraging results in a fundamental shift so that the physical-leading yields to the vital-leading, and the vital-leading yields to the mental-leading. Hence:

$$
\begin{gathered}
\text { Knowledge } \\
\downarrow \\
\text { Leverage }
\end{gathered}\left[\begin{array}{c}
P_{I, C} \\
V_{I, C} \\
M_{I, C}
\end{array}\right] \rightarrow\left[\begin{array}{lll}
P_{L} & \rightarrow & V_{L} \\
V_{L} & \rightarrow & M_{L}
\end{array}\right]
$$

Since this behavior may exist across any Time-Space continuum an equation for system-knowledge, Equation 4.2.3, is suggested:

System $_{K} \equiv \operatorname{TS}_{0 \rightarrow N}\left[\begin{array}{c}\text { Knowledge } \\ \downarrow \\ \text { Leverage }\end{array}\left[\begin{array}{c}P_{I, C} \\ V_{I, C} \\ M_{I, C}\end{array}\right] \rightarrow\left[\begin{array}{lll}P_{L} & \rightarrow & V_{L} \\ V_{L} & \rightarrow & M_{L}\end{array}\right]\right.$

## Eq 4.2.3: System Knowledge

Moving to the Observation \#4, reviewed in Section 2.1.4: 'The fact that this progress happens so that the degrees-of-freedom of the associated actors is continually increased points to an implicit-nurturing'.

This characteristic of implicit-nurturing may be referred to as 'system-nurturing'. Like the other characteristics it is suggested to exist across a Time-Space continuum. This is depicted by:
$T S_{0 \rightarrow N}$
There is an action of nurturing such that any state is always advanced to a higher level. This is depicted by:
$\coprod_{\text {Nurturing }}\left(\begin{array}{cc}P_{-} & M_{+} \\ V_{-} & V_{+} \\ M_{-} & P_{+}\end{array}\right)$
Hence, there is a 'union', depicted by 'U' that 'nurtures' the negatives towards their positives.

Further, there is an increasing action of nurturing such that the possibility of integration is always increased to form a larger and larger basis. This increasing basis is depicted as being modulated by the polar coordinates ' $r$ ' and ' $\theta$ ', where $r$ is the radius which increases from an initial value of ' 0 ', and ' $\theta$ ' is an angle from ' 0 ' to '360'.

This notion of an increasing of ' $r$ ' and ' $\theta$ ' is reinforced by the relatively recent phenomena of 'Swift Trust' as a form of trust occurring in temporary organizational structures, which can include quick starting groups or teams. It was first explored by Debra Meyerson and colleagues in 1996 (Meyerson and colleagues, 1996). In swift trust theory, a group or team assumes trust initially, and later verifies and adjusts trust beliefs accordingly. Verification of trust is facilitated by software systems that enable self-organization through control loops and observations (Kantert et al, 2016). Traditionally, trust has been examined in the context of longterm relationships. The establishment of trust has been thought to rely largely on the history of a group and the interactions between members. This traditional view of trust generally assumes that trust builds over time. However, this view is becoming problematic with the increase in globalization, change in technologies, and an increased reliance on temporary teams by organizations. Meyerson et al. propose that swift trust provides the necessary, initial, cognitive confidence for a temporary team to interact as if trust were present. However, swift trust requires an individual to verify that a team can manage vulnerabilities and expectations. Hence, the equation of system-nurturing, Equation 4.2.4, is depicted as:

System $_{N} \equiv \operatorname{TS}_{0 \rightarrow N}\left(\underset{\text { Nurturing }}{ } \int_{\left.\left(\begin{array}{cc}P_{-} & M_{+} \\ V_{-} & V_{+} \\ M_{-} & P_{+}\end{array}\right) \bmod (r, \theta)\right), ~(1)}\right.$

Eq 4.2.4: System Nurturing
It is suggested that these four characteristics exist across any system, and to denote this it is generalized that every point in any system is embedded with this four-fold intelligence. It is suggested that this four-fold intelligence is resident in every instant-spot of the system. It is suggested that to be able to leverage or activate this four-fold intelligence at will is the ultimate act of innovation. The relevancy of these suggestions is explored in detail in Chapter 5, Theoretical Model Applications.

Such a depiction of models, as Bar-Yam states in "From Big Data to Important Information" (Bar-Yam, 2016), 'is "valid" only because of the irrelevance of details...If we want to say anything meaningful about a system-meaningful in the sense of scientific replicability or in terms of utility of knowledge-the only description that is important is one that has universality, that is, is independent of details. There is no utility to information that is only true in a particular instance.

Thus, all of scientific inquiry should be understood as an inquiry into universalitythe determination of the degree to which information is general or specific.'

### 4.3 Architectural Forces

The characteristics embedded in a point suggest a level of innovation that is hard to fathom. One can only glimpse the extraordinary nature embedded in a point. And yet it can be suggested that this extraordinary nature is barely visible unless the right analytical lens of the sort being suggested here is first set up. Further, it is suggested that this extraordinary nature is responsible for a broader set of architectural forces that exist behind the visible face of things.

Hence, system-presence, system-power, system-knowledge, and system-nurturing that define the nature of every point in our system, become more tangible as a broader set of architectural forces that emanate from each of them.

Considering system-presence, here is a characteristic that appears to be everywhere (Malik, 2009) at the service of all the constructs that develop within it. There is a diligence and perseverance by which any opportunity for progress is seized.
Further, if one considers the extraordinary detail that appears in any construct, whether an atom, a body, a planet, or a galaxy, one is struck by the high degree of perfection that surfaces in this presence.

So if one contemplates the nature of this system-presence there is a set of forces that surface. Depicting such a set as $S_{S y s t e m_{P r}}$, one can arrive at elements such as
Service, Perfection, Diligence, Perseverance, amongst others, that are part of this set. Hence, the set can be described by Equation 4.3.1:
$S_{\text {System }_{P r}} \ni$ [Service, Perfection, Diligence, Perseverance, ...]

## Eq 4.3.1: Set of System Presence

Similarly, considering the characteristic of system-power, one can hypothesize that there is a family of forces that emanates from it. The kinds of forces may be thought of as Power, Courage, Adventure, Justice, amongst others. The set for system-power can hence be depicted by Equation 4.3.2:
$S_{\text {System }_{P}} \ni$ [Power, Courage,Adventure,Justice, ...]
Eq 4.3.2: Set of System Power
Similarly, considering the system-knowledge as the root of various powers that emanate from it, one may characterize the set for system-knowledge by Equation 4.3.3:
$S_{\text {System }_{K}} \ni$ [Knowledge,Wisdom,Law Making,Spread of Knowledge ...]

## Eq 4.3.3: Set of System Knowledge

The set for system-nurturing is depicted by Equation 4.3.4:
$S_{\text {System }_{N}} \ni$ [Love,Compassion,Harmony,Relationship ...]

## Eq 4.3.4: Set of System Nurturing

Section 4.4 describes how unique signatures or seeds for any type of organization can be built by leveraging the elements in the sets of architectural forces.

### 4.4 Uniqueness of Organizations

Any system is suggested to be highly innovative (Kaufmann, 1995). The four properties explored in the Section 4.2 define the source of that innovation. From this source emanate 4 sets of forces that suggest the boundaries of that innovation. It is also observed that no two organizations, regardless of scale, are alike (refer to Section 1.4). This uniqueness can be thought of as an additional mechanism of any system to ensure innovation.

In fact, a working hypothesis as per the discussion in Section 1.4 on the core mathematical framework is that every organization, whether a person, team, corporation, market, or country is unique and that this uniqueness can be specified in terms of elements of the derived sets for system-presence, system-power, system-knowledge, and system-nurturing. Specifically, it may be said that an organization's fount of uniqueness derives from one of the four properties.

Assuming then that the fount of uniqueness is system-presence, discussed in Section 4.3, a general equation for organizations that belong to the family of systempresence can be derived. Such uniqueness can be depicted as Sig $_{x}$ where the subscript ' $x$ ' refers to the source family, and 'Sig' or signature to 'uniqueness'. Hence the uniqueness of an organization in the family of system-presence would be notated by Sig $_{\text {System }_{P r}}$.

In line with the development of properties of a point and the precipitating architectural forces as discussed in Sections 4.2 and 4.3, an approach to constructing such uniqueness is to assume a primary factor X that drives the uniqueness that belongs to the set $S_{S y s t e m_{P r}}$. Further, assume that the uniqueness is qualified by a number of secondary factors Y that may belong to any of the 4 sets -
$S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}$. The primary factor X would have a greater weightage than any of the secondary factors $Y$. The weightage of $X$ hence could be depicted by the number ' a ', and the weightage of Y a number ' $b_{o-n}$ ', such that $\mathrm{a}>\mathrm{b}$.

Further, the secondary element can repeat from ' 0 - n' times, and is hence depicted as $\overline{Y b_{0-n}}$.

The equation, Equation 4.4.1, hence for a unique organization derived from the family of system-presence is:


## Eq 4.4.1: System Presence Based Unique Organization

Similarly, an equation, Equation 4.4.2, for a unique organization derived from the family of system-power is:


## Eq 4.4.2: System Power Based Unique Organization

An equation, Equation 4.4.3, for a unique organization derived from the family of system-knowledge is:

Eq 4.4.3: System Knowledge Based Unique Organization
An equation, Equation 4.4.4, for a unique organization derived from the family of system-nurturing is:

Sig $_{I}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 4.4.4: System Nurturing Based Unique Organization

The four preceding equations can be generalized by Equation 4.4.5:

Sig $=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers; } a>b\end{array}\right]$

Eq 4.4.5: Generalized Equation for Unique Organization
Having considered the structure of uniqueness, the next question is how does such uniqueness emerge? This is discussed in Section 4.5.

### 4.5 Emergence of Uniqueness

While the uniqueness of organizations as represented by the Signature is a seed, like any seed there is a process for its emergence (Kaufmann, 1995; Portugali, 2012; Yates, 2012), and the uniqueness will often be hidden or very much behind the scene until certain conditions are fulfilled (Malik, 2009).

The implicit nature of Time and Space suggest a universal developmental model that provides a cue as to the process for emergence (Deep Order Mathematics Videos, 2016). In this model the four sets of architectural forces already described form a pool in space, as it were, from which possibility arises. Possibility itself is unique from point to point and is governed by the Equation for Uniqueness (Equation 4.4.1 through 4.4.5) described in the previous section. The emergence of uniqueness is governed by the process identified when discussing the implicit order in Time, in Section 2.1.4.

Hence it is observed that initially the uniqueness takes a 'physical' form, moving on to a 'vital' form, and then onto a 'mental' form. Once the orientations implicit in each of these phases are assimilated, then the uniqueness takes on an 'integral' form. The integral form is a threshold phase, and allows the uniqueness suggested by the Signature to emerge in fuller force or in its 'force' form. The final phase is the 'contextual form' that allows the signature to act with impunity within a considered context.

Mathematically, if an organization exists at the physical phase, it may be suggested that its signature or uniqueness is modulated by the constant ${ }^{\prime} \pi^{\prime} . \pi$ is the seed of a circle or sphere and can be thought of as defining behavior that is tightly bound. Within such a tightly bound volume it will likely not even be apparent what the uniqueness of an organization necessarily is. Assuming the uniqueness to be defined by the derived question Sig, the physical-level (P) behavior can be described by the following equation-segment where 'mod' signifies modulated-by:

P: Sig* $\bmod (\pi)$

If an organization exists at the vital level, it may be suggested that its uniqueness is modulated by the Euler-constant ' e '. e is at the root of exponential behavior. The vital by definition is about assertive and aggressive growth the symbol of which is 'e'. Hence vital-level (V) modulation (represented by 'mod') can be described by the following equation-segment:
$V: S i g * \bmod (e)$
If an organization exists at the mental level, it may be suggested that its uniqueness is modulated by the Gaussian Distribution ' $G$ '. G summarizes rational behavior with a key direction followed by most, and directions more on the edge followed by outliers. Mental-level dynamics are arguably quite similar, and it can be suggested are best modeled by such a distribution (Salkind, 2007). Mental-level (M) modulation (mod) can hence be described by the following equation-segment:

M: Sig* $\bmod (G)$
The physical, the vital, and the mental levels are orientations in which patterns of perceiving, being, behaving are set in their ways. Each pattern has its purpose and its limitation and it can be argued that being able to learn from each orientation and yet being able to move beyond that, is the next logical step in any developmental model. The integral level hence, is about being able to leverage each of the patterns that naturally arise at the three preceding levels at will, and about further, being able to integrate these and arrive at new ways of perceiving and being.

Mathematically such behavior may be represented as being an integrative function $\left(\int x\right)$ where ' x ' is the ability to move between the patterns emanating from $\mathrm{G}, \mathrm{e}, \pi$, at will, represented by $\overline{G, e, \pi}$. Integral-level (I) modulation (mod) of uniqueness (Sig) can hence be represented by the following equation-segment:
$I: \operatorname{Sig} * \bmod \left(\int \overline{G, e, \pi}\right)$
The condition of overcoming any fixed and limiting patterns is the prerequisite for the emergence of 'Force' or for entering into the force-level. At this level the uniqueness behind the particular development being considered can emerge in its purity and become a truly creative dynamic. This aspect of creativity that is in a sense not bound by circumstance may be represented by the constant ' $c$ ', the speed of light in a vacuum (Perkowitz, 2011) which is an upper limit of the layer that systems practically operate in (will be discussed in more detail in Chapter 5 and Appendix 1). Force-level (F) modulation (mod) of uniqueness (Sig) can hence be represented by the following equation-segment:
$F: S i g * \bmod (c)$

Once the signature of an organization arises and continues to exercise itself in its purity, it achieves contextual-mastery ( C ) and is able to exercise itself as though the context it is acting in, that can vary in scale and complexity, were all of the same substance as itself. This equality may be represented by the integrative function ' $\int=1$ '. The equation-segment that notates this contextual-level ( C ) modulation (mod) applied to organizational uniqueness (Sig) is hence:
$C: \operatorname{Sig} * \bmod \left(\int=1\right)$
Piecing all the equation-segments together the equation for the emergence of uniqueness ( $\operatorname{Sig}_{E}$ ), where ' X ' can be any of the discussed modulations at the respective development-model levels (P, V, M, I, F, C), is hence summarized by Equation 4.5.1:

$$
\operatorname{Sig}_{E}=X\left[\begin{array}{c}
C: \operatorname{Sig} * \bmod \left(\int=1\right) \\
F: \operatorname{Sig} \bmod (c) \\
I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\
M: \operatorname{Sig} * \bmod (G) \\
V: \operatorname{Sig} * \bmod (e) \\
P: \operatorname{Sig} * \bmod (\pi)
\end{array}\right]
$$

Eq 4.5.1: Emergence of Uniqueness
The power of virtual worlds in engaging people and promoting learning as described in "Why Virtual World Matter" (Thomas \& Brown, 2009) can be seen as parallel to the emergence of uniqueness as described by Equation 4.5.1. As described by the authors there is a process of selectively leaving behind part of oneself (movement through the $\mathrm{P}, \mathrm{V}, \mathrm{M}$ levels) to recreate one's identity (movement through level I), and then engaging with others in a shared discourse and culture (movement through levels F, C), which becomes very meaningful for people.

### 4.6 Varying Culture of Organizations

Having defined an equation for the uniqueness of an organization it now becomes possible to also derive equations for the culture of an organization. Understanding culture in terms of its ability to be a foundation of innovation is important since some cultures cause stagnation while some cause innovation. Useful approaches to and frameworks on innovation at varying levels of organizational complexity are covered in some works of Sri Aurobindo (Sri Aurobindo, 1971). Additionally, works such as Ogbonna \& Harris (Ogbonna \& Harris, 2000) on the links between leadership, performance, and culture suggest that it is only particular kinds of
cultures, those that promote innovativeness as opposed to bureaucracy, for example, where performance improves.

The essential variables in the equation for uniqueness are the X and Y elements (as described in Section 4.4), where X is the primary element that defines that organization's uniqueness and $Y$ are the secondary elements that support or nuance the primary element. X is an element of the $X_{S e t}$, which is the set comprised of elements from $S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}$, and $S_{\text {System }_{N}}$. The Y elements are derived from the $Y_{S e t}$ which is also a set comprised of elements from $S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}$, and $S_{\text {System }_{N}}$.

In thinking about variation in culture derived from the same primary element, the variable factor is hence the number of elements in the $Y_{\text {Set }}$. The hypothesis is that the more the number of secondary elements, the more diverse and hence the more stable and innovative the organization will be. If an organization is thought of as having multiple levels, then an equation, Equation 4.6.1, which will define a monoculture is hence the following:

Monoculture: elements $_{Y_{\text {set }_{(n)}}}=$ elements $_{\text {Set }_{(1)}}$

## Eq 4.6.1: Monoculture

What this equation is basically stating is that at some level ' $n$ ' the number and makeup of elements are exactly the same as the initial seed-state from which the organization culture emerged. Such an organization may do well in a very specific circumstance, and probably the one it was created for. However the reality is that change is the only constant and therefore when faced with a different set of circumstances this organization will not have the source of innovation to allow it to respond differently. It will likely destabilize and go extinct.

By contrast, the equation, Equation 4.6.2, for a diverse culture is:
Diverse Culture: elements ${Y_{\text {set }_{(n)}} \propto(1+g)^{n}, ~(1)}$

## Eq 4.6.2: Diverse Culture

In this equation the number and make-up of elements at some level ' $n$ ' in the organization is different than any level that precedes it. The difference is related to an exponential function where ' g ' is a growth factor that will increase, likely linearly, as the organization gets more and more complex, and ' $n$ ' is the level down in the organization starting from level ' 1 ' which is the executive level. The diverse culture will have many more sources of innovation when faced with change because of the plethora of secondary elements that support the primary element.

### 4.7 Inherent Dynamics of Any System

So far the inherent innovation that exists at the system level and summarized by the nature of a point has been considered. Further, how this deep fount of innovation is present everywhere and how sets that make more practical the range of creative forces available in each of the four components of a point; have also been considered. These architectural forces further define the possibility inherent in any system. Leveraging these sets of forces an equation for the uniqueness of an organization, regardless of scale, was arrived at. Further, it was suggested what made for an innovative versus a stagnant culture.

In some sense the precipitation of innovation from the barely perceptible nature of the ubiquitous point, to how this reveals a play of forces, to how organizations take their seed and grow from these forces, has been traced.

It is useful to now turn full-circle to return to the initial orientations that allowed so much to be suggested about the nature of innovation in the first place. It is useful to look deeper into the nature of the physical, the vital, the mental, and the integral, and to derive equations that in effect will provide further insight into the dynamics of innovation inherent in these orientations.

In Section 4.1 the negative and positive sides of the physical, the vital, and the mental were suggested in the first set of equations that approximated the conditions of Stagnation, Stability, Entropy, Energy, Fragmentation, and Sustainability.

Here a deeper look at the dynamics inherent in each of these orientations that can allow the shift from the negative to positive, or in other words, that reveal the process of innovation in these fundamental orientations is taken. Note that these orientations are related to the notion of inclusion of meta-levels in Systems Dynamics suggested in Section 2.1.1.

Such shifts will increase the spread in taxonomies of novelty of innovation to span from the incremental, such as changing packaging on existing products, to the radical and truly revolutionary, such as the microchip (Henderson and Clark, 1990). The shift to the positive will allow birthing of radical innovations that seem to offer the greatest opportunity for performance differences (Marsili and Salter, 2005). Tushman and Anderson classify radical innovation in terms of 'competence enhancing' (positive) or 'competence-destroying' (negative), reflecting the different ways novel innovations alter patterns of industrial competition among firms working within the industry (Tushman and Anderson, 1986; Anderson and Tushman, 1990).

Hence, starting with the physical, an equation, Equation 4.7.1, is summarized as:


$$
\begin{aligned}
& \text { Physical } \\
& =\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{P r} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {Systemi }_{P r}} \\
\left(\uparrow \text { Sig }_{P_{r}} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{P} \\
\left(\uparrow>P_{P)}\right. \\
U \rightarrow \text { Physical }_{U}
\end{array}\right] T C \rightarrow \text { Physical }_{T}, \text { where }\left[\begin{array}{c} 
\\
\text { Physical }_{U} \quad \ni[\text { inertia, lethargy, status quo, } \ldots] \\
\text { Physical }_{T} \ni[\text { adaptability, durability, strength, } \ldots]
\end{array}\right]
\end{aligned}
$$

Eq 4.7.1: Inherent Dynamism in Physical
Essentially this equation is laying out the conditions of moving from the untransformed or negative physical state represented by Physical $_{U}$ to the transformed or positive physical state represented by Physical $_{T}$.

The first matrix should be read from the bottom to the top:

$$
\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{P r} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{P r}} \\
\left(\uparrow \text { Sig }_{3} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{P} \\
\left(\uparrow>P_{P)}\right. \\
U \rightarrow \text { Physical }_{U}
\end{array}\right]
$$

Hence, at the bottom is the starting point ' $U \rightarrow$ Physical $_{U}$ ' which identifies the default or untransformed ( U ) level of the physical. The next row up, ( $\uparrow>P_{P}$ ), states that when the patterns of the untransformed physical $\left(P_{P}\right)$ have been overcome ( $>$ ), movement to the next level ( $\uparrow$ ) is facilitated. Breaking through to the next level, $M_{1} \rightarrow \operatorname{Sig}_{P}$, allows its dynamics to become active. Hence, the signature or uniqueness of the physical ( $\operatorname{Sig}_{P}$ ) becomes active at meta-level $1\left(M_{1}\right)$. As this signature becomes more like a Force ( $\operatorname{Sig} \rightarrow F$ ), the conditions for breakthrough ( $\uparrow$ ) to the next level are achieved. This next level is referred to as meta-level $2\left(M_{2}\right)$, and indicates that the architectural forces represented by the set of system-presence $\left(S_{S_{\text {stem }}^{P r}}\right)$ have become more consciously active. When this Force becomes Integral $(F \rightarrow I)$ then the conditions for breakthrough ( $\uparrow$ ) to the next level are achieved. The next level is notated as $M_{3}$ for meta-level 3, and the dynamics here indicate that the equation for system-presence becomes active. Becoming active basically means that the respective meta-level dynamic begins to act at the once 'untransformed' level (U) further modifying it. Modification or transformation began when $M_{1}$ became active. Transformation is accelerated when $M_{2}$ becomes active, and even further accelerated when $M_{3}$ becomes active. Note that the matrix is essentially semiotic in
that adjacent levels exist in paired relationships and inter-related functionality (Usó-Doménech et al, 2016).

The notion of meta-layers is being explored by contemporary physicists and Erwin Laszlo in his book Self-Actualizing Cosmos (Laszlo, 2014) summarizes some of these developments: "Physicists describe the domain that underlies and embeds the particles, fields, and forces of the universe variously as quantum vacuum, physical spacetime, nuether, zero-point field, grand-unified field, cosmic plenum, or stringnet liquid." Note that 'nuether' refers to a sub-quantum level of reality (Pearson, 1997). Laszlo goes on to describe a revolutionary discovery by Nika Arkani-Hamed of Princeton's Institute of Advanced Study, of a geometrical object, the amplituhedron (Arkani-Hamid et al, 2012), which is not in space-time but governs space-time so that it "appears that spatio-temporal phenomena are the consequence of geometrical relationships in a deeper dimension of physical reality". A deeper dimension of a physical layer suggests synonymity with a meta-layer.

The rate of the transformation can be better envisioned when considering action of the Transformation Circle, or TC. The TC can be thought of as 4 concentric circles, with $M_{3}$ at the center. $M_{3}$ is surrounded by $M_{2}$, which is surrounded by $M_{1}$. The outer circle is $U$. If TC is considered to be a clock, than at time ' $t=0$ ', the physical' can be thought of as being entirely in $U$. The clock starts ticking only when some initial patterns $P_{P}$ are overcome ( $>P_{P}$ ). From this point on as time proceeds the conditions for breakthrough become riper, and a sinusoidal wave begins to integrate more of the concentric circles together. The sinusoid wave (sin) is itself modulated by an euler function, $e^{x}$, where ' x ' is determined by the strength to overcome patterns ( $\uparrow$ ) which will likely vary over time but will likely tend to be positive once the clock has started ticking because of the joy experienced with progressive movement. Being that the limit is the outer boundary of the concentric circles, there is further modulation by $\pi$ until the 4 concentric circles have been integrated. TC, hence, may be represented by Equation 4.7.2:
$T C \equiv\left(>P_{P}\right) \rightarrow \bmod \left(\sin , e^{x}, \pi\right)$
Eq 4.7.2: Transformation Circle
Hence, the initial nature of the physical that may be characterized by the set comprising of elements such as inertia, lethargy, acceptance of the status quo, amongst other such elements (Physical ${ }_{U} \ni$ [inertia,lethargy, status quo, ...]), transforms into a physical more characterized by elements such as adaptability, durability, strength, and so on (Physical ${ }_{T} \ni$ [adaptability, durability, strength, ...). This transformation represents the inherent innovation-dynamic within the Physical.

Similarly, the equation for the 'Vital', Equation 4.7.3, also shows the built-in transformation that represents the innovation-dynamic within the vital:

Vital


Eq 4.7.3: Inherent Dynamism of Vital

The equation for the 'Mental', Equation 4.7.4, is similarly summarized as:


Eq 4.7.4: Inherent Dynamism of Mental

The equation for the 'Integral', Equation 4.7.5, is similarly summarized as:

$$
\begin{aligned}
& \text { Integral }
\end{aligned}
$$

Eq 4.7.5: Inherent Dynamism of Integral
The preceding equations can be generalized by Equation 4.7.6:

$$
\text { Innovation }_{\text {orientation }-x}=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \operatorname{Sig}_{\boldsymbol{T}} \rightarrow F\right) \\
M_{1} \rightarrow \operatorname{Sig}_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

Eq 4.7.6: Generalized Equation of Innovation
In this generalized equation, Innovation orientation $-x^{\text {, refers to the inherent }}$ innovation within a specific orientation. Orientation refers to the physical, the vital, the mental, or the integral.

Further, the notion of a core-matrix that will be referred to in subsequent sections can be summarized by the following equation, Equation 4.7.7:

$$
\text { Core_matrix }=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow \text { S System }_{X} \\
\left(\uparrow \text { Sig }_{\text {S }} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right]
$$

## Eq 4.7.7: Core Matrix

One of the corollaries of the Generalized Equation of Innovation is that if the source of innovation is more influenced by a meta-level there will be simultaneity of innovation that becomes apparent at $U$. The higher the meta-level, the more likely that this simultaneity will be wider spread. In his book Where Good Ideas Come From: The Natural History of Innovation (Johnson, 2010) Johnson states: "A brilliant idea occurs to a scientist or inventor somewhere in the world, and he goes public with his remarkable finding, only to discover that three other minds had independently come up with the same idea in the past year." He refers to an essay "Are Inventions Inevitable" (Ogburn \& Thomas, 1922) which uncovered 148 instances of similar yet independent innovation, most of them occurring within the same decade. Some examples include sunspots that were simultaneously discovered in 1611 by four scientists in four different countries, and the law of conservation of energy that was formulated separately four times in the late 1840s, amongst numerous other example.

### 4.8 Equations for Stagnation and Dynamic Growth

Considering the equations for the Physical, the Vital, the Mental, and the Integral, presented in the previous section, the implicit reality of innovation present within them may become apparent. This sense of innovation is embedded within these fundamental orientations. But as the Transformation Circle, TC, indicates, each of the respective untransformed sets transforms to each of the respective transformed sets only if TC becomes active.

An easy way, hence, to characterize stagnation (or the lack of innovation) and dynamic growth (the presence of innovation) is through the following generalized equations where ' $x$ ' can be thought as the Physical, the Vital, the Mental, or the Integral.

The generalized equation for Stagnation, Equation 4.8.1, hence is:
$\frac{d\left(T C_{x}\right)}{d t} \leq 0$
Eq 4.8.1: Generalized Equation for Stagnation

The generalized equation for Dynamic Growth, Equation 4.8.2, is:
$\frac{d\left(T C_{x}\right)}{d t}>0$

Eq 4.8.2: Generalized Equation for Dynamic Growth
As a corollary it may also be suggested that rather than being universal, the second law of thermodynamics is perhaps active under conditions of stagnation, as in Equation 4.8.3. Of course there is currently no proof of this.

Hence,
IF $\frac{d\left(T C_{x}\right)}{d t} \leq 0$ THEN $\mathrm{dS}>=0$ (where S is Entropy)

## Eq 4.8.3: Suggested Equation for Entropy

If, as being suggested in the mathematical model being derived here that there is a fundamental process for the precipitation of innovation, that there are fundamental orientations implicit with dynamism, and further that there are conditions for growth and stagnation, then the notion of randomness in CAS will have to be revisited. This is done in the next section, 4.9.

### 4.9 Qualified Determinism of Complex Adaptive Systems

The model and mathematics enumerated in this work hence, suggests though that the randomness being projected by CAS scientists (Dodder \& Dare, 2000) does not really exist. Let us look at aspects of the mathematics to substantiate this.

A new function, Dynamic Interaction (DI) that has a 'vertical' and a 'horizontal' component is introduced here. The vertical component is designated as $D I_{V}$ and the horizontal component as $D I_{H}$. Several equations (Section 4.7) to capture the inherent dynamism that exists in each orientation or state have already been derived. These included equations for the dynamism in the physical, the vital, the mental, and the integral. The derived equations propose a model to give insight into how innovation occurs by changing the fundamental states that an organization is subject to. CAS scientists such as Prigogine (Prigogine, 1977) and others are proposing that a system can bifurcate in unpredictable ways to create an emergent property that cannot be predicted. DI is going to propose that in fact there is a 'qualified determinism' as opposed to randomness that occurs.

This qualified determinism is the result of the relative strengths of the levels within core-matrix identified in the derived equations. To summarize, the generalized core-matrix, already introduced in Section 4.7, is the following:
$\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow \text { S System }_{X} \\ (\uparrow \text { Sig } \rightarrow F) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x)}\right. \\ U \rightarrow x_{U}\end{array}\right]$

The application of the vertical component of the new function being proposed, $D I_{V}$, to this core matrix will yield the nature or 'strength' of the state ( x ) or orientation under consideration. If the untransformed or $U$ layer is strongest, implying that the habitual patterns that keep an organization locked into its untransformed way of operation are still very active, then the nature of the output of $D I_{V}$, notated by xstate, will be $x_{U}$. If the habitual patterns have been overcome then the strength of the x-state increases since it is the dynamics of $M_{1}$ or $S i g_{x}$ that are now active. In this case the x -state will be $\operatorname{Sig}_{x}$. If the unique 'signature' has become a 'force', then the conditions for activation of $M_{2}$ have been put in place and the x-state will be even higher, $S_{\text {System }_{X}}$. The architectural forces active in $M_{2}$ are by definition more powerful than $\operatorname{Sig}_{x}$ that is a derivation of a set of such architectural forces. If the 'force' so acting becomes impersonal so that an organizational ego-state is overcome, then the x -state will have the most strength and is characterized by System $_{X}$ active at $M_{3}$. Hence, $D I_{V}$ applied to a core-matrix will yield the 'strength' in terms of the x -dynamic that is active. This is illustrated by the following equation,

Equation 4.9.1 which can be considered to be a deductive proof in the context of this model:
$D I_{V}\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow \text { S System }_{X} \\ \left(\uparrow \text { Sig }_{\text {a }} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x)}\right. \\ U \rightarrow x_{U}\end{array}\right]=>\mathrm{x}$-state $\in\left(x_{U}\right.$, Sig $_{x}$, S $_{\text {System }_{X}}$, System $\left._{X}\right)$
Where: $\operatorname{Strength}\left(\right.$ System $\left._{X}\right)>\operatorname{Strength}\left(S_{\text {System }_{X}}\right)>\operatorname{Strength}\left(\right.$ Sig $\left._{x}\right)>\operatorname{Strength}\left(x_{U}\right)$
Eq 4.9.1: Illustrating Action of Dynamic Interaction - vertical component
What is to be noted here is that while the action of $D I_{V}$ yields a relative strength and therefore a 'single' value for the core- or x-matrix under consideration yet each x matrix in itself could have an infinite number of possibilities. This should be clear in looking at how $x_{U}, \operatorname{Sig}_{x}, S_{\text {System }_{X}}$, and System $_{X}$, were initially defined.

Hence, taking the example where $\mathrm{x}=$ physical:
Physical $_{U} \ni$ [inertia, lethargy, status quo, ...]
As can be seen Physical $_{U}$, defined in Section 4.7, is already an infinite set with qualities similar to the ones already specified.

Similarly, Sig $_{P}$, defined in Section 4.4, also has an infinite variation:
Sig $_{P}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\left.S_{S y t e m_{P r}}\right]}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r} r}, \\ a, b \text { are integers } ; a>b\end{array}\right]\end{array}\right]$
$S_{S_{s t e m_{P r}}}$, defined in Section 4.3, is also an infinite set with forces of the nature specified in the following equation:
$S_{S y s t e m_{P r}} \ni$ [Service,Perfection,Diligence,Perseverance, ...]
And recall that in Section 4.2, System Pr has been defined as:

So in essence $D I_{V}$ is really giving us a summary assessment of the 'level' of the xmatrix under consideration with all its infinite potentiality. An example will follow shortly.

The other component of DI, as suggested earlier in this section, is the horizontal component, $D I_{H}$. Just as $D I_{V}$ yields a summary assessment of the level that an xmatrix is operating at, similarly $D I_{H}$ yields a summary assessment of the direction that a system or organization under consideration is going to continue its development in considering the physical, the vital, the mental, and the integral orientations to be the choices.

Assuming that any organization or system is inherently unique, as this mathematical model proposes, and assuming that the infinite sets of $x_{U}$ and $S_{S y s t e m_{X}}$ applied across the physical, vital, mental, and integral orientations respectively will account for any state that an organization can experience, then at a certain point in time any organization under consideration is going to have a direction-bias in one of the possible physical, vital, mental, or integral directions. Hence, $D I_{H}$ will yield the summary direction that is going to lead an organization into its future given the current states active in it.

This summary direction is going to be yielded by considering the relative strengths of the separate core x-matrices - the physical, the vital, the mental, the integral under consideration. The assumption is that there will be one core-matrix that will be stronger than the others.

Hence, as an example, first applying $D I_{V}$ across all four x-matrices may, for example, yield the following results, with the strongest level within each x-matrix highlighted and bolded:
$\left[\begin{array}{c}\text { System }_{P r} \\ \boldsymbol{S}_{\text {System }_{P r}} \\ \text { Sig }_{P} \\ \text { Physical }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{P} \\ \text { System }_{P} \\ \text { Sig }_{V} \\ \text { Vital }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{K} \\ S_{\text {System }_{K}} \\ \text { Sig }_{M} \\ \text { Mental }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{N} \\ \text { Stestem }_{N} \\ \text { Sig }_{I} \\ \text { Integral }_{U}\end{array}\right]$

Since by definition the strength of $S_{y s t e m_{x}}$ is greater than $S_{S_{y s t e m}^{x}}$, which is greater than $\operatorname{Sig}_{x}$, which is greater than $x_{U}$, applying $D I_{H}$, as in Equation 4.9.2, across these $x$-matrices, as in the example following it will then yield the strongest direction, which in this example is the Physical:
$D I_{H}\left(\left[\begin{array}{c}\text { System }_{P r} \\ S_{\text {System }_{P r}} \\ \text { Sig }_{P} \\ \text { Physical }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{P} \\ S_{\text {System }_{P}} \\ \text { Sig }_{V} \\ \text { Vital }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{K} \\ S_{\text {System }_{K}} \\ \text { Sig }_{M} \\ \text { Mental }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{N} \\ S_{\text {System }_{N}} \\ \text { Sig }_{I} \\ \text { Integral }_{U}\end{array}\right]\right)=$ Orientation $_{\text {Strongest }}$

Eq 4.9.2: Illustrating Action of Dynamic Interaction - horizontal component
Example:
$D I_{H}\left(\left[\begin{array}{c}\text { System }_{P r} \\ \boldsymbol{S}_{\text {System }_{P r}} \\ \text { Sig }_{P} \\ \text { Physical }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{P} \\ \text { System }_{P} \\ \text { Sig }_{V} \\ \text { Vital }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{K} \\ S_{\text {System }_{K}} \\ \text { Sig }_{M} \\ \text { Mental }_{U}\end{array}\right]\left[\begin{array}{c}\text { System }_{N} \\ S_{\text {System }_{N}} \\ \text { Sig }_{I} \\ \text { Integral }_{U}\end{array}\right]\right)=$ Physical

Hence, DI function will yield the following organizational direction, as in Equation 4.9.3, where 'x_matrix' is used interchangeably with 'orientation':
 $x_{-}$matrix $_{\text {strongest }} @$ level $_{\text {strongest }}$

Eq 4.9.3: Organizational Direction
Generalizing, as in Equation 4.9.4, where Org_Dir is organizational direction:
Org_Dir $=D I\left(\left[\begin{array}{c}M_{3} \rightarrow \operatorname{System}_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow S_{\text {System }_{X}} \\ \left(\uparrow \operatorname{Sig}_{\boldsymbol{T}} \rightarrow F\right) \\ M_{1} \rightarrow \operatorname{Sig}_{x} \\ \left(\uparrow>P_{P)}\right. \\ U \rightarrow x_{U}\end{array}\right]{ }_{x=p, v, m, i}\right) \rightarrow x_{-}$matrix $_{\text {strongest }} @$ level $_{\text {strongest }}$

## Eq 4.9.4: Generalized Equation for Organizational Direction

Hence, this mathematical model is suggesting that any situation, rather than having a random outcome, has a 'qualified deterministic' outcome. In the introduction to his book "Where is Science Going?" (Planck, 1933), James Murphy points out that the reason Planck spent so much of his time giving lectures on causation was because of the trend of physicists at the time, which has continued to the modern
day, to overthrowing the principle of causation following the development of quantum theory, which he felt was misplaced. "Planck would claim", he wrote, "and so would Einstein, that it is not the principle of causation itself which has broken down in modern physics, but rather the traditional formulation of it." Murphy also quotes James Jeans (Jeans, 1932) to suggest the issue associated with causation and determinism: "Einstein showed in 1917 that the theory founded by Planck appeared, at first sight at least, to entail consequences far more revolutionary than mere discontinuity", and here he is referring to the finding that radiant energy is not emitted in a continuous flow, but in integral quantities, or quanta, which can be expressed in integral numbers. Continuing: "It appeared to dethrone the law of causation from the position it had therefore held as guiding the course of the natural world. The old science had confidently proclaimed that nature could follow only one road, the road which was mapped out from the beginning of time to its end by the continuous chain of cause and effect; state A was inevitably succeeded by state B. So far the new science has only been able to say that state A may be followed by state B or C or D or by innumerable other states. It can, it is true, say that B is more likely than C, C than D, and so on; it can even specify the relative probabilities of B, C, and D. But, just because it has to speak in terms of probabilities, it cannot speak with certainty which state will follow which; this is a matter which lies on the knees of the gods - whatever gods there may be."

While under the apparent dynamics at the quantum level there may appear to be randomness and a dethroning of the principle of causation, the notion of a multiplicity of levels, each having its impact on the strength of an orientation and further on the consequent direction from a multiplicity of possible orientations, is being suggested here as determining the direction of any CAS, while still allowing infinite variation in the details that may define its. Hence, the positions of Planck and Einstein are vindicated when considering Equation 4.9.4.

Further, assuming any CAS where multiple elements are active, connected, interdependent, and emergent, it may be possible to understand, through application of calculus, as to which level is the source for change.

Hence, where N may be source of change, the rate of change of N will resolve into one of $P_{U}, V_{U}, M_{U}, I_{U}, P_{T}, V_{T}, M_{T}$, or $I_{T}$. This may be summarized by Equation 4.9.5, where y is either U or T :
$\frac{d N}{d t} \rightarrow\left[\begin{array}{cc}P_{U} & P_{T} \\ V_{U} & V_{T} \\ M_{U} & M_{T} \\ I_{U} & I_{T}\end{array}\right] \rightarrow x_{y}$, where $y \in(U, T)$
Eq 4.9.5: Establishing the Nature of the Change

If T, implying that the action of one of the meta-levels has caused transformation, then application of one of the following integrals will determine which level is the likely source for change.

Hence, for $M_{1}$, if the integral of $\frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}$ across a limited area ' $a$ ' in the vicinity of the change, is greater than some threshold value Threshold signature, then the signature dynamics are likely the source of change. This is summarized by Equation 4.9.6:
$\int_{0}^{a} \frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}>$ Threshold $_{\text {Signature }}$
Eq 4.9.6: Signature Dynamics as the Source of Change
For $M_{2}$, if the integral of $\frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}$ across a larger area ' $b$ ' extending beyond the vicinity of the change, is greater than some threshold value Threshold ${ }_{\text {ArchitecturalForces }}$, then the architectural forces are likely the source of change. This is summarized by Equation 4.9.7:
$\int_{0}^{b} \frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}>$ Threshold ArchitecturalForces
Eq 4.9.7: Architectural Forces as the Source of Change
For $\mathrm{M}_{3}$, if the double integral of $\frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}$ across the CAS specified by ' A ', and across some time ' $t$ ', is greater than some threshold value ThresholdsSystemProperty, then the system properties are likely the source of change. This is summarized by Equation 4.9.8:
$\int_{0}^{t} \int_{0}^{A} \frac{\partial\left(x_{U} \rightarrow x_{T}\right)}{\partial t}>$ Threshold $_{\text {SystemProperty }}$
Eq 4.9.8: System Properties as the Source of Change

### 4.10 Framing Organizational Transitions at Layer U

It was suggested earlier that the basic dissipative inequality provides insight into the observable frames within which organizational transitions express themselves. Recall the inequality - Prigogine's Dissipative Structure Inequality - reviewed in Section 2.1.5.1:

$$
\frac{d V(x(t))}{d t} \leq u(t) \cdot y(t)
$$

Where $\mathrm{x}(\mathrm{t})$ are states in the structure, $\mathrm{u}(\mathrm{t})$ are inputs, $\mathrm{y}(\mathrm{t})$ are outputs, and $\mathrm{V}(\mathrm{x})$ is a storage function.

Building of the previous hypothesis the 'context' of the dissipative structure being created may also be considered in further distinguishing the type of structure being created. Since there are many steps in the journey from $x_{U}$ to $x_{T}$ this in effect may frame the nature of the transition that is taking place. The general equation for the emergence of uniqueness, derived in Section 4.5, provides a framework for the possible steps in such a transition. Recalling:

$$
\operatorname{Sig}_{E}=X\left[\begin{array}{c}
C: \operatorname{Sig} * \bmod \left(\int=1\right) \\
F: \operatorname{Sig} \bmod (c) \\
I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\
M: \operatorname{Sig} * \bmod (G) \\
V: \operatorname{Sig} * \bmod (e) \\
P: \operatorname{Sig} * \bmod (\pi)
\end{array}\right]
$$

$\operatorname{Sig}_{E}$ is suggested as the model for emergence of the physical, the vital, the mental, and the integral orientations. At the very least therefore there would be 24 different storage functions, $\mathrm{V}(\mathrm{x})$, that would be possible and each would have a different nature. These 24 possibilities are the product of the six states in Sig $_{E}$ times the four orientations. Orientation may be represented by 0 , and the states within these by S . Hence:

$$
O S \epsilon\left[\begin{array}{c}
\text { System }_{P r} \\
S_{S_{S y s t e m_{P r}}} \\
\text { Sig }_{P} \\
\text { Physical }_{U m} \\
\text { Physical }_{U v} \\
\text { Physical }_{U p}
\end{array}\right]\left[\begin{array}{c}
\text { System }_{P} \\
S_{\text {System }_{P}} \\
\text { Sig }_{V} \\
\text { Vital }_{U m} \\
\text { Vital }_{U v} \\
\text { Vital }_{U p}
\end{array}\right]\left[\begin{array}{c}
\text { System }_{K} \\
S_{\text {System }_{K}} \\
\text { Sig }_{M} \\
\text { Mental }_{U m} \\
\text { Mental }_{U v} \\
\text { Mental }_{U p}
\end{array}\right]\left[\begin{array}{c}
\text { System }_{N} \\
S_{\text {System }_{N}} \\
\text { Sig }_{I} \\
\text { Integral }_{U m} \\
\text { Integral }_{U v} \\
\text { Integral }_{U p}
\end{array}\right]
$$

In this case the basic dissipative inequality may be enhanced, as suggested in Equation 4.10.1, to:
$\frac{d V_{O S}(x(t))}{d t} \leq u(t) \cdot y(t)$

Further, recall the innovative dynamism in our system captured by the general equation derived in Section 4.7:

The essential transition in this innovative dynamism is occurring from $x_{U}$ to $x_{T}$. As an organization approaches $x_{T}$ there will be a higher degree of information and order associated with it and it may be suggested that in its transition from $x_{U}$ to $x_{T}$ the storage energy $V_{O S}(x)$ will simultaneously increase. This may be expressed by Equation 4.10 .2 as:

$$
\frac{d V_{O S}\left(x_{U}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{T}(t)\right)}{d t}
$$

Eq 4.10.2: Transition Inequality from $x_{U}$ to $x_{T}$
Note that in replacing $\mathrm{x}(\mathrm{t})$ in the original form of the inequality by $x_{g}(t)$ where g is u or $t$, allows for multi-layer causal dynamics that is absent from a framing where only $\mathrm{x}(\mathrm{t})$ is considered. Philosophically this framing transcends the limit to explain phenomena at surface-layer $U$ by mechanics at the same layer only.

Further, and keeping in mind the six states of progression identified by $\operatorname{Sig}_{E}$, it may be suggested that in its journey from $x_{U}$ to $x_{T}$ progressively more complex storage functions mapped to each of these states are reached and traversed, as identified by the multiple inequalities depicted in Equation 4.10.3:

$$
\begin{aligned}
\frac{d V_{O S}\left(x_{U}(t)\right)}{d t} & \rightarrow \frac{d V_{O S}\left(x_{T}(t)\right)}{d t}: \\
\frac{d V_{O S}\left(x_{P}(t)\right)}{d t} & <\frac{d V_{O S}\left(x_{V}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{M}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{I}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{F}(t)\right)}{d t} \\
& <\frac{d V_{O S}\left(x_{C}(t)\right)}{d t}
\end{aligned}
$$

Eq 4.10.3: Transition Inequality Depicting Multi-Layer Causal Dynamics

### 4.11 Framing and Modeling Shifts in Innovation at Layer U

If innovation is assumed as generally being the breaking of certain existing patterns and the formation of new ones then it is should be possible to leverage Turing's activator-inhibitor equations (Turing, 1952) to frame shifts in innovation at the surface level, $U$. The notion of innovation as being the adoption of new patterns is consistent with Cognitive Psychologist Sternberg's model of creativity (Sternberg, 2012). In the 'Assessment of Creativity: An Investment Based Approach' he writes: "Creativity is a habit. Behind all innovations one finds creativity, so innovations arise from a habit. When I speak of a habit, I refer to 'an acquired behavior pattern regularly followed until it has become almost involuntary'. That is, creativity becomes a way of life that one regularly utilizes so that one is hardly aware one is engaging in it."

Turing suggested the use of an activator ( u ) and an inhibitor (v). The activator stimulates production of both entities, whereas the inhibitor inhibits it. Applying his equations $u$ may be thought of as being the 'new innovative pattern' and $v$ as being the 'existing pattern(s)'. The relevant system of equations, where the diffusion rates for $u$ and $v$ are $D_{u}$ and $D_{v}$ is then:

$$
\begin{aligned}
& \frac{\partial u}{\partial t}=f(u, v)+D_{u} \nabla^{2} u \\
& \frac{\partial v}{\partial t}=g(u, v)+D_{v} \nabla^{2} v
\end{aligned}
$$

In light of the preceding discussion it may be suggested that a shift between any of the ordered states at layer U may be 'framed' rather than 'modeled' by the preceding Turing equations. 'Modeling' will require integration or a mapping of the action of the meta-layers into the shifts being observed and is intended to capture the fuller set of causal dynamics. 'Framing' is a relatively static orientation that only captures the observed surface effects.

Recall the equation of innovative dynamism derived in Section 4.7:

$$
\text { Innovation }_{\text {orientation }-x}=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \operatorname{Sig}_{\mathrm{A}} \rightarrow F\right) \\
M_{1} \rightarrow \operatorname{Sig}_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T} \text {, where }\left[\begin{array}{ccc}
x_{U} \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

It may be suggested that the general transition from $x_{U}$ to $x_{T}$ can be framed at the layer $U$ by the Turing activator-inhibitor equations. Hence, replacing the ' $u$ ' or activator, or new innovative pattern by $x_{T}$, and the ' $v$ ' or inhibitor, or existing pattern by $x_{U}$ an equation, Equation 4.11.1, to frame shift in innovation at U is arrived at:

Shift $_{\text {innovation }}=\left(\frac{\partial x_{T}}{\partial t}=f\left(x_{T}, x_{U}\right)+D_{x_{T}} \nabla^{2} x_{T}, \frac{\partial x_{U}}{\partial t}=g\left(x_{T}, x_{U}\right)+D_{x_{U}} \nabla^{2} x_{U}\right)$
Eq 4.11.1: Shift in Innovation Leveraging Turing Activator-Inhibitor Equations
Note the similar philosophical point here as in the previous section. The Turing activator-inhibitor equations had been defined as operating at the surface layer only. Hence, the cause for the shifts occurring at $U$ are suggested as the actions of $u$ and $v$ which are themselves surface-layer causal-agents. By contrast the modified form that appears in Shift innovation $^{\text {contains in }}$ it the action implicit at meta-layers by replacing $u$ and $v$ by $x_{T}$ and $x_{U}$ respectively.

### 4.12 Framing Complexity

As per the discussion in Section 2.1.5.3 it is proposed that complexity of CAS is a function of the level of activity in a system, which will therefore increase as metalevels become active. As per the Working Hypothesis 2.1.5.3.1 generated earlier in Section 2.1.5.3, complexity in a system may be thought of as a function of multiplicity (the number of interacting components), interdependence (how connected those elements are), and diversity (the degree of heterogeneity). Hence:

$$
\text { Complexity } \left._{\text {System }}=f \text { (Multiplicity, Interdependence, Diversity }\right)
$$

Recall that the general scheme of meta-levels is encapsulated by the core-matrix:
$\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow S_{\text {System }_{X}} \\ \left(\uparrow \text { Sig }^{\rightarrow} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x)}\right. \\ U \rightarrow x_{U}\end{array}\right]$
If the untransformed or U layer is strongest, implying that the habitual patterns that keep an organization locked into its untransformed way of operation are still very active, then the nature of the system will be $x_{U}$.

With $x_{U}$ active the complexity of the system can be thought of in the following manner:

- Low diversity: Untransformed physical, vital, mental, and integral elements that are the actors in this system are basically homogeneous since habitual patterns keep behavior locked in place. Taking the example where $\mathrm{x}=$ physical, this homogeneity is described in the following manner:

$$
\text { Physical }_{U} \ni[\text { inertia,lethargy, status quo, ...] }
$$

- Low multiplicity: Being homogeneous the number of interacting components is lower than what it can be.
- Low interdependence: These elements are not sufficiently unique to have become organizing forces, and therefore the level of interdependence will also be low

If the habitual patterns have been overcome the dynamics of $M_{1}$ or $\operatorname{Sig}_{x}$ are now active. The dynamics of $\operatorname{Sig}_{X}$ have now become active which opens the possibility to a vast number of heterogeneous elements. These elements are all organized enough that they will cause interdependence. Considering the case where $\mathrm{x}=$ physical:

Sig $_{P}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Where $S_{S_{S S t e m_{P r}}}$ is an infinite set with forces of the nature specified in the following manner:
$S_{S y s t e m_{P r}} \ni$ [Service,Perfection, Diligence, Perseverance, ...]
Hence, the complexity of the system will increase with the increase in multiplicity, interdependence, and diversity.

If the unique 'signature' has become a 'force', then the conditions for activation of $M_{2}$ have been put in place and the system state will be $S_{\text {System }_{X}}$. The architectural forces active in $M_{2}$ are by definition more powerful than $\operatorname{Sig}_{x}$ that is a derivation of a set of such architectural forces.

If the 'force' so acting becomes impersonal so that an organizational ego-state is overcome, then the system state will have the most strength and is characterized by System $_{X}$ active at $M_{3}$.

Hence, it is seen that in general as the higher meta-level becomes active, the system becomes more complex as measured by the function Complexity System .

But a function $D I_{V}$, had already been derived in Section 4.9 that indicates which level in a system is active:
$D I_{V}\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow S_{\text {System }_{X}} \\ \left(\uparrow \text { Sig }_{\text {a }} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x}\right. \\ U \rightarrow x_{U}\end{array}\right]=>$ x-state $\in\left(x_{U}\right.$, Sig $_{x}$, S System $_{X}$, System $\left._{X}\right)$
As a reminder the application of $D I_{V}$ will yield the nature or 'strength' of the state ( x ) or orientation under consideration. If the untransformed or U layer is strongest, implying that the habitual patterns that keep an organization locked into its untransformed way of operation are still very active, then the nature of the output of $D I_{V}$, notated by x-state, will be $x_{U}$. If the habitual patterns have been overcome then the strength of the x-state increases since it is the dynamics of $M_{1}$ or $\operatorname{Sig}_{x}$ that are now active. In this case the x -state will be $\mathrm{Sig}_{x}$. If the unique 'signature' has become a 'force', then the conditions for activation of $M_{2}$ have been put in place and the x-state will be even higher, $S_{S_{s y t e m_{X}}}$. The architectural forces active in $M_{2}$ are by definition more powerful than $\operatorname{Sig}_{x}$ that is a derivation of a set of such architectural forces. If the 'force' so acting becomes impersonal so that an organizational egostate is overcome, then the x -state will have the most strength and is characterized by System $_{X}$ active at $M_{3}$.

Hence, a modified equation, Equation 4.12.1, for system complexity can be arrived at, where:

Complexity $_{\text {System }} \propto D I_{V}$
Eq 4.12.1: System Complexity
Now, relating this to the notion of complicated versus complex systems, it may be said that complex systems (CAS) are basically systems in which meta-levels have become active. In complicated systems the degree of interdependence is by definition low (Miller \& Page, 2007). The degree of interdependence in turn could be low because no element has yet become a sufficient center of organization. In complicated systems it is likely that meta-levels are not active, even though there may be an incubation to make them more active. Or it could be the case that they were once active but have since become inactive because of having reached a plateau of stability. Technological dynamism creates high network connectedness versus technological stability that creates low network connectedness (Tatarynowicz et al., 2015). As meta-levels become active it can be seen that multiplicity, interdependence, and diversity naturally increase. Complicated
systems could be viewed as an elementary stage in the formation of complex systems.

Considering the function $\operatorname{Sig}_{E}$ this level of maturity may be linked to the emergencematrix within Sig $_{E}$. Recalling Sig $_{E}$ :
$\operatorname{Sig}_{E}=X\left|\begin{array}{c}C: \operatorname{Sig} * \bmod \left(\int=1\right) \\ F: \operatorname{Sig} \bmod (c) \\ I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\ M: \operatorname{Sig} * \bmod (G) \\ \text { V:Sig } \bmod (e) \\ P: \operatorname{Sig} * \bmod (\pi)\end{array}\right|$
The Emergence-Matrix, as in Equation 4.12.2, can be thought of as:
Emergence-Matrix $=\left|\begin{array}{c}C: \operatorname{Sig} * \bmod \left(\int=1\right) \\ F: \operatorname{Sig} \bmod (c) \\ I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\ M: \operatorname{Sig} * \bmod (G) \\ V: \operatorname{Sig} * \bmod (e) \\ P: \operatorname{Sig} * \bmod (\pi)\end{array}\right|$
Eq 4.12.2: Emergence-Matrix
Keeping this emergence-matrix in mind, it may be said that a complicated system is one in which the signature is at a $\mathrm{P}, \mathrm{V}$, or M level. Hence, as depicted by Equation 4.12.3, where ' $I$ ' refers to the 'integral' level in equation 4.12.2:

Complicated System $=\operatorname{System}\left(\operatorname{Sig}_{E}: X<I\right)$
Eq 4.12.3: Complicated System
By contrast a complex system, as in Equation 4.12.4, can be thought of as:
Complex $_{\text {System }}=$ System $\left(\right.$ Sig $\left._{E}: X \geq I\right)$
Eq 4.12.4: Complex System
4.13 Summary

In this chapter a conceptual analytical framework to define a mathematical kernel of innovation in CAS is constructed. The mathematical kernel is expressed as a series of derived equations arrived at inductively. The range of equations varies from the micro, defining the nature of a point in any CAS system, to the macro, encompassing dynamism and innovation in any CAS taken as a whole. Given that subsequent equations are built on previous ones there is a high degree of cohesiveness between the set of equations used to enumerate innovation in CAS.

Further, existing notions of emergence and randomness already described in Sections 1.1, 1.2, and 1.3, are reframed through proposing a 'qualified determinism' and a mathematical working out of how this may operate in Section 4.9. Existing equations leveraging Prigogine's inequality are enhanced by including mathematical distinctions in arriving at the model for innovation in CAS to suggest subtleties in organizational transitions in general. Similarly Turing's activator-inhibitor equations are also enhanced to suggest how the process of shift in innovation may occur when perceived from level $U$ only.

The derived equations and mathematical contributions in this chapter are summarized in Figure 4.13.1:

| $\#$ | Summary of Derived Equations \& Mathematical Contribution |
| :--- | :--- |
| 1 | Bias of Complex Adaptive Systems |
| 2 | Nature of a Point in CAS |
| 3 | Architectural Forces in CAS |
| 4 | Uniqueness of Organizations |
| 5 | Emergence of Uniqueness of Organizations |
| 6 | Varying Culture of Organizations |
| 7 | Inherent Dynamics of CAS |
| 8 | Stagnation \& Dynamic Growth in CAS |
| 9 | Qualified Determinism in CAS |
| 10 | Framing Organizational Transitions Leveraging Prigogine's Inequality |
| 11 | Framing \& Modeling Shift in Innovation Leveraging Turing's Activator- <br> Inhibitor Equations |
| 12 | Framing Complexity in CAS |

Figure 4.13.1 Summary of Derived Equations \& Mathematical Contributions in Chapter

These mathematical equations potentially shed insight into key attributes of the desired theory summarized in Figure 1.3.1:

- CAS as being characterized by qualified determinism at each level of organizational complexity
- This qualified determinism as being orchestrated by a cohesive mathematical framework
- This mathematical framework as explaining the spectrum of possibility from stagnation to sustainability / progress
- The mathematical framework and the derived equations as providing a basis for innovation in CAS
- The field of innovation as being advanced by creating a series of mathematical equations to better understand innovation
- The mathematical framework as providing insight into further potential development at any level of organizational complexity
- The mathematical framework as applicable to multiple-levels of complexity, from the quantum to the astrophysical

By virtue of the derived set of equations being applied to CAS at various scale and levels of complexity, ranging from the quantum level to the living cell and beyond, it is proposed that the derived set of equations is universal across CAS and in Chapter 5 and beyond these equations will be applied to gain further insight into each of the areas and cases under consideration.


CHAPTER 5: Theoretical Model Application
The model developed in the previous chapter is applied to several CAS domains in this chapter. Since the developed mathematical model for innovation in CAS is derived inductively, and therefore is subject to the failing of monotonicity (Vickers, 2016), application areas where viable insights are suggested by the use of deductive logic arguably becomes more important. Such an approach of induction followed by deduction is implicit in theory and model creation and has reportedly been employed by scientific thinkers such as Newton and Einstein. In Newton's case the theory of gravitation is linked to an actual event of an apple falling from a tree (Stukely, 1752). In Einstein's case, a compass needle pointing 'North' made him imagine a unified field that existed behind observable phenomena (Isaacson, 2008). Einstein, through thought experiments often originating in such observations, laid down the theoretical foundations of the modern age through quantum and relativity theory, and these were only proven by further experiment in specific domains often decades later. In this dissertation the application of the theoretical model to the target domains allows for some confidence building in the model itself.

CAS is suggested to exist at multiple levels of scale and complexity as already discussed in Sections 1.1, 1.2, and 1.3. Hence, the following criteria are used in the approach to selecting possible CAS application domains:

1. First start with a CAS application domain that has already been subject to a highlevel of research. While the living cell and the related areas of medicine and health continue to receive mega-funding, as apparent by the increase in the US NIH funding (Kaiser, 2015) it is interesting to see that even in areas such as Computer Science, research related to the living cell is emerging as a top area. For example, in an article in Forbes on top funded areas in Computer Science, bio-informatics appears as number three (Markov, 2015). In Wired magazine (Marlow, 2013) an article on the ten 'hottest' fields of science research points to DNA-related research as number three to better understand the code of life that necessarily intersects with information processing and computer science. The Living Cell is hence selected as the starting point for the application of the mathematical model of innovation derived in Chapter 4.
2. Move to other domains at different scales so that it can be shown that the mathematical model for innovation in CAS exists at multiple scale and levels of complexity. Hence the quantum level is selected, first for an application of some aspect of the mathematical model of innovation to properties considered to be true of the quantum level in general. Second to explore an alternative scheme for the characterization of quantum particles that is consistent with the suggestions that emerge when considering the cellular level.
3. Move to a level of complexity between the quantum and cellular level. This would be the atomic level and specifically the properties of elements as characterized by the Periodic Table, where also a large amount of research has already been conducted. The Wired magazine article on the ten 'hottest' fields of science research (Marlow, 2013), for example, points to energy and superconducting both involving atom-level research.
4. Having potentially observed similarity across these three areas of increasing scale and complexity - quantum, atomic, cellular - select the meta-area of CAS itself to draw insight into additional properties of CAS.

Hence, considering the target application domains of the cellular level, the quantum level, the architecture of quantum particles, the periodic table, the drawing of additional insight into CAS, and the further derivation of general properties of CAS themselves there are 6 "test" cases used in applying the model as summarized in detail in Sections 3.1, 3.2, and 3.3. Note that the mathematical model of innovation in CAS can also be applied to other areas of increasing scale and complexity. Hence in Chapter 7 it is applied to the corporate level with a case study at Stanford University Medical Center. This case study will illustrate how innovation can be framed and its management approached to thereby practically suggest an approach to the management of innovation.

As a reminder the research instrument being focused on in this chapter is "Simplification", as highlighted in Figure 5.1. Figure 5.1 adapted from Figure 4.1, laid out the relationship of the research instrument to the mathematical model.

| \# | Research Instrument | Code | Relationship <br> to Chapter 4 |
| :---: | :--- | :--- | :--- |
| RI1 | Cohesiveness of derived <br> mathematical equations | "Cohesiveness" | Model derived <br> (Chapter 4) |
| RI2 | Further insight and simplification of <br> properties in target domains | "Insight and <br> Simplification" | Dependent <br> (This chapter) |
| RI3 | Relationship of adjustable parameters <br> to system innovation | "Parameter <br> Relationship" | Dependent <br> (Chapter 6) |
| RI4 | Ability of derived mathematical <br> equations to frame organizational <br> change and innovation | "Frame <br> Change" | Dependent <br> (Chapter 7) |
| RI5 | Cohesiveness of integration of piece- <br> meal mathematics into main model | "Integration" | Dependent <br> (Chapter 8) |
| RI6 | Addressing of generated hypotheses <br> in the literature review | "Addressing |  |
| Hypotheses" | Dependent <br> (Chapter 8) |  |  |
| RI7 | Ease with which established theory <br> suggests structure of mathematical <br> model | "Established |  |
| Theory" | Dependent <br> (Chapter 8) |  |  |

Figure 5.1 Highlighting "Insight \& Simplification" as the Research Instrument of Chapter 5

An assessment of the ability of the mathematical model derived in this dissertation to simplify the domains under consideration will be made in Chapter 8, Evaluation.

### 5.1 Application of Generalized Equation of Innovation at the Cellular Level

This examination will begin by looking at the molecular structure at the cellular level. In 'The Machinery of Life', Goodsell, an Associate Professor of Molecular Biology at the Scripps Research Institute (Goodsell, 2010) suggests that every living thing on Earth uses a similar set of molecules to eat, to breathe, to move, and to reproduce. There are molecular machines that do the myriad things that distinguish living organisms that are identical in all living cells. This nanoscale machinery of cells uses four basic molecular plans with unique chemical personalities: nucleic acids, proteins, lipids, and polysaccharides.

It is useful to take a deeper look into each of these molecular plans.
Nucleic acids basically encode information. They store and transmit the genome, the hereditary information needed to keep the cell alive. They function as the cell's librarians and contain information on how to make proteins and when to make them.

They are hence, the keepers of a cell's knowledge, its wisdom, its ability to make laws, the vehicle to spread knowledge within cells and to the next generation of cells. Being so, one can see that there is similarity with the set for systemknowledge highlighted earlier in Section 4.3. Reproducing Equation 4.3.3:
$S_{\text {System }_{K}} \ni$ [Knowledge,Wisdom, Law Making,Spread of Knowledge ...]
Nucleic acids can therefore be thought of as a precipitation of system-knowledge at the cellular level.

Proteins are the cells work-horses. Look anywhere in a cell and one will see proteins at work. Proteins are built in thousands of shapes and sizes, each performing a different function. As Goodsell describes, "some are built simply to adopt a defined shape, assembling into rods, nets, hollow spheres, and tubes. Some are molecular motors, using energy to rotate, or flex, or crawl. Many are chemical catalysts that perform chemical reactions atom-by-atom, transferring and transforming chemical groups exactly as needed." With their wide potential for diversity, proteins are constructed to perform most of the everyday tasks of the cells. In fact human cells build around 30,000 different kinds of proteins to execute on the diverse array of cellular level tasks.

Proteins hence, exist for service, to bring about perfection at the level of the cell, are characterized by extreme diligence and perseverance, and so on. Being so, one can see that there is similarity with the set for system-presence highlighted earlier in Section 4.3. Reproducing Equation 4.3.1:
$S_{S y s t e m_{P r}} \ni$ [Service,Perfection,Diligence,Perseverance, ...]

Proteins can therefore be thought of as a precipitation of system-presence at the cellular level.

Lipids by themselves are tiny molecules, but when grouped together form the largest structures of the cell. When placed in water lipid molecules aggregate to form huge waterproof sheets. These sheets easily form boundaries at multiple levels and allow concentrated interactions and work to be performed within a cell. Hence, the nucleus and the mitochondria are contained within lipid-defined compartments. Similarly, each cell itself is contained within a lipid-defined boundary.

Lipids are therefore promoters of relationship, of harmony in the cell, of nurturing the cell-level division of labor, of allowing specialization and uniqueness to emerge, hence perhaps of earlier forms of compassion and love, and so on. The notion of such early forms of compassion is consistent with the biologist's perspective that at some point a gene for compassion was developed in pre-human species (Wright, 2009). Being so, one can see that there is similarity with the set for systemnurturing highlighted earlier in Section 4.3. Reproducing Equation 4.3.4:
$S_{\text {System }_{N}} \ni$ [Love,Compassion,Harmony, Relationship ...]
This function of harmonization suggests that lipids can therefore be thought of as a precipitation of system-nurturing at the cellular level.

Polysaccharides are long, often branched chains of sugar molecules. Sugars are covered with hydroxyl groups, which associate to form storage containers. As a result polysaccharides function as the storehouse of cell's energy. In addition polysaccharides are also used to build some of the most durable biological structures. The stiff shell of insects, for example are made of long polysaccharides.

Polysaccharides function to create energy, power, courage, strength thereby readying the cell for adventure, and so on. Being so, one can see that there is similarity with the set for system-power highlighted previously in Section 4.3. Reproducing Equation 4.3.2:
$S_{\text {System }_{P}} \ni$ [Power,Courage, Adventure,Justice, ...]
Providing energy and strength, polysaccharides can be thought of as a precipitation of system-power at the cellular level.

Hence, one can see that the four-fold intelligence that exists in a point appears to also order the functioning of a cell, and therefore it is possible that the generalized equation for innovation may also apply at the level of the cell. As explained by Cohen and Harel in "Explaining a complex living system: dynamics, multi-scaling
and emergence" (Cohen \& Harel, 2007), 'The environment of the living system is a most critical source of information. True, DNA serves as a special repository of information because it is replicated and transmitted across generations, but DNA is meaningless without the proteins and other molecules that selectively activate segments of the DNA sequence in variable and alternative ways to create genes. The activation of specific genes emerges from the dynamic state of the cell. One could argue that DNA is just as much a servant of the cell's state as it is the cell's master.'

If so, a further precipitation of the model for innovation after the 'architectural forces' specified by the sets, would be as unique organizational signatures of the cellular-based nanoscale machinery.

Hence, nanoscale machinery could have a generalized signature derived from the generalized equation of organizational uniqueness. Note that such an equation is function rather than form based and suggests that any nanoscale machinery is driven by a unique combination of functionality that will in turn determine how it is assembled using the right molecular plans. Reproducing Equation 4.4.5:

Sig $=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{S A S t e m ~_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers; } a>b\end{array}\right]$

Hence, a protein could have a generalized signature, as in Equation 5.1.1, derived from the system-presence family:

Sig $_{\text {protein }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\left.S_{s y t e m_{P r}}\right]}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{S_{y s t e m_{P}}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq 5.1.1: Generalized Signature of Protein
This could yield a vast number of functional proteins. In fact it may be possible that the 30,000 or so known proteins created by the human cell could each be specified by a signature equation of this nature. It may be possible to map existing proteins to functionality as suggested by the four sets of molecular plans.

Using the same logic it is also possible that there could be more nucleic acids, more lipids, more polysaccharides, and even more proteins than are currently known or currently exist.

Hence, a nucleic acid could have a generalized signature, as in Equation 5.1.2, derived from the system-knowledge family:


Eq 5.1.2: Generalized Signature of Nucleic Acid
Lipids could have a generalized signature, as in Equation 5.1.3, derived from the system-nurturing family:

Sig $_{\text {lipid }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.1.3: Generalized Signature of Lipid
Polysaccharides could have a generalized signature, as in Equation 5.1.4, derived from the system-power family:

Sig $_{\text {polysaccharide }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.1.4: Generalized Signature of Polysaccharide

It could be that the genome's reportedly vast tracts of non-coding DNA is reserved for the advances that will inevitably take place as more and more possibilities expressed by such generalized equations come to bear.

This notion of further adaptability implies that the set $x_{U}$ in the "untransformed" layer U , is likely a 'relatively' as opposed to an absolutely untransformed set, and therefore the starting point for a more sophisticated transformation, which will in turn become the starting point for a further transformation. If so, then the generalized form of the equation, as in Equation 5.1.5, would become:

## Innovation orientation $-x$

$$
=\left(\left[\begin{array}{c}
\left(\uparrow \rightarrow \text { System }_{X}\right. \\
(\uparrow \rightarrow I) \\
M_{2} \rightarrow \text { S System }_{X} \\
\left(\uparrow \operatorname{Sig}_{3} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] T C \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
$$

Eq 5.1.5: Evolving Form of Generalized Equation of Innovation
Here $\left\langle x_{U} \mid x_{T}\right\rangle$ signifies that $x_{T}$ becomes the new starting point $x_{U}$ as the process of adaptability continues.

A key difference at the cellular level will likely be in the way the Transformation Circle, TC, functions. The TC was introduced and discussed in Section 4.7. Essentially it models a process for the varying rate of transformation with which the dynamics of innovation of any system may proceed.

Hence, for complex behavioral systems whether humans, teams, corporations, markets, or systems, the transformation of the untransformed set, $x_{U}$, proceeds in the direction from the untransformed layer, U , through the meta-levels, to $M_{3}$. The trigger for this journey is the overcoming of habitual patterns for a specific orientation: $>P_{x}$. The notion of choice or free-will is therefore inherent. No transformation begins without the organization initiating the trigger conditions. Further, as already suggested in Section 4.7, the synthesis or integration is modeled as a function of $\sin , \mathrm{e}$, and $\pi$.

At the cellular level though it seems that the synthesis between layers happens from $M_{3}$, through the meta-levels, to the outer or untransformed layer, U. One can conclude this because there already appears to be a high degree of perfection at the cellular level, and this is put in place automatically, as it were. There does not appear to be free-will or choice as the initiating trigger, in the way that it occurs for a complex organization.

Hence, the Transformation Circle, TC, appears to fulfill the synthesis in another way. Using a system of polar coordinates, since the TC is envisioned as 4 concentric circles, then the radius ' $r$ ', measuring from the core to the untransformed layer, indicates the extent of the synthesis. With a shorter ' $r$ ' extending only across one or two layers the synthesis is incomplete. When extended across all concentric circles, the synthesis is complete. The time taken to achieve this synthesis is measured by
$\theta$. If it is instantaneous, then $\theta=0$. If it occurred in full-cycle it would be $\theta=360$, and so on.

Once the synthesis is complete, then the outer layer automatically follows certain rules imposed on it by the meta-layers. This action in organizations, such as the cell, is in stark contrast to the complex organization where the rules need to be changed through deliberation.

The quaternary basis of cell operation suggests that the generalized mathematics of innovation may also be true at the cellular level, which after all is also an instance of organization. If so, there are several implications:

1. There are slight adjustments to the generalized equation for innovation so that the untransformed layer and set are 'relatively' as opposed to absolutely untransformed or transformed, depending on which end of the spectrum the process of transformation is perceived from. In other words, transformation of the untransformed set happens iteratively
2. The Transformation Circle (TC) suggested as the process by which synthesis of different layers of organization occurs proceeds slightly differently at the cellular as opposed to the complex behavioral organizational level
3. All nanoscale machinery operative at the cellular level can be thought of in terms of function, which in turn will specify form. The function itself can be derived from cellular-level 'architectural forces' that parallel the general sets of architectural forces as modeled for the larger containing system
4. Organizational signature equations seeded on cellular-level architectural forces will further likely specify a range of nanoscale cellular machinery that may not exist today. Adaption of the genome and of the constituents of the known quaternary molecular plans will likely express the suggested 'missing' machinery with time 5. Quaternary based mathematics of innovation may also suggest advances to cellular level medical technology some time in the future
5. Quaternary based mathematics of innovation may further suggest construction of synthesized nanobots some time in the future

### 5.2 Application at the Quantum Level

A quaternary basis for organization was the starting point for arriving at a generalized equation of innovation for organizations first introduced by Equation 4.7.6 in Section 4.7, and then further modified by Equation 5.1.5 in Section 5.1. The organizations being considered were larger common organizations of various types. In Section 5.1 it was suggested though that the cell too may have a quaternary basis of operation where each basis is similar to the bases in the generalized equation. Based on this insight it is suggested that the generalized equation for innovation can also be applied at the cellular level. A general logical question then is can this same equation for innovation be applied orders of magnitude smaller, at the quantum level as well as suggested in Figure 1.3.1 that summarizes the key attribute of the desired theory?

In his book QED: The Strange Theory of Light and Matter (Feynman, 1985), Feynman states that the theory he presents will not explain why or how Nature acts the way it does, but will explain with very high accuracy the probability that a photon emitted from a monochromatic light source is detected by a photon detector. Brian Clegg, in his book, The Quantum Age (Clegg, 2014) relates how Feynman in a public lecture about quantum particles says: "You think I am going to explain it to you so you can understand it? No, you are not going to be able to understand it. Why, then, am I going to bother with all this? Why are you going to sit here all this time, when you are not going to be able to understand what I am going to say? It is my task to persuade you not to turn away because you do not understand it. You see, my physics students don't understand it either. This is because I don't understand it. Nobody does."

Feynman jokingly goes on to suggest that the particles route could be absurd - going around Jupiter, to the local hot-dog stand, before reaching the detector. But this absurdity has been generalized and has become an edifice by which quantum nature is now framed and understood. Even Einstein has said, "God does not play dice with the Universe". If theories and models have to be invented in which it appears that dice is being played, then perhaps the models are not quite correct. This may warrant looking at alternatives to explain some of the quantum-level explanations that have been proposed, whether for fusion in the sun, or enzyme action at the level of the cell, amongst others.

Figure 2.2.1, illustrating the link between the key attributes of the desired theory and the working hypotheses and design principles, highlights a number of working hypotheses related to the key desired attribute of meta-level functionality bringing about adaptiveness at the surface layer. These hypotheses suggest that at the heart of the absurdities lies one related proposition: this is simply that all of life is purely physical in its orientation and by corollary that science cannot remain science unless it explains all things solely on the basis of what the eye can see, even if through a microscope or telescope or some other similar single-level lens.

Physics has, as has all of science, tended to take a solely physical approach to explaining phenomena. Hence it is believed that everything is emergent and therefore in an almost randomized way, as it would likely have to be if the source of causes is conceived as arising from a single and sole layer of reality, as opposed to a possible set of distinct layers. Absurdity has to arise when there is the artificial compression of meta-level action into one level without even recognizing the impetuses that exist from possible meta-levels. Probability functions and uncertainty principles and equations have to be erected when distinctions of action caused by principles of organization that are non-physical in nature are suppressed or ignored. Ignorance can be easily swept under a probability or uncertainty function that is then erected as new knowledge. It is then easy to come to believe that the nature of reality is the nature of the last model that has been proposed to deal with this lack of knowledge, and therefore questionable edifice upon
questionable edifice continues to be built often without going back to first principles. No matter how sophisticated or accurate any model may prove to be it will be easier to advance knowledge if it is remembered that any knowledge is itself a model. As Nassim Taleb in the Black Swan (Taleb, 2010) suggests: "You view the world from within a model."

This notion of building edifice upon edifice is somewhat reminiscent of observations made by Joseph Weizenbaum, the MIT computer scientist, in his book Computer Power and Human Reason in writing about his experiments with ELIZA, a natural language processor he had developed (Weizenbaum, 1967). He states: "This reaction to ELIZA showed me more vividly than anything I had seen hitherto the enormously exaggerated attributions an even well-educated audience is capable of making, even strive to make, to a technology it does not understand."

But as is being proposed in the generalized equation for innovation in Section 4.7, there is a multi-layered action and meta-levels that have a profound impact on physical phenomena. If once the existence of these meta-layers is admitted, and of the dynamics of organization that prevail for example at $M_{1}, M_{2}$, and $M_{3}$, as proposed in this dissertation, then there are a range of alternative models and theories that can be created, and it is likely that a vastly different set of implications for the management of technology involving quanta may arise.

By way of summary, the following is the core-matrix at the heart of the generalized equation for innovation already derived in Section 4.7, which will be referred to in proposing alternative explanations:

$$
\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \text { Sig }^{\rightarrow} F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right]
$$

Hence, it is being suggested that just behind the untransformed or physical layer, U , there is a meta-level, $M_{1}$, in which the uniqueness of any organization is specified. The specification itself is derived from further meta-levels, $M_{2}$ and $M_{3}$, which precipitate in infinite ways yet assuring overall system coherence.

This notion of the uniqueness of organization as determined by a meta-level, even in the case of the quantum world, can perhaps be brought home by a thought experiment:


Consider the simplest of atoms - the hydrogen atom. The nucleus comprises one proton around which encircles one electron. Envisioning this atom one can think of a baseball stadium as being representative of the atom. But the central nucleus, in this case the proton, is the size of a baseball. The electron is flying around this baseball at high speed to create an image of the stadium. But one can see in this visualization that the atom mainly comprises empty space. If it is being proposed that the diversity of individual elemental characteristics is determined by the simple possible combinations of protons and electrons and other possible fundamental particles that mainly occupy empty space, then can't it also be proposed that similar structures such as baseball stadiums with a single baseball at the center, or rather all the different structures from the micro to the macro with similar spatial characteristics would also then express different functionality that would then create our reality?

But that then would create complete chaos as one diverse configuration and therefore functionality countered another. This is clearly not the case and therefore form by itself should not determine function. In fact in several design disciplines such as architecture (DeZurko, 1952) and software engineering (Martin, 2002), form is intended to follow function. And if any CAS has a mathematical kernel of innovation as is being proposed in this dissertation then developments in that system are the outcome of such design and form hence will likely follow function.

Rather keeping the core-matrix in mind perhaps it may be the case that the unique signature of an element, as determined by $\operatorname{Sig}_{x}$ at $M_{1}$, precipitates in a self-similar configuration fixed for it by the unique atomic configuration for an element as specified by its atomic weight (number of protons in the nucleus). The unique configuration of an atom as determined by its atomic weight then becomes a "switch" by which the associated function from any possible number of functions as determined by $\operatorname{Sig}_{x}$ precipitates and determines the character of the element.

As things get very small so that limits are approached it stands to reason that if there are indeed multiple layers of organization, something of the influence of the meta-levels should be more directly observable even under conventional techniques. If system models are still strapped to a sole single physical level, then any impetus or influence from a meta-level is therefore likely going to more easily be misinterpreted or misunderstood. Compensation mathematics and equations will then have to be erected to explain the misunderstandings away.

Schrodinger's equation, which seeks to model how a quantum state of a quantum system changes with time, is perhaps an example of this.

Consider the original time-dependent form (Paul, 2008):
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi=\widehat{H} \psi$
$\psi$ depicts a wave form and can be thought of as a probable cloud of possible states. $\widehat{H}$ is the Hamiltonian operator which is a focusing function, and in its essence what the equation may be suggesting is that the way a wave form changes over time is equivalent to some expressible state of the possibilities inherent in the cloud of possible states. Note that the expression for time is implicit in this equation but can be expressed in various ways depending on how many particles, ranging from 1 to many, in how many dimensions, ranging from 1 to many, one is seeking to model.

But the cloud of possible states is another way of saying that one really does not know what is going on because possibly, how a meta-level may influence action at the observed level has never been considered. If the existence of the meta-levels, and in this case, of $\operatorname{Sig}_{x}$ at $M_{1}$, is considered possible, then it is far more reasonable to admit that form is configured by function and the very dynamics of what appears to be random may now appear to be far more logical. There is now more context to interpreting observation at the quantum level.

Schrodinger himself had misgivings about the applicability of this equation that seemed to apply at the quantum level, to the macro-world (Stewart, 2012). To bring his misgivings to light he invented a thought experiment concerning a cat. This cat would be in a superposed state in a quantum black-box. A radioactive particle, a decaying-particle detector, and a flask of poison, were the other inhabitants of the black-box. At some point the particle will decay, be detected, and as in the thoughtexperiment at that point, triggered by the decaying particle, the poison in the flask would be released. The cat would then die. But in the meanwhile the cat would be in a superposed states of being both dead and alive. Only when the box was opened would the wave function collapse and a single definite state emerge.

Schrodinger was hoping to highlight the absurdity of the application of having a cat in both a dead and alive state at the macro-level. Instead physicists found this thought experiment to be sensible, even at the macro-level, and began to generalize the findings based on this. Hence, for example the idea of superposition at the physical level began to be thought of as real. It is interesting to note that in his lectures on Schrodinger's equation Feynman (Gottlieb, 2013) has stated "Where did he get that [equation] from? Nowhere. It is not possible to derive it from anything you know. It came out of the mind of Schrödinger".

Even if Schrodinger believed his equation to explain aspects of quantum nature, he definitely did not believe that it was applicable at the macro-level. Here even the interpretation of this accepted equation in explaining quantum behavior is being questioned. The hypothesis here, and as per the math presented in this dissertation, is that the whole issue of increasing absurdity that Feynman referred to could have been avoided or minimized if the action of meta-levels as per the proposed model in this dissertation were somehow accounted for in initial modeling.

The following sections suggest alternative explanations to now commonly accepted quantum nature features in the light of the meta-level mathematical model proposed in this dissertation.

### 5.2.1 Dual wave-particle nature

Schrodinger's equation can be interpreted a little differently than proposed in the preceding section, Section 5.2: the wave aspect $\psi$ may actually be an indication that the unique function at the meta-level $M_{1}$ as specified by $S i g_{x}$, is going to assure itself one way or another, as apparent by the probability distribution of appropriate particles specified by $\psi$. In other words, whatever particles need to manifest to assure that the meta-level function is fulfilled, will manifest. The wave-particle nature is incomplete without reference to $\operatorname{Sig}_{x}$ at $M_{1}$. The wave and the particle are a child of the meta-level function and are in this way of looking at it incidental to what the meta-level function must achieve.

In "The Interpretation of Quantum Mechanics" (Schrodinger, 1995) a series of unpublished papers and talks, published posthumously by Schrodinger's daughter, he suggests that a wave has both a surface and rays, and these move together. The surface suggests the transverse movement of the wave, while the wave-ray the longitudinal movement observed as the particle. They are never separate and exist always together. This suggestion appears to be consistent with the interpretation of wholeness characterized by possible meta-level function that always accompanies the 'observed' or 'incidental' particle(s). In other words, a meta-level math model such as developed in this dissertation suggests that the layers $\mathrm{U}, M_{1}$, $M_{2}$, and $M_{3}$ exist simultaneously and the "duality" may be a "quadrality".

### 5.2.2 Independent states as specified by superposition

Given that emergent phenomena are in reference to a meta-level context, the superposition that Schrodinger's equation suggests does not define and set into motion manifest independent states, but only possibilities that the meta-level function may cause in fulfilling its implicit intent. The notion of multiverses and alternative histories of universe is in this interpretation of Schrodinger's equation, unnecessary. Therefore this may appear to be one of the unnecessary generalizations of quantum behavior at the macro-level that a number of physicists have assumed as true (Saunders et al., 2012).

### 5.2.3 Quantum tunneling

Quantum tunneling has been proposed as a mechanism by which quantum-sized particles can 'penetrate' boundaries that classical physics says it should not be able to (Clegg, 2014). This ability of a particle to manifest in a region as defined by the
wave-function that defines it, rather than by the apparent physical forces of attraction and repulsion that surround it, whether in a cell or the Sun for example, is what is termed quantum tunneling. It has been likened to a quantum teleportation of sorts.

Enzymes, for example, speed up chemical reactions so that processes can be completed orders of magnitude faster inside living cells. Quantum tunneling has been proposed as the way in which this happens so that electrons and protons can vanish from one position in a biomolecule and apparently rematerialize in another without passing through the gap in between(Clegg, 2014).

But suppose there exists a meta-level function, $\mathrm{Sig}_{x}$, which is the organizational signature of a necessary cellular level energy creation and monitoring function, Sig cellular-energy-type 1 in this case. In order to fulfill itself, it may be suggested that this function which exists just behind any surface visible range, oversees the movement or manifestation of electrons and protons and monitors cellular energy 'type1'. If protons and electrons are the visible sign or precipitation of this fundamental energy required for cell function, they can then be thought of as 'mapping' the path of this meta-level principle of organization. The wave hence depicts the very meta-level principle of energy-organization manifest as the probability that electrons and protons that serve that principle will show up in the locations suggested by the wave.

As Al-Khalili expresses in his book on Quantum Biology (Al-Khalili, 2014), that it is difficult to figure out how a packet of energy captured by a cell in the process of photosynthesis actually makes its way so unerringly through chlorophyll molecules to a structure called the reaction centre where its energy is stored. Khalili reports that in an experiment conducted in Berkeley in 2007 laser light was fired at photosynthetic complexes. The research proposed that the energy packets do not hop about randomly, but through quantum effects behave like a spread-out wave, sampling all possible paths, to thereby find the quickest one. But this precisely suggests wholeness as in Schrodinger's own view in his quantum mechanical interpretations (Schrodinger, 1994) and as suggested by the mathematical model for innovation in this dissertation.

Here hence a fine-tuned hypothesis is offered based on the generalized core-matrix. Basically when dealing with limits in nature, and here of the microscopic limits approaching Planck length, $l_{P}$, where the structure of space-time is dominated by quantum effects, this is where $M_{1}$, precipitates to the visible layer $U$. Hence many of the phenomena attributed to the physical layer $U$, are suggested to be the result of an infinite variety of organizational-functions as specified by $\operatorname{Sig}_{x}$ and occurring at $M_{1}$. Hence, the integrity of the physical layer, U , is intact, and there is nothing 'weird' or 'absurd' about it. What is being seen or observed though, are the effects of the meta-layers, where 'physics' operates differently, and is modulated by another
set of laws. It is perhaps fair to say that Science still has a way to go to uncover all those laws.

In his book "What is Life?" (Schrodinger, 1944) Schrödinger suggests that life is based on a principle whereby its macroscopic order is a reflection of quantum-level order, rather than the molecular disorder that characterizes the inanimate world (the idea of molecular disorder is consistent with the suggested equation, 4.8.3, for entropy in Section 4.8). He called this principle "order from order". He suggested that, unlike inanimate matter, living organisms reach down to the quantum domain and utilize its strange properties in order to operate the extraordinary machinery within living cells. This notion of reaching into or mobilizing a deeper-level order is consistent with the meta-level organizational principles presented here, and summarized in the generalized core-matrix. Only, in the model presented in this dissertation it is the deeper level order that mobilizes the surface order, especially when the orders are characterized by a higher degree of automaticity as discussed in Section 5.1, as in the case of the cell, and arguably in the case of quantum particles as well.

### 5.2.4 Canceling out of quantum dynamics

In his model on Quantum Electro Dynamics (QED) Feynman suggests that photons do not have to travel in straight lines. In fact, when emitted from a monochromatic light source, they will travel in every direction possible, while arriving at the photon detector. There is a reality of superposition in which all possible paths are traversed by photons. His model allows for combining all the paths together through vector addition to arrive at the path of the straight line recognized by classical physics. In other words, the quantum dynamics cancel themselves out so that one path emerges.

So whether at the micro or macro level vector addition results in a single path to which molecular, atomic, or photonic movement is subject. It is also interesting to note that at the macro-level there is a similar canceling out effect of random molecular motion that yet leaves a containing entity subject to some observed law. For example, if a gas is heated up, inspite of all movement of molecules that cancel one another out, yet the gas will expand in proportion to the applied heat and not in proportion to the apparent molecular motion. Such connection from microscopic behavior to macroscopic properties is the subject of statistical thermodynamics that deals with average properties of the molecules, atoms, or elementary particles in random motion in a system of many such particles (Ebeling \& Sokolov, 2005).

These observations are consistent with the idea of a possible existence of an 'organizational function', say 'movement from A to B in an apparent straight line' or the 'equivalence of applied energy', belonging to $M_{1}$ as the realm of signatures or organizational functions, to which organizations whether at the photonic, atomic molecular levels at $U$ are subject.

### 5.2.5 Traveling faster than the speed of light

It has been proposed that information at the quantum level is shared faster than the speed of light (Brumfiel, 2008). But the speed of light is a limit at the physical level, U. At $M_{1}$ though, it may be suggested that there exists a general organizationalfunction, $\operatorname{Sig}_{x}$, not limited by C, which may be a prime organizing factor in quantumparticle dynamics.

Philosophically, C sets a limit on the ability to transcend space-time. This is suggested by the equations of the alteration of time and space as a body approaches the speed of light. In his book on the special and general theory of relativity (Einstein, 1995), Einstein describes the effect, captured by these equations. Hence, as a body approaches the speed of light, it is perceived by an observer in another frame of reference to be contracting. This contraction - Einstein's Length Contraction - is specified by the factor:

Length $_{\text {contraction }}=$ Length $\left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)$
Hence, as the speed of an object, v, increases, the perceived body contracts in dimension. At v=c, the body basically disappears. In other words it can be thought of as having transcended the space continuum, or broken into another space-reality specific to the meta-levels. The length contraction, suggested therefore as being specific to $U$, is suggested by Equation 5.2.5.1. Recall that $U$, in the mathematical model derived in this dissertation, refers to the untransformed or visible layer which is subject to dynamics not only at the level of $U$, but also from each of the meta-layers, $M_{1}$ through $M_{3}$ :

Length $_{\text {contraction }-U}=$ Length $_{U}\left(\sqrt{1-\frac{v_{U}^{2}}{c_{U}^{2}}}\right)$
Eq 5.2.5.1: Length Contraction at Layer U (Leveraging Einstein Theory of Relativity)
Similarly, as a body approaches the speed of light, as perceived by an observer in a 'stationary' frame of reference, there is a time elongation, so that time moves much slower in the moving frame. This is specified at $U$ by the following equation, Equation 5.2.5.2:

Time $_{\text {elongation }-U}=\frac{\operatorname{Time}_{U}}{\sqrt{1-\frac{v_{U}{ }^{2}}{c_{U}{ }^{2}}}}$
Eq 5.2.5.2: Time Elongation at $U$ (Leveraging Einstein Theory of Relativity)

Here too, as v approaches c, time elongates representing a slow down in time at U , also similarly indicating the urge to transcend time as it is experienced at U .

From these equations it is also clear that space and time vary together and are 'one'. Strictly they need to be referred to as a space-time continuum. C is constant while time and space can vary. This also gives some insight into the nature of light, that it may be proposed creates the 'context' for operation at $U$. Hence it is that C sets a limit on the ability to transcend space-time.

In considering operations at $U$, it can be summarized that while time and space can vary based on velocity of a body, or in other words space-time experience can change relative to each body or organization at $U, C$ sets the limit to how much the experience of space-time can vary.

It can further be suggested that perhaps ' C ' is different at different levels in the mathematical model derived in this dissertation: that is, ' C ' at U is different than ' C ' at $M_{1}$ and so on. This inequality of $C$ can be specified, as in Equation 5.2.5.3, as:

$$
\text { Inequality }_{C}: C_{U}<C_{M_{1}}<C_{M_{2}}<C_{M_{3}}
$$

## Eq 5.2.5.3: Inequality of $C$

In "Slow Light" Perkowitz's recent treatment of today's breakthroughs in the science of light (Perkowitz, 2011) he states: "Although relativity implies that it's impossible to accelerate an object to the speed of light, the theory may not disallow particles already moving at speed C or greater." In the 1960's, Olexa-Myron P. Bilaniuk of Swarthmore College and E.C. George Sudarshan at Syracuse University began considering how to fit what they called "metaparticles" with speeds greater than C into the relativistic scheme. The approach was extended in 1967 by Gerald Feinberg (Feinberg, 1970), or Rockefeller and Columbia Universities, in his theoretical paper "Possibility of Faster-Than-Light-Particles," Feinberg also introduced the wonderful name "tachyons" for these hypothetical particles, from the Greek word "tachys" meaning swift." Perkowitz goes on to say how a flurry of papers have continued to appear about tachyons.

Such a variation in the speed of light would allow for different dynamics at each level in the model as proposed in Chapter 4. Hence a finite speed C by definition necessitates the reality of a past, a present, and a future. An infinite speed of light would allow for transcendence of the notion of space and time as experienced at $U$. Appendix 3, Integrating Light into the Equation of Space-Time Emergence, explores this link of multiple speeds of light with the model created in this dissertation. The resulting equation, A3.10, is reproduced here for illustration:


### 5.2.6 Entanglement

This quantum property has been invoked in describing how birds navigate across the earth (Al-Khalili, 2014). Assume though that there is an organizational-function $\operatorname{Sig}_{x}$ where $\mathrm{x}=$ 'flock migration in winter', for example. This is an $M_{1}$ dynamic and particular organizations at the physical layer are subject to it. Quantum particles may or may not have to be involved in this. As Khalili relates though, 'studies of the European robin suggest that it has an internal chemical compass that utilizes entanglement. This phenomenon describes how two separated particles can remain instantaneously connected via a quantum link. The current best guess is that this takes place inside a protein in the bird's eye, where quantum entanglement makes a pair of electrons highly sensitive to the angle of orientation of the Earth's magnetic field, allowing the bird to "see" which way it needs to fly.'

But also there have been experiments where a photon of light is used to create two entangled photons (Vivoli, 2016). These then share properties when separated. Once entangled, by whatever mechanism, it could be suggested that it is an organizational-function such as $\operatorname{Sig}_{x}$ that takes over. Space-time constraints are therefore changed, and a 'quantum' link is now in effect.

### 5.2.7 Going backward in time

In his book QED: The Strange Theory of Light and Matter, Feynman (Feynman, 1985) suggests that there are particles that can move backward in time as suggested in Figure 5.2.2. Illustrated is an electron, Electron-1, and a photon, Photon-1, moving towards one another. At some time Electron-1 decays into Photon-2, and Positron-1 (an electron with a positive charge) which then moves backward in time. It then appears to interact with Photon-1, and a new Electron-2 is created.


Figure 5.2.2 Particle Moving Backward in Time (Feynman, 1985)

If however, the existence of a meta-level is assumed, as per the mathematical model developed in this dissertation then spontaneous particle generation from a metalevel organizational-function (in this case organizational-function = "Positron-1 moving forward in time") could remove the notion of particles moving backward in time. As Stephen Hawking has said in A Brief History of Time (Hawking, 1988), if backward time travel were possible, why aren't there any visitors from the future?

### 5.2.8 Quantum Fluctuations

In his book, The Little Book of String Theory, Princeton University's Gubser (Gubser, 2010) describes the effect on approaching absolute zero temperature on molecules. He takes the example of water molecules and relates that one cannot make the water molecules colder than absolute zero, -273.15 Celsius, because there is no more thermal energy to suck out at that temperature. However, quantum uncertainty, the phenomenon which relates the momentum and location of electrons in atoms necessitates that the water molecules will still vibrate. Gubser suggests this by considering Heisenberg's uncertainty relation - Heisenberg's Uncertainty Principle:
$\Delta p X \Delta x \geq \frac{h}{4 \pi}$
Where $\Delta p$ is the uncertainty in a particle's momentum, $\Delta x$ is the uncertainty in the particle's location, and $h$ is the Planck's constant. In frozen water crystals it is precisely known where the water molecules are, and therefore $\Delta x$ is fairly small. This means that $\Delta p$ has to be considerably larger, and therefore that the water molecules are still vibrating even though they are at absolute zero. This innate
vibration, known as 'quantum zero-point' energy, expresses the phenomenon of quantum fluctuations.

As suggested earlier in Section 5.2.3, the Planck's constant order of magnitude (10${ }^{34}$ ) suggests the boundary between U and $M_{1}$ and the quantum fluctuations, the uncertainty relation, and the quantum zero-point energy could be an expression of the essential Signature function, $\operatorname{Sig}_{x}$, that is posited as a key formative force behind organization at $U$. In this interpretation the thermal energy describes the essential energy at $U$, while the uncertainty relation may suggest the phenomenon of innovation-precipitation "physically" linking $M_{1}$ and U . In this case it may be suggested that integration of meta-levels with the surface level, $I_{U}^{M}$, is indicated by the uncertainty relation, as in Equation 5.2.8.1:
$I_{U}^{M} \rightarrow \Delta p X \Delta x \geq \frac{h}{4 \pi}$
Eq 5.2.8.1: Integration of Levels (Leveraging Heisenberg's Uncertainty Relation)

### 5.2.9 Summary

So it appears to be possible, at least from the qualitative discussions and inferred inductions from literature, to interpret these afore-mentioned quantum effects superposition and existence of multiverses, dual-wave particle nature, quantum tunneling, traveling faster than the speed of light, entanglement, going backward in time, and quantum fluctuations, using an alternative model as presented here. The use of the alternative model allows physical reality to maintain its integrity regardless of scale, since the "weirdness" - superposition, dual-wave particle nature, etc. - experienced at the quantum level is possible through admitting the existence of meta-layers with a different "physics" existing at each meta-layer. It could be suggested therefore that the generalized equation of innovation arrived at the cellular level, Equation 5.1.5 (reproduced below), also applies at the quantum level, since the equation relates dynamics at the untransformed layer $U$ to dynamics at several meta-layers.

## Innovation orientation $-x$



If that is the case, then the next thing to consider is if the architecture of quantumlevel particles may also parallel the quaternary system being suggested here. This will be explored in the next section.

### 5.3 Architecture of Quantum Particles

The Standard Model of particle physics is a theory concerning the electromagnetic, weak, and strong nuclear interactions, as well as classifying all the subatomic particles known (Cottingham, 2007).

When sub-atomic particles were first discovered, progressively it was found that there were hundreds of them. Subsequently these particles were categorized by four properties: mass, spin, charge, and life-time to yield three fundamental types. These types were quarks, leptons, and bosons, as illustrated Figure 5.3.1:


Figure 5.3.1: Architecture of Quantum Particles

Quarks and leptons were believed to be the fundamental constituents of matter. There were six quarks and six leptons. Hence, a proton, for example, was composed of two "up" quarks and one "down" quark. Quarks have unusual names - up, down, charm, strange, top, bottom, with each respective pair belonging to a different generation. A neutron, for example, was composed of two "down" quarks and one "up" quark.

Electrons, part of the lepton family, in combination with the nucleus form all atoms. Hence the claim was made that quarks and leptons are the constituents of matter. All known matter particles are composed of quarks and leptons (Olive, 2014).

But if the classification is considered a little differently it can be observed that quarks are the only fundamental particles that contribute to creating the nucleus. Atomic number is specified by the number of protons in the nucleus. Atomic number in turn uniquely identifies the element from the periodic table. Hence, an
atomic number of 47 , for example, specifies that the element is Silver. In other words it can be suggested that the unique properties of an element, the knowledge of what it is and how it will behave in the universe, is related to the quark. It may be suggested that quarks, therefore, are associated with the precipitation of systemknowledge in the quantum world.

Even though electrons in combination with the nucleus constitutes all atoms, it therefore may make sense to separate electrons and therefore the whole family of leptons, since they are physically similar, into a different class inspired by the quaternary architecture. If the apparent characteristics of leptons are considered they appear to be point-like particles without internal structure (Olive, 2014). While quarks only exist in composite particles with other quarks, leptons are solitary particles.

The best-known lepton is the electron. In his book, Representing Electrons: A Biographical Approach to Theoretical Entities, Arabatzis (Arabatzis, 2006) details the characteristics of electrons. The electron may be considered as a surrogate for the lepton class. The electron appears to be the associated with the flow of energy and power. Further they appear to be the adventurers easily leaving the atom they are a part of. They also lock or form bonds with other atoms through the force of attraction and repulsion. In some sense they seem to be a representation or precipitation of system-power.

The Bosons are thought of as force-carriers. They are what allow all known matter particles to interact. The three fundamental bosons in this category are the photon, the W and Z bosons, and the gluon. The carrier particle of the electromagnetic force is the photon. The carrier particle of the strong nuclear force that holds quarks together is the gluon. The carrier particle for the weak interactions, responsible for the decay of massive quarks and leptons into lighter quarks and leptons, are the W and Z bosons.

Bosons can be thought of as the precipitation of what created relationship and harmony at the quantum level. Hence they can be thought of as the precipitation of system-nurturing.

This leaves the other discovered fundamental particle the Higgs-Boson. In ordinary matter, most of the mass is contained in atoms, and the majority of the mass of an atom resides in the nucleus, made of protons and neutrons. Protons and neutrons are each made of three quarks. It is the quarks that get their mass by interacting with the Higgs field (Olive, 2014). Hence the Higgs-Boson can be thought of as the mass-giver. In other words it is what gives presence to the quarks and it can be thought of as the precipitation of the system-presence. Just as there are multiple particles in each of the other 'families' it is likely that there will be multiple particles in the Higgs-Boson family. Recent research at CERN indicates that the Higgs-Boson may have a cousin (Overbye, 2015).

A suggested scheme based on the quaternary architecture hence, is illustrated in Figure 5.3.2:


Figure 5.3.2 Quaternary Architecture of Quantum Particles

This quaternary architecture has some possible implications for further particle research. Assuming that the quaternary architecture is a function of $M_{2}$, this implies the following for the other levels in the model:

1. There may be a Master-Particle that embodies the quaternary intelligence in itself, and is representative of $M_{3}$. Perhaps this is a material precipitation of a key organizing principle around which the quaternary aspects arrange and expand themselves. If this were the case recent thoughts on supersymmetry suggesting the deeper-level symmetrical equivalence of bosons and fermions (Kane, 2013) may be pushed even further to suggest the deeper-level symmetrical equivalence of all four categories of quantum particles.
2. Just as at the cellular level one can hypothesize a Sig function, perhaps there is a Sig function, Sig particle as in Equation 5.3.1 that would give more insight into the myriad of existing and yet undiscovered particles:

Sig $_{\text {particle }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{S_{\text {Sstem }}^{K}}, S_{S y s t e m_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{S y s t e m_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq 5.3.1: Generalized Signature of Particle

Hence, it could be that the signature for the family of quarks, as in Equation 5.3.2, is:

Sig $_{\text {quarks }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.\end{array}\right]$

Eq 5.3.2: Generalized Signature of Quarks

The signature for the family of leptons, as in Equation 5.3.3, is:
Sig $_{\text {leptons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq 5.3.3: Generalized Signature of Leptons

The signature for the family of gauge bosons, as in Equation 5.3.4, is:
Sig $_{\text {bosons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq 5.3.4: Generalized Signature of Bosons

The signature for the Higgs-boson and any other similar particle, as in Equation 5.3.5, is:

Sig $_{\text {Higgs-boson }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.3.5: Generalized Signature of Higgs-Boson
3. Meta-levels suggest that existence is functional. This also may imply that even the bases of matter at the very quantum-level is not fixed but subject to adaptability. New particles may manifest depending on function to be expressed. Hence the following equation of innovation may also be true:

Innovation $_{\text {orientation-x }}$

$$
=\left(\left[\begin{array}{c}
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow \text { S System }_{X} \\
\left(\uparrow \text { Sig }_{\text {M }} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] T C \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
$$

The notion of adaptability at the quantum-level is perhaps suggested by numerous experiments where it is proposed that the act of observing particles changes the outcome (Weizmann, 1998). When the level of observation increases then light behaves as particles in double-slit experiments. When the level of observation decreases then light behaves more as a wave with interference being observed. While this behavior may have to do with the constraints of measurement and the nature of light, as explored further in Section A1.1, Nature of Light and Its Impact on Quantum Levels, at the same time the wholeness and even the mystery of matter and light is yet to be fully explored and understood. As Einstein is known to have written to his friend Besso towards the end of his life :"All these fifty years of pondering have not brought me any closer to answering the questions, What are light quanta?" (Stone, 2013).

Philosophically the existence of such a quaternary system even at the quantum particle level perhaps suggests that it is "turtles all the way down". The notion of turtles all the way down as popularized by William James, the father of American Psychology (Wilson, 1983). It had been reported that James met a lady at a talk he was giving, and the lady insisted that the earth was resting on a turtle. James asked "and what is that turtle resting on", and she replied that it was "turtles all the way down". This is not as absurd as it sounds though. Mandelbrot's Set (Mandelbrot, 1982) is a pervasive set that comprises turtle-like objects that exist regardless of scale in the set. In Colors of Infinity Arthur C. Clarke and Nigel Lesmoir-Gordon (Lesmoir-Gordon, 2004) suggest that this set is the "thumb-print of God" and illustrate how everything around us can be perceived as emanating from this ' M Set'.

Figure 5.3.3 illustrates the essential turtle, with myriad turtles emanating from it. As one penetrates the M-Set it is found that there are infinite number of turtles regardless of scale:


Figure 5.3.3 Mandelbrot Set with "Turtles All the Way Down" (Mandelbrot, 1982)
Metaphorically, as the mathematics being derived here can be thought of as a foundation for CAS regardless of level, turtles may be important. The four feet of a turtle can be thought of as the quaternary basis of the mathematics that creates a unique structure or shell, with the all-seeing and fully retractable head, or four-fold intelligence embedded in a point, in front.

### 5.4 The Periodic Table

It appears that several layers of organization, from the macro to the micro, as suggested by the explorations in previous sections, at the large-system, cellular-, and quantum-levels may be organized in a similar quaternary manner. CAS though exist at multiple levels of scale and complexity, and the question is whether such a quaternary basis may also exist at some level of complexity between quanta and the cell. There has been substantial research done at the atomic or element level, and therefore it may prove useful to study structure of the Periodic Table.

After all, as Scerri suggests in "The Periodic Table: A Very Short Introduction" (Scerri, 2011) the urge to systematize and mathematize the periodic table has existed for at least a couple of centuries. He describes the efforts of Wolfgang Dobereiner in the early 1800s to create triads of elements in which one element had both the average of chemical elements and atomic weights of two other elements.

For example, Lithium, Sodium, and Potassium are all softish grey metals with low densities. While Pottasium is highly reactive with water, Lithium is not. But Sodium, whose atomic weight, 23, is intermediate between Lithium, 7, and Potassium, 39, also shows intermediate reactivity with water. This was a highly significant discovery because it showed a numerical regularity at the heart of the relationship between the nature and the properties of the elements.

A regular view of the Periodic Table follows in Figure 5.4.1:


Figure 5.4.1 Regular View of Periodic Table (Dayah, 1997)

There are several implicit organizational devices employed in this view. Hence in one view the elements appear as non-metals, noble gases, alkali metals, alkaline earth metals, metalloids, halogens, metals, transitional metals, lanthanides, and actinides. In another view they appear as ordered by atomic number (the number of protons in the nucleus). In a third view they appear as 18 columns such that the elements in each column have the same number of electrons in their outer shell. This last view has been accepted as perhaps the most significant (Gray, 2009) as all elements in the same column have similar chemical properties.

But further, when viewed from the outer shell or orbital of an atom, the Periodic Table can be split into four groups as illustrated in Figure 5.4.2:


Figure 5.4.2 Orbital View of Periodic Table (Dayah, 1997)

These groupings are by probability clouds, S, P, D, and F. The first type of probability cloud, $S$-orbital, is symmetrical - meaning an electron is equally likely to be in one direction as in any other. The P-orbital has two lobes and indicates that an electron is equally likely to be found on one side or the other of a nucleus. The respective probability clouds (UCDAVIS-SEO, 2015) are illustrated in Figure 5.4.3:


Figure 5.4.3 Orbitals or Probability Clouds Around a Nucleus(source: UC Davis Chem Wiki)

Since the S-orbital is a sphere there is only one of it. Since the P-orbital appears as 2 lobes, there are three possible configurations of it. Since the D-orbital appears as 4 lobes, there are 5 possible configurations of it. Since the F-orbital appears as 6 lobes, there are 7 possible configurations of it. Since every single orbital can have only 2 electrons with opposite spin (Pauli, 1964), one can compute that these 4 types of valence or outermost orbitals gives rise to the 18 columns of the periodic table. Further, multiple size shells for each of the S, P, D, F probability clouds constructs the entire set of elements of the periodic table.

The question, related to key attribute \#8 in Figure 2.2.1, Link between 'Key Attributes of Desired Theory' \& Design Principles and Working Hypotheses, is whether such an S, P, D, F quaternary basis lends itself to the mathematical model being developed in this dissertation? This is a significant question, since in his book "The Periodic Table: Its Story and Significance", Scerri (Scerri, 2007) suggests that Mendeleev, the leading discoverer of the Periodic Table was able to make the progress he did because he contemplated and returned to the philosophical ideas behind the elements formulated at least as early as the time of Aristotle. Hence a meta-function view as the basis of the mathematics here may be significant if one
can show some mapping between the four architectural sets
$S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}$, and the S, P, D, and F groupings.
The general suggestion therefore is that each element in the periodic table would have a unique signature as determined by:

Element $_{\text {sig }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{S_{\text {System }}^{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\left.{S y y s t e m_{N}}\right]}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

### 5.4.1 The S-Group

Exploring the S-group, one sees that it consists primarily of alkali metals and alkali earth metals. These groups are extremely electropositive easily losing electrons and forming positive ions and releasing a lot of energy while doing so. In his book, Essential Elements (Tweed, 2003), Tweed refers to these groups as the "violent world of the s-block". Gray, in The Elements, points out that stars shine because they are transmuting vast amounts of hydrogen into helium, both of which are sblock elements. This characteristic of easily released energy that the elements of this group share suggests that the S-group may parallel or be a precipitation of the architectural set, $S_{\text {System }_{P}}$.

It is also known that all elements are created in stars. In his article on "The Many Looks of the Periodic Table" (Katz, 2008) Katz refers to the 'Chemical Galaxy' illustration created by Philip Stewart from Oxford University to reflect the fact that all elements are created in the stars. His visual follows in Figure 5.4.1.1:


Figure 5.4.1.1 Chemical Galaxy [source: (Stewart, 2007)]
Philosophically the s-orbital as a probability cloud indicates the equal likelihood that an electron can be anywhere in a symmetrical sphere around a nucleus. Since all other orbitals can be thought of as occurring within the cloud specified by the sorbital, in some sense this is like an imprint or precipitation from meta-levels that allows future and more varied meta-functions to more easily precipitate at the level of $U$. The elements that are part of the S-group may be thought of as the adventurers with courage who venture into a brave new world to create some foundation by which all other element-creations can follow. The fact that H and He constitute $98 \%$ of the Universe (Heiserman, 1991) relative to other elements therefore makes sense in this view, especially since H and He provide the fuel with which the star-furnaces manufacture all other elements.

Hence, a series of equations linked to $S_{\text {System }_{P}}$ as the prime set can be suggested, starting with the s-orbital mapping, as in Equation 5.4.1.1:

Element $_{S \text {-orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.1.1: S-Orbital Element

Further, the equivalent mapping between traditional element groupings and $S_{\text {System }_{P}}$ as in Equations 5.4.1.2 and 5.4.1.3 can be specified:


Eq 5.4.1.2: Alkali Metal Element
Element $_{\text {Alkali earth metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq 5.4.1.3: Alkali Earth Metal Element

Further, as a representative element belonging to the Alkali Metal group the equation for Lithium (Li), as in Equation 5.4.1.4, would be:

$$
\begin{gathered}
\text { Element }_{\text {Lithium }}= \\
X a+\overline{Y b_{0-n}} \text { where }
\end{gathered}\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

## Eq 5.4.1.4: Lithium

### 5.4.2 The P-Group

In the group of elements with a valence shell specified by the p-orbital, recall from Section 5.4 that the probability of an electron is equally likely on either side of the nucleus. There are some very significant elements in this group that are part of the metal, metalloid, non-metal, halogen, and noble gas sub-groupings. Carbon, Nitrogen, Oxygen, and Silicon are some of the sample elements.

In some sense this grouping summarizes all the element possibilities within it. It is perhaps that the possibility of ideas behind all elements has precipitated in this group and one can hypothesize that this group may be a reflection of the Set of Knowledge, $S_{\text {System }_{K}}$, forming archetypes from which all other elements are created.

Philosophically, the one probability cloud (S) becoming two (P) signifies an essential polarity created within a unit space. If the hypothesis that the form is a 'switching' function that attracts function into form is true, then this dual manifestation may be viewed as the prerequisite condition by which a larger number of such 'switches' also come into being. This 'essential two' created along three dimensions of space may allow a threshold meta-function experimentation to come into being. Being the first instance of this variability in space it could be that it therefore becomes an attractor for all the essential element-archetypes to precipitate.

But further, the essential elements that allow both thinking and virtual thinking machines to come into being, are also contained within this group. Carbon is the basis of DNA and of all life. The fact the Silicon (Si), directly below it in the periodic table and therefore sharing essential qualities, is considered the basis of all virtual thinking machines is therefore perhaps significant and may reinforce the notion that the P-group is a precipitation of $S_{S y s t e m_{K}}$.

Hence, a series of equations linked to $S_{S y s t e m_{K}}$ as the prime set can be suggested, starting with the p-orbital mapping, as in Equation 5.4.2.1:

$$
\begin{gathered}
\text { Element }_{P-\text { Orbital }}= \\
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{{S y s t e m_{K}}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{gathered}
$$

## Eq 5.4.2.1: P-Orbital Element

Further, the equivalent mapping between traditional element groupings and $S_{\text {System }_{K}}$ as in Equations 5.4.2.2 through 5.4.2.6 can also be specified:

Element $_{\text {Metal }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m_{K}}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right]}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq 5.4.2.2: Metal Element
Element $_{\text {Metalloid }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.2.3: Metalloid Element

Element $_{\text {Non-Metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq 5.4.2.4: Non-Metal Element
Element $_{\text {Halogen }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.2.5: Halogen Element
Element $_{\text {Noble Gas }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.2.6: Noble Gas Element
Further, as a representative element belonging to the Non-Metal group the equation for Carbon (C), as in Equation 5.4.2.7, would be:

Element $_{\text {Carbon }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.2.7: Carbon

### 5.4.3 The D-Group

The D-group comprises the Transition Metals. These metals are generally hard and strong, exhibit corrosive resistance, and can be thought of as workhorse elements. Many industrial and well-known elements sit in this group: Titanium, Chromium, Manganese, Iron, Cobalt, Nickel, Copper, Zinc, Silver, Platinum, and Gold, amongst others.

The d-orbital itself is a probability space characterized by four lobes around the nucleus. Four lobes occurring in 5 possible planes around the nucleus will likely
create a space of stability, since there is a possibility of four lobes creating the four vertices of a tetrahedron that has been implicitly positioned as one of the most stable shapes (Fuller, 1982). Work done in Crystal Field Theory (UCDAVIS-CFT 2015) reinforces this concept. The general stability of the transition metals is reinforced by the d-orbital arrangement. The following illustration gives an example of the inherent creation of an octahedral arrangement (comprised of 2 tetrahedrons) and its dynamics with approaching forces as a reason for this general stability:


An Octahedral Arrangement of Six Negative Charges around a Metal Ion Causes the Five d Orbitals to Split into Two Sets with Different Energies. (a) Distributing a charge of -6 uniformly over a spherical surface surrounding a metal ion causes the energy of all five d orbitals to increase due to electrostatic repulsions, but the five $d$ orbitals remain degenerate. Placing a charge of -1 at each vertex of an octahedron causes the $d$ orbitals to split into two groups with different energies: the $d_{x^{2}-y^{2}}$ and $d_{z^{2}}$ orbitals increase in energy, while the, $d_{x y}$. $d_{x z}$, and $d_{y z}$ orbitals decrease in energy. The average energy of the five $d$ orbitals is the same as for a spherical distribution of a -6 charge, however. Attractive electrostatic interactions between the negatively charged ligands and the positively charged metal ion (far right) cause all five $d$ orbitals to decrease in energy but does not affect the splittings of the orbitals. (b) The two eg orbitals (left) point directly at the six negatively charged ligands, which increases their energy compared with a spherical distribution of negative charge. In contrast, the three $t_{2 g}$ orbitals (right) point between the negatively charged ligands, which decreases their energy compared with a spherical distribution of charge.

Figure 5.4.3.1 D-Orbital Octahedral Arrangement (UCDAVIS-CFT, 2015)
Much of the constructed world around us is created from these elements. Further, most of the series in the group easily lose one or more electrons to form a vast array of compounds. Continuing to draw a link with the quaternary architecture that is the foundation of the mathematical model being developed in this dissertation, it can be seen that these metals exist for service, to help bring about perfection in the
constructed world, to help much of the machinery in which they are used, and to assist the processes dependent on them to be completed with diligence. Hence, these transition metals appear to be a precipitation of System-Presence.

Therefore, a series of equations linked to $S_{S y s t e m_{P r}}$ as the prime set can be suggested, starting with the d-orbital mapping, as in Equation 5.4.3.1:


## Eq 5.4.3.1: D-Orbital Element

Further, the equivalent mapping between traditional element groupings and $S_{\text {System }_{P r}}$, as in Equation 5.4.3.2, can also be specified:

Element $_{\text {Transition Metal }}=\begin{gathered}X \in\left[S_{\text {System }_{P r}}\right] \\ X a+\overline{Y b_{0-n}} \text { where }\end{gathered}\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq 5.4.3.2: Transition Metal Element

Further, as a representative element belonging to the Transition-Metal group the equation for Gold (Au), as in Equation 5.4.3.3, would be:


Eq 5.4.3.3: Gold

### 5.4.4 The F-Group

The F-group comprises the Lanthanides and Actinides. Philosophically, the f-orbital, consisting of 6 probability lobes around the nucleus in 7 different planes, implicitly suggesting the notion of extended relationship and collectivity: the attempt to build larger and larger bonds within a small space. Continuing to draw the link with the quaternary architecture, it is likely that this group is a precipitation of SystemNurturing.

Thinking about Lanthanides, some interesting facts may reinforce this notion:

- First, the spin of electrons in the valence shell is aligned, creating a very strong magnetic field. The notion of creating a strong magnetic field seems to be consistent with the notion of engendering a collectivity through the ordered attraction and repulsion of elements.
- Second, these elements curiously occur together in nature often in the same ores and are chemically interchangeable (Gray, 2009) also suggesting the notion of forming a tight intra-group collectivity.

Thinking about Actinides, the following illustration in Figure 5.4.4.1 is useful:


Figure 5.4.4.1 Actinides: Radioactive \& Human-Made Elements (Mrsi, 2012)
In Figure 5.4.4.1 the following observations as it relates to Actinides can be made:

1. The pink in the bottom row indicates those elements that are inherently radioactive. This implies that these elements have inherently crossed a threshold of stability and have the urge, over their own half-lives, to decompose into other elements. This natural urge may suggest some boundary conditions on the notion of collectivity and nurturing, giving insight into these conditions.
2. The purple group indicates human-made elements that are also radioactive. While there is the urge to create larger collectivity, again this may give insight into the boundaries of this condition.
3. The entire Actinide group, as opposed to the Lanthanide group that is inherently stable, is unstable. It is curious that both these should be part of the f-group, and they must provide insight into boundary conditions into the notion of collectivity in elements.

Hence, a series of equations linked to $S_{S y s t e m_{N}}$ as the prime set can be suggested, starting with the f-orbital mapping, as in Equation 5.4.4.1:

Element $_{F-\text { Orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.4.1: F-Orbital Element
Further, the equivalent mapping between traditional element groupings and $S_{\text {System }_{N}}$, as in Equations 5.4.4.2 and 5.4.4.3, can also be specified:

Element $_{\text {Lanthanide }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq 5.4.4.2: Lanthanide Element
Element ${ }_{\text {Actinide }}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

## Eq 5.4.4.3: Actinide Element

Further, as a representative element belonging to the Lanthanide group the equation for Lanthanum (La), as in Equation 5.4.4.4, would be:

$$
\left.\begin{array}{c}
\text { Element }_{\text {Lanthanum }}= \\
X a+\overline{Y b_{0-n}} \text { where }
\end{array}\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]\right) ~\left[\begin{array}{c}
\end{array}\right]
$$

In studying the structure of the Periodic Table four primary groupings emerge - Sgroup, P-group, D-group, and F-group. These are suggested to be manifestations of the quaternary system central to the derived mathematical model for innovation for CAS. The reinforced perception that such a quaternary system exists at multiple scales - the quantum level, the atomic level, and the cellular level - suggests that there may be something to learn about sustainability of CAS at each of these levels, since indeed organization at these levels appears to endure and become the foundation for an array of phenomena at that level. Section 5.5 will explore sustainability of such CAS systems.

### 5.5 Sustainability of CAS Systems

It appears that several layers of organization, from the macro to the micro, as suggested by the preceding explorations at the large-system, cellular-, atomic-, and quantum-levels may be organized in a similar manner. What insight does this provide into organization and into the mathematics of innovation? It is time to explore this here.

Summarizing organization at the level of the quantum, it can be seen that the integrity and functioning of the atom depends on the integration of all four architectural forces, as discussed in Section 5.3. Hence quarks, representative of the family of Knowledge, leptons, representative of the family of Power, gauge bosons, representative of the family of Nurturing, and the Higgs-boson, representative of the family of Presence, act together to create the structure and functionality of every single atom.

As a reminder the associated equations follow:
Sig $_{\text {quarks }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{{S y s t e m_{K}}^{\prime}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.\end{array}\right]$
Sig $_{\text {leptons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m_{P}}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Sig $_{\text {bosons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Sig $_{\text {Higgs-boson }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{S y s t e m_{P r}}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},} S_{S_{\text {System }}^{P}}, S_{{S y s t e m_{K}}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

What this suggests is that every single thing may have been created through some integration and balance of the four sets of architectural forces.

Going up the organizational scale to the next level of complexity, to the level of the atom, the same pattern is found again: every single atom belongs to one of four groups, and in unison these four groups orchestrate the set of combinations of known compounds, as discussed in Section 5.4.

Hence, Alkali Metals and Alkali Earth Metals configured by the S-Group, from the family of System-Power, Metals, Metalloid, Non-Metals, Halogens, and Noble Gases configured by the P-Group, from the family of System-Knowledge, Transition Metals configured by the D-Group, from the family of System-Presence, and Lanthanides and Actinides configured by the F-Group, from the family of System-Nurturing, act together to create the complex array of compounds that form the entire material from which the physical world around us is constructed.

Key resulting suggested equations are the following:

$$
\begin{gathered}
\text { Element }_{\text {Alkali metal }}= \\
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{gathered}
$$

Element $_{\text {Alkali earth metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{{S y s t e m_{K}}^{\prime}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

$$
\text { Element }_{\text {Metal }}=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

Element $_{\text {Metalloid }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

$$
\begin{gathered}
\text { Element }_{\text {Non-Metal }}= \\
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{gathered}
$$

$$
\begin{aligned}
& \text { Element }_{\text {Halogen }}= \\
& X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right] \\
& \begin{array}{c}
\text { Element }_{\text {Noble Gas }}= \\
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{\text {System }_{\text {Pr }}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{array} \\
& \begin{array}{c}
\text { Element }_{\text {Transition Metal }}= \\
X a+\overline{Y b_{0-n}} \text { where }
\end{array}\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P r}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}},\right. \\
a, b \text { are integers } ; a>b
\end{array}\right] \\
& \left.\left.\begin{array}{cc}
\text { Element }_{\text {Lanthanide }}= & X \in\left[S_{\text {System }_{N}}\right] \\
X a+\overline{Y b_{0-n}} \text { where }
\end{array}\right] \begin{array}{c} 
\\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers; } a>b
\end{array}\right]
\end{aligned}
$$

Element ${ }_{\text {Actinide }}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

Going up the organizational scale to the next level of complexity, to the level of the cell, the same pattern is found again: basically, as already summarized in Section 5.1, every single cell of every single living creature that has ever been studied by humankind also has a similar balance and integration of these four sets of forces acting together.

Hence, proteins, from the family of Presence, nucleic acids, from the family of Knowledge, polysaccharides, from the family of Power, and lipids, from the family of Nurturing, work together to create the balanced functioning of every living cell.

As a reminder the associated equations follow:
Sig $_{\text {protein }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m}^{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ S_{\text {System }_{P}}, S_{S y s t e m_{K}}, \\ a, b \text { are integers } ; ~\end{array}, b>b\right.\end{array}\right]$


$$
\begin{aligned}
& \text { Sig }_{\text {nucleic acid }}=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{S_{\text {System }}^{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right] \\
& \text { Sig }_{\text {lipid }}=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right] \\
& \text { Sig } \text { poysaccharide }= \\
& X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{aligned}
$$

The hypothesis hence, is that even at larger scale, in considering the most innovative organization at the level of teams, corporations, or markets, a similar integration and balance of the four sets of forces may yield the best results. Recent developments in sustainability investment models ranging from Socially Responsible Investing (Logue, 2008), the Global Reporting Initiative (Willis, 2003), the Dow Jones Sustainability Index (Hope \& Fowler, 2007), the Principles of Responsible Investing (Harvard-Edu, 2014), all reinforce the concept of investment criteria becoming broader to be based not only on economic, but on environmental, social, and governance factors as well. Further evidence suggests that financial returns on such broad-based investment models continually beat financial returns on regular funds such as the S\&P 500, for instance (Openshaw, 2015). In their study of major transitions in evolution Smith and Szathmary (Smith \& Szathmary, 1995) chronicle the development of life toward increasing complexity as an application of successful collaboration and even co-evolution by which species evolve by changing together as system pressures increase. This hypothesis can be summarized by the following graph:


Figure 5.5.1 Sustainability of CAS
A brief look at history will also reinforce this same hypothesis. Those civilizations that have endured typically have a balance of all four families (Sri Aurobindo, 1971). Civilizations that have become extinct typically have had a focus on few drivers of innovation. Jared Diamond proposes five interconnected causes of collapse that may reinforce each other: non-sustainable exploitation of resources, climate changes, diminishing support from friendly societies, hostile neighbors, and inappropriate attitudes for change (Diamond, 2005). But these five sources may also be thought of as symptoms that arise due to the failure to adopt the catholicity of the sources of innovation emanating from each of the four sets of families proposed in this dissertation. Further, the historian Toynbee suggested that societies decay because of their over-reliance on structures that helped them solve old problems (Toynbee, 1961). It can be interpreted that being thus biased they are unable to adopt the catholicity of the sources of innovation emanating from each of the four sets of families.

An equation for the sustainability of systems, Sustainability Systems , where the interaction between the four families of forces is instrumental can be created. Hence, as in Equation 5.5.1:

Sustainability $_{\text {Systems } \propto \text { Interaction }\left(S_{\text {System }_{\text {Pr }}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right) .}$
Eq 5.5.1: Sustainability of Systems

### 5.6 General CAS Mathematical Operators

Chapter 2 had referenced several 'properties' thought to be true of complex adaptive systems. These properties primarily emanate from the prevalent bottomup view of CAS, summarized in Section 2.1.2 and also from the rule-based view of

CAS, Section 2.1.3. Here, and based on the mathematics that is being developed in this dissertation 12 general properties or more precisely 'mathematical operators', arrived at deductively and thought to be true of CAS will be suggested. In contrast to the bottom-up and the rules-based views these operators are the result of considering some of the dynamics of each of the levels and integration of the multiple levels of the mathematical model presented in this dissertation. These are non exhaustive, but rather are indicative of the nature of CAS.

Leveraging the quaternary basis of the mathematical model developed in Chapter 4 these mathematical operators can be summarized in four categories that parallel the four-fold scheme: Presence, Power, Knowledge, and Nurturing. Since it is being suggested that any CAS is organized by such a quaternary system it perhaps also makes sense to consider the mathematical operators using such a lens.

In Section 2.1.4 it has been suggested that there is an implicit order to Time and an implicit order to Space. Hence, any organization and the causality that it is subject to has to be reconsidered in light of the implicit time-space order that define the very fabric that animates all organization. If Space is plotted on the $x$-axis, and Time on the $y$-axis, and a general curve constructed characterized by a time-space boundary-n, as in Figure 5.6.1, then it can be assumed that the area, Time-Space Continuum, under the time-space boundary-n, is subject to the mathematics and properties being developed here.


Figure 5.6.1 Time-Space Continuum

The following operators have to be considered in context to the General Equation for Innovation, Equation 4.7.6, derived in Section 4.7. This equation suggests that any CAS has implicit in it the urge to transform the untransformed layer, $U$, by opening to the influence of the meta-layers, $M_{1}, M_{2}$, and $M_{3}$. In so doing the very sources of innovation are altered and the visible characteristics of systems are
transformed by the action of these sources of innovation. Several sets have already been suggested that explore these sources of innovation - $S_{S_{S S t e m_{P} r}}, S_{S_{s s t e m_{P}}}$, $S_{\text {System }_{K}}$, and $S_{\text {System }_{N}}$, and the resultant characteristics of CAS - Physical ${ }_{T}$, Vital $_{T}$, Mental $_{T}$, and Integral $_{T}$. The following sections leverage various aspects of the already derived mathematical model to frame sets of mathematical operators along each of the four basis of CAS that components or elements of CAS will be subject to in their journey toward transformation.

### 5.6.1 Presence-Based Mathematical Operators

The notion of 'presence' as an organizing principle for CAS is introduced in Section 2.1.4 and further developed in Chapter 4, and in particular in Section 4.7 that derives an equation, Equation 4.7.1, for the dynamism related to presence inherent in any CAS. Mathematical operators associated with presence would shed insight into the essential nature of any CAS element. Three representative presence-based mathematical operators include Fullness, Equality, and Uniqueness, and are described below. This presence-based set can be summarized by Equation 5.6.1.1:

Presence_based Mathematical_operators $\ni$ [Fullness, Equality,Uniqueness, ...]
Eq 5.6.1.1: Set of Presence-based Mathematical Operators
Fullness refers to the possibility that every single point in the time-space continuum is informed by the four-fold fullness derived earlier in Section 4.2. Hence, in equation form it may be suggested that Fullness is the union (U) of System ${ }_{P r}$, System $_{P}$, System $_{K}$, and System ${ }_{N}$. The full form of Fullness, as in Equation 5.6.1.2, would hence be:


Eq 5.6.1.2: Full Form of Fullness

Equation 5.6.1.2 can be read from the top of the matrix. The first row highlights the dynamics of the system-presence, System $_{P r}$, aspect of a point already derived in Section 4.2. The second row highlights the dynamics of the system-power, System $_{P}$, aspect of a point. The third row highlights the dynamics of the system-knowledge, System $_{K}$, aspect of a point. The fourth row highlights the dynamics of the systemnurturing, System $_{N}$, aspect of a point. The union, U , of all four rows signifies the fullness existing in every single point in any system.

A short-form of the equation for Fullness, as in Equation 5.6.1.3, would be:
Fullness $=\bigcup\left[\begin{array}{l}\text { System }_{P r} \\ \text { System }_{P} \\ \text { System }_{K} \\ \text { System }_{N}\end{array}\right]$

## Eq 5.6.1.3: Short Form of Fullness

In general for any two points ' A ' and ' B ' it can be suggested that the Fullness behind $A$ is the same as the Fullness behind B, as in Equation 5.6.1.4. This suggestion may also be arrived at by considering Einstein's General Theory of Relativity (Einstein, 1995) that states: "All bodies of reference K, K1 etc. are equivalent for the description of natural phenomena (formulation of the general laws of nature), whatever may be their state of motion." If we shrink these bodies of reference or coordinate systems to infinitesimal dimensions, thus approaching a 'point', this suggests that there is equivalence in that the general laws of nature are equally valid at any two points. Hence:

Fullness $_{A} \equiv$ Fullness $_{B}$

## Eq 5.6.1.4: Equivalence of Fullness

Equality refers to the possibility that since every point in a time-space continuum is always some expression of the underlying four-fold fullness, hence every point is equal to any other point. This implies that all expressions and developments share a fundamental equality with all other points. Depicting a point 'a' by $\operatorname{Point}_{A}$ and a point 'b' by Point $_{B}$, then an equation for Equality, as in Equation 5.6.1.5, would be:

Equality: Point $_{A} \equiv$ Point $_{B}$

## Eq 5.6.1.5: Equality

Uniqueness refers to the possibility that any point is fundamentally unique. An easy way to envision this is to see that any point in a time-space continuum is a result of a unique time-space intersection. Hence, two points within a time-space continuum, $A$ and $B$, can always be envisioned as having unique time and space coordinates:


Figure 5.6.1.1 Uniqueness in Time-Space Continuum

But even for any organization, which can itself be considered a further development of a point or points through the action of time and space, the already proposed equation in Section 4.4 that governs organizational signatures may be restated as an equation for uniqueness, as in Equation 5.6.1.6:

Uniqueness: Sig $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{S_{S y s t e m_{K}}}, S_{S_{y s t e m_{N}}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{S_{\text {Sy®tem }}^{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq 5.6.1.6: Uniqueness

### 5.6.2 Power-Based Mathematical Operators

Power-based mathematical operators pertain to the fount of dynamism that will tend to determine an organization's practical action. The notion of 'power' as an organizing class within CAS is introduced in Section 2.1.4 and further developed in Chapter 4, and in particular in Section 4.7 that derives an equation, Equation 4.7.3, for the dynamism related to power inherent in any CAS. Mathematical operators related to 'power' give insight into how an organization will tend to meet circumstance and other organizations (Malik, 2009). Representative operators include Direction, Fractal, and Intersection and in contrast to Knowledge-based operators discussed in Section 5.6.3, tend to create the more visceral and immediate reactions to circumstance. Knowledge-based operators on the other hand tend to create action more in line with long-term or strategic plans. The integration between these different types of action and the notion of building such
ambidexterity in organizations that combine incremental with perhaps more revolutionary shift is being recognized as increasingly important (Platzek et al, 2014). This power-based set can be summarized by Equation 5.6.2.1:

Power_based $_{\text {Mathematical_operators }} \ni$ [Direction, Fractal,Intersection ...]

## Eq 5.6.2.1: Set of Power-based Mathematical Operators

Direction refers to the possibility that direction at any possible system bifurcation point is not random and not determined either. Rather it can be thought of as a qualified determinism and is a function of applying $D I_{V}$ and $D I_{H}$ to the set of possibilities existing at a bifurcation point. As per the functioning of $D I_{V}$ and $D I_{H}$ discussed in detail in Section 4.9, it is the strongest or most 'powerful' possibility that will tend to determine what will emerge as an organization meets circumstance and other organizations. The equations have already been defined previously in Section 4.9, as in Equation 5.6.2.2:

Direction: Org_Dir $=D I\left(\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow S_{\text {System }_{X}} \\ \left(\uparrow \text { Sig }_{X} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{P}\right. \\ U \rightarrow x_{U}\end{array}\right] \quad x=p, v, m, i\right) \rightarrow$ $x_{-}$matrix $_{\text {strongest }} @ l e v e l_{\text {strongest }}$

## Eq 5.6.2.2: Direction

Fractal refers to the possibility that as organizational complexity increases, the base orientation, orientation- x , where x could be physical, vital, mental, or integral, of an average organization at some level of complexity ' $n$ ', will tend to determine the orientation of an organization at a level of complexity ' $n+1$ '. Likewise the orientation or an organization at level of complexity ' $n+1$ ' will tend to determine the orientation of an organization at level of complexity ' $n$ '. This is summarized by Equation 5.6.2.3:

Fractal: Orientation $n_{x} @$ Complexity $_{n} \leftrightarrow$ Orientation $_{x} @$ Complexity $_{n+1}$

## Eq 5.6.2.3: Fractal

Organizational complexity refers to an order of magnitude change as in from a person to a team, of from a team to a business unit, and so on, for example. In Nature such fractal arrangements abound in the way the human body is constructed to the very structure of galaxies (Briggs, 1992). This notion has been suggested to exist in complex behavioral systems as well as described in some detail in books
such as The Fractal Organization (Malik, 2015), The Fractal Organization (Hoverstadt, 2008), and The Misbehavior of Markets (Mandelbrot \& Hudson, 2006). This notion relates to 'power' in that it is the patterns at one level that will tend to determine the patterns at another level, often preempting what may be a more logical choice based on reason. This kind of behavior has been suggested as causing cyclic fluctuations in stock and other markets where greed and fear often trump more intelligent and rational choices (Frost, 2005). Greed rises until fear sets in. Fear rises until greed sets in.

Intersection occurs when two organizations shift orientations to the next successive level due to the shock of interaction. Hence if an organization is at a physical orientation, it may be shifted to a vital orientation (Platzek, 2012) when intersection occurs, as in Equation 5.6.2.4, where the function 'Next Element' extracts the next element from the Set S comprising of the elements (physical, vital, mental, integral). Examples of such phenomenon abound where failure to make a shift results in shock of conflict repeating itself endlessly. Such a process with applicability at multiple levels of complexity has been captured by the series of books on crucial conversations (Patterson et al, 2011).

Intersection: Organization Orientation $\rightarrow$ Next Element $(S)$

## Eq 5.6.2.4: Intersection

### 5.6.3 Knowledge-Based Mathematical Operators

Knowledge-based mathematical operators have to do with how organizations tend to develop by existing or toward increasing knowledge over time. The notion of 'knowledge' as an organizing class for CAS is introduced in Section 2.1.4 and further developed in Chapter 4, and in particular in Section 4.7 that derives an equation, Equation 4.7.4, for the dynamism related to knowledge inherent in any CAS.
Representative knowledge properties discussed in this section include Alternative, Flowering, and Higher, which as suggested in Section 5.6.2 are of a different nature than 'power' or dynamism-based properties that tend to determine an organization's visceral or immediate reaction to the market place. This knowledgebased set can be summarized by Equation 5.6.3.1:

Knowledge_based Mathematical_operators $\ni$ [Alternative, Flowering, Higher ...]
Eq 5.6.3.1: Set of Knowledge-based Mathematical Operators
Alternative refers to an alternative narrative that an organization will tend to embed itself in. These alternative narratives relate to the physical, the vital, the mental and the integral orientations introduced in Section 2.1.4 and further elaborated by means of the derived equations in Section 4.7. These narratives can easily become fixed and can strongly influence the entire internal and external orientation of an
organization. The book 'The Fractal Organization: The Future of Enterprise' suggests a theory of such narratives with their consequent effect on practical action (Malik, 2015). The alternative narratives are best described using the generalized equation derived for Innovation in Section 4.7, as in Equation 5.6.3.2. Hence:

## Alternative: Innovation orientation $-x$

$$
=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
(\uparrow \text { Sig } \rightarrow F) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] T C \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

## Eq 5.6.3.2: Alternative

Where, ' $x$ ' can be thought of as an element from the Set of Orientations:
$x \in($ physical, vital, mental, integral $)$
' X ' can be thought of as an element from the Set of System-level architectural forces. Hence:
$X \in$ (Presence, Power, Knowledge, Nurturing)
Flowering refers to the possibility that any time-space boundary-n, depicted as $T S_{n}$ will have more potential or possibility associated with it than a time-space boundary-(n-1), depicted as $T S_{n-1}$. Putting this into equation format as in Equation 5.6.3.3:

Flowering: Possibility ${ }_{T s_{n}}>$ Possibility $_{T s_{n-1}}$

## Eq 5.6.3.3: Flowering

Higher refers to the possibility that over time the direction will always tend to move to a higher meta-level. This can be summarized by using the notion of the corematrix introduced in Section 4.7 and summarized by Equation 4.7.7. Hence, as in Equation 5.6.3.4:

Higher: Upward $\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow \text { System }_{X} \\ (\uparrow \text { Sig } \rightarrow F) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x}\right. \\ U \rightarrow x_{U}\end{array}\right]$

## Eq 5.6.3.4: Higher

Note also that while many organizations are practically at the untransformed level for a long time there is still movement within that level that can generally be depicted as a change from a predominantly physical-orientation to a more mentalorientation as was discussed in Section 2.1.4. In general this shift in orientation is implied, as discussed in Section 4.7, as more and more fixed patterns are overcome: $\uparrow>P_{x}$, where $P_{x}$ refers to patterns along an orientation ' $x$ ' where $x$ is an element from the set: (physical, vital, mental, integral). Section 4.7 has a detailed discussion of all the notations in Equation 5.6.3.4.

### 5.6.4 Nurturing-Based Mathematical Operators

Nurturing-based mathematical operators have to do with the nature of relationship within CAS. These relationships are posited as being of a nurturing nature and emanate from the notion of 'nurturing' as an organizing class for CAS as introduced in Section 2.1.4 and further developed in Chapter 4, and in particular in Section 4.7 that derives an equation, Equation 4.7.5, related to the dynamism of nurturing inherent in any CAS. Representative mathematical-operators include Remember, Linking, and Relate. This nurturing-based set can be summarized by Equation 5.6.4.1:

Nurturing_based Mathematical_operators $^{\ni}$ [Remember,Linking,Relate ...]

## Eq 5.6.4.1: Set of Nurturing-based Mathematical Operators

Remember has to do with remembering that there is something in each organization that existed before the existence of any organization, and further, that there is something in each organization that exists in every other organization. This can be thought of going back to a time-space moment of zero, and subsequently of expanding into the Time-Space Continuum keeping that connection in mind. There is something in each organization that exists in every organization and highlights a special way to relate to the underlying system:



Figure 5.6.4.1 Remembrance in Time-Space Continuum

This may be depicted by Equation 5.6.4.2:
Remember: Ubiquity $\left[\begin{array}{l}\stackrel{\leftrightarrows S=0}{\leftrightarrows} \\ T S>0\end{array}\right]$

## Eq 5.6.4.2: Remember

Where the condition of going back ( $\leftarrow$ ) before any organization existed, TS $=0$, is invoked as all developments proceed ( $\rightarrow$ ), TS $>0$, to create a sense of ubiquity. The sense of ubiquity is a remembrance. This notion may be akin to the concept that everything that is in the universe emanated from the Big Bang and that we are all recycled stardust (Swimme, 2001).

Link refers to the condition whereby in any time-space coordinate, irrespective of the level of untransformed reality ( $U$ ), any present state can be consciously linked to the underlying ubiquitous system. This can be depicted by Equation 5.6.4.3 where the conditions that usually need to be in place for a meta-layer to actively influence layer $U$ disappear. There may be an attitude or receptiveness on the part of the element at $U$ that allows such linking to take place. Hence:

Link: $\left[\begin{array}{rl}M_{3} & \rightarrow \text { System }_{X} \\ M_{2} & \uparrow \downarrow \\ & \uparrow S_{\text {System }_{X}} \\ & \uparrow \downarrow \\ M_{1} & \rightarrow \text { Sig }_{x} \\ & \uparrow \downarrow \\ U & \rightarrow x_{U}\end{array}\right]$

## Eq 5.6.4.3: Linking

Relate is a way to relate to the System so as to offer or surrender activities any kind of organization is involved in, to the System, and hence the Fullness or intelligence embedded in every point. This can be depicted by an Offer function, such that the first or relatively untransformed element in the function, depicted by $x_{U}$, is being offered to the second one, the union of the four-fold intelligence embedded in each point and depicted by the union function U[ ], as in Equation 5.6.4.4:


Eq 5.6.4.4: Relate

### 5.7 Summary

In this chapter the general mathematical model for innovation in CAS was applied to a number of target areas. These target areas are CAS themselves and represent domains of increasing complexity and scale. Hence there were two applications of the model at the quantum-level, one at the atomic-level, and one at the cellular level. Note that the length scale at the quantum level starts at $10^{-35} \mathrm{~m}$ (Mastin, 2009). Length scale at the atomic level is approximately $10^{-11} \mathrm{~m}$ (Mastin, 2009). Length scale at the cellular level is approximately $10^{-6} \mathrm{~m}$ (Wikipedia Order of Magnitude, 2016). The differential from the quantum to the cellular level in terms of length scale hence scans approximately 29 orders of magnitude. The estimated diameter of the Universe is $10^{27} \mathrm{~m}$ (Mastin, 2009). The differential therefore from the quantum level to the size of the Universe in terms of length scale is 58 orders of magnitude. It is interesting to note that the general mathematical model for innovation has been applied to instances across half the length scale of the Universe in this chapter.

The application of the mathematical model in each of these domains suggested a cohesive view and operation tied to the quaternary architecture underlying the model for innovation. The quaternary architecture is suggested to drive emergence at subsequent level of complexity so that while the form such architecture takes is clearly different as evident by the form of quantum particles versus atoms versus cells, for instance, the underlying principles of organization are the same regardless of scale and complexity. Such relation of architecture across emergent layers suggests enhancement to generally accepted views of emergence (Dodder \& Dare, 2000) and complexity (Bar-Yam, 2002) in CAS.

Further, the mathematical model was then applied to the constructs of CAS itself, first suggesting an additional insight into the creation of sustainable CAS, and secondly, suggesting sets of mathematical operators that elements in CAS could be subject to.

The application of a single model for innovation to six instances of CAS seems to suggest an increasing validity to the model and Chapter 8 will look at this more objectively. It would however also be useful to simulate the generalized equation for innovation that stands at the summit of the mathematical model, to see what further insight is gained into the model, and to further test the validity of the model, and some practical manifestation of it. This is the focus of the next chapter, Chapter 6, Simulation.

## CHAPTER 6: System Simulation of Quaternary Model

In Chapter 5 through a process of deductive logic the derived mathematical model has been applied to further the understanding of CAS domains at increasing scale and levels of complexity. In this chapter Equation 4.7.6 (reproduced below for convenience), the Generalized Equation of Innovation, is simulated to descriptively research how innovation in CAS may be altered through manipulating key parameters contained in the equation. The research instrument being focused on in this chapter therefore is "Parameter Relationship", as highlighted in Figure 6.1, adapted from Figure 4.1:

| $\#$ | Research Instrument | Code | Relationship to <br> Chapter 4 |
| :---: | :--- | :--- | :--- |
| RI1 | Cohesiveness of derived <br> mathematical equations | "Cohesiveness" | Model derived <br> (Chapter 4) |
| RI2 | Further insight and simplification of <br> properties in target domains | "Insight and <br> Simplification" | Dependent <br> (Chapter 5) |
| RI3 | Relationship of adjustable parameters <br> to system innovation | "Parameter <br> Relationship" | Dependent <br> (This chapter) |
| RI4 | Ability of derived mathematical <br> equations to frame organizational <br> change and innovation | "Frame <br> Change" | Dependent <br> (Chapter 7) |
| RI5 | Cohesiveness of integration of piece- <br> meal mathematics into main model | "Integration" | Dependent <br> (Chapter 8) |
| RI6 | Addressing of generated hypotheses <br> in the literature review | "Addressing |  |
| Hypotheses" | Dependent <br> (Chapter 8) |  |  |
| RI7 | Ease with which established theory <br> suggests structure of mathematical <br> model | "Established | Dependent <br> (Chapter 8) |

Figure 6.1 Highlighting "Parameter Relationship" as the Research Instrument of Chapter 6

In Chapter 8, Evaluation, the ability of the simulation in shedding additional insight between the parameters in Equation 4.7.6 and System Innovation, will be evaluated.

Equation 4.7.6 has been selected since it is a summary of the mathematical model of innovation in CAS and is suggested to have applicability across domains of increasing scale and complexity. Reproducing Equation 4.7.6:

$$
\text { Innovation }_{\text {orientation }-x}=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \operatorname{Sig}_{X} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

### 6.1 Overview of Modeling

Before getting into the details of the simulation, some general concepts of modeling (Section 6.1.1), the Generalized Equation for Innovation (Section 6.1.2), and relevant modeling concepts in Vensim (Section 6.1.3) will be reviewed.

### 6.1.1 General Concepts of Modeling

Holland (Holland, 1992) suggests that for an effective simulation of CAS, the simulation should directly mimic the parallel interactions of complex adaptive systems, and that there should be a visual, game-like interface that provides natural controls for exploring such systems. The model developed in the Vensim simulator will be shown to possess these characteristics in this chapter. As discussed further in Section 6.1.3, Vensim was selected because of the ease with which it allows the development, exploration, analysis and optimization of simulation models in an interactive software environment.

Models can be used for multiple purposes. In Models and Science (Frigg \& Hartman, 2012) it is stated: "Models can perform two fundamentally different representational functions. On the one hand, a model can be a representation of a selected part of the world...Depending on the nature of the target, such models are either models of phenomena or models of data. On the other hand, a model can represent a theory in the sense that it interprets the laws and axioms of that theory. These two notions are not mutually exclusive as scientific models can be representations in both senses at the same time." The mathematical model developed in this dissertation may be thought of as a model that represents a theory. The Vensim model may be thought of as interpreting some key laws of that mathematical model.

Generally system models have several fundamental components built into them (Forrester, 1996). These include causal links and loops, stocks and flows, and time delays. Causal links are illustrated by the arrows in Figure 6.1.3.1 and indicate how variables link together to create an understanding of the structure of a system under
consideration. Causal loops generally indicate how feedback between variables may reinforce or balance certain structures. Causal links and loops help to understand how system behavior will change over time. Stocks, flows, and time delays can be thought of as the specific constructs that bring causal links and loops to life. Hence, stocks can be thought of as variables whose value accumulate or deplete over time. Stock variables are illustrated by the various 'descriptors' that appear to float or are in boxes in Figure 6.1.3.1. Flows can be thought of as the rate of change of a stock. Flow variables are depicted by the 'hour-glass' superimposed on an arrow in Figure 6.1.3.1. Time delays seek to introduce an element of reality by modeling delays between cause and effect. For simplicity causal loops and time delays will not initially be modeled. Enhanced models using these constructs will be proposed as subjects of further research.

### 6.1.2 Review of Generalized Equation of Innovation

The model mimics the dynamics at the four levels in equation 4.7.6. Hence there are varying dynamics by layer as discussed in more detail in Section 4.7. The default layer is the surface or untransformed layer, U. As per Equation 4.7.6, the dynamics at this layer are typified by the following sub-equation:
$U \rightarrow x_{U}$
$x_{U}$ is the untransformed set where $x \ni[p h y s i c a l$, vital, mental, integral $]$
Hence:
Physical $_{U} \ni$ [inertia,lethargy,status quo, ...]
Vital $_{U} \ni$ [aggression, self centeredness, exploitation, ...]
Mental $_{U} \ni$ [fixation,fundamentalism,fragmentation, ...]
Integral $_{U} \ni[$ possession, usurpation, hidden agendas, ...]
In other words the sources of system innovation derive from untransformed sets that would by definition hardly be enduring and sustainable.

It is possible though for the untransformed patterns to be overcome, as signified by the sub-equation:
$\uparrow>P_{x}$
When this begins to happen then the dynamics of meta-level $1, M_{1}$, become more active as per the sub-equation:

$$
M_{1} \rightarrow \operatorname{Sig}_{x}
$$

In this case the dynamics are governed more by the Generalized Signature equation, Equation 4.4.5:

Sig $=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{\text {Stem }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}},\right. \\ \left.S_{\text {System }_{P}}, S_{S_{y s t e m_{K}}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
As per the equation there are then a large number of positive and unique sources of innovation that can become active. For the sake of simplicity, when this meta-level becomes active then rather than simulate the signature equation (Eq 4.4.5) to feed values into the innovation equation ( Eq 4.7.6), a contribution to system innovation is assumed that will have a variable positive monthly rate to the Vensim-defined variable "Innovation Contribution". This will be described in more detail in Section 6.3.

As discussed in Section 4.7 when the signatures at $M_{1}$ become strong, signified in the model by "becoming a force", then the conditions for the activation of the dynamics of $M_{2}$ to become active are put in place.

The signature becoming strong is represented by the sub-equation:
$\uparrow \operatorname{Sig} \rightarrow F$
The dynamics of $M_{2}$ are signified by the sub-equation:
$M_{2} \rightarrow S_{\text {System }_{X}}$
In this case it is the primary set of architectural forces themselves, as discussed in Section 4.3 that become active. Note that in contrast at $M_{2}$ it is a signature that is a combination of the elements of the sets of architectural forces that become active.

For the sake of simplicity, when this meta-level becomes active then rather than simulate the sets of architectural forces, defined by Equations 4.3.1 through 4.3.4, to feed values into the innovation equation (Eq 4.7.6), a contribution to system innovation is assumed that will have a variable positive monthly rate to the Vensimdefined variable "Innovation Contribution" that will be higher than the rate of innovation contribution of $M_{1}$. This will be described in more detail in Section 6.4.

When the forces themselves become "integral" as discussed in Section 4.7, then the dynamics of $M_{3}$ can influence the untransformed layer, $U$, as well. The forces become integral is represented by the sub-equation:
$\uparrow F \rightarrow I$

The dynamics of $M_{3}$ are represented by the sub-equation:
$M_{3} \rightarrow$ System $_{X}$
Recall from Section 4.2 that $M_{3}$ represents the more dynamic activation of "intelligence at each point" in any system.

For the sake of simplicity, when this meta-level becomes active then rather than simulate the nature of a system-point, defined by Equations 4.2.1 through 4.2.4, to feed values into the innovation equation (Eq 4.7.6), a contribution to system innovation is assumed that will have a variable positive monthly rate to the Vensimdefined variable "Innovation Contribution" that will be higher than the rate of innovation contribution of $M_{2}$. This will be described in more detail in Section 6.5.

### 6.1.3 Relevant Features and Modeling in Vensim

Vensim allows some very useful features that help set up a model that in fact mimics the dynamics at each of the layers, $\mathrm{U}, M_{1}, M_{2}$, and $M_{3}$. These include:

1. Setting up a clock that allows a step-based approach from some initial value to a final value based on a predefined incremental step. The simulation is based on setting into motion the clock.
2. Setting up variables that can be calculated in real-time as the clock proceeds.
3. Setting up consistent units for variables and equation outcomes, as depicted in Figure 6.1.3.2.
4. Setting up complex equations that connect various variables together through causal links and loops, including 'rate of change' variables. In Figure 6.1.3.1 the arrows connect variables that are tied together in single equations. Hence Figure 6.1.3.2 depicts how the variable 'System Innovation' ties together every other box (or variable) in Figure 6.1.3.1. Rate of change variables are depicted by the hourglass symbol in Figure 6.1.3.1.
5. Equations can be set up using a range of mathematical functions and logic statements. This model heavily uses the integral $\left(\int t\right)$ function, rate of change $\left(\frac{d y}{d t}\right)$ function, and IF-THEN logic.
6. Rates of change variables can also function as real-time 'dials' that allow the simulation to proceed mimicking more real-life conditions. This addresses Holland's suggestion (Holland, 1992) for an effective simulation of CAS having a visual, game-like interface that provides natural controls for exploring such systems. 7. As Seen in Figure 6.1.3.1, parallel interactions of CAS can be set up with each vertical line of boxed-variables potentially proceeding in parallel with other vertical lines of boxed-variables, hence also addressing Holland's second suggestion of what contributes to an effective simulation for CAS.

Note that each vertical line in Figure 6.1.3.1 represents one of the quaternary pillars fundamental to this mathematical model of innovation in CAS. For the sake of simplicity detailed explanation for one of the pillars - the physical - will be elaborated in the following sections in this chapter. The same logic will then be assumed to apply to the remaining pillars, that then results in the summary model as represented by Figure 6.1.3.1.

Note that there are 36 Vensim-equations that together define the single Generalized Equation of Innovation, Equation 4.7.6. Only some of these equations will be illustrated in the following sections. The whole set of Vensim-equations though, appear in detail in Appendix 5. These equations are structured in the following way to be able to simulate Equation 4.7.6:

1. Time variables
2. Overall System Innovation
3. Initial states at untransformed layer, $U$
4. Meta-layer influence at untransformed layer, U
5. Activating meta-layer 1
6. Signature-based dynamics at meta-layer 1
7. Activating meta-layer 2
8. Architectural force-based dynamics at meta-layer 2
9. Activating meta-layer 3
10. Point-based dynamics at meta-layer 3

The overall Vensim-based model is summarized in Figure 6.1.3.1:


Figure 6.1.3.1 Model of Generalized Equation for Innovation in Vensim Simulation

The overall equation for System Innovation is built into Vensim as in Figure 6.1.3.2:


Figure 6.1.3.2 Equation for System Innovation in Vensim Simulation

Units of System Innovation are setup as "Innovation Contribution" units in Vensim as shown in Figure 6.1.3.2.

The equation captured for the Vensim-based System Innovation in Figure 6.1.3.2 hence is as depicted in Equation 6.1.3.1, where the units are defined as 'Innovation Contribution':

System Innovation ="Integral (+)"+"Integral (-)"+"Mental (+)"+"Mental (-)"+"Physical (+)"+"Physical (-)"+"Vital (+)"+"Vital (-)"
Units: Innovation Contribution

## Eq 6.1.3.1: Vensim-based Equation System Innovation

The mathematical depiction of Equation 6.1.3.1 is similar to the Vensim depiction as illustrated by Equation 6.1.3.2:

System Innovation
$=$ Integral $(+)+\operatorname{Integral}(-)+\operatorname{Mental}(+)+$ Mental $(-)+$ Physical $(+)+$ Physical $(-)+$ Vital (+) + Vital (-)

## Eq 6.1.3.2: Mathematical Depiction of Vensim-based System Innovation

The x-dimension in Equation 4.7.6, the Generalized Equation for Innovation reproduced earlier in the introduction to this chapter, represents the orientation Physical, Vital, Mental, or Integral. Any of the orientations followed by (-) represent innovation at the untransformed layer $U$. Any of the orientations followed by (+) represent innovation under the influence of one or more meta-levels. The sum of the $(-)$ and the $(+)$ components along a dimension suggest the innovation along a that particular dimension, and the overall innovation in a system is computed by adding the $(-)$ and $(+)$ contributions from all four dimensions.

At time T = 0, System-Innovation is depicted by the graph on top of the 'simulation' in Figure 6.1.3.3:


Figure 6.1.3.3 System Innovation at $T=0$
Hence, the initial System Innovation at the start of the simulation is 0 as indicated by the graph in Figure 6.1.3.3.

### 6.2 Only Untransformed Layer Active

With only $U$, the untransformed layer active, key sources of innovation are the untransformed physical, untransformed vital, untransformed mental, and untransformed integral, depicted by 'Physical(-)’, 'Vital(-)’, Mental(-), Integral(-)
respectively, in the untransformed layer in the Vensim model as illustrated in Figure 6.2.1:


Figure 6.2.1 Untransformed Layer in Vensim Model

The Innovation-Contribution due to the untransformed physical, "Physical (-)", over time has been set to be a random uniform number between the ranges of -2 (first argument of function below) and +5 (second argument of function below) as defined in Figure 6.2.2. Further the seed number (third argument in function in Figure 6.2.2 below) is set to 0 to create a specific stream of random numbers, which will be the same stream on subsequent simulation runs. The same seed is chosen for the purposes of gaining greater understanding in the initial stages of subsequent simulation runs. The -2 to +5 range depicts that in general even though the sources of innovation are untransformed as suggested by Equation 4.7.6 and as summarized in Section 6.1.2, yet there may be a general positive contribution to innovation. However, because of the untransformed nature of the physical there is no surety that this contribution will be positive and hence it can also be negative. Note that these are experimental values fed into the simulation to understand the outcome when such initial conditions are assumed, and can be altered as the understanding due to this research approach increases.


Figure 6.2.2 Equation for Physical (-) in Vensim Simulation

The outcome of this random function in terms of the variable contribution to innovation due to "Physical ( - )" is suggested by the Vensim-generated graph in Figure 6.2.3 and appears in general to vary randomly from a contribution of -2 Innovation-Contribution units to a contribution of +5 Innovation-Contribution units.


Figure 6.2.3 Physical (-) Innovation Contribution

At the untransformed level it can be argued that the contribution to innovation from the physical, the vital, the mental, and the integral are all approximately equal since there are canceling and countering effects always active. Therefore for the sake of simplicity the equations and graphs for each of these sources are assumed to be the same.

Hence Innovation Contribution due to the untransformed vital, "Vital (-)", is depicted by the graph in Figure 6.2.4:


Figure 6.2.4 Vital (-) Innovation Contribution

Similarly, Innovation Contribution due to the untransformed mental is depicted by Figure 6.2.5:


Figure 6.2.5 Mental (-) Innovation Contribution

Innovation Contribution due to the untransformed integral is depicted by Figure 6.2.6:


Figure 6.2.6 Integral (-) Innovation Contribution

Under such conditions, with only "Physical (-)", "Vital (-)", "Mental (-), and "Integral $(-)$ " active, overall System-Innovation is depicted by Figure 6.2.7:


Figure 6.2.7 System Innovation from Physical (-), Vital (-), Mental (-), Integral (-)
So over 10 years, the contribution to innovation fluctuates between -5 to +15 Innovation-Contribution units. This is consistent with the discussion in Section 2.1.4 and Design Principle 2.1.4.1 that suggests that there is a general underlying direction in all CAS that basically moves from apparent disorder to increasing order and that can manifest as increasing innovation, where recall that innovation is suggested to be a movement from the four untransformed sets (as reviewed in Section 6.1.2) to transformed sets (refer to Chapter 4 for the detailed discussion).

Further, as per the meta-layered quaternary mathematical model being developed in this dissertation, innovation is positioned as potentially being infinite. The activation of the meta-layers is what makes this so. The fact that the quaternary organization has scaled at least 29 orders of magnitude from the quantum levels to the manifest cellular levels, as suggested in Section 5.7, with constantly increasing functionality and capability still organized along the quaternary dimensions suggests this infinite potentiality (this trend is further summarized in Section A1.3, Complexification of Four-Foldness). As per the discussion in Section 2.1.5.3 on complicated versus complex systems (Sargut \& McGrath, 2011) and the subsequent framing of the equation in Section 4.12, complexity of a system may be summarized by:

$$
\text { Complexity }_{\text {System }}=f(\text { Multiplicity, Interdependence, Diversity })
$$

But for a system to maintain such multiplicity, interdependence, and diversity implies that it has become inherently innovative, as is also being suggested by the notion of transforming sets as per the mathematical model constructed in this dissertation.

Even compressed into the frame of decades the notion of ever-increasing innovation and returns is suggested by Brian Arthur in his Harvard Business Review article 'Increasing Returns and the New World of Business' (Arthur, 1996). Based on his study of the balance sheets of high-tech companies in the 1990s Arthur noticed that their returns were increasing over time based on the increase of marginal money over time. He writes: "Increasing returns are the tendency for that which is ahead to get further ahead...They are mechanisms of positive feedback that operate within markets, businesses, and industries - to reinforce that which gains success or aggravate that which suffers loss."

With the meta-levels becoming active, "Physical (+)", "Vital (+)", "Mental (+), and "Integral (+)" will become active, thereby likely increasing the overall System Innovation. While this will be explored in subsequent sections in this chapter, note that the detailed Vensim-based equation for "Physical (+)" a key 'parallel-line' along the physical-dimension that contributes to System-Innovation is captured in Equation 6.2.1:
"Physical (+)"= INTEG ("Physical (sig)"+"Force (presence)"+"Point (presence)", 0)

## Eq 6.2.1: Vensim-based Equation "Physical ( + )"

The complete mathematical depiction of Equation 6.2.1 that captures all changes with each incremental time-step as the clock proceeds from time $=0$ to time $=\mathrm{t}$, is as depicted by Equation 6.2.2:
"Physical $\left(+\right.$ )" $=\int_{0}^{t}[$ "Physical (sig)" + "Force (presence)" + "Point (presence)" $] d t$

Eq 6.2.2: Mathematical Depiction of Vensim-based "Physical (+)"
Note that the functions "Vital (+)", "Mental (+)", and "Integral (+)" parallel "Physical $(+)$ " and their Vensim-forms appear in Appendix 5.

### 6.3 With Meta-Level $M_{1}$ Active

$M_{1}$ becomes active with the overcoming of negative physical, vital, mental, and integral patterns. $\mathrm{M}_{1}$ is depicted by Figure 6.3.1:


Figure 6.3.1 Meta-Layer 1 in Vensim Model

Specifically, the patterns to be overcome are summarized in Equation 4.7.1:
Inherent Dynamism in the Physical, Equation 4.7.3: Inherent Dynamism in the Vital, Equation 4.7.4: Inherent Dynamism in the Mental, and Equation 4.7.5: Inherent Dynamism in the Integral. Respectively the patterns to be overcome in each of the dimensions are called out in the Physical ${ }_{U}$, Vital $_{U}$, Mental $_{U}$, and Integral ${ }_{U}$ sets in the introduction to this chapter and summarized here for convenience:

Physical $_{U} \ni$ [inertia,lethargy,status quo,...]
Vital $_{U} \ni$ [aggression, self centeredness, exploitation, ...]
Mental $_{U} \ni[$ fixation,fundamentalism,fragmentation, ...]
Integral $_{U} \ni[$ possession, usurpation, hidden agendas, ...]
As these patterns are overcome, $\uparrow>P_{x}$, where $x \ni$ [physical, vital, mental, integral], unique signatures are actively tapped into, which is modeled as a rate of change function that contributes to innovation at a constant rate of 1 unit per month. Note that this inherently assumes that innovation can continue to grow in a positive
direction. It further assumes that if it has not, this is because habitual restrictive patterns are constantly in play. With the overcoming of the restrictive patterns deeper sources of innovation are tapped which allows more rapid growth of innovation. Mathematically this will be depicted as in Equation 6.2.1:
$M_{1}$ Generalized Innovation Contribution/Month: $\frac{d\left(\uparrow>P_{x}\right)}{d t}=1$
Eq 6.3.1: Mathematical Depiction of Generalized Innovation Contribution/Month @ $M_{1}$
The same equation that illustrates the overcoming of negative physical patterns, specifically, is modeled in Vensim as in Figure 6.3.2:


Figure 6.3.2 Equation for Overcoming Physical Patterns in Vensim Simulation

With the physical patterns overcome, the Innovation-Contribution per month feeds into the value of the signature for the physical as per Equation 6.2.2:
"Physical (sig)" $=\int_{0}^{t}\left[\frac{d\left(\uparrow>P_{\text {physical }}\right)}{d t}\right] d t$
Eq 6.3.2: Mathematical Depiction of 'Physical (Sig)"
The equivalent form in Vensim is depicted in Equation 6.3.3:
"Physical (sig)"= INTEG ("Overcome (physical patterns)",0)

Eq 6.3.3: Vensim equivalent of 'Physical (Sig)"

The initial value in the simulation, when the physical patterns are not yet overcome, is depicted by the slider being at zero in the simulation snap-shot in Figure 6.3.3. Note that the sliders are like 'dials' that can be changed to alter the input values in the simulation.


Figure 6.3.3 Innovation Contribution when no Physical Patterns Overcome

As physical-patterns are overcome, so that the slider depicted in Figure 6.3.3 is moved to the right the level of Innovation Contribution increases. Figure 6.3.4 depicts the level of System Innovation with just the physical patterns continually being overcome at the highest rate. One can see that all other sliders or controls at $M_{1}$ and even at $M_{2}$ and $M_{3}$ are at zero:


Figure 6.3.4 System Innovation with Physical Patterns Overcome

As can be seen in the simulated graph in Figure 6.3 .4 overall System Innovation has gone up to 700 Innovation-Contribution units over 10 years from a level of approximately 15 Innovation-Contribution units. Hence, it has increased by a factor of about 47 .

As other categories of patterns at $U$, specifically the patterns contained in sets Vital $_{U}$, Mental $_{U}$, and Integral ${ }_{U}$, are overcome System Innovation goes up by another factor of 3 from a value of approximately 700 Innovation-Contribution units to approximately 2000 Innovation-Contribution units over 10 years as illustrated in Figure 6.3.5. Note that the simulated mechanism for overcoming patterns is the variable sliders as in Figure 6.3.5. As a slider moves to the right it signifies the degree to which patterns are overcome:


Figure 6.3.5 System Innovation with All U-Level Patterns Overcome

### 6.4 With Meta-Level $\mathrm{M}_{2}$ Active

If the signature that was activated at $\mathrm{M}_{1}$ becomes a Force then the conditions for the activation of $\mathrm{M}_{2}$ are put into place. As discussed in Section 4.5 on the emergence of uniqueness, the signature becomes a Force when limiting patterns in the way a signature is exercised are overcome. The mechanism in the simulation that emulates such overcoming of patterns is the variable slider, and movement to the right indicates the strength with which limiting patterns may have been overcome. $\mathrm{M}_{2}$ is depicted by the rectangle in Figure 6.4.1:


Figure 6.4.1 Meta-Layer 2 in Vensim Model

At $\mathrm{M}_{2}$ the rate of change of Innovation Contribution is set at double than that at $\mathrm{M}_{1}$. As per discussion in Section 4.7 this is meant to signify the enhanced power of innovation at $\mathrm{M}_{2}$. This is an estimated value for the purposes of generally exploratively understanding the simulation and can be fine-tuned with additional learning and is captured by the Vensim-equation as specified by Figure 6.4.2:


Figure 6.4.2 Innovation Contribution at $M_{2}$

Mathematically this rate of contribution is modeled by Equation 6.4.1:
$M_{2}$ Generalized Innovation Contribution/Month: $\frac{d(\uparrow s i g \rightarrow F)}{d t}=2$

Eq 6.4.1: Mathematical Depiction of Generalized Innovation Contribution/Month @ $M_{2}$
At $M_{2}$ Signatures become Forces. But signatures can only become forces if they have already begun to act with impunity in a range of circumstances as per discussion in Chapter 4. For the sake of simplicity this change in the nature of signatures is represented by the movement of the sliders to the right at layer $\mathrm{M}_{2}$ as in Figure 6.4.1, and is represented logically by an IF-THEN function in Vensim. So, not only does a signature have to have a positive value to begin with, but further the slider for 'Physical (sig) becomes Force' has also to be activated. When this is true then "Force (Presence)" will grow as per the integral function modeled in Vensim in Figure 6.4.3 below:


Figure 6.4.3 Modeling of "Force (Presence)"

The IF statement for Force (presence) modeled in Vensim will therefore take the value Physical (sig) becomes Force, if Physical (sig) > 0 . Else it will take the value 0 . Mathematically "Force(Presence)" can be depicted by Equation 6.4.2:
"Force (Presence)" $=\int_{0}^{t}\left[\frac{d(\uparrow \text { sig } \rightarrow F)}{d t}\right] d t$

## Eq 6.4.2: Mathematical Depiction of "Force (Presence)"

With the Physical Signature as a Force, but no other Force active, System Innovation can go up by a factor of 2 from the previous level in Figure 6.3.4 at approximately 2000 Innovation-Contribution units over 10 years, to approximately 4000 Innovation-Contribution units as depicted in Figure 6.4.4 below:


Figure 6.4.4 System Innovation when Physical (Sig) becomes Force

When all 'signatures become forces' becomes active the System Innovation goes up by a further factor of 2 from $\mathrm{M}_{1}$, from approximately 4000 Innovation-Contribution units to approximately 8000 Innovation-Contribution units over 10 years as depicted in Figure 6.4.5 below:


Figure 6.4.5 System Innovation when All Signatures become Force
6.5 With Meta-Level $\mathrm{M}_{3}$ Active

Dynamics at $\mathrm{M}_{3}$ imply that all Forces are fully active. Layer $\mathrm{M}_{3}$ is depicted by the rectangle in Figure 6.5.1:


Figure 6.5.1 Meta-Layer 3 in Vensim Model

With all Forces active, any architectural family can become Impersonal, and the Innovation Contribution per month is then modeled as being triple the rate of the Innovation Contribution per month at $\mathrm{M}_{1}$ as in Figure 6.5.2. This value is meant to depict the relatively accelerated rate of innovation contribution at $\mathrm{M}_{3}$ relative to $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$. With explorative learning following on-going research using this model these values may later be changed.


Figure 6.5.2 Innovation Contribution at M3

Mathematically this rate of contribution is modeled by Equation 6.5.1:
$M_{3}$ Generalized Innovation Contribution/Month: $\frac{d(F \rightarrow I)}{d t}=3$

Eq 6.5.1: Mathematical Depiction of Generalized Innovation Contribution/Month @ $M_{3}$
At $\mathrm{M}_{3}$ Forces become impersonal. But for this to be true, all four categories of forces have first to be positive. This is the condition of impersonality as implied by the discussion in Chapter 4 where one impersonal force has to lead other impersonal forces. When this is true then "Point (Presence)" is modeled to grow as per the integral function modeled in Vensim in Figure 6.5.3 below:


Figure 6.5.3 Modeling All Forces as Active

The If-Then statement in Figure 6.5 .3 is basically saying that if each of the conditions, Force (presence), Force (power), Force (knowledge) and Force
(nurture), has a value greater than zero, only then will 'Presence become impersonal'. Else Point (presence) will remain zero.

Mathematically "Point (Presence)" can be depicted by Equation 6.5.2:
"Point (Presence)" $=\int_{0}^{t}\left[\frac{d(F \rightarrow I)}{d t}\right] d t$

## Eq 6.5.2: Mathematical Depiction of "Point (Presence)"

When an architectural family becomes impersonal that family leads the four-fold intelligence in a point. An example of the modeling considering the family of Presence is the following. Also illustrated is the dynamic innovation contribution due to the activation of a point being led by Presence as in Figure 6.5.4:


Figure 6.5.4 Innovation Contribution When Point is Led by Presence

With just the Point being led by Presence the overall System Innovation increases by a factor of 0.25 from the previous meta-level at approximately 8000 InnovationContribution units to approximately 10000 Innovation-Contribution units over 10 years. Note that all other 'Points' are still at zero as in Figure 6.5.5:


Figure 6.5.5 System Innovation When Point is Led by Presence

With all Points fully active so that intelligence is being variously led by each of the architectural families, the level of System Innovation goes up by a factor of 1.8 from 10,000 Innovation-Contribution units to approximately 18,000 InnovationContribution units over 10 years as in Figure 6.5.6:



Figure 6.5.6 System Innovation When All Families Active at M3

### 6.6 Activation of Point Summary

In the simulation, with Force (Presence) at zero, Point (Presence) cannot be activated. This is illustrated in the next two graphs in Figure 6.6.1 and Figure 6.6.2.

Hence, Force (Presence) at zero, as illustrated by Figure 6.6.1, will result in Point (Presence) also being zero, as illustrated by Figure 6.6.2:


Figure 6.6.1 Point (Presence) not Activated when Force (Presence) $=0$


Figure 6.6.2 Point $($ Presence $)=0$ when Force $($ Presence $)=0$

This in turn will not allow $\mathrm{M}_{3}$ to become active since all Forces have to become impersonal for this to be the case, and System Innovation will be just as Innovation Contribution from $\mathrm{M}_{2}$ even though the sliders have been pushed to maximum level on all "dials" at $\mathrm{M}_{3}$ as depicted in Figure 6.6.3:


Figure 6.6.3 M3 Inactive if a Single Force $=0$

It is only when the dial for Force (Presence) at $\mathrm{M}_{2}$ has been turned on by moving the slider to the right that an Innovation Contribution per month kicks in as in Figure 6.6.4:


Figure 6.6.4 M3 Active if all Forces Active

This in turn allows Innovation Contribution from Point (Presence) as in Figure 6.6.5:


Figure 6.6.5 Innovation Contribution from Point (Presence) in Active M3

This in turn allows $M_{3}$ to contribute to overall System Innovation as in Figure 6.6.6:


Figure 6.6.6 Conditions for Active Contribution to System Innovation from M3

### 6.7 Simulation Summary

The simulation presented in the previous section reveals that System Innovation changes in the following manner from level to level as depicted in Figure 6.7.1:

| LEVEL | STATE | SYSTEM <br> INNOVATION |
| :--- | :--- | :--- |
| U | With negative patterns active | 15 x |
| M1 | With physical patterns overcome | 700 x |
| M1 | With all U-level patterns overcome | $2,000 \mathrm{x}$ |
| M2 | When physical signature becomes force | $4,000 \mathrm{x}$ |
| M2 | When all signatures become forces | $8,000 \mathrm{x}$ |


| M3 | When point is led by presence | $10,000 \mathrm{x}$ |
| :--- | :--- | :--- |
| M3 | When point is led by all families | $18,000 \mathrm{x}$ |

Figure 6.7.1 Summary System Innovation Changes from Level to Level

This is with the following general rates of change for Innovation Contribution per month as discussed in the previous sections and summarized in Figure 6.6.2:

| Level | Innovation <br> Contribution/Month |
| :--- | :--- |
| M1 | 1 |
| M2 | 2 |
| M3 | 3 |

Figure 6.7.2 Rates of Innovation Contribution Change from Level to Level
There is clearly a larger relative leap when M1 becomes active, from a factor of 15 to 700: approximately 47 times. The subsequent overcoming of further categories of patterns continues to have a large absolute impact, though in relative terms with the previous jump as a baseline is less than the first jump from $U$ to M 1 . This can be explained by the fact that the overcoming of a whole category of habitual patterns likely signifies a significant break from the norm, and begins to alter the way that innovation precipitates.

The rates of Innovation-Contribution used in the Vensim modeling are experimental to simulation research output and more research has to be done to understand the effect of overcoming the sets of patterns that allow higher meta-levels to become active. It is possible to set up causal balancing feedback loops in Vensim so that the random function that generates Innovation-Contribution varies depending on the degree to which the meta-layers become active. This will be one of the areas of further research, but for the case presented in this thesis this is considered adequate at this point. Further, sensitivity analyses will need to be performed to evaluate what the relative change to Innovation Contribution/Month should be at successive meta-levels.

The question is to what extent can the equations simulated in this chapter that form the generalized equation of innovation be used practically as well. The next chapter, Chapter 7, will explore a case where the equations at $U$ simulated in this chapter, will be explored from such a practical aspect.

## CHAPTER 7: Stanford University Medical Center Case Study

The researcher and author of this dissertation was part of Stanford University Medical Center's Organizational Development group for 5 years until 2014. He was given license and permission to experiment under the aegis of the Director of the Department (refer to Appendix 6 - A, for communication with the Director at Stanford University Medical Center) and attempted to bring about a higher degree of organizational sustainability (as defined in section 5.4 - Sustainability of CAS Systems, and summarized by the equations in Section 7.1 below) through the use of frameworks, methodologies, and software he had developed. Note that a key output and culmination of this work was a practical "field guide" developed jointly by the author of this dissertation and the Director of the Department, which appeared in the book, The Fractal Organization (Malik, 2015), and is briefly reviewed in Section 7.5 (refer to Appendix 6 - B, for author's acknowledgment of Stanford University and the Director of Organizational Development in the development of this work). All the frameworks, methodologies, and software were consistent with the underlying Mathematics of Innovation for CAS as defined in this dissertation.

Note that this case study follows an ex post facto research approach to attempt to validate the efficacy of the organizational innovation framework as framed by the mathematical equations derived in this dissertation. The investigator interpreted some of the attitudes, values, perceptions, and behaviors of individuals and groups at Stanford University Medical Center as case study research over a multi-year period in context of some of the mathematical equations derived in this dissertation. A primary disadvantage of the ex post facto research approach is that the researcher working in retrospect, and therefore lacking control of the experiments already conducted, may be led to incorrect interpretation. The advantage of the ex post facto research approach is that a controlled inquiry can lead to useful interpretations in situations such as a complex real-time medical environment, where it is often difficult to conduct true experiments (Sevilla at al, 1992).

The research instrument being focused on in this chapter is "Frame Change", as highlighted in Figure 7.1, adapted from Figure 4.1:

| \# | Research Instrument | Code | Relationship to <br> Chapter 4 |
| :---: | :--- | :--- | :--- |
| RI1 | Cohesiveness of derived <br> mathematical equations | "Cohesiveness" | Model derived <br> (Chapter 4) |
| RI2 | Further insight and simplification of <br> properties in target domains | "Insight and <br> Simplification" | Dependent <br> (Chapter 5) |
| RI3 | Relationship of adjustable parameters <br> to system innovation | "Parameter <br> Relationship" | Dependent <br> (This chapter) |
| RI4 | Ability of derived mathematical <br> equations to frame organizational <br> change and innovation | "Frame | Dependent <br> Change" |

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| RI5 | Cohesiveness of integration of piece- <br> meal mathematics into main model | "Integration" | Dependent <br> (Chapter 8) |
| :--- | :--- | :--- | :--- |
| RI6 | Addressing of generated hypotheses <br> in the literature review | "Addressing <br> Hypotheses" | Dependent <br> (Chapter 8) |
| RI7 | Ease with which established theory <br> suggests structure of mathematical <br> model | "Established |  |
| Theory" |  |  |  |$\quad$| Dependent |
| :--- |
| (Chapter 8) |

Figure 7.1 Highlighting "Frame Change" as the Research Instrument of Chapter 7
In Chapter 8, Evaluation, the ability of the derived mathematical equations to frame change and innovation, will be assessed.

### 7.1 The Objectives and The Equations

Hence, the objective of the work at Stanford University Medical Center, to engender a higher degree of organizational innovation, may be summarized by the Generalized Equation of Innovation, Equation 4.7.6, reproduced below for convenience. As discussed in detail in Chapter 4, layers $U, M_{1}, M_{2}$, and $M_{3}$ may be thought of as the dynamic sources of innovation. By definition $U$ is an untransformed layer and to the extent that it can open to the influence of the metalayers, the untransformed sets $x_{U}$ will progress toward the transformed sets $x_{T}$ along the physical, the vital, the mental, and the integral dimensions and innovation will have deemed to occur.

Reproducing Equation 4.7.6:

$$
\text { Innovation }_{\text {orientation }-x}=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \text { Sig }_{\text {S }} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T}, \text { where }\left[\begin{array}{ccc}
x_{U} \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

By default it is generally the untransformed layer, $U$ that is first active. Activation of the other layers requires overcoming of habitual patterns, and this was the thrust of much of the work at Stanford. The levers for overcoming the habitual patterns increase with the activation of the meta-layers along each dimension of the being, represented by the equations for the physical (Equation 4.7.1), the vital (Equation 4.7.3), the mental (Equation 4.7.4), and the integral (Equation 4.7.5). Hence, reproducing Equation 4.7.1:

Physical


## Reproducing Equation 4.7.3:

Vital


Reproducing Equation 4.7.4:

Mental

Reproducing Equation 4.7.5:

$$
\begin{aligned}
& \text { Integral }
\end{aligned}
$$

Basically Physical $_{U}$ has to be progressively replaced by Physical $_{T}$, Vital $_{U}$ has to be progressively replaced by Vital $_{T}$, Mental $_{U}$ progressively replaced by Mental ${ }_{T}$, and Integral $_{U}$ by Integral ${ }_{T}$.

Summarizing, the aim of the work represented in this case study was to get employees, teams, and departments to overcome habitual patterns at their respective levels to allow access to deeper founts of innovation as defined by the equations for the physical, the vital, the mental, and the integral, to thereby allow a greater range of sources of innovation to come into being. While it is difficult to measure such a vast range of possible sources of innovation, in Section 7.3 an attempt to measure a single source, Synergy, belonging to the set of Nurturing, was made through a controlled experiment.

### 7.2 The Beginning of the Work

At the start of the journey the core organizational group, referred to as the Organizational Development (OD) group / team, studied the theory of fractal organizations that frames in words what is happening in the process of innovation as captured by the mathematical equations derived in this dissertation. This was done through a detailed study of the organizational fractal primer, Connecting Inner Power with Global Change (Malik, 2009). Equations 4.7.1 through 4.7.6 reproduced in Section 7.1 to frame the work at Stanford are fractal in nature in that they frame the workings of innovation of CAS at any scale. Further, they suggest that the nature of CAS at a level of complexity as dictated by the set of equations that frame the kernel of innovation, will influence CAS at a subsequent level of complexity as implied by Equation 5.6.2.3 reproduced here for convenience:

Fractal: Orientation ${ }_{x}$ @ Complexity $\leftrightarrow$ Orientation $_{x}$ @ Complexity ${ }_{n+1}$
Fractal effects are also implied by the simulation presented in Chapter 6 in that the nature of the results are consistent with the dynamics of the active underlying layer from U through $M_{3}$. When only U is active the untransformed behaviors result in overall system innovation continually fluctuating between a narrow-band as illustrated in Figure 6.2.7. When the meta-layers become active system innovation can begin to increase substantially as illustrated by Figures 6.3.4, 6.3.5, 6.4.4, 6.4.5, 6.5.5, and 6.5.6.

Following the study circle the OD Team consisting of six people, went through a halfday workshop on the Emotional Intelligence Builder software (Deep Order Technologies Team-Analytics, 2016). This software helps one become aware of some habitual physical, vital, and mental emotion-based patterns people and teams are easily subject to. Note that many of the elements in the untransformed sets at layer $U$ referred to in Equations 4.7.1, 4.7.3, 4.7.4, and 4.7 .5 are accompanied by such physical, vital, and mental emotions that are bound up with the elements. The Emotional Intelligence Builder software focuses on the management of these

emotions and therefore of the untransformed elements specified by the $X_{U}$ untransformed sets that often will not allow a person to be objective if they are seized by an emotion. Daniel Goleman refers to this phenomenon as 'Amygdala Hijack' in his book on emotional intelligence (Goleman, 2005).

Such awareness creates the possibility of choice, as one may more consciously choose to invest in another response when the restricting pattern under consideration arises. The moment-by-moment choice making facilitates the movement from the untransformed sets $X_{U}$ to the transformed sets $X_{T}$, which is the implicit goal of Equations 4.7.1, 4.7.3, 4.7.4, and 4.7.5. The menu of habitual physical, vital, and mental emotional-patterns available through the Emotional Intelligence Builder software are summarized by the six columns in Figure 7.2.1 below:


Figure 7.2.1 Summary of States of Being
Following the OD Team's exposure to the software in facilitating the movement from $X_{U}$ to $X_{T}$ as laid out in Equations 4.7.1, 4.7.3, 4.7.4, and 4.7.5, the next logical step was to have a wider audience within Stanford experiment with the software as well. Hence, the next stream of work took place with the Stanford Center of Education and Professional Development (SCEPD). Several custom courses hinged
around the Emotional Intelligence Builder software were developed and offered primarily to nurses at Stanford University Medical Center (refer to Appendix 6 - C).

These initial successes resulted in the launch of a larger program focused on a core leadership group at Stanford Hospital \& Clinics.

### 7.3 Results of Work at Stanford Hospital \& Clinics Leadership Academy

There is a value in holding up a mirror to teams as suggested by Lencioni's 'The Five Dysfunctions of a Team' (Lencioni, 2002). Planning and tracking plans-whether tasks completed, milestones met, outcomes realized-is part of this, but only captures a fraction of the image. As important is the capturing of softer, feelingsbased team dynamics, as covered by the Emotional Intelligence Builder software: "What makes team members excited, or complacent, or fearful, or angry? What is the intensity of a feeling? What team dynamics may have caused it? Did a feeling persist beyond the boundary of a team meeting, to perhaps keep the team-member up at night? It is often the nature of persistent feelings that at the end of the day will make or break a team." Appendix 6 - D illustrates software instructions aimed at capturing such 'softer' dynamics. Equations 4.7.1, 4.7.3, 4.7.4, and 4.7.5 suggest that the nature of U can be transformed by influences from the meta-levels, $M_{N}$, where N $=1,2$, or 3 . This transformation can be assessed by the changing nature of the patterns being captured in the Emotional Intelligence Builder software. Such change in the nature of patterns may indicate the phenomena of neuroplasticity whereby connections between neurons are changed bringing about change in response patterns given the same stimuli (Bavelier et al., 2012).

Given the initial success with the OD Team and with the courses offered through SCEPD, this web-based team-development software was used by Stanford Hospital \& Clinics to accelerate team development through its 6-month Leadership Academy program. The web-based tool computes fractal patterns in real-time and illustrates the true and often unstated dynamics occurring at the level of teams. The results obtained from this Stanford program are used ex post facto to reinforce the further efficacy of the Equation 4.7.6, The Generalized Equation of Innovation, which was also simulated in Vensim as reported in Chapter 6.

Insight into fractal patterns are important because it is the "small" behaviors, attitudes, perceptions at the individual or team level that correlate with and often determine larger outcomes at the team, unit, and corporate level. The Leadership Academy was an annual program in which close to 50 leaders from across the institute participated to develop and put into practice key leadership skills identified as critical to its future. Leaders were placed into teams that worked together on strategic projects.

The nature of the patterns that each team member experienced was self-captured in the web-based tool, anonymously or openly, depending on how transparent a team
chose to be. Team members had to pause to really get into the nature of what they were experiencing and it is likely that such a pause and self-look began to build a set of muscles not often used (Felicia, 2011), by virtue of which a sustainable awareness field then continues to grow in richness.

Tracking such information sheds light on the culture of a team in process of being created. Focusing on feelings is critical because failure to do so erodes team environment and accelerates destructive dynamics. Successfully managing potentially destructive feelings, on the other hand, allows the team to scale the team maturity curve faster as implied in Tuckman's classic article 'Developmental Sequence in Small Groups' (Tuckman, 1965). Hence, changing the nature of interaction between team members allows the team to shift through the forming-storming-norming-performing stages of team development at an accelerated pace to thereby increase team productivity.

Such team-development software can help teams move through the forming-storming-norming-performing stages of team development at an accelerated pace. Oftentimes project teams get stuck at the forming or storming stages. In reality very few teams make it through to the norming and performing stages. Through tracking issues and accompanying states of being the software draws attention to patterns that cause the team to stagnate around a particular state of dysfunction. Identification of such patterns unequivocally identifies the stage of development that a team is at, and sets the bases for the team to begin to work away from such patterns to more desirable ones.

If there is a prevalent pattern of states such as 'synthesizing', 'reasoning', 'calmness', 'patience', and 'enthusiasm', punctuated by only instances of states such 'haste', and 'fear' for example, this would seem to indicate that a team is operating at the norming stage. A norming stage is where the team has begun to develop and follow rational contracts to govern itself (Stein, 2016). The software allows teams to become more aware of the patterns holding them up, allows them to begin to surface and address issues in a safer way, provides insights into the particular circumstances that typically cause such patterns of dysfunction, suggests numerous ways in which to begin to move to better patterns of functioning, and allows tracking and shifting of such patterns in real-time.

The approximately 50 leaders divided into 7 project teams, worked on strategic hospital-based initiatives, and were asked to track team dynamics on a regular basis. In fact, they were given a choice of whether to track team dynamics or not to. While individuals in those teams who chose to use the tool reported increased sensitivity to feelings-based team dynamics, the question is did the use of the tool actually increase productivity at the team level? Results from a simulated subarctic survival exercise (Human Synergistics, 2013) that each of the 7 teams went through, indicated this to be the case. In this simulation, involving a plane crash, each team member is individually asked to rank 15 items necessary for their survival. The team then collectively ranks the same items. If the team score is better than the best
individual score, then synergy is deemed to have occurred. If the team score is worse that the best individual score then synergy is deemed not to have occurred. A lower score implies a higher degree of team synergy.

The experiment revealed a couple of clear correlations (Aurosoorya, 2011). First, those teams whose use of the team dynamics tool was low exhibited lower synergy as suggested by Figure 7.3.1. Second, those teams whose use of the team dynamics tool is at the ideal level even for a period of time registered a higher degree of synergy as measured by the survival exercise. This is likely because the nature of interaction between the team members was positively impacted by becoming aware of, calling out, and acting on negative dynamics in real-time. In other words, and ex post facto, the untransformed set Integral $_{U}$ began to move towards the transformed set Integral ${ }_{T}$ as specified in Equation 4.7.5. Synergy, which is contained in this equation, was positively affected. It is also likely that the very act of becoming aware of and calling out positive dynamics tended to reinforce them.

Figure 7.3.1 summarizes the correlation between teams that more actively used the team dynamics tool and their level of synergy as measured by the subarctic survival simulation:


Figure 7.3.1 Stanford Hospital \& Clinics and Author's Correlation of Emotional Intelligence Builder Software with Increased Team Synergy

As can be seen, a couple of correlations are suggested. First, those teams whose use of the team dynamics tool is low (max level = medium, in the yellow graphic in Figure 7.3.1) exhibited lower synergy. Note that the use of the tool was measured over a four-month period on the dates indicated on the x -axis of the graphs. Second, those teams whose use of the team dynamics tool is at the ideal level (indicated by
the blue line in the yellow graphic in Figure 7.3.1) even for some period of time registered a higher degree of synergy as measured by the survival exercise in the blue graphic in Figure 7.3.1. This was at least in part reportedly caused by their subsequently internalizing the mechanisms learned through use of the Emotional Intelligence Builder tool.

### 7.4 Work With the Pediatric Intensive Care Unit

Following the program with the Leadership Academy, the leadership teams of several departments at Stanford University Medical Center engaged in multi-month projects to improve the management of their departments through the activation of additional sources of innovation. This section covers a representative piece of work conducted at the Pediatric Intensive Care Unit (PICU) at Stanford University's Lucille Packard Children's Hospital.

The PICU is an area within the hospital specializing in the care of critically ill infants, children, and teenagers. Because of the acuity of PICU patients and the risk of lifethreatening complications the ratio of professionals to patients is generally higher than in other parts of a hospital. Further, complex technology and equipment such as mechanical ventilators and patient monitoring devices is often in use and makes for a more complex sociotechnical environment. This means that the PICU generally has a larger operating budget as compared to other areas of a hospital, and typically has more organizational development related issues. Further, as related by the leaders in dialog with the researcher of this dissertation, the PICU could do with an improvement in Employee Engagement and Patient Satisfaction scores. Studies indicate a direct correlation between these scores and future revenues (Al-Mailam, 2005).

The Director had heard about the approach employed by the author and researcher of this dissertation in helping leaders and departments and wanted to engage in a 3month project, as another case study for this research. The project was kicked off with a 1-day training and orientation in which the PICU Leadership Team was initiated into the world of fractals and innovation, and the real choice they had in accepting or changing their operating reality by making small personal changes in their active states of being, and in alignment with Equation 4.7.6.

The 1-day orientation grounds participants in self- and team-awareness. The day starts with some hypothetical exercises. A typical scenario may be the following: "Imagine you are all working for a very demanding and self-serving boss who wants more than anything else to meet their own goals, regardless of what that implies for their staff. Now imagine there was a fire in one of the local schools and many children are being rushed to be treated at the hospital. The PICU rapidly fills up, and stress-levels amongst nurses quickly escalate. This affects decision-making and continues to put the department and hospital at higher than normal risk. You have to get the environment back to normalcy as quickly as possible."

The participants all have access to the Emotional Intelligence Builder web-based environment as described in the previous section, and record their perceptions of self and of others in the leadership team. The framing questions to record such observations may be of the following nature : "What are you experiencing? What do you perceive your colleagues on this team are experiencing? What is the nature of decision-making? How are you handling conflict as a team?" All entries are anonymous, but contribute toward a single "team dashboard" projected in real-time. There are over 40 emotion-based patterns to choose from as illustrated in Figure 7.2.1, ranging from lethargy, boredom, fear, anger, frustration, to joy, courage, synergy, and so on. Appendix 6 - E illustrates representative information captured for the team.

Since such patterns are visceral they tend to influence the working culture of the team much more powerfully than ideas or thought in the short-term (Goleman, 2005). As participants continue to reflect on the questions in the context of the hypothetical situation definite patterns emerge which are immediately reflected on the summary team screen. After 30 minutes of entry the team then switches gear and begins to analyze this data. What is the Direction-pattern indicating about the situation? Since patterns will repeat themselves on different scale, that is, on the individual, and the department level, what is the pattern going to indicate about how the department is likely to operate in this scenario? Different patterns indicate different operating realities and by looking at a pattern one can tell the nature of the operating reality and the stage of development of the team/department quite conclusively (Aurosoorya, 2015).

Now that the team has had their feet wet with the idea of patterns, self- and otherawareness, and the fractal imprint, the second half of the day switches to real scenarios. Typical scenarios latch onto real and critical projects or changes that the department is currently going through, or on the nature of leadership that is being felt by the team. For this first-day orientation the PICU Leadership Team decided to focus on a department restructuring they were currently in the midst of.

Usually the first day is concluded by analyzing the patterns being displayed. This offers a quick visceral insight into the nature of the problems and issues and the summary gestalt being experienced because of the restructure. Typical questions that participants get insight into include: "What are people confident about? What do they fear are the big obstacles? Are there personalities coming in the way of the work? What is the general mood in the environment? How much conflict is being swept under the carpet to perhaps surface in other and often more destructive ways?"

As with most teams, the summary pattern being displayed indicated that the team was somewhere between the forming and storming stage (Tuckman, 1965), even though the leadership team had worked together for over 1 year, and the project was already a few months past the launch phase. This was a less than positive
prognosis and quickly generated a number of hypotheses into how employee engagement and patient satisfaction scores could improve.

At the end of the day the team contracted to use the Emotional Intelligence Builder SaaS environment on a regular basis. They were advised to engage with the SaaS environment every time they had a team meeting, or an interaction with one another. It typically takes only a few minutes to enter one's feelings and thoughts. It was agreed to meet at the end of the month and analyze the data together. The author and researcher of this dissertation also entered into a coaching arrangement with several of the leaders to help them see the reality they were creating through who and what they were or were not being and doing.

The PICU leadership team went through a few iterations of this cycle-of the team entering observations for a month and being coached - and then collectively analyzing the patterns and their implications. This approach forced the leaders to see things that they would not normally focus on, and forced them to take actions they would not earlier have taken. As a result the team maturity improved, as did the active management of the restructuring project. The foundation for sustainably improving employee engagement and patient satisfaction scores was arguably put in place through more systematically exercising several additional sources of innovation. These additional sources of innovation included 'team synergy', from the set of system nurturing as illustrated by Equation 4.3.4, 'objective decisionmaking' from the set of system knowledge as illustrated by Equation 4.3.3, and a higher degree of 'diligence' from the set of presence as illustrated by the Equation 4.3.1. These in turn, and ex post facto, further activated the transformations captured by the Equations $4.7 .5,4.7 .4$, and 4.7 .1 respectively to put in place a more sustainable reality of innovation.

### 7.5 The Fractal Organization Field Guide

Following the multi-year experimentation the author of this dissertation and the Director of the Organizational Development department sought to create a practical fractal-based methodology hinged on the physical, the vital, and the mental, and subsequently embarked on a multi-year journey to create a field guide to be used by any aspiring department or corporation seeking to bring about sustainable innovation.

This field guide was a key part of the Sage published, The Fractal Organization (Malik, 2015), with a foreword by Dean Dipak Jain of Kellogg Graduate School of Management, INSEAD, and Sasin (refer to Appendix 6 - F). Further this book, encapsulating the practical field guide and the fractal-based theoretical foundation (refer to Appendix 6 - G, for TOC of The Fractal Organization that contains the "field guide"), was endorsed by several of the Stanford University Medical Center leaders (Appendix 6-H).

### 7.6 Summary

The framing and subsequent enhancement of innovation, as captured by Equations 4.7.1 through 4.7.6, began with a small group - the core OD Team, and progressed to a series of courses offered through the Stanford Center of Education and Professional Development. Subsequently a leadership cadre used the same program over a six-month period. This led to several department leadership teams also going through up to a multi-month innovation engagement to improve performance at the department level. This work culminated in a practical field guide that was published in The Fractal Organization (Malik, 2015) that sought to capture key underlying aspects of the multi-year experimentation to-date.

This work had a degree of success as captured and reported by sources other than the author and researcher of this dissertation. In particular in 2011 a Forbes reporter (Skibola, 2011) wrote an article capturing the essence of this work in a piece, "Mindfulness as a Tool of Organizational and Social Change". Further, the Society of Human Resource Management did a report (SHRM, 2011) on "'Advancing Sustainability: HR's Role" where an expert view on changing the culture at Stanford University Medical Center (SUMC) was provided by the Director of Organizational Development at Stanford University Medical Center, on the fractal effects of small changes at the individual level on larger issues of the discipline of Sustainability.

In the Director's words (SHRM, 2011): "Change programs are often initiated at the policy level. Such top-down orchestration is important, but it does not always work. To make change sustainable, regardless of the type of change, it would be most effective to supplement top-down change with actual shifts in attitudes, behaviors and even perceptions. This is what I am focusing on at SUMC. To make it happen, we have initiated a series of organizational interventions and courses that focus on shifting the individual point of view from the physical to the vital to the mental. Such interventions and courses focus on team-building, conducting crucial conversations, conflict resolution, coaching, among others. These are critical in shifting individual behavior. The 'physical' refers to old, established ways of doing things that have often outlived their utility but continue to be followed because of force of habit. The 'vital' refers to a lot of experiment, often led by feeling and emotion, and is critical as an organization tries to break away from established ways of doing things. The 'mental' refers to well-thought out and holistic ideas that, in fact, must become the engine of decision-making, as opposed to habit or emotion. To build sustainable organizations, it is essential that employees begin to operate at the mental level. But equally the well-thought-out ideas have to be supported by the vital-the emotion and feeling-and the physical-the past capital and infrastructure that are the result of the organization's historical success."

Further details on the multi-year journey at Stanford and its spread into other areas, are captured by the author of this dissertation on Medium (Malik, 2015). Such
spreading is consistent with the influence from multi-levels suggested by Equations 4.7.1 through 4.7.6 as discussed in Chapter 4.

So far, from Chapter 1 through Chapter 7, this dissertation has proceeded to:

1. Establish key attributes of the desired theory that became overall guiding principles for the research and are suggested by fundamental departures from the existing body of CAS thinking as reviewed in Chapter 1.
2. Reviewed representative literature of existing mathematical modeling as it relates to innovation in CAS, and surfaced existing design principles and suggested working hypotheses in Chapter 2.
3. Constructed a conceptual analytical framework expressed as a series of equations that defined a mathematical kernel of innovation in CAS in Chapter 4.
4. Applied the mathematical framework to sample CAS domains to arrive at relevant conclusions about the domains to further understanding of these domains, while also suggesting the efficacy of the derived model, and initial validation of working hypotheses in Chapter 5.
5. Simulated the primary mathematical equation for innovation using Vensim Simulator, to descriptively research how innovation in CAS may be altered through manipulating key parameters in the equation in Chapter 6.
6. In ex post facto manner applied some of the derived equations for CAS innovation to a multi-year case study at Stanford University Medical Center to gain further insight into a process of organizational innovation in this chapter.

Having considered a case study to frame innovation, Chapter 8 will focus on evaluating all the streams of research in this dissertation.

CHAPTER 8: Evaluation

The purpose of this chapter is to evaluate the research conducted in the "Link Chapter" against the research instruments established in Chapter 3 and depicted by the "Research Instrument Code" in Figure 8.1. Hence, Section 8.1 as detailed in the second row of Figure 8.1, for example, will evaluate the "cohesiveness" of the derived mathematical equations. Note that this table is an enhanced version of the original in Chapter 3, Figure 3.3.1.

| Section <br> $\#$ | Area of <br> Evaluation | RI\# | Research <br> Instrument <br> Description | Research <br> Instrument <br> Code | Link Chapter |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 8.1 | Mathematical <br> Equations | RI1 | Cohesiveness <br> of derived <br> mathematical <br> equations | "Cohesiveness" | 4,5 |
| 8.2 | CAS Target <br> Application <br> Domains | RI2 | Further <br> insight and <br> simplification <br> of properties <br> in target <br> domains | "Insight and <br> Simplification" | 5 |
| 8.3 | System <br> Simulation | RI3 | Relationship <br> of adjustable <br> parameters to <br> system <br> innovation | "Parameter <br> Relationship" | 6 |
| 8.4 | Stanford Case <br> Study | RI4 | Ability of <br> derived <br> mathematical <br> equations to <br> frame <br> organizational <br> change and <br> innovation | "Frame" <br> Change" | 7 |
| 8.5 | Existing CAS <br> Math | RI5 | Cohesiveness <br> of integration <br> of piece-meal <br> mathematics <br> into main <br> model | "Integration" | 4 |

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$\left.\begin{array}{|l|l|l|l|l|c|}\hline 8.6 & \begin{array}{l}\text { Literature } \\ \text { Review }\end{array} & \text { RI6 } & \begin{array}{l}\text { Addressing of } \\ \text { generated } \\ \text { hypotheses in } \\ \text { the literature } \\ \text { review }\end{array} & \begin{array}{l}\text { "Addressing } \\ \text { Hypotheses" }\end{array} & 2 \\ \hline 8.7 & \begin{array}{l}\text { Proof of } \\ \text { Mathematical } \\ \text { Structure }\end{array} & \text { RI7 } & \begin{array}{l}\text { Ease with } \\ \text { which } \\ \text { established } \\ \text { theory } \\ \text { suggests } \\ \text { structure of } \\ \text { mathematical } \\ \text { model }\end{array} & \text { "Established } & \text { Theory" }\end{array}\right] 4,9$

Figure 8.1 Research Evaluation

### 8.1 Evaluation of Mathematical Equations

The primary research instrument RI1, as summarized in Figure 8.1, is the cohesiveness of the derived mathematical equations. In Chapter 4 a conceptual analytical framework to define a mathematical kernel of innovation in CAS was constructed. The mathematical kernel was expressed as a series of derived equations arrived at inductively. The range of equations varies from the micro, defining the nature of a point in any CAS system, to the macro, encompassing dynamism and innovation in any CAS taken as a whole. Some further equations were also derived in Chapter 5 in the course of applying existing equations to the chosen target domains. Given that subsequent equations are built on previous ones there is a high degree of cohesiveness between the set of equations used to enumerate innovation in CAS. The range of derived mathematical equations and a summary evaluation of cohesiveness is as summarized by Figure 8.1.1:

| $\#$ | Area | Purpose | Cohesiveness |
| :--- | :--- | :--- | :--- |
| 1 | System Bias | Initial approximation of <br> system bias based on 3 <br> fundamental states | Provides a basis for evaluation |
| 2 | Nature of a Point | Derived equations for four- <br> fold intelligence inherent in a <br> point | Provides foundational basis |
| 3 | Architectural Forces | Four sets of the sources of <br> innovation | Four sets describe more fully <br> four-fold source of innovation in <br> a point in \#2 |
| 4 | Uniqueness of <br> Organizations | Equation for organizational <br> uniqueness | Equations are built on <br> architectural forces in \#3 |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 5 | Emergence of Uniqueness | Equation for surfacing of organizational uniqueness | Equations are built on 3 fundamental states used in \#1 |
| 6 | Varying Culture of Organizations | Describes conditions for diversity of culture | Equations based on Y-Sets identified in \#4 |
| 7 | Inherent Dynamics of our System | Describe the inherent innovativeness in our system | Built on areas \#1 through \#4 |
| 8 | Stagnation \& Dynamic Growth | Describes conditions for stagnation and growth | Builds on insights provided by \#7 |
| 9 | Qualified Determinism in CAS | Describes relationship between determinism and randomness in our system | Builds on \#7 |
| 10 | Framing Organizational Transitions at Layer U | Suggests further distinctions in Prigogine's Dissipative Structure Inequality | Utilizes insights provided by \#1, \#5, \#7 to seamlessly integrate equation for dissipative structure inequality into constructed mathematical model |
| 11 | Framing \& Modeling Shift in Innovation at Layer U | Framing of shifts in innovation based on Turing ActivatorInhibitor equations | Utilizes insights provided by \#7 to integrate this set of equations into constructed mathematical model |
| 12 | Framing Complexity | Framing of complexity based on generally accepted measures in the literature | Utilizes \#5 and \#9 to provide enhanced framing of complexity |
| 13 | Molecular Plans at Cellular Level | Explores equations for nature of proteins, nucleic acids, lipids, and polysaccharides | Utilizes to \#4 and \#7 to build equations for the 4 molecular plans and provides additional insight into the possible structure of the cell |
| 14 | Alteration of Space and Time as Suggested by Theory of Relativity | Specifies alterations to space and time at $U$ | Suggested by \#5 |
| 15 | Integration of $U$ and $M_{x}$ | Suggests an equationdepiction of integration of $U$ and $M_{x}$ | Utilizes \#1 through \#4 |
| 16 | Architecture of Quantum Particles | Explores equations for the nature of quantum particles, quarks, leptons, bosons, and the Higgs-boson | Utilizes \#4 and \#7 to build equations for a possible architecture of quantum particles while also providing insight into possible missing pieces of quantum architecture |
| 17 | Periodic Table Element equations | Hypothetical equations for elements in the Periodic Table | Leverages \#4 |
| 17 | Sustainability of CAS | Suggests an equation for achieving sustainability of CAS | Builds on \#3 to provide possible insight into a possible way to |

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|  |  |  | enhance sustainability of any CAS |
| :--- | :--- | :--- | :--- |
| 18 | General CAS <br> Mathematical <br> Operators | Definition of some CAS <br> mathematical operators in <br> light of the mathematical <br> model constructed here | Suggests a set of CAS <br> mathematical operators based on <br> the constructed mathematical <br> model |

Figure 8.1.1 Cohesiveness of Derived Mathematical Equations

### 8.2 Evaluation of CAS Target Application Domains

The primary research problem of this dissertation as stated in Section 1.2 is: Can an engine of innovation within CAS be framed such that it will usefully apply across all CAS regardless of level of complexity or scale?

Given this several CAS target domains that vary in complexity and scale were selected. As each target domain is considered the general test will be the ability of the constructed mathematical model to provide insight and simplify some existing properties in the domain as summarized by RI2 in Figure 8.1. Recall from Chapter 5 that the following criteria are used in the approach to selecting target CAS domains:

1. First start with a CAS domain that has already been subject to a high-level of research. As summarized in Chapter 5, based on funding received the Living Cell is such a CAS domain.
2. Move to other domains at different scales so that it can be shown that the mathematical model for innovation in CAS exists at multiple scale and levels of complexity. Hence the quantum level is selected, first for an application of some aspect of the mathematical model of innovation to properties considered to be true of the quantum level in general. Second to explore an alternative scheme for the characterization of quantum particles that is consistent with the suggestions that emerge when considering the cellular level.
3. Move to a level of complexity between the quantum and cellular level. This would be the atomic level and specifically the properties of elements as characterized by the Periodic Table, where also a large amount of research has already been conducted.
4. Having potentially observed similarity across these three areas of increasing scale and complexity - quantum, atomic, cellular - select the meta-area of CAS itself to draw insight into additional properties of CAS.

Summarizing, the target CAS domains that have been considered include:

1. The cellular level
2. The quantum level
3. The architecture of quantum particles
4. The periodic table
5. Sustainability in CAS

## 6. General CAS properties

### 8.2.1 Application of Generalized Equation of Innovation at Cellular Level

As summarized by RI2 in Figure 8.1 the application of the mathematical model derived here to the cellular level will be evaluated in relation to:

1. Its ability to provide additional insight at the cellular level
2. Its ability to simplify existing properties at the cellular level

As summarized in Section 5.1 some insight gained by applying the model to the domain of the cell includes:

- All nanoscale machinery operative at the cellular level can be thought of in terms of function, which in terms will specify form.
- The function itself can be derived from cellular-level 'architectural forces' that parallel the general sets of architectural forces suggested to be true for the larger containing system as summarized by Equations 4.3.1, 4.3.2, 4.3.4, and 4.3.4.

Hence, reproducing Equation 5.1.1 that builds on the architectural forces specified by Equation 4.3.1:

$$
\text { Sig }_{\text {protein }}=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P r}}\right] \\
Y \in\left[S_{\text {System }_{P r} r} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers; } a>b
\end{array}\right]
$$

Reproducing Equation 5.1.2 that builds on the architectural forces specified by Equation 4.3.3:

Sig $_{\text {nucleic acid }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m}^{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Reproducing Equation 5.1.3 that builds on the architectural forces specified by Equation 4.3.4:

Sig $_{\text {lipid }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers; } a>b\end{array}\right]$
Reproducing Equation 5.1.4 that builds on the architectural forces specified by Equation 4.3.2:

$$
\begin{aligned}
& \text { Sig }_{\text {poysaccharide }}= \\
& X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}},}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{aligned}
$$

Further, the evolving form of the generalized equation of innovation Equation 5.1.5 reproduced here can specify innovation at the cellular level:

$$
\begin{aligned}
& \text { Innovation }_{\text {orientation-x }} \\
& \qquad\left(\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
(\uparrow \operatorname{Sig} \rightarrow F) \\
M_{1} \rightarrow \operatorname{Sig}_{x} \\
\left(\uparrow>P_{x}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T} \text {, where }\left[\begin{array}{c}
x_{U} \ni[\ldots] \\
x_{T} \\
\exists[\ldots]
\end{array}\right]\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
\end{aligned}
$$

Beyond offering insight the reproduced equations also suggest simplification to cellular operation by summarizing all dynamics at the cellular level by these five equations.

Further insight also surfaced in Section 5.1 was summarized as areas of potential research:

1. Organizational signature equations seeded on cellular-level architectural forces will further likely specify a range of nanoscale cellular machinery that may not exist today. Adaption of the genome and of the constituents of the known quaternary molecular plans will likely express the suggested 'missing' machinery with time 2. Quaternary based mathematics of innovation may also suggest advances to cellular level medical technology some time in the future
2. Quaternary based mathematics of innovation may further suggest construction of synthesized nanobots some time in the future

### 8.2.2 Application at the Quantum Level

As summarized by RI2 in Figure 8.1 the application of the mathematical model derived here to the quantum level is evaluated in relation to:

1. Its ability to provide additional insight at the quantum level
2. Its ability to simplify existing properties at the quantum level

It appears to be possible to explain the following quantum effects - superposition and existence of multiverses, dual-wave particle nature, quantum tunneling,
traveling faster than the speed of light, entanglement, and going backward in time, using an alternative model as presented here. The use of the alternative model allows physical reality to maintain its integrity regardless of scale. The "weirdness" experienced at the quantum level is possible through admitting the existence of meta-layers with a different "physics" existing at each meta-layer, and is only brought into focus because of the relatively high degree of control that observation of a few particles allows.

Note that the weirdness as experienced at the quantum-level also exists at the untransformed, or physical level, U. Such weirdness, as in the case at the quantumlevel is a function of the meta-levels and as is being discussed in this dissertation, exists at any scale, even that of 'normal' existence.

### 8.2.2.1 Dual wave-particle nature

As suggested earlier in Section 5.2.1, Schrodinger's equation can be interpreted differently: the wave aspect $\psi$ is actually an indication that the unique function at the meta-level $M_{1}$ as specified by $\operatorname{Sig}_{x}$, is going to assure itself one way or another as apparent by the probability distribution of appropriate particles specified by $\psi$.

### 8.2.2.2 Independent States as Specified by Superposition

As per the discussion in Section 5.2 .2 given that emergent phenomena are in reference to a meta-level context, the superposition that Schrodinger's equation suggests does not define and set into motion manifest independent states, but only possibilities that the meta-level function may cause in fulfilling its implicit intent.

### 8.2.2.3 Quantum Tunneling

As per the discussion in Section 5.2.3 quantum tunneling may be replaced by the existence of a meta-level function, $\operatorname{Sig}_{x}$, which is the organizational signature of a necessary cellular level energy creation and monitoring function, Sig cellular-energy-type 1 in this case. In order to fulfill itself, it may be suggested that this function which exists just behind any surface visible range, oversees the movement or manifestation of electrons and protons and monitors cellular energy 'type1'. If protons and electrons are the visible sign or precipitation of this fundamental energy required for cell function, they can then be thought of as 'mapping' the path of this meta-level principle of organization. The wave hence depicts the very meta-level principle of energy-organization manifest as the probability that electrons and protons that serve that principle will show up in the locations suggested by the wave.

### 8.2.2.4 Canceling out of Quantum Dynamics

As discussed in Section 5.2.4, in his model on Quantum Electro Dynamics (QED) Feynman suggests that photons do not have to travel in straight lines. In fact, when emitted from a monochromatic light source, they will travel in every direction possible, while arriving at the photon detector. There is a reality of superposition in which all possible paths are traversed by photons. His model allows for combining all the paths together through vector addition to arrive at the path of the straight line recognized by classical physics. In other words, the quantum dynamics cancel themselves out so that one path emerges.

So whether at the micro or macro level vector addition results in a single path to which molecular, atomic, or photonic movement is subject. It is also interesting to note that at the macro-level there is a similar canceling out effect of random molecular motion that yet leaves a containing entity subject to some observed law. For example, if a gas is heated up, inspite of all movement of molecules that cancel one another out, yet the gas will expand in proportion to the applied heat and not in proportion to the apparent molecular motion. Such connection from microscopic behavior to macroscopic properties is the subject of statistical thermodynamics that deals with average properties of the molecules, atoms, or elementary particles in random motion in a system of many such particles (Ebeling \& Sokolov, 2005).

These observations are consistent with the idea of a possible existence of an 'organizational function', say 'movement from A to B in an apparent straight line' or the 'equivalence of applied energy', belonging to $M_{1}$ as the realm of signatures or organizational functions, to which organizations whether at the photonic, atomic, or molecular levels at $U$ are subject.

### 8.2.2.5 Traveling Faster Than the Speed of Light

As per the discussion in Section 5.2.5 it has been proposed that information at the quantum level is shared faster than the speed of light (Brumfiel, 2008). But the speed of light is a limit at the physical level, U . At $M_{1}$ a general organizationalfunction, $\operatorname{Sig}_{x}$, modeled to be the organizing factor in quantum-particle dynamics, is not limited by C, the speed of light. Philosophically, C sets a limit on the ability to transcend space-time.

This is brought out more explicitly from the equations of the alteration of time and space as a body approaches the speed of light. Adaptations to these equations are specified as:

Length $_{\text {contraction }-U}=$ Length $_{U}\left(\sqrt{1-\frac{v_{U}{ }^{2}}{c_{U}^{2}}}\right)$

Time $_{\text {elongation }-U}=\frac{\text { Time }_{U}}{\sqrt{1-\frac{v_{U}^{2}}{c_{U}^{2}}}}$

Modeling for this possibility ' C ' can be proposed as different at different levels: that is, ' $C$ ' at $U$ is different than ' $C$ ' at $M_{1}$ and so on. This inequality of $C$ can be specified as:

$$
\text { Inequality }_{C}: C_{U}<C_{M_{1}}<C_{M_{2}}<C_{M_{3}}
$$

8.2.2.6 Entanglement

As discussed in Section 5.2 .6 entanglement has been invoked in describing how birds navigate across the earth. Assume though that there is an organizationalfunction Sig $_{x}$ where $\mathrm{x}=$ flock migration in winter, for example. This is an $M_{1}$ dynamic and particular organizations at the physical layer are subject to it. Quantum particles may or may not have to be involved in this.

But also there have been experiments where a photon of light is used to create two entangled photons. These then share properties when separated. Once entangled, by whatever mechanism, as per the mathematical model developed in this dissertation a $M_{1}$ organizational-function such as $\operatorname{Sig}_{x}$ takes over. Space-time constraints are therefore changed, and a 'quantum' link is now in effect.

### 8.2.2.7 Going Backward in Time

In his book QED: The Strange Theory of Light and Matter, Feynman suggests that there are particles that can move backward in time. As discussed in Section 5.2 .7 if however a meta-level is assumed then spontaneous particle generation from a metalevel organizational-function (in this case Positron-1 moving forward in time) could remove the notion of particles moving backward in time.

### 8.2.2.8 Quantum Fluctuations

As discussed earlier in Section 5.2.8 at the Planck's constant order of magnitude (10${ }^{34}$ ) boundary conditions between U and $M_{1}$ are being experienced, and the quantum fluctuations, the uncertainty relation, and the quantum zero-point energy could be an expression of the essential Signature function, $\operatorname{Sig}_{x}$, that is posited as a key
formative force behind organization at $U$. In this interpretation the thermal energy describes the essential energy at $U$, while the uncertainty relation suggests the phenomenon of innovation-precipitation.

This integration of meta-levels with the surface-level, $I_{U}^{M}$, may be suggested by Equation 5.2.8.1, Integration of Levels (Leveraging Heisenberg's Uncertainty Relation, and reproduced here for convenience:
$I_{U}^{M} \rightarrow \Delta p X \Delta x \geq \frac{h}{4 \pi}$

### 8.2.3 Architecture of Quantum Particles

As summarized by RI2 in Figure 8.1 the application of the mathematical model derived here to the architecture of quantum particles will be evaluated in relation to:

1. Its ability to provide additional insight into the architecture of quantum particles 2. Its ability to simplify the architecture of quantum particles

As discussed in Section 5.3 an alternative yet simple and coherent architecture to quantum particles is suggested by the mathematical model as summarized in Figure 5.3.2, Quaternary Architecture of Quantum Particles, and reproduced here for convenience:


Further, this quaternary architecture provides insight into further possible particle research. Assuming that the quaternary architecture is a function of $M_{2}$, this implies the following for the other levels in the model:

1. There may be a Master-Particle that embodies the quaternary intelligence in itself, and is representative of $M_{1}$. Perhaps this is a material precipitation of a key organizing principle around which the quaternary aspects arrange and expand themselves.
2. Just as at the cellular level one can hypothesize a Sig function, perhaps there is a Sig function, Sig particle that would give more insight into the myriad of existing and yet undiscovered particles. Reproducing Equation 5.3.1 for convenience:

Sig $_{\text {particle }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{{S y s t e m_{K}},}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Hence, it could be that the signature for the family of quarks, as represented by Equation 5.3.2 and reproduced here for convenience, is:

Sig $_{\text {quarks }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{{S H y s t e m_{K}}^{\prime}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.\end{array}\right]$

The signature for the family of leptons, as represented by Equation 5.3.3 and reproduced here for convenience, is:

Sig $_{\text {leptons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m_{P}}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

The signature for the family of gauge bosons, as represented by Equation 5.3.4 and reproduced here for convenience, is:

Sig $_{\text {bosons }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
The signature for the Higgs-boson and any other similar particle, represented by Equation 5.3.5 and reproduced here for convenience, is:

Sig $_{\text {Higgs-boson }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{S y s t e m_{P}}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

3. Meta-levels suggest that existence is functional. This also may imply that even the bases of matter at the very quantum-level is not fixed but subject to adaptability. New particles may manifest depending on function to be expressed. Hence the following equation of innovation, Equation 5.1.5 reproduced here for convenience, is shown to be true:

$$
\begin{aligned}
& \text { Innovation } \text { orientation }-x \\
& \left.=\left(\left[\begin{array}{c}
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \text { Sig }^{\rightarrow} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] T C \rightarrow x_{T} \text {, where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]\right)\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
\end{aligned}
$$

### 8.2.4 The Periodic Table

As summarized by RI2 in Figure 8.1 the application of the derived mathematical model to the periodic table will be evaluated in relation to:

1. Its ability to provide additional insight into the architecture of the periodic table
2. Its ability to simplify the architecture of the periodic table

Hence as discussed in Section 5.4, the insight and simplicity afforded by the mathematical model when applied to the periodic table is in suggesting a relationship between key categories of elements - Alkali Metals, Alkali Earth Metals, Metals, Metalloids, Non-Metals, Halogens, Noble Gases, Lanthanides, and Actinides and the informing meta-level functions.

Hence, reproducing equations from Section 5.4:

$$
\begin{aligned}
& \text { Element }_{\text {Alkali metal }}=\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
X a+\overline{Y b_{0-n}} \text { where }
\end{array}\left[\begin{array}{c} 
\\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right] \\
& \text { Element }_{\text {Alkali earth metal }}= \\
& X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{aligned}
$$

Element $_{\text {Metal }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m_{K}}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.\end{array}\right]$
Element $_{\text {Metalloid }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Non-Metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Halogen }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element ${ }_{\text {Noble Gas }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{\text {Pr }}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Transition Metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$\left.\left.\begin{array}{cc}\text { Element }_{\text {Lanthanide }}= & X \in\left[S_{\text {System }_{N}}\right] \\ X a+\overline{Y b_{0-n}} \text { where }\end{array}\right] \begin{array}{c} \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r} r}, \\ a, b \text { are integers }\end{array} ; a>b\right.\end{array}\right]$
Element ${ }_{\text {Actinide }}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.
\end{array}\right]
$$

Further insight is similarly provided by similarly mapping each element of the periodic table may to functional equations of the same type.

### 8.2.5 Sustainability of CAS Systems

As summarized by RI2 in Figure 8.1 the application of the mathematical model derived here to the sustainability of CAS will be evaluated in relation to:

1. Its ability to provide additional insight into the sustainability of CAS systems
2. Its ability to simplify the modeling of sustainability of CAS systems

As discussed in Section 5.5 the interaction between the four families of forces is instrumental, to enhance sustainability of CAS. Equation 5.5.1, Sustainability of CAS Systems, is reproduced here for convenience:

Sustainability $_{\text {Systems }} \propto$ Interaction $^{\left(S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right)}$
This relationship can be depicted graphically as in Figure 5.5.1 reproduced here for convenience:


The equation and the graph provide insight while capturing the simplicity of sustainability when thought of in terms of the four underlying sets of architectural forces as discussed in Section 4.3.

### 8.2.6 General CAS Properties

As summarized by RI2 in Figure 8.1 the application of the mathematical model derived here to the area of CAS properties will be evaluated in relation to:

1. Its ability to provide additional insight into CAS systems by virtue of properties generated by the mathematical model
2. Its ability to simplify the modeling of CAS systems through leveraging properties generated by the mathematical model

As discussed in Section 5.6, Chapter 2 had referenced several 'properties' thought to be true of complex adaptive systems. These properties primarily emanate from the prevalent bottom-up view of CAS, summarized in Section 2.1.2 and also from the rule-based view of CAS, Section 2.1.3. Here, and based on the mathematics that is being developed in this dissertation 12 general properties or more precisely 'mathematical operators', arrived at deductively and thought to be true of CAS will be suggested. In contrast to the bottom-up and the rules-based views these operators are the result of considering some of the dynamics of each of the levels and integration of the multiple levels of the mathematical model presented in this dissertation. These are non exhaustive, but rather are indicative of the nature of CAS.

Leveraging the quaternary basis of the mathematical model developed in Chapter 4 these mathematical operators can be summarized in four categories that parallel the four-fold scheme: Presence, Power, Knowledge, and Nurturing. Since it is being suggested that any CAS is organized by such a quaternary system it perhaps also makes sense to consider the mathematical operators using such a lens.

In Section 2.1.4 it has been proposed that there is an implicit order to Time and an implicit order to Space. Hence, any organization and the causality that it is subject to has to be reconsidered in light of the implicit time-space order that define the very fabric that animates all organization. If Space is plotted on the x-axis, and Time on the $y$-axis, and a general curve constructed characterized by a time-space boundary-n, as in Figure 5.6.1, reproduced here for convenience, then it can be assumed that the area, Time-Space Continuum, under the time-space boundary-n, is subject to the mathematics and properties being developed here.


The 12 operators have to be considered in context to the General Equation for Innovation, Equation 4.7.6, derived in Section 4.7. This equation suggests that any CAS has implicit in it the urge to transform the untransformed layer, $U$, by opening
to the influence of the meta-layers, $M_{1}, M_{2}$, and $M_{3}$. In so doing the very sources of innovation are altered and the visible characteristics of systems are transformed by the action of these sources of innovation. Several sets have already been suggested that explore these sources of innovation - $S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}$, and $S_{\text {System }_{N}}$, and the resultant characteristics of CAS - Physical $_{T}$, Vital $_{T}$, Mental $_{T}$, and Inte gral $_{T}$. Section 5.6 leveraged various aspects of the already derived mathematical model to frame sets of mathematical operators along each of the four basis of CAS that components or elements of CAS will be subject to in their journey toward transformation thus adding further insight into the area of CAS properties in general, and further simplifying the understanding of accelerating the journey of any CAS to a higher degree of innovativeness.

### 8.3 Evaluation of System Simulation

As summarized by RI3 in Figure 8.1 the basis of evaluation of the system simulation is in the way the adjustable parameters of the system simulation shed insight into resulting system innovation.

Figure 6.7.1 Summary System Innovation Changes from Level to Level, reproduced here for convenience, summarizes the level of relative system innovation achieved with a change in the adjustable parameters:

| LEVEL | STATE | SYSTEM <br> INNOVATION |
| :--- | :--- | :--- |
| U | With negative patterns active | 15 x |
| M1 | With physical patterns overcome | 700 x |
| M1 | With all U-level patterns overcome | $2,000 \mathrm{x}$ |
| M2 | When physical signature becomes force | $4,000 \mathrm{x}$ |
| M2 | When all signatures become forces | $8,000 \mathrm{x}$ |
| M3 | When point is led by presence | $10,000 \mathrm{x}$ |
| M3 | When point is led by all families | $18,000 \mathrm{x}$ |

As discussed in Section 6.7, there is clearly a larger relative leap when M1 becomes active, from a factor of 15 to 700: approximately 47 times. The subsequent overcoming of further categories of patterns continues to have a large absolute impact, though in relative terms with the previous jump as a baseline is less than the first jump from $U$ to $M 1$. This can be explained by the fact that the overcoming of a whole category of habitual patterns likely signifies a significant break from the norm, and begins to alter the way that innovation precipitates.

### 8.4 Evaluation of Stanford Case Study

As summarized by RI4 in Figure 8.1 the basis of evaluation of the Stanford case study is in the ability if the derived mathematical equations to frame organizational change and innovation.

As discussed in Section 7.1 the objective of the work at Stanford University Medical Center, to engender a higher degree of organizational innovation, was summarized by the Generalized Equation of Innovation, Equation 4.7.6, reproduced below for convenience. As discussed in detail in Chapter 4, layers U, $M_{1}, M_{2}$, and $M_{3}$ may be thought of as the dynamic sources of innovation. By definition $U$ is an untransformed layer and to the extent that it can open to the influence of the metalayers, the untransformed sets $x_{U}$ will progress toward the transformed sets $x_{T}$ along the physical, the vital, the mental, and the integral dimensions and innovation will have deemed to occur.

Reproducing Equation 4.7.6:

$$
\text { Innovation }_{\text {orientation }-x}=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{X}} \\
\left(\uparrow \operatorname{Sig}_{\boldsymbol{l}} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

By default it is generally the untransformed layer, $U$ that is first active. Activation of the other layers requires overcoming of habitual patterns, and this was the thrust of much of the work at Stanford. The levers for overcoming the habitual patterns increase with the activation of the meta-layers along each dimension of the being, represented by the equations for the physical (Equation 4.7.1), the vital (Equation 4.7.3), the mental (Equation 4.7.4), and the integral (Equation 4.7.5). Hence, reproducing Equation 4.7.1:
Physical

Reproducing Equation 4.7.3:

> Vital
> $=\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{P} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow S_{\text {System }_{P}} \\ \left(\uparrow \text { Sig }_{\text {a }} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{V} \\ \left(\uparrow>P_{V)}\right. \\ U \rightarrow \text { Vital }_{U}\end{array}\right]$ TC $\rightarrow$ Vital $_{T}$, where $\left[\begin{array}{c}\left.\text { Vital }_{U} \ni[\text { aggression, self centeredness, exploitation, } \ldots]\right] \\ \text { Vital }_{T} \ni[\text { energy, support, adventure, enthusiasm, } \ldots]\end{array}\right]$

Reproducing Equation 4.7.4:

$$
\begin{aligned}
& \text { Mental }
\end{aligned}
$$

## Reproducing Equation 4.7.5:

$$
\begin{aligned}
& \text { Integral } \\
& =\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{N} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{N}} \\
\left(\uparrow \text { Sig }_{\sim} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{I} \\
\left(\uparrow>P_{I)}\right. \\
U \rightarrow \text { Integral }_{U}
\end{array}\right] \text { TC } \rightarrow \text { Integral }_{T}, \text { where }\left[\begin{array}{l}
\text { Integral }_{U} \ni[\text { possession, usurpation, hidden agendas, } \ldots] \\
\text { Integral }_{T} \ni[\text { appreciation, shift POV, MPV, synthesis, }, . .]
\end{array}\right]
\end{aligned}
$$

${\text { Basically } \text { Physical }_{U} \text { has to be progressively replaced by Physical }}_{T}$, Vital $_{U}$ has to be progressively replaced by Vital $_{T}$, Mental $_{U}$ progressively replaced by Mental ${ }_{T}$, and Integral $_{U}$ by Integral ${ }_{T}$.

Summarizing, the aim of the work was to get employees, teams, and departments to overcome habitual patterns at their respective levels to allow access to deeper founts of innovation as defined by the equations for the physical, the vital, the mental, and the integral, to thereby allow a greater range of sources of innovation to come into being. While it is difficult to measure such a vast range of possible sources of innovation, in Section 7.3 an attempt to measure a single source, Synergy, belonging to the set of Nurturing, was made through a controlled experiment.

The framing and subsequent enhancement of innovation, as captured by Equations 4.7.1 through 4.7.6, began with a small group - the core OD Team, and progressed to a series of courses offered through the Stanford Center of Education and Professional Development. Subsequently a leadership cadre used the same program over a six-month period. This led to several department leadership teams also going through up to a multi-month innovation engagement to improve performance at the department level.

As discussed in Section 7.5 this work had a degree of success as captured and reported by sources other than the author and researcher of this dissertation. In particular in 2011 a Forbes reporter (Skibola, 2011) wrote an article capturing the essence of this work in a piece, "Mindfulness as a Tool of Organizational and Social Change". Further, the Society of Human Resource Management did a report (SHRM, 2011) on "'Advancing Sustainability: HR's Role" where an expert view on changing the culture at Stanford University Medical Center (SUMC) was provided by the Director of Organizational Development at Stanford University Medical Center, on the fractal effects of small changes at the individual level on larger issues of the discipline of Sustainability.

In the Director's words (SHRM, 2011): "Change programs are often initiated at the policy level. Such top-down orchestration is important, but it does not always work. To make change sustainable, regardless of the type of change, it would be most effective to supplement top-down change with actual shifts in attitudes, behaviors and even perceptions. This is what I am focusing on at SUMC. To make it happen, we have initiated a series of organizational interventions and courses that focus on shifting the individual point of view from the physical to the vital to the mental. Such interventions and courses focus on team-building, conducting crucial conversations, conflict resolution, coaching, among others. These are critical in shifting individual behavior. The 'physical' refers to old, established ways of doing things that have often outlived their utility but continue to be followed because of force of habit. The 'vital' refers to a lot of experiment, often led by feeling and emotion, and is critical as an organization tries to break away from established ways of doing things. The 'mental' refers to well-thought out and holistic ideas that, in fact, must become the engine of decision-making, as opposed to habit or emotion. To build sustainable organizations, it is essential that employees begin to operate at the mental level. But equally the well-thought-out ideas have to be supported by the vital-the emotion and feeling-and the physical-the past capital and infrastructure that are the result of the organization's historical success."

### 8.5 Evaluation of Existing Math in CAS

As summarized by RI5 in Figure 8.1 the basis of evaluation of the mathematical model derived in this dissertation is the degree to which existing math in CAS may be cohesively integrated into it. If the integration is relatively seamless this may
result in a higher degree of confidence that the model developed in this dissertation is deep and wide enough to be a basis of thinking about a more general mathematics for CAS.

The piece-meal areas that have been considered as part of the existing math in CAS literature review include:

- Prigogine's Structure Inequality, as discussed in some detail in Section 4.10, Framing Organizational Transitions at Layer U
- Turing's Activator-Inhibitor Equations, as discussed in some detail in Section 4.11, Framing and Modeling Shifts in Innovation at Layer U
- Measure of Complexity, as discussed in some detail in Section 4.12, Framing Complexity

Exploration in each of these areas obviously started with equations framed independently of the math model being developed in this dissertation.
Subsequently each of equations were slightly modified in Sections 4.10, 4.11, and 4.12 respectively, so that they gave further insight into CAS while yet maintaining the framing of the math model derived in this dissertation.

Hence, Prigogine's Dissipative Structure inequality was slightly modified to provide additional insight into the nature of the storage functions $\mathrm{V}(\mathrm{x})$ that develop through the dynamics of CAS:
$\frac{d V_{O S}(x(t))}{d t} \leq u(t) \cdot y(t)$
Where:
$O S \epsilon\left[\begin{array}{c}\text { System }_{P r} \\ S_{\text {System }_{P r}} \\ \text { Sig }_{P} \\ \text { Physical }_{U m} \\ \text { Physical }_{U v} \\ \text { Physical }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{P} \\ S_{\text {System }_{P}} \\ \text { Sig }_{V} \\ \text { Vital }_{U m} \\ \text { Vital }_{U v} \\ \text { Vital }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{K} \\ \text { Stystem }_{K} \\ \text { Sig }_{M} \\ \text { Mental }_{U m} \\ \text { Mental }_{U v} \\ \text { Mental }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{N} \\ \text { System }_{N} \\ \text { Sig }_{I} \\ \text { Integral }_{U m} \\ \text { Integral }_{U v} \\ \text { Integral }_{U p}\end{array}\right]$

Further, the relationship between a $x_{U}$ and $x_{T}$ storage function was suggested leveraging the notion of 'untransformed' and 'transformed' as per the Equation of Innovation 4.7.6:
$\frac{d V_{O S}\left(x_{U}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{T}(t)\right)}{d t}$
And further between generated storage functions as they evolve from $x_{U}$ to $x_{T}$ :

$$
\begin{aligned}
\frac{d V_{O S}\left(x_{U}(t)\right)}{d t} & \rightarrow \frac{d V_{O S}\left(x_{T}(t)\right)}{d t}: \\
\frac{d V_{O S}\left(x_{P}(t)\right)}{d t} & <\frac{d V_{O S}\left(x_{V}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{M}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{I}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{F}(t)\right)}{d t} \\
& <\frac{d V_{O S}\left(x_{C}(t)\right)}{d t}
\end{aligned}
$$

The Turing activator-inhibitor equations were modified using the framing derived in the math to get insight into shifting innovation:

Shift $_{\text {innovation }}=\left(\frac{\partial x_{T}}{\partial t}=f\left(x_{T}, x_{U}\right)+D_{x_{T}} \nabla^{2} x_{T}, \frac{\partial x_{U}}{\partial t}=g\left(x_{T}, x_{U}\right)+D_{x_{U}} \nabla^{2} x_{U}\right)$

Further a generally accepted measure of complexity was leveraged, namely:

Complexity $_{\text {System }}=f$ (Multiplicity,Interdependence,Diversity)
To gain new insight into framing complexity:
Complexity System $\propto D I_{V}$

Emergence-Matrix $=\left|\begin{array}{c}C: \operatorname{Sig} * \bmod \left(\int=1\right) \\ F: \operatorname{Sig} \bmod (c) \\ : \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\ M: \operatorname{Sig} * \bmod (G) \\ V: \operatorname{Sig} * \bmod (e) \\ P: \operatorname{Sig} * \bmod (\pi)\end{array}\right|$

Complicated $_{\text {System }}=$ System $\left(\right.$ Sig $\left._{E}: X<I\right)$

Complex $_{\text {System }}=$ System $\left(\operatorname{Sig}_{E}: X \geq I\right)$

### 8.6 Evaluation in Relation to Generated Hypotheses from Literature Review

As summarized by RI6 in Figure 8.1 the basis of evaluation of the mathematical model derived in this dissertation is the degree to which the design principles and working hypotheses generated in the literature review conducted in Chapter 2 have

been addressed by this model. As summarized in Figure 1.5.1 design principles are based on existing insight, and working hypotheses are based on possible new insight pointed to in this dissertation. This section therefore reviews how the design principles and hypotheses generated from the representative literature review have been incorporated into the constructed mathematical model.

### 8.6.1 System Dynamics

The following tree suggests how gaps in the field of System Dynamics surfaced in Section 2.1, have been addressed by the Mathematics of Innovation for CAS derived in this dissertation:

1. The first-level in the tree summarizes hypotheses generated when reviewing some representative literature on System Dynamics
2. The second-level summarizes how these gaps have been incorporated into the mathematics


Figure 8.6.1.1 System Dynamics Hypotheses Tree

Note that the derived mathematics will likely strengthen the field of System Dynamics by inclusion of meta-layers as conceptualized here. Further, the mathematics is in general made more relevant by consideration of and addressing suggested gaps in System Dynamics.

### 8.6.2 Bottom-Up Approach

The following tree suggests how gaps in the bottom-up approach to CAS surfaced in Section 2.1.2, have been addressed by the Mathematics of Innovation for CAS derived in this dissertation:

1. The first-level in the tree summarizes hypotheses generated when reviewing some representative literature on the prevalent bottom-up approach to CAS 2. The second-level summarizes how these gaps have been incorporated into the mathematics


Figure 8.6.2.1 Bottom-Up Approach Hypotheses Tree

Note that the derived mathematics will enhance the conceptualization of CAS even when considered from the bottom-up. Further, the mathematics is in general made more relevant by consideration of and addressing suggested gaps in the bottom-up view.

### 8.6.3 Rule-Based Systems

The following tree suggests how gaps in Rule-Based Systems surfaced in Section 2.1.3, have been addressed by the Mathematics of Innovation for CAS derived in this dissertation:

1. The first-level in the tree summarizes hypotheses generated when reviewing some representative literature on the Rule-Based Systems approach to CAS
2. The second-level summarizes how these gaps have been incorporated into the mathematics


Figure 8.6.3.1 Rule-Based Systems Hypotheses Tree

Note that the derived mathematics will enhance the conceptualization of Rule-Based approach to CAS. Further, the mathematics is in general made more relevant by consideration of and addressing suggested gaps in the Rule-Based approach.

### 8.6.4 Top-Down Approach

The following tree summarizes how hypotheses generated when considering the top-down approach surfaced in Section 2.1.4, have been addressed by the Mathematics of Innovation for CAS derived in this dissertation:

1. The first-level in the tree summarizes hypotheses generated when reviewing some representative literature on mathematics in CAS
2. The second-level summarizes how these hypotheses have been incorporated into the derived mathematics in this dissertation


Figure 8.6.4.1 Top-Down Approach Hypotheses Tree

### 8.6.5 Mathematics

The following tree suggests how gaps in existing mathematics of CAS surfaced in Section 2.1.5, have been addressed by the Mathematics of Innovation for CAS derived in this dissertation:

1. The first-level in the tree summarizes hypotheses generated when reviewing some representative literature on mathematics in CAS
2. The second-level summarizes how these gaps have been incorporated into the derived mathematics in this dissertation


Figure 8.6.5.1 Mathematics in CAS Hypotheses Tree

### 8.7 Evaluation in Relation to Einstein's Theory of Relativity

As summarized by RI7 in Figure 8.1 the basis of evaluation of the mathematical model derived in this dissertation is the ease with which established theory, in this case Einstein's Theory of Relativity, suggests the structure of the mathematical model. Structure is to be thought of in contrast to the detail contained in the model.

Einstein's Theory of Relativity is pertinent to this model in that the model looks at organization that must exist in time and space with implicit order in it. The implicit space-time order is the starting point of this mathematical model. The Theory of Relativity too is looking at the essential nature of the space-time continuum.

The Theory of Relativity basically states that laws of physics are the same in all uniformly moving frames of reference, and further that the speed of light, C, is constant, but time moves differently in different frames of reference.

In his book on relativity (Einstein, 1995) states that "If relative to K, K1 is a uniformly moving coordinate system devoid of rotation, then natural phenomena run their course with respect to K1 according to exactly the same general laws as with respect to K. This is the Principle of Relativity."

He offers a simple argument for the validity of the Theory of Relativity:
"Now in virtue of its motion in an orbit around the sun, our earth is comparable with a railway carriage traveling with a velocity of about 30 kilometers per second. If the principle of relativity were not valid we should therefore expect that the direction of motion of the earth at any moment would enter into the laws of nature, and also that physical systems in their behavior would be dependent on the orientation in space with respect to the earth. For owing to the alteration in direction of the velocity of revolution of the earth in the course of a year, the earth cannot be at rest relative to the hypothetical system $\mathrm{K}[0]$ throughout the whole year. However the most careful observations have never revealed such anisotropic properties in terrestrial physical space, i.e., a physical non-equivalence of different directions. This is a very powerful argument in favor of the principle of relativity."

The Special Theory of Relativity did not include the effects of gravitation and so Einstein formulated the General Theory of Relativity. This he stated as:
"All bodies of reference K, K1 etc, are equivalent for the description of natural phenomena (formulation of the general laws of nature), whatever may be their state of motion."

Now given this as true, it may be said that every coordinate system from $K_{0}$ through $K_{\infty}$ has to have implicit in it the ability to formulate any general law of nature. Yet in systems observations apparent infinite variety is noticed. Therefore every coordinate system from the quantum-particle level to the macro-system level has to have a way to express what is implicit in every space-time point-instant in its infinite possibility. Fullness has to exist potentially in every part. In other words, every possible coordinate system that exists at U, from $K_{0}$ through $K_{\infty}$, has to have implicit in it a mathematical model that can express infinite potentiality.

But this is precisely what the mathematical model derived in this dissertation does. It has constructed a multi-layered system of equations that progressively expresses
infinite diversity by assuming some simple basic principles. It therefore may be suggested that implicit in every coordinate system or every organization, exists a mathematical model that connects via equations, the dynamics of the unseen with the dynamics of the seen in a system of progressively more concrete layers ranging from $M_{3}$ through $M_{2}$ through $M_{1}$ through $U$.

Why though assume a system of multi-layers. Einstein's equation on energy-mass equivalence suggests why.

In this equation,
$E=\frac{m c^{2}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
The minimum amount of energy that accompanies any point-instant of mass can be thought of as a field of C x C. It has already been suggested that C is a limit tied to the implicit nature of $U$. It specifies the possible space-time behavior that can exist in one dimension. C x C perhaps can be thought of as the possible space-time behavior that exist in two dimensions or a field such as $U$.

Now if the 'velocity' of the point-instant represented by m increases, then as can be seen the field of energy will transcend the limit set by U. But if the limit possible at $U$ has been transcended then the suggestion is that some in-view or dynamic representative of meta-layers must have been revealed. Therefore the point-instant represented by m may be thought of as a precipitation from other space-time realms and a system of multi-layers suggested as existing behind each point-instant.

A question is how can the 'velocity' of a point-instant increase? This may be tied to rate of change of the organization emanating from a point-instant. If that rate of change increases due to a rapid change in patterns then basically it is implied that the limits for transcendence have been put in place, and that innovation is bound to happen.

### 8.8 Summary

The preceding sections of Chapter 8 have evaluated the research as per Figure 8.1. Figure 8.9.1, Summary of Research Evaluated, builds on Figure 8.1 to summarize the results of this evaluation.

| Section <br> $\#$ | Area of <br> Evaluation | RI\# | Research <br> Instrument <br> Description | Research <br> Instrument <br> Code | Link <br> Chapter | RI <br> Successfully <br> Addressed |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.1 | Mathematical | RI1 | Cohesiveness | "Cohesiveness" | 4,5 | Yes |

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|  | Equations |  | of derived mathematical equations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.2 | CAS Target Application Domains | RI2 | Further insight and simplification of properties in target domains | "Insight and Simplification" | 5 | Yes |
| 8.3 | System Simulation | RI3 | Relationship of adjustable parameters to system innovation | "Parameter Relationship" | 6 | Yes |
| 8.4 | Stanford Case Study | RI4 | Ability of derived mathematical equations to frame organizational change and innovation | "Frame Change" | 7 | Yes |
| 8.5 | Existing CAS Math | RI5 | Cohesiveness of integration of piece-meal mathematics into main model | "Integration" | 4 | Yes |
| 8.6 | Literature Review | RI6 | Addressing of generated hypotheses in the literature review | "Addressing Hypotheses" | 2 | Yes |
| 8.7 | Proof of Mathematical Structure | RI7 | Ease with which established theory suggests structure of mathematical model | "Established Theory" | 4,9 | Yes |

Figure 8.9.1 Summary of Research Evaluated

Conclusions and contributions are summarized in the next and final chapter, Chapter 9.

CHAPTER 9: Conclusions

### 9.1 Overview

The research in this dissertation suggests that all systems can be considered as parts of Complex Adaptive Systems (CAS). As per discussions around simple, complicated, and complex systems in Sections 2.1.5.3 and 4.12, simple systems, with few interactions and high predictability may be thought of as newly formed or young systems just beginning their journey toward maturity. Complicated systems may be thought of as systems where multiplicity, interdependence, and diversity are beginning to increase, as per derived Equation 4.12.3 (Complicated $_{\text {system }}=$ System ( $\left.\operatorname{Sig}_{E}: X<I\right)$ ) with active orientations still of the untransformed physical, vital, and mental natures. Complex systems can be thought of as systems characterized by a high degree of multiplicity, interdependence, and diversity, as
 orientations of the integral, force, and contextual natures, which by definition tend to be more transformed, and under existing paradigms of causality, practically renders any further development unpredictable. The evolution of CAS is summarized by Figure 9.1.1.


Figure 9.1.1 Relationship Between Maturity of CAS and Nature of System
When systems are viewed from the bottom-up it is difficult to separate the trees from the forest and the nature of the system will more easily be characterized by properties discussed in Section 1.2 such as distributed control, connectivity, coevolution, dependence on initial conditions, emergence, paradox, unpredictability, amongst others. To begin to derive a "systems" mathematics for innovation though, as has been done in this dissertation, it is perhaps easier to begin to view systems from the outside-in, which will also naturally yield additional properties unviewable when looking at systems from the bottom-up. Such properties have been
derived and applied to distinct domains in Chapter 4 and 5 and are summarized in Section 9.2.

The systematic-ness of innovation begins with it as being conceptualized to exist in every single space-time point in a system. The notion of 'every single space-time point in a system' has been referred to as Meta-Layer $3, M_{3}$, and is characterized by a four-fold intelligence. By a process of precipitation this innovation expresses itself through a series of architectural forces that become the various sources of innovation. These series or arrays of forces further precipitate by informing organizational signatures. This layer of organizational signatures has been referred to as Meta-Layer 1, $M_{1}$, Organizations, therefore, can be thought of as formations with a unique signature at their center, and can vary in complexity and scale.

There is a further precipitation or reflection that happens as the four-fold intelligence at $M_{3}$ informs the surface level, so that Presence informs the physical, Power informs the vital, Knowledge informs the mental, and Nurturing informs the integral. In this process though, the unique signature for each organization is hidden by common surface dynamics, and "to innovate" is to work through and to change the habitual and common patterns, primarily at the surface layer $U$, in order to allow the deeper founts of innovation to become active at the surface level. When this happens, then it is suggested that innovation occurs.

Once that is more clearly seen then even the erected probabilistic and uncertainty functions assumed to be true of the fundamental layers of nature, may be relegated to their place as interim devices in model building as suggested by the discussion on Qualified Determinism and subsequent derived equations in Section 4.9.

In fact, as suggested by the Light-Enhanced Unified Field equation, Equation A1.4.2 (reproduced below), everything, from unseen energy fields, to quantum particles, to atoms, to molecules, to cells, and therefore to all animate and even inanimate and even unseen objects, and therefore even any CAS system regardless of scale would have a high-degree of quaternary intelligence embedded in it and exist simultaneously. Re-quoting Schrodinger (Schrodinger, 1989): "What we observe as material bodies and forces are nothing but shapes and variations in the structure of space. Particles are just schaumkommen (appearances). The world is given to me only once, not one existing and one perceived. Subject and object are only one. The barrier between them cannot be said to have broken down as a result of recent experience in the physical sciences, for this barrier does not exist."

This leads to a more refined understanding of CAS. First, they are complex systems. Complex systems imply that what appears in space-time at layer $U$ has not been expressed in it before. The complexity arises because there is four-fold intelligence implicit in every point-instant at U . This implies that the action of $M_{3}$ is continually happening at U . Adaptive systems imply that apparently bounding circumstances at $U$ are often transcended by organization at $U$ so that these organizations adapt. But this adaptation can be perceived as a function of the unique signature behind every organization that may use circumstance to further express its truth, to thereby allow something more of itself to manifest at U . Adaptation is therefore a function of the action of $M_{1}$ and $M_{2}$ at U . System implies that there are several parts that together define what may happen. The system can therefore be thought of as the function of the dynamics at $M_{1}, M_{2}, M_{3}$, and U in combination. Further, CAS is ubiquitous and exists at every scale in the universe, from Planck's distance to Gigaparsecs.

This implies that even the CAS properties suggested in Section 1.2 will need to be rethought, as summarized in Section 9.3 on Further Research.

The properties, operators, and applications of such a mathematics of innovation to multiple domains of increasing scale and complexity also suggest that the derived mathematical model of innovation may be thought of as a framework to view the unified nature that exists across scales of time and space, starting from the initial Big Bang to now, along the time-dimension, and from the quantum to cosmic levels along the space-dimension.

Thinking about CAS as purposeful, and animated by a mathematically-framed engine of innovation, allows additional solutions to a host of complex problems regardless of scale - at the quantum-particle, atomic, cellular, human, organizational, sociotechnical, market, economy, political, and social levels - to be conceptualized, designed, elaborated, and managed differently.

As Schrodinger says in his book 'My View of the World’ (Schrodinger, 1961), "the development of metaphysics leads to physics". Considering the metaphysical view of an all-pervasive four-fold intelligence, then the mathematical model presented here essentially develops the mathematics by which innovation makes itself tangible at any and perhaps at all scale.

### 9.2 Contributions

This research leverages the point of view that any system is by its fundamental nature deeply innovative. As discussed in Section 2.1.4, empirical observation regardless of field, indicates the natural movement from the physical, to the vital, to mental orientations. This gives insight into the implicit nature of Time in our system. The question as to what the implicit order in Time tells us leads to the hypothesis of four macro-characteristics true regardless of location, and hence point to an implicit nature of Space in our system. The need to consider Time and Space differently must give rise to a different set of causal organizational and system models.

The starting point for reconsidering any such causal model is the nature of each point. Each point is embedded with system-presence, system-power, systemknowledge, and system-nurturing. This implicit nature is the fount of a large set of architectural forces that seek to seed themselves into any developmental effort. Hence any organization regardless of scale is unique, and its uniqueness is determined by the combination of architectural forces. While these three sets of conditions, M1 being the uniqueness of an organization, M2 being the set of architectural forces from which uniqueness is determined, and M3 being the implicit nature of a point, stand behind each practical orientation, each practical orientation (the physical, the vital, the mental, the integral) begins by being more untransformed, and through the action of the meta-levels gradually becomes more transformed.

This movement from the untransformed to the transformed orientations reinforce the inherent innovation prevalent in our system. Hence innovation is observed as precipitating from the barely visible level, M3, to the closely practical level, the untransformed, U . These observations and the equations derived suggest a vastly different way of mobilizing and managing innovation and therefore technology as well, in general.

Key attributes of the derived mathematical model include:

- Any system is characterized by qualified determinism at each level of organizational complexity
- This qualified determinism is orchestrated by a cohesive mathematical framework
- This mathematical framework explains the spectrum of possibility from stagnation to sustainability / progress
- The mathematical framework and the derived equations provide a basis for innovation
- The mathematical framework will advance the field of innovation by creating a series of mathematical equations to better understand innovation
- The mathematical framework for innovation will apply to organizations at different levels of complexity from the 'simple' to the more 'complex'. Hence it will provide insight into how innovation happens at the atomic/quantumparticle level, the level of the biological cell, the level of the human being, the
level of a team, the level of the corporation, the level of the market, amongst other levels
- The mathematical framework will provide insight into further potential development at each level of organizational complexity
- This framework separates the nature of functionality by meta-layers and suggests key dynamics operative at each layer. Under certain conditions the respective meta-level functionality and operations become active to bring about complexity and adaptiveness in the visible, surface layer
- The mathematics of innovation suggested in this research constructs a generalized equation of innovation that may exist at multiple-levels of complexity ranging from the quantum-particle level, through the cellular level, to the larger organizational, market, and system levels. The mathematics constructs functions that highlight key operations for each relevant layer and the interaction between layers to bring about the emergent adaptability and complexity visible in the surface layer.
- This approach integrates a "bottom-up" and "top-down" approach to CAS. The bottom-up approach is characteristic of key CAS schools of thought such as MIT and Santa Fe Institute, amongst others. The top-down approach has been the focus of my previous research and is elaborated in a series of books: Connecting Inner Power with Global Change (Malik, 2009), Redesigning the Stock Market (Malik, 2011), and The Fractal Organization (Malik, 2015).

The range of derived equations in the constructed mathematical model are summarized in the following Table:

| $\#$ | Area | Purpose |
| :--- | :--- | :--- |
| 1 | System Bias | Initial approximation of <br> system bias based on 3 <br> fundamental states |
| 2 | Nature of a Point | Derived equations for four- <br> fold intelligence inherent in a <br> point |
| 3 | Architectural Forces | Four sets of the sources of <br> innovation |
| 4 | Uniqueness of <br> Organizations | Equation for organizational <br> uniqueness |
| 5 | Emergence of <br> Uniqueness | Equation for surfacing of <br> organizational uniqueness |
| 6 | Varying Culture of <br> Organizations | Describes conditions for <br> diversity of culture |
| 7 | Inherent Dynamics of <br> our System | Describe the inherent <br> innovativeness in our system |
| 8 |  <br> Dynamic Growth | Describes conditions for <br> stagnation and growth |
| 9 | Qualified | Describes relationship |

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|  | Determinism in CAS | between determinism and <br> randomness in our system |
| :--- | :--- | :--- |
| 10 | Framing <br> Organizational <br> Transitions at Layer <br> U | Suggests further distinctions <br> in Prigogine's Dissipative <br> Structure Inequality |
| 11 | Framing \& Modeling <br> Shift in Innovation at <br> Layer U | Framing of shifts in innovation <br> based on Turing Activator- <br> Inhibitor equations |
| 12 | Framing Complexity | Framing of complexity based <br> on generally accepted <br> measures in the literature |
| 13 | Molecular Plans at <br> Cellular Level | Explores equations for nature <br> of proteins, nucleic acids, <br> lipids, and polysaccharides |
| 14 | Space and Time <br> Alteration as per <br> Theory of Relativity | Specifies equations for <br> alteration of space and time at <br> U |
| 15 | Integration of U and <br> M using Quantum <br> fluctuation | Suggests an equation-based <br> depiction of the integration of <br> meta-levels with the surface <br> level |
| 16 | Architecture of <br> Quantum Particles | Explores equations for the <br> nature of quantum particles, <br> quarks, leptons, bosons, and <br> the Higgs-boson |
| 23 | Dark Matter, Dark <br> Energy | Relation of Dark Matter and <br> Dark Energy in terms of <br> derived mathematical model |
| 17 | Periodic Table <br> Element equations <br> Principle | Hypothesizes equations to <br> describe Periodic Table <br> elements |
| 18 | Sustainability of CAS | Suggests an equation for <br> achieving sustainability of CAS |
| 20 | Equations for EM <br> Sperators | Definition of some CAS <br> mathematical operators in <br> light of the mathematical <br> model constructed here |
| 21 | Wave Equations | Suggested equations for <br> electro-magnetic spectrum |
| 22 | Uncertainty Principle equations for different <br> systems |  |
| Further qualification of |  |  |
| Seisenber's Uncertainty |  |  |$|$


| 24 | Quantum Computing | Non-Probabilistic Superposed <br> Quantum State in Qubits |
| :--- | :--- | :--- |
| 25 | Emergence of CAS in <br> Universe | A unified field equation for the <br> emergence of CAS in space <br> and time, for all scales |

Figure 9.2.1 Range of Derived/Enhanced Mathematical Equations
In the Universe in a Nutshell (Hawking, 2001) Stephen Hawking suggests the positivist approach in which a scientific theory is a mathematical model that describes and codifies observations. He suggests, "A good theory will describe a large range of phenomena on the basis of a few simple postulates and will make definite predictions that can be tested". The research in this dissertation formulates a model based on such initial observations and simple postulates and describes a large range of phenomena.

Through deriving mathematical equations, and by further applying these to various domains ranging from the quantum, to the atomic, to the cellular, to the astrophysical, this research has been able to provide mathematical contributions to the theory of CAS and to various CAS application areas.

With respect to the theory of CAS, mathematical contributions have been made to understanding the underlying directional bias of CAS activity, understanding the nature of each point in any CAS, and creating mathematical sets for architectural forces that are posited to be behind the development of any CAS. Further, mathematical contributions have been made to understanding the inherent dynamics in any CAS, the dynamics of stagnation and growth in CAS, and the balance of randomness and determinism of any CAS. Mathematical contributions also extend to framing complexity in CAS, understanding what can drive sustainability of CAS, and arriving at a general set of mathematical operators true of any CAS. This category of contributions derives from considering CAS from the outside-in instead of from the bottom-up, and allows a corresponding mathematics of innovation of CAS to be framed based on that.

In terms of application areas in the organizational space, mathematical contributions have been made to understanding uniqueness of organizations, the emergence of uniqueness in organizations, and what constitutes varying culture of organizations. Further, existing work done by Nobel Laureate Ilya Prigogine and Alan Turing have been leveraged to further frame organizational transitions, and to frame and model shifts in innovations, respectively. Contributions to knowledge in this category derive from a common functional four-foldness that allows a simple yet diverse framing of a range of organizations.

Further mathematical contributions have been made in a range of CAS areas at different scale and level of complexity. Hence, a series of equations have been derived for the electromagnetic spectrum. Quantum, atomic, and cellular wave
equations have been derived building off Schrodinger's existing Wave Equation. Further qualifications have been derived for Heisenberg's Uncertainty Principle and an equation has been derived for the integration of different layers of CAS also using Heisenberg's Uncertainty Principle. Equations for space and time alteration as per Einstein's Theory of Relativity have also been derived. The contributions to knowledge in this category derive from the framing of the interplay of different layers allowing for a logical separation of otherwise confusing dynamics.

Additionally, equations for the architectures of quantum particles, periodic table elements, and molecular plans at the cellular level have also been derived. The contribution to knowledge derives from leveraging a common four-foldness and action of multiple layers with different fundamental dynamics.

In all, over 225 equations in 25 different areas have been derived in this dissertation.

### 9.3 Further Research

This dissertation also suggests several potential areas of further research. In particular:

1. The theory presented here suggests enhancements to the field of System Dynamics and Systems Thinking. Some of these enhancements were surfaced in the literature review and have been integrated into the derived mathematical model. Further research would need to be conducted to understand the self-organizing characteristics of system-power, systemknowledge, system-presence, and system-nurturing identified in this dissertation.
2. While equations have been derived for the physical, vital, mental, and integral, these suggest the need for additional research to systematically understand the process by which $X_{U} \rightarrow X_{T}$, that is, the process by which the untransformed set becomes the transformed set. Related to this, research on the best way in which meta-levels precipitate into the untransformed levels would also need to be structured and conducted.
3. While stability in general is perceived as being enhanced by diversity, the equations for mono-culture and diverse-culture with their implications on stability need to be backed by research.
4. The equation for uniqueness and emergence of uniqueness need to be validated with additional research.
5. The notion that stagnation and dynamic growth are functions of how successfully an organization is propagating innovativeness will need to be further researched.
6. Several of the equations derived in this dissertation have been expressed in the 'mod', modulated by, or function, 'f', forms. Further research to derive
more precise relationships between the participating variables will need to be conducted.
7. While the Stanford case leverages the innovation framework described here through the use of SaaS-based software, the lasting effects on innovation through the use of such SaaS-based software will also need to be researched.
8. Further research needs to be conducted to explore the trinary-based fractalrepresentation approach to innovation.
9. The supposition that the derived mathematical framework can be applied to any domain where a quaternary basis is observed will have to be further researched.
10. The supposition that there is a limit to randomness will need to be further researched in light of the derived mathematical framework.
11. Research will need to be conducted to more precisely relate the existing key qualities of Complex Adaptive Systems - distributed control, interconnectivity, sensitive dependence on initial conditions, emergent behavior, paradox of order and chaos - with the alternative classes of qualities suggested by this mathematical model.
12. Application of the mathematical model to the CAS domain of the biological cell suggests the following additional research needs to be conducted:
a. The notion that innovation happens iteratively as opposed to absolutely and finally, as suggested by the iterative-form of the General Equation for Innovation
b. The difference in the action of Transformation Circle (TC) between the cell and more complex CAS as it integrates the layers of organization
c. The functional specification of all nanoscale machinery operative at the cellular level and its relationship to the proposed cellular-level 'architectural forces'
d. The surfacing of further nanoscale cellular machinery that may not exist today, but is tied to Organizational Signature equations seeded on cellular-level architectural forces
e. Adaption of the genome to likely express the suggested 'missing' nanoscale machinery with time
f. Advances to cellular level medical technology based on quaternary based mathematics of innovation
g. Construction of synthesized nanobots based on the quaternary based mathematics of innovation
13. This quaternary architecture has some possible implications for further quantum particle research:
a. Existence of a Master-Particle that embodies the quaternary intelligence in itself, and is representative of $M_{1}$. Perhaps this is a material precipitation of a key organizing principle around which the quaternary aspects arrange and expand themselves.
b. Further Sig functions, that would give more insight into the myriad of existing and yet undiscovered particles
14. The rates of Innovation-Contribution used in the Vensim modeling are experimental to simulation research output and more research has to be done to understand the effect of overcoming the sets of patterns that allow higher meta-levels to become active. It is possible to set up causal balancing feedback loops in Vensim so that the random function that generates Innovation-Contribution varies depending on the degree to which the metalayers become active. Further, sensitivity analyses will need to be performed to evaluate what the relative change to Innovation Contribution/Month should be at successive meta-levels.
15. The suggestion that the derived mathematical model may frame a unified field (as elaborated in Appendix 1) existing from the initial birth of the universe through all space and time needs to be further researched.
16. Research into alternative schemes of superposition and quantum computing as suggested by Appendix 2 needs to be conducted, based on the notion of superposition suggested by the mathematical model in this dissertation.
17. As suggested by Appendix 3, research needs to be conducted into the integration of different layers of light in a possible unified equation for spacetime emergence.
18. Quantitative validation of all working hypotheses since this research uses working hypotheses to arrive at a conceptual analytical framework.


## APPENDIX 1: APPLICATION OF THE DERIVED MATHEMATICAL MODEL TOWARD THE FRAMING OF A UNIFIED FIELD THEORY

There is a fundamental paradox that has accompanied the development of Quantum Physics. A number of Nobel Laureates, of which a few have been mentioned in Chapter 1 - Prigogine, Gell-Mann, Feynman - have through their work reinforced this paradox. It is bought out clearly in encounters between Heisenberg and Einstein that Heisenberg describes in "Encounters with Einstein" (Heisenberg, 1983).

Heisenberg describes how after a lecture he delivered in Berlin Einstein invited him to his house for a deeper discussion on quantum mechanics. Heisenberg relates, that Einstein found it strange that Heisenberg had not included the electron path in a description of the interior of an atom. Heisenberg stated that he had not done so because it could not be observed. Einstein then stated that every theory has unobservable quantities. This epitomizes a fundamental difference between the two, and perhaps between two schools of thought.

In considering the meta-layered mathematical model of innovation at the heart of any CAS derived in this dissertation though, both schools of thought can complement each other and are perhaps necessary to a deeper understanding. Heisenberg and the long line of Nobel Laureates dealing with quantum physics have diligently described their observations of quantum weirdness. For example, Heisenberg's Uncertainty principle as it relates to the position and momentum of a particle, in his own words is stated as: "The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa" (Heisenberg, 1927). Further as he states in Encounters with Einstein, "The laws of nature were dealing with temporal change of the possible and the probable...which can be registered only in statistical fashion, and are no longer predictable". Section 5.2 elaborates other common observations at the quantum level. On the other hand the Einstein-like intuitive leaps that constructs conceptual models is equally necessary in making sense of "weirdness".

The problem would be if the former reductionist-type scientist made models based only on what can be observed, and refused to admit of causal models built on nonobservables. Another problem would be if the intuitive-type scientist built conceptual models based on concept alone without relating them to observables. In this vein Heisenberg's uncertainty principle, as previously described in Section 5.2.8 on Quantum Fluctuations, seems to capture the inherent presence of meta-levels, and statistics and uncertainty can be thought of as a way, in the interim, to describe such presence. A paradox therefore is that the apparently more reductionist-type scientist such as Heisenberg has provided this approach by which to describe that which fundamentally exceeds any limit and generally escapes attempts at reducibility.

At its crux, therefore, any unified field theory must be about integrating that which can be seen with that which cannot be seen:

- Hence, a unified field theory (UFT) should integrate the possibility that exists in any field with the emerging particles, atoms, molecules, and other constructs of matter that emerge in that field. This suggestion is inspired by Einstein's own thought that the best way to eliminate the need for an ether as separate from matter would be to find the elusive UFT (Isaacson, 2008). Section A1.2, The Electromagnetic spectrum as a Manifestation of Four-Fold Intelligence, explores the electromagnetic spectrum as an instance of fourfoldness.
- But further, a UFT should also integrate the dual aspects of wave and particle such as is suggested to define light or even Nobel Laureate Broglie's "matter waves" (De Broglie, 1929). Some interpretations of wave mechanics already suggest such a UFT as summarized by Schrodinger (Schrodinger, 1989) in his quote: "What we observe as material bodies and forces are nothing but shapes and variations in the structure of space. Particles are just schaumkommen (appearances). The world is given to me only once, not one existing and one perceived. Subject and object are only one. The barrier between them cannot be said to have broken down as a result of recent experience in the physical sciences, for this barrier does not exist." Section A1.1, The Nature of Light and Its Impact on the Quantum Level, discusses the integration of the dual aspects of wave and particle by considering fourfoldness associated with light.
- Further as just suggested, the paradox of uncertainty and randomness should also be integrated with that of causality. Interpretations of the quantum state and wave mechanics (Hawking, 2008) suggest that a certain form of determinism may still exist contrary to the notion that quantum weirdness eliminates any possibility of a deterministic universe. Such a 'qualified determinism' was derived in Section 4.9 based on the mathematical model of innovation in CAS developed in this dissertation.
- Further the inherent quantum-modeling dichotomy as epitomized by the points of view of Einstein and Heisenberg should also be integrated as just suggested. That is, concept or abstraction needs to be given as important a place as observables and held as real unless proven otherwise. Chapter 4 on creation of a multi-layered math model with conceptual or abstract layers held as real and informing visible layer addresses such quantum-modeling dichotomy.
- But further, in its emergence the various forms of matter as it 'complexifies' should be an outcome of any UFT. Section A1.3, Complexification of FourFoldness traces four-foldness from the level of the electromagnetic spectrum through quantum particles, atoms, and cells.
- In Reinventing the Sacred (Kaufmann, 2008) makes the point that the prevalent scientific approach of reductionism is fundamentally incompatible with emergent function that can never appear in any equation because the possibilities of the 'adjacent possible' are simply not known. Therefore
'function' as opposed solely to 'form' should also be integral in any UFT for completeness, and since these too can thought of as causative agents in determining an outcome in a field.

Summarizing, the suggested conditions for a UFT appear as in Figure A1.1, along with the specific section in which it has /will be addressed:

| Number | Condition | Where Addressed |
| :--- | :--- | :--- |
| 1 | Integration of field with emerging <br> particles and other forms of matter <br> 2 | Integration of dual aspects of wave and <br> particle |
| spectrum as on EM <br> manifestation of four- <br> fold intelligence |  |  |
| 3 | Integration of randomness with <br> causality | Section A1.1 on nature of <br> light and impact on <br> quantum level |
| 4 | Integration of quantum-modeling <br> dichotomy, integrating points of view of <br> Einstein and Heisenberg | Section 4.9 on Qualified <br> Determinism |
| Chapter 4 on creation of <br> a multi-layered math <br> model with conceptual <br> or abstract layers held as <br> real and informing <br> visible layer |  |  |
| 5 | Complexification of matter as an <br> outcome of UFT | Section A1.3 outlining <br> complexification of four- <br> foldness /matter in its <br> emergence |
| 6 | 'Function' integral in any UFT | Chapter 4 on creation of <br> a multi-layered math |
| model with conceptual |  |  |
| or abstract layers, |  |  |
| specifically Section 4.4 |  |  |
| on signatures or |  |  |
| functions, held as real |  |  |
| and informing visible |  |  |
| layer |  |  |$|$| lay |
| :--- |

Figure A1.1 Some Suggested Conditions for a Unified Field Theory
Note that Section A1.4 builds off the previous sections in this chapter to derive a unified field equation.

## A1.1 The Nature of Light and its Impact on Quantum Levels

The mathematical model derived in this dissertation assumes as a starting point an implicit order in time and space (Malik 2009, 2015). Summarizing from Section 2.1.4, in observations of many trends regardless of area, a similar pattern of emergence surfaces. This pattern takes on three aspects that have been referred to as the physical, the vital, and the mental. Since these patterns are observed at the surface-layer, U , regardless of scale and regardless of organization being considered, the possibility of an implicit order in space surfaces. After all what is it about systems that causes the emergence of similar patterns regardless of scale and area? Four macro properties were suggested that have formed the whole bases of the mathematics that has been derived in this dissertation. Further, it is observed that what appears in time is related to the properties in space. Hence time and space are intricately linked, even when viewed from the perspective of naturally surfacing trends, and as has been the practice in Physics, would be better referred to as a space-time continuum.

Consider this emergence in organizations from the point of view of light. At the surface layer, U , light is known to have a fixed velocity C . This is not an infinite velocity, but a finite velocity of 186,000 miles/second in a vacuum. But the very fact that it is finite means that past, the present, and the future occurs because of its finiteness. Further, this finiteness in speed effectively creates separation as any entity begins to perceive other entities as apart from itself. If $C$ were infinite, then time and space would be experienced vastly differently. The sense of a present moment only would become the abiding reality, as everything would be connected with everything else instantaneously.

Now considering the pattern implicit in time, it can be seen that really the physical, the vital, and the mental, are themselves representative of the past, the present, and the future respectively. Hence the nature of any organization that arises in a spacetime continuum could be thought of as related to or perhaps even arising as it does because of the way light propagates in that system.

But this is only looking at surface layer, $U$. In the mathematical model derived in this dissertation that there are multiple layers with different fundamental dynamics associated with them (refer to Chapter 4). It must be that the propagation of light in these different layers is different. The mathematical model developed in this dissertation assumes an intelligent, purposeful system, and has started the mathematical formulation by assuming a four-fold intelligence as elaborated in Section 4.1. This four-fold intelligence is a system-presence, a system-knowledge, a system-power, and a system-nurturing. It acts with an implicit wisdom, knowing what to do when, and how. But if considered from the point of view of light, fourfold intelligence can only be true if the speed of light were infinite (please refer to Appendix 3 for a more detailed discussion of this).

Hence, it could be said that the speed of light is critical in setting up the foundation for space-time and emerging possibility in any CAS. It can be suggested that light reveals a different aspect of its nature at each layer in our multi-layered system.

The following inequality for C has already been considered earlier in Section 5.2.5, on traveling faster than the speed of light:

$$
\text { Inequality }_{C}: C_{U}<C_{M_{1}}<C_{M_{2}}<C_{M_{3}}
$$

$C_{U}$ is known to be 186,000 miles/second. It may be suggested that $C_{M_{3}}=\infty$ miles/second. The varying speed of light by layer is fundamental in creating the nature of that layer, and the hypothesis is that time and space and what arises in time through space is intimately related to $C_{X}$, where $\mathrm{X} \in\left(\mathrm{U}, M_{1}, M_{2}, M_{3}\right)$. In other words the play of space and time and what emerges in a certain space-time continuum is intimately connected with $C_{X}$.

Since C is finite and therefore there is past, present, and future implied by it, this implies that at $U$ a point has to become quanta. This is implicit in the notion of finiteness. Since light takes a finite amount of time to get from A to B, a "unit" of light will require a finite time to traverse that. Quanta at the subatomic level is perhaps related to this finite time and distance for a unit of light to be expressed.

Planck's discovery that energy at the subatomic level acts as quanta therefore makes sense. It is to be noted though that Planck's treatment of quanta was more as a mathematical convenience that allowed the derivation of an equation that explained the curve of radiation wave-lengths at varying temperatures of a heated black-body (Isaacson, 2008). Einstein though postulated quanta as a fundamental property of light itself, rather than as something that arose in the interaction of light with matter as Planck thought. Einstein's theory produced a law of the photoelectric effect where the energy of emitted electrons would depend on the frequency of light. Einstein received the Nobel Prize for this discovery (Isaacson, 2008).

If $C$ is the upper limit of the layer $U$, then it makes sense that the lower limit $h$ (Planck's constant) should be inversely proportional to C. Hence: $h \propto \frac{1}{C}$

This relationship is in fact substantiated by combining two well-known equations: the first is the electromagnetic equation connecting speed of light with wavelength and frequency (elaborated in next section), and the second is Einstein's photoelectric equation connecting energy with frequency of light:
(1) $C=v \lambda$
(2) $E=h v$

Yields:
$h=\frac{E \lambda}{C}$
About h, H.A. Lorentz the Dutch scientist has commented in The Science of Nature (Lorentz, 1925): "We have now advanced so far that this constant not only furnishes the basis for explaining the intensity of radiation and the wavelength for which it represents a maximum, but also for interpreting the quantitative relations existing in several other cases among the many physical quantities it determines. I shall mention a few only, namely the specific heat of solids, the photo-chemical effects of light, the orbits of electrons in the atom, the wavelengths of the lines of the spectrum, the frequency of the Roentgen rays which are produced by the impact of electrons of given velocity, the velocity with which gas molecules can rotate, and also the distances between the particles which make up a crystal. It is no exaggeration to say that in our picture of nature nowadays it is the quantum conditions that hold matter together and prevent it from completely losing its energy by radiation."

So just as C sets up the past-present-future experience and reality of $U$, h suggests that this experience will take place in shells of matter. In the absence of the limit h, as pointed out by Lorentz, only radiation, and no matter would exist. Hence there appears to be a four-foldness implicit in the nature of light as well. This is discussed in further detail in Appendix 3.

But further, on observing the pattern of four-foldness emerging at various scales as per the research in this dissertation, this also suggests an orderliness of sorts even in the realm of quanta, and further, that the notion of unification of energy-mass-electricity-magnetism-gravity perhaps also holds here.

The previous discussion of the variance in the speed of C by meta-layer may throw some further light on the quantum realm. First, summarizing:

1. At $U$ the speed of light in a vacuum, $C_{U}$, is finite at 186,000 miles $/ \mathrm{sec}$. This finiteness creates the reality and experience of past-present-future, and further a sense of fragmentation and separation. Further, assuming that $C_{U}$ is a fundamental upper-limit at $U$, the inverse of it , $\frac{1}{C_{U}}$, must define some fundamental lower limit at U . This is indeed the case as Planck's constant, h , is proportional to this. ' h ' allows for matter to be sustained, as it fundamentally limits the dispersion of energy as suggested by Lorentz.
2. At $M_{3}$, the speed of light, $C_{M_{3}}$, is suggested as being $\infty$ miles/sec. This allows a reality of 'oneness' and the possibility of a suggested four-fold intelligence existing in every point-instant of space-time (as discussed in Appendix 3).
3. As discussed in Section 5.2 the quantum world, here designated by Q, because it is at boundary of $U$ accesses and interrelates with the meta-levels. As such, the speed of light, $C_{Q}$, will appear as a hybrid as in the following figure. Note though
that it is really the speed of light at the native or resident layer that becomes active, and that this is simply being represented as $C_{Q}$ for convenience:


Figure A1.1.1 Speed of Light at Quantum Level

Note that research on the speed of light also indicates that it may go faster than $\mathrm{C}_{\mathrm{U}}$. While the speeds suggested currently through experimental research, and summarized below, may be only incrementally higher than $C_{U}$ the notion that $C_{U}$ can be exceeded appears to be put in place:

1. The Heisenberg uncertainty principle already suggests that photons can travel at any speed, even exceeding $C_{U}$, for short periods.
2. Notion of different space-time realities, also known as meta-levels, suggests that light can travel differently in a layer different from the four-dimensional space-time that apparently defines our observable world (Hawking, 1988)
3. In his book QED Feynman (Feynman, 1985) says "...there is also an amplitude for light to go faster (or slower) than the conventional speed of light. You found out in the last lecture that light doesn't go only in straight lines; now, you find out that it doesn't go only at the speed of light! It may surprise you that there is an amplitude for a photon to go at speeds faster or slower than the conventional speed, c." In research conducted at Humboldt University (Chown, 1990), Scharnhorst has made calculations using the theory of quantum electrodynamics to reveal the possible existence of "faster-thanlight" photons. This is known as the Scharnhorst effect.
4. As discussed in Section 5.2 .5 on traveling faster than the speed of light, Perkowitz makes the point that the theory of relativity does not disallow particles already moving at C or greater.

The point is that the reality at $Q$ is going to be different than the reality at $U$. This should be apparent from considering the relation of $C_{X}$ to the consequent reality. In Q the fundamental lower limit, h , which allows matter to sustain itself, is itself going
to fluctuate. Hence, as X tends to $\mathrm{M}_{3}, \mathrm{C}_{\mathrm{X}}$ will tend to infinity, and h will become a fraction of itself. As it becomes a fraction of itself the quantization effect will be lowered, and matter will get dispersed more and more easily to in effect take on a wave-like appearance.

This notion has perhaps already been suggested by Schrodinger's equation, which fundamentally models matter as a wave rather than a particle, and shows how this wave propagates (Stewart, 2012). Recall the equation introduced in Section 5.2:
$\mathrm{i} \frac{\mathrm{h}}{2 \pi} \frac{\partial}{\partial \mathrm{x}} \psi=\widehat{\mathrm{H}} \psi$
Consider this in light of the discussion on Q . ' i ' is a complex number and suggests the interplay of two dimensions, one being real, and one being 'imaginary'. But the 'imaginary' dimension could be thought of as none other than the meta-levels implicit in this mathematical model, and suggested to be real at $Q$. Further, $\frac{h}{2 \pi}$ is in line with the suggestion just made that h will have to become a fraction of itself as C increases. Hence, the change in the wave function, $\frac{\partial}{\partial \mathrm{x}} \psi$, is intimately related to i and $\frac{h}{2 \pi}$, and perhaps only makes sense when considered in the context of $i x \frac{h}{2 \pi} x \frac{\partial}{\partial x} \psi$, which has to be the case when dealing with the integration of dynamics of multiple levels.

Further, the change in the wave function, $\frac{\partial}{\partial x} \psi$, is related to $\widehat{H} \psi$, and suggests that there is some system "energy", represented by the Hamiltonian, $\widehat{H}$, that when applied to the existing wave, $\Psi$, will indicate how the wave will be expressed going forward.

But as discussed in Section 5.2, at Q the dynamics of $\operatorname{Sig}_{X}$ become real, and in fact is a fundamental organizing principle for all organization at $U$, and starting at the dimension of $h$.

It may be the case that as the level of complexity of organization at the micro-level increases, as in from quantum, to atomic, to cellular, $\widehat{H}$, itself complexifies as it houses further nuances or "superpositions" of $\operatorname{Sig}_{X}$. $\operatorname{Sig}_{X}$ may be thought of as having more components as the complexity at such micro-levels of organization increases. As discussed in Section 5.2 since all complexity or innovation is due to the integration of the dynamics of meta-levels with $U$, the representation of wave functions or $\psi$ at the boundary world, Q , may be enhanced by consideration of $\operatorname{Sig}_{X}$. Hence, $\widehat{\mathrm{H}}$ may be further qualified by notating $\widehat{\mathrm{H}}_{Q}$ or $\widehat{\mathrm{H}}_{A}$ or $\widehat{\mathrm{H}}_{C}$ for quantum, atomic, and cellular respectively, as per the following inequality in Equation A1.1.1:
$\widehat{\mathrm{H}}_{Q}<\widehat{\mathrm{H}}_{A}<\widehat{\mathrm{H}}_{C}$ (where: $\widehat{\mathrm{H}}_{X} \propto$ Energy $_{\text {Sig }_{X}}$ )

## Eq A1.1.1: Hamiltonian Inequality

This inequality is specified by the number of components of $\operatorname{Sig}_{X}$ which it is assumed will have a direct effect on potential and kinetic energy of the wave-system (which are considered to be how $\widehat{\mathrm{H}}$ is measured) as in Equation A1.1.2:

Energy $_{\text {Sig }_{Q}}<$ Energy $_{\text {Sig }_{A}}<$ Energy $_{\text {Sig }_{C}}$
Eq A1.1.2: Quantum, Atomic, Cellular Energy Inequality
Similarly the following wave equations for quantum, atomic, and cellular respectively, as in Equations A1.1.3, A1.1.4, and A1.1.5 may be distinguished:
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{Q}=\widehat{H}_{Q} \psi_{Q}$
Eq A1.1.3: Hamiltonian at Quantum Level
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{A}=\widehat{H}_{A} \psi_{A}$
Eq A1.1.4: Hamiltonian at Atomic Level
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{C}=\widehat{H}_{C} \psi_{C}$
Eq A1.1.5: Hamiltonian at Cellular Level

But further, it may also be suggested that the uncertainty principle itself is only valid at $U$, and that, because of the finiteness of $C$. This finiteness as already suggested implies h , which implies that if the position of a particle is going to be observed by shining light on it, the light has to have at least a quantum of energy. But to determine the position of a particle accurately light of a shorter wavelength would have to be used (Hawking, 1988) which would have to have a minimum amount of energy, which in turn would interfere with the velocity and hence momentum of the particle. The uncertainty in measuring the momentum may therefore be thought of as a consequence of the finiteness of the speed of light, C.

If $C_{U}$ were to approach $C_{Q}$ though, the quantum would be smaller and the uncertainty in measuring position or momentum would be reduced. At $C_{M_{3}}$ there would be no uncertainty since light would accurately tell both position and momentum definitively.

Hence, the uncertainty principle may be further qualified, as in Equations A1.1.6, A1.1.7, and A1.1.8:
$@_{\mathrm{U}}: \Delta p X \Delta x \geq \frac{h}{4 \pi}$
Eq A1.1.6: Uncertainty Principle at $U$
$@ \mathrm{C}_{\mathrm{Q}}: \Delta p X \Delta x \rightarrow 0$
Eq A1.1.7: Uncertainty Principle at $Q$
$@ C_{M_{3}}: \Delta p X \Delta x=0$

## Eq A1.1.8: Uncertainty Principle at M3

The notion of position and momentum becoming finite at $U$ also may imply that space, time, and quanta are emergent rather than absolute properties. This is the conclusion of Arkani-Hamed of the Institute of Advanced Studies in the following thought experiment (Wolchover, 2013):
'Locality says that particles interact at points in space-time. But suppose you want to inspect space-time very closely. Probing smaller and smaller distance scales requires ever higher energies, but at a certain scale, called the Planck length, the picture gets blurry: So much energy must be concentrated into such a small region that the energy collapses the region into a black hole, making it impossible to inspect. "There's no way of measuring space and time separations once they are smaller than the Planck length," said Arkani-Hamed. "So we imagine space-time is a continuous thing, but because it's impossible to talk sharply about that thing, then that suggests it must not be fundamental - it must be emergent."

Unitarity says the quantum mechanical probabilities of all possible outcomes of a particle interaction must sum to one. To prove it, one would have to observe the same interaction over and over and count the frequencies of the different outcomes. Doing this to perfect accuracy would require an infinite number of observations using an infinitely large measuring apparatus, but the latter would again cause gravitational collapse into a black hole. In finite regions of the universe, unitarity can therefore only be approximately known.'

## A1.2 The Electromagnetic Spectrum as a Manifestation of the Four-Fold Intelligence

Einstein's Cross, a quasar in the Pegasus constellation, is an excellent example of gravitational lensing (Redd, 2015). The quasar is about 8 billion light-years from Earth, and sits behind a galaxy that is 400 million light-years away. Gravitational lensing is the phenomenon by which light traveling around a massive object, such as
a black hole, is bent, causing the object to act as a lens for the things that lie behind it. Astronomers routinely use this method to study stars and galaxies behind massive objects. Hence, four images of the quasar appear around the galaxy because the intense gravity of the galaxy bends the light coming from the quasar.

This is one instance of the many proofs of Einstein's General Theory of Relativity. In this theory one begins to get a glimpse of a possible unified theory of physical phenomena. Electromagnetism, mass, energy, gravitation are unified in this theory, certainly at the astronomical levels.

Looking at this more deeply, because of mass-energy equivalence, as suggested by $E=m c^{2}$, an object with a high mass will have a high energy. Energy itself signifies the presence of an electromagnetic field. And as the electromagnetic (EM) spectrum perhaps suggests, a field has ingrained in it the ability to alter any space-time continuum. This is further elaborated by the following equation that links C with the frequencies and wavelengths in the EM spectrum. Hence:
$C=\nu \lambda$

Where $v$ represents the frequency and $\lambda$ represents the wavelength of an electromagnetic spectrum. The frequency $v$ may be thought of as symbolic of time, and the wavelength $\lambda$ as symbolic of space. The variation in $v$ and $\lambda$ are depicted by the following table (ASTR161, 2015):

| Spectrum of Electromagnetic Radiation |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Region | Wavelength <br> (Angstroms) | Wavelength <br> (centimeters) | Frequency <br> $(\mathbf{H z})$ | Energy <br> $(\mathbf{e V})$ |
| Radio | $>10^{9}$ | $>10$ | $<3 \times 10^{9}$ | $<10^{-5}$ |
| Microwave | $10^{9}-10^{6}$ | $10-0.01$ | $3 \times 10^{9}-3 \times 10^{12}$ | $10^{-5}-0.01$ |
| Infrared | $10^{6}-7000$ | $0.01-7 \times 10^{-5}$ | $3 \times 10^{12}-4.3 \times 10^{14}$ | $0.01-2$ |
| Visible | $7000-4000$ | $7 \times 10^{-5}-4 \times 10^{-5}$ | $4.3 \times 10^{14}-7.5 \times 10^{14}$ | $2-3$ |
| Ultraviolet | $4000-10$ | $4 \times 10^{-5}-10^{-7}$ | $7.5 \times 10^{14}-3 \times 10^{17}$ | $3-10^{3}$ |
| X-Rays | $10-0.1$ | $10^{-7}-10^{-9}$ | $3 \times 10^{17}-3 \times 10^{19}$ | $10^{3}-10^{5}$ |
| Gamma Rays | $<0.1$ | $<10^{-9}$ | $>3 \times 10^{19}$ | $>10^{5}$ |

Figure A1.2.1 Variation of $v$ and $\lambda$ in the EM Spectrum (source: Dept of Astronomy, University of Tennesse)

Hence, already implicit in the EM spectrum is a vast range of time-space possibility, ranging from the smallest theoretical distance at U , the Plank distance ( $1.6 \times 10^{-35}$ m ), to one of the largest units of length, the gigaparsec ( 3.26 billion light-years).

Further, energy E, can be specified as:
$E=h v$

Where $h$ is Planck's constant, the elementary quantum of action, and $v$, is the frequency of the EM spectrum. This also implies the vast energy gradient implicit in the EM spectrum and as illustrated in the preceding figure.

When mass increases, then the field around that mass intensifies, and space-time will curve as the General Theory of Relativity suggests. Black holes corresponding to greatly compressed volumes of mass specified by a 'Schwarzchild radius' are regions in space where gravity is so strong that Nature as it is understood breaks down (Stein, 2011). Space-time curves on itself and light cannot escape the effect of gravity. These predictions of the general theory represent limits and are actually found in the universe. In his book on string theory Gubser further suggests that gravity is nothing other than the urge for time that is moving fast to essentially slow down (Gubser, 2010) further reinforcing the notion that such 'fundamentals' as time, space, gravity, mass are deeply interconnected with one another. In a brief account of an extension to the theory of gravitation (Einstein, 1950) in Scientific American, Einstein suggests that since the theory of general relativity implies the representation of physical reality by a continuous field, the concept of particles or material points cannot play a fundamental part. The particle can only appear as a limited region in space in which the field strength or the energy density is particularly high.

What is referred to as "light" is the visible portion of the EM spectrum. So it may be said that it is really the EM spectrum through its impact on space-time that influences the nature of a specific layer, as in $U$, through the meta-layers.

Just as it can be seen that implicit in the EM spectrum is a vast range of space-time possibility and a vast energy gradient, it may be suggested that mass, with its implicit energy equivalence, and further, representative of any object at $U$, is a precipitation, or because of the presence of a diverse energy gradient implied by its energy equivalence, a shell that can more easily receive the characteristic dynamics of each of the meta-layers $M_{1}$ through $M_{3}$ which therefore, under the right conditions as laid out in the mathematics developed in this dissertation, allows further action of the meta-layers to occur at U .

Hence, even though science has proceeded in piece-meal fashion, yet it can be seen that it is perhaps approaching the formulation of a unified theory of everything. But what if a unified field has been assumed to begin with: that is, oneness, as represented by the four-fold intelligence implicit in every space-time point-instant, that progressively precipitates as per the suggested dynamics already laid out for $M_{3}, M_{2}, M_{1}$, and U? Then as suggested earlier, the prime difference would be that the drive toward a unified theory would be approached from the top-down, instead of from the bottom-up. But the interesting thing is that the bottom-up appears to be reaching the same point.

This dissertation has hinged on the four-fold scheme that is suggested as being the basis of all systems, and therefore of any sustainable CAS. The following subsections explore the EM Spectrum as another manifestation of this essential fourfold scheme.

## A1.2.1 Electromagnetic Spectrum System-Nurturing Equations

As suggested, the speed of light or speed of the EM spectrum, C, sets up the nature of dynamics possible at $U$. Past, present, future, and the notion of separation, the very stuff that influences an organization's orientation in the world is therefore potentially set up by C. It may be said that C therefore architects the possibility of interaction in the system and can therefore be thought of as a precipitation of system-nurturing. Further, as discussed in Section A1.1, the inverse of C, h, sets up the lower limit of a layer, in this case deciding the presence of shells of matter.

So being, it can be suggested that the nature of the resultant interactions allows matter-based organizations regardless of scale to come into their own, to grow into their boundaries, and to form bonds based on the sense of being separated from other perceived organizations. In equation form, as in Eq A1.2.1.1:
$\begin{array}{cc}E M \_ \text {Spectrum }_{\text {Speed }} & = \\ X a+\overline{Y b_{0-n}} \text { where }\end{array}\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq A1.2.1.1: Speed of EM Spectrum

This also implies that as the speed of $C$ changes, the nature of relationship will also change. Perhaps it is that there are several different relationships, or types of harmonies possible.

Leveraging the previously defined inequality for $C_{X}$,

Inequality $_{C}$ : $C_{U}<C_{M_{1}}<C_{M_{2}}<C_{M_{3}}$
It may be suggested that the nature of the 'layer' will change. Hence, as in Equations A1.2.1.2 through A1.2.1.5:
$[\operatorname{Nature}(U)]_{C_{U}}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.1.2: Nature of $U$ as a Function of $C(U)$
$\left[\text { Nature }\left(M_{1}\right)\right]_{C_{M_{1}}}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.1.3: Nature of M1 as a Function of C(M1)
$\left[\operatorname{Nature}\left(M_{2}\right)\right]_{C_{M_{2}}}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.1.4: Nature of M2 as a Function of C(M2)
$\left[\operatorname{Nature}\left(M_{3}\right)\right]_{C_{M_{3}}}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers; } a>b
\end{array}\right]
$$

Eq A1.2.1.5: Nature of M3 as a Function of C(M3)
This idea will be further explored in the next section, A1.3.

## A1.2.2 Electromagnetic Spectrum System-Knowledge Equations

The EM spectrum itself, with its vast range of natures from gamma rays through visible light through radio waves, with its implicitness of time-space possibility as suggested by $v$ and $\lambda$, may be thought of as an arrangement of archetypes of what is possible in systems, and therefore is perhaps a precipitation of system-knowledge, as in Equation A1.2.2.1:

EM_Spectrum $_{\text {Structure }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

## Eq A1.2.2.1: Structure of EM Spectrum

What this also implies is that the significance or intent of the different types of waves that exist can also be expressed by this general equation where the X and Y elements will vary. What precisely these elements are will need to be worked out. Hence, as in Equations A1.2.2.2 through A1.2.2.18:

Gamma Rays $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.2: Gamma Rays Intent
$X-$ Rays $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.3: X-Rays Intent

Ultraviolet $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.4: Ultraviolet Intent

White Light Intent $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.5: White Light Intent

Violet Light ${ }_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.6: Violet Light Intent
Indigo Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{K}}\right] \\ Y \in\left[S_{S_{\text {Stem }_{P r} r},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.7: Indigo Light Intent

Blue Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.8: Blue Light Intent


Eq A1.2.2.9: Green Light Intent
Yellow Light ${ }_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{K}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ \left.S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]\end{array}\right.$
Eq A1.2.2.10: Yellow Light Intent

Orange Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.11: Orange Light Intent

Red Light ${ }_{\text {Intent }}=$

$$
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

Eq A1.2.2.12: Red Light Intent
Infrared $_{\text {Intent }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{S y s t e m_{K}}}\right] \\ Y \in\left[S_{\text {System }_{\text {Pr }},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.13: Infrared Intent

Millimeter Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.14: Millimeter Waves Intent
Microwaves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.2.15: Microwaves Intent

FM Radio Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.16: FM Radio Waves Intent

Short Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.2.17: Short Waves Intent

AM Radio Waves $_{\text {Intent }}=$

$$
X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{K}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

Eq A1.2.2.18: AM Radio Waves Intent

## A1.2.3 Electromagnetic Spectrum System-Power Equations

The energy-gradient implicit in the EM spectrum suggests the power and energy with which knowledge moves and is perhaps a precipitation of system-power, as in Equation A1.2.3.1:


Eq A1.2.3.1: EM Spectrum Energy

This energy as suggested is directly proportional to the frequency, of which there is an infinite range as predicted by Maxwell's equations. Different frequencies have different penetration profiles (HyperPhysics, 2016) and it may be suggested that the nature of the energy is also a precipitation of a range of to be determined metafunctions as suggested in Equations A1.2.3.2 through A1.2.3.18. Hence:

Gamma Rays Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.2: Gamma Rays Nature of Energy

$$
\begin{aligned}
& X-\text { Rays }_{\text {Nature of Energy }}= \\
& X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{P}}\right] \\
Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{S_{y s t e m_{N}}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
\end{aligned}
$$

Eq A1.2.3.3: X-Rays Nature of Energy
Ultraviolet $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.4: Ultraviolet Nature of Energy

White Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.5: White Light Nature of Energy
Violet Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {Stem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.6: Violet Light Nature of Energy
Indigo Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.7: Indigo Light Nature of Energy

Blue Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.8: Blue Light Nature of Energy
Green Light Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.9: Green Light Nature of Energy
Yellow Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {Stem }_{P r} r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.10: Yellow Light Nature of Energy

Orange Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.11: Orange Light Nature of Energy
Red Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.12: Red Light Nature of Energy
Infrared $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.13: Infrared Nature of Energy


Millimeter Waves ${ }_{\text {Nature of Energy }}=$
$X \in\left[S_{\text {System }_{P}}\right]$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.14: Millimeter Waves Nature of Energy
Microwaves $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.15: Microwaves Nature of Energy
FM Radio Waves Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.3.16: FM Radio Waves Nature of Energy

Short Waves $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.17: Short Waves Nature of Energy
AM Radio Waves Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.3.18: AM Radio Waves Nature of Energy

## A1.2.4 Electromagnetic Spectrum System-Presence Equations

Mass can be thought of as a container at $U$ within which all possibility happens. In other words it can be thought of as a precipitation of system-presence as depicted in Equation A1.2.4.1:


Eq A1.2.4.1: EM Spectrum Mass Possibility
If the frequencies are infinite, then the possibility of the 'types' of masses or matter is also infinite. The mystery of 'Dark Matter' suggested by scientists to be $27 \%$ of our universe, as opposed to 5\% of visible matter (NASA-darkmatter, 2016) may have some relation to this as further discussed in section A1.3.

Hypothetically the Equations A1.2.4.2 through A1.2.4.18 depict a range of mass possibilities:

Gamma Rays Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.2: Gamma Rays Mass Possibility
$\left.\begin{array}{l}X-\text { Rays }_{\text {Mass_Possibility }}=\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ X a+\overline{Y b_{0-n}} \text { where }\end{array}\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]\right.\end{array}\right]$
Eq A1.2.4.3: X-Rays Mass Possibility

Ultraviolet $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.4: Ultraviolet Mass Possibility

White Light Mass_Possibility $=$ $X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ S_{\text {System }_{P}}, S_{S y s t e m_{K}}, \\ a, b \text { are integers } ;\end{array} a>b\right.\end{array}\right]$

Eq A1.2.4.5: White Light Mass Possibility

Violet Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.6: Violet Light Mass Possibility
Indigo Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ S_{\text {System }_{P}}, S_{\text {System }_{K}}, \\ a, b \text { are integers } s ; a>b\end{array}\right]\end{array}\right]$

Eq A1.2.4.7: Indigo Light Mass Possibility

Blue Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.8: Blue Light Mass Possibility
Green Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{S_{y s t e m}^{K}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.9: Green Light Mass Possibility
Yellow Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.4.10: Yellow Light Mass Possibility

Orange Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {Stem }_{P r}},}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.2.4.11: Orange Light Mass Possibility

Eq A1.2.4.12: Red Light Mass Possibility
Infrared $_{\text {Mass_Possibility }=}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ S_{\text {System }_{P}}, S_{\text {System }_{K}}, \\ a, b \text { are integers } s ; a>b\end{array}\right]\end{array}\right]$
Eq A1.2.4.13: Infrared Mass Possibility

Millimeter Waves mass_Possibility $=$
$X \in\left[S_{S H y s t e m_{P r}}\right]$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.14: Millimeter Waves Mass Possibility
Microwaves Mass_Possibility $=$

Eq A1.2.4.15: Microwaves Mass Possibility
FM Radio Waves Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.16: FM Radio Waves Mass Possibility


Eq A1.2.4.17: Short Waves Mass Possibility
AM Radio Waves $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.2.4.18: AM Radio Waves Mass Possibility

## A1.3 Complexification of Four-Foldness

The following graphic from the Lawrence Berkeley Laboratory of University of California, Berkeley (Particle Data Group, 2015) illustrates the history of the universe from the Big Bang to the present:


Figure A1.3.1 History of the Universe (source: Particle Data Group, LBNL)

Now imagine that initial point. There was perhaps a high-intensity of "light" that exploded out. At time, $\mathrm{t}=10^{-36} \mathrm{~s}$, this high-intensity of light is depicted by the cumulative energy of photons, $\mathrm{E}=10^{16} \mathrm{GeV}$. Photons, recall from Section 5.3, are the carriers of the EM Spectrum. Photons are quanta and themselves a result of the ubiquitous action of $h$. Therefore it may be suggested that already at that point there was the four-fold wholeness implicit in the EM Spectrum.

This four-fold fullness was expressed by the EM Spectrum (system-knowledge), its implicit energy-gradient (system-power), the speed C with which is propagated (system-nurturing), and the potential for mass (system-presence) also implicit in it. This reinforces the ubiquity of the proposed four-fold intelligence that exists at every point-instant of possible space-time, and which existed concretely since the very beginning. It was a fundamental organizing principle that appears to have existed at the very beginning of the Universe.

In a 'Brief History of Time’ Hawking (Hawking, 1988) states that the eventual goal of science is to provide a single theory that describes the whole universe. Scientists will usually separate this problem into two parts. First, focusing on laws that suggest how the universe changes with time. Second, focusing on the initial state of the universe. Hawking relates how some people feel that science should be concerned only with the first part. Since however, there appears to be very regular laws that govern how the universe changes with time, it must be equally reasonable to suppose that there are laws governing the initial state, he argues.

The existence of the four-fold intelligence as an initial organizing principle is consistent with Hawking's argument. Only, in this mathematical model, it is the same set of laws that exists at both the initial starting point, and the subsequent development of CAS in the Universe.

In looking at the nature of the four-fold order it is apparent that the four-fold wholeness is more tightly related to a smaller set of fundamental physical attributes. Summarizing, as in Equations A1.3.1 through A1.3.4:

System $_{\text {Nurturing }} \propto C_{U}$

## Eq A1.3.1: System Nurturing at Electromagnetic Level

System $_{\text {Power }} \propto h v$, where v is the frequency of the EM wave
Eq A1.3.2: System Power at Electromagnetic Level
System $_{\text {Knowledge }} \propto[f(v) \& f(\lambda)]$
Eq A1.3.3: System Knowledge at Electromagnetic Level
And since, $E=m c^{2}$ or $m=\frac{E}{c^{2}}$, and substituting hv for $E$,
System $_{\text {Presence }} \propto h v / c^{2}$
Eq A1.3.4: System Presence at Electromagnetic Level
But, $C \propto 1 / h$
Hence, the general relationships of the four-fold wholeness may be understood by knowing just two variables, $C$, and either, $\lambda$ or $v$.

Now at time, $\mathrm{t}=10^{-10} \mathrm{~s}$, as depicted in Figure A1.3.1, the four-fold fullness begins to express itself in a series of particles. Quarks, leptons, bosons, which also may imply
the presence of the Higgs-boson that gives quarks their mass are observed. But as discussed in Section 5.3 quarks are a precipitation of system-knowledge, leptons of system-power, bosons of system-nurturing, and the Higgs-boson of systempresence. Hence it is again observed that the four-fold fullness has expressed itself at a different order of complexity.

Einstein has suggested that particles are an excitation of the underlying field. The EM spectrum gives insight into a field that propagates at $C$ at different frequencies. But there may be other fields too. The Higgs Field has been suggested as being one of these (Jepsen, 2013). The Standard Model suggests that quarks, leptons, bosons, and the Higgs-boson are different fundamental particles, which implies that the four-fold wholeness has now a more complex basis of its operation, since as compared with the four-fold wholeness of the EM spectrum the number of independent basis appears to have increased as summarized in Figure A1.3.3, Complexification of Matter in Terms of Levels of Implicit Wholeness, below.

Summarizing, as in Equations A1.3.5 through A1.3.8:
System $_{\text {Nurturing }} \propto f$ (bosons)
Eq A1.3.5: System Nurturing at Particle Level
System $_{\text {Power }} \propto f($ leptons $)$
Eq A1.3.6: System Power at Particle Level
System $_{\text {Knowledge }} \propto f($ quarks $)$
Eq A1.3.7: System Knowledge at Particle Level
System $_{\text {Presence }} \propto f$ (Higgs_boson)
Eq A1.3.8: System Presence at Particle Level
While it is known that there is a fundamental relationship between one of the bosons - photons - and the EM spectrum, and further that there is a relationship between a possible Higgs Field and the Higgs-Boson, there are similarly other fields of which fundamental particles are excitations (Jepsen, 2013). Assuming that the four-fold wholeness of the EM Field may provide some insight into the nature of such other fields, it follows that the organization of particles likely therefore has at least two four-fold wholenesses implicit in it - one at the field level, manifest as something akin to the EM field, and one at the particle level itself. Hence it appears that with the flow of time, there is likely a complexification of matter. This suggestion will continue to be explored here.

As discussed in Section 5.4 the Periodic Table itself is also an expression of the fourfold wholeness. Hence, elements configured by the d-orbital are an instance of system-presence, by the p-orbital an instance of system-knowledge, by the s-orbital and instance of system-power, and by the f-orbital of system-nurturing. As per Figure A1.3.1, History of the Universe, this emergence likely began at $\mathrm{t}=3 \times 10^{5}$ years when the atom is suggested to have emerged, and continued at least to $t=10^{9}$ years with the emergence of stars which it is already known are the furnaces in which heavier atoms were created.

In an article published in the Santa Fe Institute Bulletin, Cherry Picking the Periodic Table (Trefil, 2008), it is suggested that life has cherry-picked certain elements on which to base itself. These are Hydrogen, Carbon, Oxygen, Iron, and Calcium and the relative presence of these by weight at three different scales are indicated as in the figure below:

| Element | Universe | Earth Crust | Human Body |
| :--- | :--- | :--- | :--- |
| Hydrogen | 74 | 1.5 | 10 |
| Helium | 24 | 0 | 0 |
| Carbon | 0.005 | 0.01 | 18 |
| Oxygen | 0.01 | 46 | 65 |
| Iron | 0.001 | 6.2 | $<0.05$ |
| Calcium | 0.0007 | 4.6 | 1.5 |

Figure A1.3.2 Distribution of Common Chemical Elements (source: Sante Fe Institute)

But in light of the mathematical model being considered here it may be that every element must be a precipitation or shell for some essentially innovative function. While more detailed form-function equations have already been derived in section 5.4, there is one more expression of these equations suggested here in Equations A1.3.9 through A1.3.13.

Element $_{\text {sig }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.3.9: Generalized Signature for Element

And more specifically:
Element $_{\text {sig }_{P}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.3.10: Generalized Signature for Physical Element
Element $_{\text {sig }_{V}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.3.11: Generalized Signature for Vital Element
Element $_{\text {sig }_{M}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Eq A1.3.12: Generalized Signature for Mental Element
Element $_{\text {sig }_{I}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{S_{\text {System }}^{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Eq A1.3.13: Generalized Signature for Integral Element

So now a third four-fold organization, in addition to the field-level organization suggested by the EM Spectrum, the particle-level organization suggested by the architecture of fundamental particles, comes into play to further complexify the architecture of matter.

Leveraging off the fundamentally different probability clouds, as in Equations A1.3.14 through A1.3.17:

System $_{\text {Nurturing }} \propto f\left(f_{\text {_orbital }}\right)$
Eq A1.3.14: System Nurturing at Atomic Level
System $_{\text {Power }} \propto f($ s_orbital $)$
Eq A1.3.15: System Power at Atomic Level
System $_{\text {Knowledge }} \propto f($ p_orbital $)$
Eq A1.3.16: System Knowledge at Atomic Level
System $_{\text {Presence }} \propto f($ d_orbital $)$
Eq A1.3.17: System Presence at Atomic Level


Atoms therefore have three four-fold wholenesses implicit in them.
At time, $\mathrm{t}=13.8 \times 10^{9}$ years, there is a fourth clear expression of the same four-fold order as the bases of an even more complex organization, that of cellular life and all that is founded on it. This can be summarized as in Equations A1.3.18 through A1.3.21:

System $_{\text {Nurturing }} \propto f$ (lipids)
Eq A1.3.18: System Nurturing at Cellular Level
System $_{\text {Power }} \propto f($ polysaccharides $)$
Eq A1.3.19: System Nurturing at Cellular Level
System $_{\text {Knowledge }} \propto f$ (nucleic acids)
Eq A1.3.20: System Knowledge at Cellular Level
System $_{\text {Presence }} \propto f($ proteins $)$
Eq A1.3.21: System Presence at Cellular Level
Cellular life therefore seems to have at least four four fold-wholenesses implicit in it to therefore further complexify the architecture of matter.

This scheme of complexification is summarized by Figure A1.3.3:

| Level | Independent "Variables" | Levels of Implicit Wholeness | Number of Levels |
| :---: | :---: | :---: | :---: |
| EM Spectrum | C, $v$ or $\lambda$ | - EM | 1 |
| Particle Level | Quarks, Leptons, Bosons, HiggsBoson | - EM <br> - Particle | 2 |
| Atomic Level | S-orbital, P-orbital, D-orbital, F-orbital | - EM <br> - Particle <br> - Atom | 3 |
| Cellular Level | Proteins, Nucleic Acids, Lipids, Polysaccharides | - EM <br> - Particle <br> - Atom <br> - Cell | 4 |

Figure A1.3.3 Complexification of Matter in Terms of Levels of Implicit Wholeness
In reference to the four sets at $M_{2}$ (introduced in Section 4.3) it may also be that as the basis of matter complexifies as it journeys through the EM spectrum and other potential fields, the elementary particles, the atom, and cellular life, as summarized in Figure A1.3.3, the sets which have been positioned to each contain infinite elements, concretely manifest more of their function-elements. This suggestion has been the basis of the derived equations in Chapter 5 . The complexity of matter may therefore be related to the number of manifested function-elements of the set of four sets. This idea is consistent with the notion of the "adjacent possible" suggested by Kaufmann (Kaufmann, 2003) in which innovation is positioned as a recombination of existing parts to create new value - or of existing sets to combine parts of themselves to create new elements based on new circumstance. If MS signifies manifested-set, so that the cardinality or number of elements in the combined set is the union of the four manifested-sets, this may be summarized by the following equation, Equation A1.3.22:

Matter $_{\text {Complexity }} \propto\left|M_{\text {System }_{P r}} \cup M S_{\text {System }_{P}} \cup M S_{\text {System }_{K}} \cup M S_{\text {System }_{N}}\right|$
Eq A1.3.22: Complexity of Matter in Terms of Manifested Set
According to NASA though, only 5\% of the matter in the universe is visible, while as much as $27 \%$ is 'dark matter', that does not form the familiar stars, galaxies, or the atoms (NASA-darkmatter, 2016). But since the architectural sets at $M_{2}$ are infinite the question is why would all functions need to manifest in the same way? As stated in an article 'Dark Energy: The Biggest Mystery in the Universe' (Panek, 2010), "Sight itself has blinded us to the Universe". It should be possible for other matterbased constructions to come into being based on the functionality they exist for. For instance, as already suggested there could be other fields besides the EM Field and the Higgs Field, that create other kinds of elementary particles, that creates other kinds of atoms, and that results in other kinds of structures not visible with our current instrumentation.

Hence also, as summarized by Equation A1.3.23:
Matter $_{\text {Complexity }} \propto[$ Visible Matter + Dark Matter $]$

## Eq A1.3.23: Complexity of Matter in Terms of Visible and Dark Matter

Further, as suggested in Chapter 4 where the foundation for this mathematical model is derived, space is not empty but contains the four-fold intelligence in it from which all organizational schemes and innovation itself are growths. This notion of space having 'amazing' properties was, according to a report on 'dark energy' by the Harvard-Smithsonian Center for Astrophysics, first suggested by Einstein (HarvardSmithsonian Center for Astrophysics, 2004). Quoting: "Einstein was the first person
to realize that empty space is not nothingness. Space has amazing properties, many of which are just beginning to be understood."

Einstein had suggested the existence of a 'dark energy' about 100 years ago (NASASupernova, 2001) as a property of space that caused the expansion of the universe. Dark Energy is estimated to comprise as much as $68 \%$ of the universe and as Figure A1.3.4 proposes, is the force causing the acceleration of the expansion of the universe.


This diagram reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. Astronomers theorize that the faster expansion rate is due to a mysterious, dark force that is pulling galaxies apart.
NASA/STSci/Ann Feild

Figure A1.3.4 Changes in Rate of Expansion of Universe (source: NASA/STSci/Ann Feild)

Keeping in mind the complexification of matter as it journeys from the field-level through the quantum-, atomic-, and cellular-levels, summarized in Figure A1.3.5, in some sense it may be possible to re-interpret the supposed expansion-contraction dynamics of cosmology in relation to the mathematical model derived in this dissertation.

| Principle | Field Level | Quantum <br> Level | Atomic <br> Level | Cellular Level |
| :--- | :--- | :--- | :--- | :--- |

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| System- <br> knowledge | EM Spectrum | Quarks | P-Orbital <br> group | Nucleic Acids |
| :--- | :--- | :--- | :--- | :--- |
| System- <br> presence | Mass | Higgs-boson | D-Orbital <br> group | Proteins |
| System-power | Energy-gradient <br> implicit in EM <br> Spectrum | Leptons | S-Orbital <br> group | Polysaccharides |
| System- <br> nurturing | C, speed of EM <br> Spectrum or Light | Bosons | F-Orbital <br> group | Lipids |

Figure A1.3.5 Complexification of Matter Related to Four-Fold Principle

First, flipping the left and right sides of the equation on Matter $_{\text {Complexity }}$ to yield Equation A1.3.24:
$\left|M S_{\text {System }_{P r}} \cup M S_{\text {System }_{P}} \cup M S_{\text {System }_{K}} \cup M S_{\text {System }_{N}}\right| \propto$ Matter $_{\text {Complexity }}$

## Eq A1.3.24: Flipped Matter-Complexity Equation

This implies that the manifested-set, MS, is growing at a certain threshold level. Assuming this threshold level, $M S_{-} G r o w t h_{\text {Threshold }}$, is a property of space related to dark energy it may be possible to restate the condition of cosmological expansion and contraction. Hence, so long as the MS is increasing at a certain rate that exceeds MS_Growth ${ }_{\text {Threshold }}$ the level of dark energy is such that the acceleration of galaxies, Acceleration $_{\text {Galaxies }}$ exceeds the contracting force of gravity at the universal level, Gravity $_{\text {Universe }}$.

Therefore, as in Equations A1.3.25 and A1.3.26:
MS_Growth $_{\text {Threshold }}:$ Acceleration $_{\text {Galaxies }}>$ Gravity $_{\text {Universe }}$
Eq A1.3.25: Expansion of Universe Related to Manifest Set Growth Threshold
Conversely:
!MS_Growth Threshold : Acceleration Galaxies $^{<}$Gravity $_{\text {Universe }}$
Eq A1.3.26: Contraction of Universe Related to Manifest Set Growth Threshold

Where the relation between $M S_{-} G r o w t h_{\text {Threshold }}$ and Dark Energy is suggested by Equation A1.3.27:

MS_Growth Threshold $\propto$ Dark Energy
Eq A1.3.27: Relation Between Manifested Set Growth Threshold and Dark Energy

In this interpretation so long as more of the infinite set at $M_{2}$ results in the manifested-set, $M S_{\text {System }}$, the universe keeps growing.

It is possible to re-figure Figure A1.3.5 with an additional Astronomical Level parallel to the Field Level since the field(s) must act at both the micro and macro scale:

| Principle | Field Level | Quantum <br> Level | Atomic <br> Level | Cellular Level | Astronomical <br> Level |
| :--- | :--- | :--- | :--- | :--- | :--- |
| System- <br> knowledge | EM <br> Spectrum | Quarks | P- <br> Orbital <br> group | Nucleic Acids | EM Spectrum |
| System- <br> presence | Mass | Higgs- <br> boson | D- <br> Orbital <br> group | Proteins | Mass |
| System- <br> power | Energy- <br> gradient <br> implicit in <br> EM <br> Spectrum | Leptons | S- <br> Orbital <br> group | Polysaccharides | Energy- <br> gradient <br> implicit in EM <br> Spectrum |
| System- <br> nurturing | C, speed of <br> EM <br> Spectrum or <br> Light | Bosons | F- <br> Orbital <br> group | Lipids | C, speed of EM <br> Spectrum or <br> Light |

Figure A1.3.6 Order of Magnitude Comparison
In Figure A1.3.6 if the relative distances between levels are considered, as previously discussed in Section 5.7, a pattern emerges. The quantum level, as discussed is dealing in distances of the order of $10^{-35} \mathrm{~m}$. The cellular level is dealing with distances of the order of $10^{-6} \mathrm{~m}$. The astronomical level is dealing with distances as large as $10^{26} \mathrm{~m}$ - the estimated size of the universe. It can be observed therefore that the cell is approximately in the center with roughly 29 orders of magnitude on either side. Given the relative range of these constructs it may be that this four-fold architecture pervades all possible space-times, and perhaps is the bases of a unified theory whether in math or in physics.

But if the suggested meta-layered mathematical model exists, then what that also implies is that there is yet a different way in which everything in the universe is connected.

But as several of the findings in quantum physics imply, such a different nature of connection indeed exists. Rather than existing at layer $U$, though, the different connections are the result of natural dynamics of the proposed meta-layers $M_{1}$ through $M_{3}$. It has already been suggested for instance, that light at $M_{3}$ travels instantly, and that this allows an implicit intelligence aware of everything in its space-time domain to exist. The different natures of the meta-layers will allow interpretations such as quantum tunneling, entanglement, superposition, traveling backward in time, dual wave-particle nature to exist, as already discussed in Section 5.2.

In this view it becomes clear that Quantum Physicists have already broken into the meta-layers and their resident dynamics without realizing it. It is this combination of layer-specific physics that is being confused as the physics that exists at $U$, the purely untransformed material layer, which is causing physicists to describe their observations as weird. The weirdness arises only because there is an expectation of the dynamics at $U$, when in fact dynamics of multiple layers is what is coalescing at the quantum level.

The notion of different 'physics' at different layers has repercussions for "qubits" and quantum computing and is taken up in Appendix 2 on Qubits and Quantum Computing.

## A1.4 A Unified Field Equation

As suggested by section A1.3 the essential equation of innovation has not only organized the emergence of every scale of the universe, but existed as the 'law' governing the initial state, $\mathrm{t}=0$. Consider how this may be the case.

Recall the equation, Equation 5.1.5, the evolving form of the generalized equation of innovation:

## Innovation orientation-x

$$
=\left(\left[\begin{array}{c}
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow \text { System }_{X} \\
\left(\uparrow \operatorname{Sig}_{\boldsymbol{M}} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] T C \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
$$

With respect to Innovation orientation $-x$ represented in this equation the following may be suggested:

- At time, $\mathrm{t}=0$ seconds, only $M_{3}$ is active. This has already been suggested and implicit in the very derivation of the mathematics presented here.
- At time, $0 \geq \mathrm{t}>\infty, M_{2}$ the set of architectural forces continually gets added to from some set of initial forces. This addition is itself a function of the emergence that is being examined here.
- At time, $0>\mathrm{t} \geq 10^{-36}$ seconds, the equation of Innovation, Innovation orientation $-x$, is such that $M_{1}$ also becomes active, while U and TC are suppressed while present. The activation of $M_{1}$ begins to result in unique expressions or signatures of the set of architectural forces, and in this case in the reality of the essentially ubiquitous EM Spectrum as a vehicle of the fourfold intelligence that expressed itself in all that existed and in all that unfolded from that point in time on.
- At time, $\mathrm{t} \sim 10^{-10}$ seconds, fundamental particles emerge as an essential material basis of the four architectural forces that frame all further development. As in the case of the EM Spectrum this implies the activity of $M_{1}$.
- At time, $\mathrm{t} \sim 3 \times 10^{5}$ years light atoms emerge, and at time $\mathrm{t} \sim 10^{9}$ years heavier atoms in the stars emerge. These also imply the continued activity of $M_{1}$.
- At time, $\mathrm{t} \sim 13.8 \times 10^{9}$ years, the fourth clear expression of the same four-fold order as the bases of an even more complex organization, that of cellular life and all that is founded on it becomes clear. This too implies the activity of $M_{1}$. But further, TC becomes active here, with an implicit direction of
operation as discussed in section 5.1 (Application of Generalized Equation of Innovation at the Cellular Level) from $M_{3}$ to $U$. Note that the sets of architectural forces specified by $M_{2}$ continue to increase the number of elements they comprise of as the complex interaction between the layers continues.
- At time $t>13.8 \times 10^{9}$ years, human-beings, and more complex social organizations emerge. Here TC acts with an implicit direction of operation from $U$ to $M_{3}$.

Based on the aforementioned description Equation A1.4.1 for Emergence true of any space-time scale may be generalized as the following:


## Eq A1.4.1: Unified Field Equation

The core part of Equation 5.1.5 is expressed in Equation A1.4.1 as the 'Space' component. This brings out explicitly what was implicit in Equation 5.1.5, pointing to the forms of four-fold intelligence contained in every point in space. A further implication of this equation, brought out more explicitly through the elaboration of the 'Time' component, is that the layers $\mathrm{U}, M_{1}, M_{2}$, and $M_{3}$ exist simultaneously. This suggests that field, wave, and particle also exist simultaneously, which is perhaps borne out by the whole notion of wave-particle duality. This equation also gives insight into the nature of matter since matter, both 'regular' and 'dark', may be governed by this equation. Since everything, from unseen energy fields, to quantum particles, to atoms, to molecules, to cells, and therefore to all animate and even inanimate and even unseen objects are proposed to be governed by this equation, even the bottom-up CAS view that supposes decentralized control, randomness, uncertainty etc. (refer to Section 1.2 for proposed properties of CAS) in effect would have a high-degree of quaternary intelligence embedded in it as suggested by this equation.

Note that it is possible to model the relationship between the variable speed of light as suggested in Section A1.2 and the existing space and time component matrices called out in Equation A1.4.1 to arrive at a light-enhanced version of the equation of
emergence. This discussion appears in detail in Appendix 3 on the integration of light in the space-time emergence equation. The resulting equation, A 3.10 , is reproduced as Equation A1.4.2 as summarized here for:


Eq A1.4.2: Light-Enhanced Unified Field Equation

## A1.5 Summary

The highlighted rows in Figure A1.8.1 summarize how the mathematical model derived in this dissertation is used as a basis to address conditions \#1, \#2, and \#5 to frame a UFT.

| Number | Condition | Where Addressed |
| :--- | :--- | :--- |
| $\mathbf{1}$ | Integration of field with emerging <br> particles and other forms of matter | Section A1.2 on EM <br> spectrum as <br> manifestation of four- <br> fold intelligence |
| $\mathbf{2}$ | Integration of dual aspects of wave and <br> particle | Section A1.1 on nature of <br> light and impact on <br> quantum level |
| 3 | Integration of randomness with <br> causality | Section 4.9 on Qualified <br> Determinism |
| 4 | Integration of quantum-modeling <br> dichotomy, integrating points of view of <br> Einstein and Heisenberg | Chapter 4 on creation of <br> a multi-layered math <br> model with conceptual <br> or abstract layers held as <br> real and informing <br> visible layer |
| $\mathbf{5}$ | Complexification of matter as an <br> outcome of UFT | Section A1.3 outlining <br> complexification of four- <br> foldness /matter from <br> the beginning to now |
| 6 | 'Function' integral in any UFT | Chapter 4 on creation of |



Figure A1.8.1 Addressing Suggested Conditions for a Unified Field Theory
Note that Section A1.4 culminates in the derivation of a Unified Field equation.


## APPENDIX 2: QUBITS AND QUANTUM COMPUTING

Quantum computing has developed to parallel the perceived weirdness at the quantum level as elaborated in Section 5.2. Hence, instead of 'bits' the architecture of such computers is based on 'qubits' or 'quantum bits' that can be thought of as a unit of quantum information. Implicit in quantum bits is the notion that information is superposed or exists as many different possible states simultaneously until the information is measured, in which case it collapses into one of the basis states that is superposed. It has also been suggested that the information in the superposed or quantum state can never be retrieved (Nielsen \& Chuan, 2010).

Linear algebra has been used to model the probability that a quantum state will collapse into a particular basis state (Aaronson, 2016). The existing quantum mathematics related to the formulation of a simple 2 -state $(0,1)$ quantum-state, $|\psi\rangle$, is given by the general probabilistic form where $\alpha_{1}$, a complex number, is the probability of the ket-state $\mid 0>$ occurring, and $\alpha_{2}$, also a complex number, is the probability of the ket-state |1> occurring. Ket-states, |n>, are the notation to represent any quantum state. Further, $\Sigma_{i}\left|\alpha_{i}\right|^{2}=1$. Hence the 2 -state quantumstate can be expressed as:
$\left|\psi>=\alpha_{1}\right| 0>+\alpha_{2} \mid 1>$
But there are a few fundamental issues with this approach. First, why should it be the case that information exists in the same format in both the quantum and nonquantum classical states? If as suggested in Section A1.1 light travels at different speeds, then by virtue of a diminishing ' $h$ ' the wave will be more real at quantum levels, and particles (which are what is trying to be measured) will only be "schaumkommen". Unless a technology that can access the information in waves is created it seems it will be impossible to truly create a 'quantum' computer.

This dissertation proposes a mathematical model that models information at layers where the speed of light is greater than C. In this model superposition does still exist, but in a different way than is currently modeled by qubit mathematics, and further there is a function, DI (Section 4.9), that allows superposed states to be retrieved in a non-probabilistic, or therefore more 'deterministic' way.

A revised form of superposition would be the non-probabilistic form:
$\left\lvert\, \psi>=D I\left[\begin{array}{c}M_{3} \\ M_{2} \\ M_{1} \\ U\end{array}\right]\right.$
Eq A2.1: Alternative Non-probabilistic Representation of Superposed Quantum State

In Equation A2.1 it is the possibilities implicit in each layer: $M_{3}, M_{2}, M_{1}$, and U , that are superposed. Depending on the application there could be up to infinite 'basis' states represented by this multi-level superposed quantum state. The superposition is resolved through the Dynamic Interaction function, DI, as elaborated in Section 4.9. Since the superposition involves a similar quaternary scheme regardless of level ( $\left.M_{3}, M_{2}, M_{1}, \mathrm{U}\right)$ as summarized in Figure A1.3.5, it should be possible to have insight into the superposed information by the device of precipitation using some future technology such as arrays of particle-detectors, atom-detectors, or molecular-plan-detectors. These detectors have not yet been built and are offered as a suggestion to a U-level technology to gain insight into a $M_{x}$ or meta-level phenomenon. The assumption is that the scheme of quaternary representation or bias has a logic by which precipitation is consistent through the levels. Basis states would increase rapidly with the complexification of matter, or as $U$ was approached, as suggested by Figure A1.3.3.

Such a scheme would not be the basis of a qubit quantum computer, but of another kind of quantum computer whose purpose would be to give insight into the deeper dynamics at play behind any apparent situation. Such a computer may offer insight into outcomes of situations given the fuller play of forces existing across $M_{3}, M_{2}, M_{1}$, and $U$. It may also offer insight to what and where dynamics at the $M_{3}, M_{2}$, or $M_{1}$ levels need to change to create an intended outcome.

## APPENDIX 3: INTEGRATING LIGHT INTO THE EQUATION OF SPACE-TIME EMERGENCE

The speed of light has significant implications on the experienced nature of reality. The finiteness, C, at 186,000 miles per second in a vacuum (Perkowitz, 2011), creates an upper bound to the speed with which any object may travel also implying that objective reality will be experienced as a past, a present, a future from the point of view of that object (Einstein, 1995). These characteristics - a past, a present, a future - are implicit in the nature of light and become part of objective reality because of the speed of light.

Further, C also creates a lower bound when inverted (1/C) being proportional to Planck's constant, $h$, that pegs the minimum amount of energy or quanta required for expression at the sub-atomic level (Isaacson, 2008). Planck's constant, h, therefore allows matter to form (Lorentz, 1925) and for the reality of nature with a past, present, and future, to also be experienced as a phenomena of connection between seemingly independent islands of matter. This characteristic of 'connection' is therefore also proposed to be implicit in the nature of light and becomes part of objective reality because of the speed of light.

But a 'past' can also be viewed as established reality as defined by what the eye or other lenses of perception can see. Such lenses see what has already 'physically' been formed in time (Malik, 2009). A 'present' implies the working out of the play of forces and suggests the 'vitality' of nature (Malik, 2009) where the most energetic or powerful force will express itself over others. A 'future' implies the notion of a cause, or seed, or direction, and suggests the 'mentality' or meaning that perhaps drives the emergence of phenomena (Malik, 2009).

These implicit characteristics of the nature of light as experienced at the layer of reality so set up by a finite speed of light may hence be summarized by Equation A3.1, where $C_{U}$ refers to the speed of light of 186,000 miles per second, that has created the perceived nature of reality, U :

## $C_{U}:[$ Physical, Vital, Mental, Connection $]$

## Eq A3.1: Implicit Characteristics of the Nature of Light

It is known however that at quantum levels the nature of reality is characterized by wave-particle duality. Light itself (Feynman, 1985) and matter (De Broglie, 1929) may be experienced as both particles and waves (Ekspong, 2014). But for matter to be experienced as waves implies that ' $h$ ' has become a fraction of itself, $\mathrm{h}_{\text {fraction. This }}$ further implies that $C$ must have become greater than itself, $C_{N}$, such that the inequality specified by Equation A3.2 holds:
$C_{N}>C_{U}$

Eq A3.2: Layer N, Layer U Inequality of Speed of Light
Note that what is implied here is that just as there is a nature of reality specified by $U$ that is the result of the speed of light being 186,000 miles per second, so too there is another nature of reality specified by N that is the result of a speed of light greater than 186,000 miles per second. This is akin to recent developments in physics with the notion of property spaces being separate from but influencing physical space as explored by Nobel Physicist Frank Wilczek (Wilczek, 2016). But further in "Slow Light" Perkowitz's recent treatment of today's breakthroughs in the science of light (Perkowitz, 2011) he states: "Although relativity implies that it's impossible to accelerate an object to the speed of light, the theory may not disallow particles already moving at speed C or greater. In the 1960's, Olexa-Myron P. Bilaniuk of Swarthmore College and E.C. George Sudarshan at Syracuse University began considering how to fit what they called "metaparticles" with speeds greater than C into the relativistic scheme. The approach was extended in 1967 by Gerald Feinberg (Feinberg, 1970), or Rockefeller and Columbia Universities, in his theoretical paper "Possibility of Faster-Than-Light-Particles," Feinberg also introduced the wonderful name "tachyons" for these hypothetical particles, from the Greek word "tachys" meaning swift." Perkowitz goes on to say how a flurry of papers have continued to appear about tachyons.

Current instrumentation, experience, and normal modes of thinking having developed as a bi-product of the characteristics so created in the layer of reality $U$ may be inadequate to access N without appropriate modification.

The notion of wave-particle duality already challenges the notion of normal thinking perhaps because as modeled in this dissertation, wave-like phenomena is a function of faster than C motion and particle-like phenomena is a function of less than or equal to C motion as discussed in some detail in Section A1.1. That these may be happening simultaneously is reinforced by principles such as complementarity in which experimental observation may allow measurement of one or another but not of both (Whitaker, 2006).

But then taking this trend of a possible increase in the speed of light to its limit, this will result in a speed of light of infinite miles per second. The question is, what is the nature of reality when light is traveling at infinite miles per second? In any space-time continuum light originating at any point will instantaneously have arrived at every other point. Hence light will have a full and immediate presence in that space-time continuum. Further, that light will know everything that is happening in that space-time completely instantaneously - that is know what is emerging, what is changing, what is diminishing, what may be connected to what, and so on - or have a quality of knowledge. It will connect every object in that space-time completely and therefore have a quality of connection or harmony. Finally nothing will be able to resist it or set up a separate reality that excludes it and hence it will have a quality of power.

These implicit characteristics of the nature of light as experienced at the layer of reality so set up by an infinite speed of light may hence be summarized by Equation A3.3, where $C_{\infty}$ refers to the speed of light of $\infty$ miles per second, that has created the perceived nature of reality, $\infty$ :

## $C_{\infty}$ : [Presence, Power, Knowledge, Harmony]

## Eq A3.3: Implicit Characteristics of Nature of Light at $\infty$ Speed

But it can also be noticed that 'physical' is related to presence, 'vital' is related to power, 'mental' is related to knowledge, and 'connection' is related to harmony.

The question then, is how do these apparent qualities at $\infty$ precipitate or become the physical-vital-mental-connection based diversity experienced at $U$ ? This may be achieved through the intervention or action of a couple of mathematical transformations acting on the implicit characteristics of nature of light at $\infty$ speed as summarized by Equation A3.3. First, the essential characteristics of Presence, Power, Knowledge, Harmony that it is posited exist at every point-instant by virtue of the ubiquity of light at $\infty$ will need to be expressed as sets with up to infinite elements. Second, elements in these sets will need to combine together in potentially infinite ways to create a myriad of seeds or signatures that then become the source of the immense diversity experienced at U . This suggests that all that is seen and experienced at $U$ may be nothing other than 'information' or 'content' of light and as such that there are fundamental mathematical symmetries at play where everything at $U$ is essentially the same thing that exists at $\infty$.

Assuming that the first transformation occurs at a layer of reality K where the speed of light is $C_{K}$, such that $C_{U}<C_{K}<C_{\infty}$, this may be expressed by Equation A3.4:
$C_{K}:\left[S_{P r}, S_{P o}, S_{K}, S_{H}\right]$

## Eq A3.4: The First Transformation at Layer K

$S_{P r}$ signifies 'Set of Presence', $S_{P o}$ signifies 'Set of Power', $S_{K}$ signifies 'Set of Knowledge', $S_{H}$ signifies 'Set of Harmony/Nurturing' as discussed and enumerated in Section 4.3.

Assuming that the second transformation occurs at a layer of reality N where the speed of light is $C_{N}$, such that $C_{U}<C_{N}<C_{K}<C_{\infty}$, this may be expressed by Equation A3.5:
$C_{N:} f\left(S_{P r} \times S_{P o} \times S_{K} \times S_{H}\right)$
Eq A3.5: The Second Transformation at Layer $N$

The unique seeds are therefore a function, f , of some unique combination of the elements in the four sets $S_{P r}, S_{P o}, S_{K}, S_{H}$ as discussed and enumerated in Section 4.4.

The relationship between the layers of light may be hypothesized by the following matrix in Equation A3.6:

Light $_{\text {Matrix }}=\left|\begin{array}{c}C_{\infty}:[P r, P o, K, H] \\ \left(\downarrow R_{C_{K}}=f\left(R_{C_{\infty}}\right)\right) \\ C_{K}:\left[S_{P r}, S_{P o}, S_{K}, S_{H}\right] \\ \left(\downarrow R_{C_{N}}=f\left(R_{C_{K}}\right)\right) \\ C_{N:} f\left(S_{P r} x S_{P o} x S_{K} x S_{H}\right) \\ \left(\downarrow R_{C_{U}}=f\left(R_{C_{N}}\right)\right) \\ C_{U}:[P, V, M, C]\end{array}\right|$

## Eq A3.6: Light Matrix

The matrix should be read from the top row down to the bottom row as indicated by the $\downarrow$ between rows, and suggests a series of transformations leading from the ubiquitous nature of light implicit in a point - presence, power, knowledge, harmony - to the seeming diversity of matter observed at the layer of reality U which is fundamentally the same presence, power, knowledge, and harmony projected into another form of itself.

The first transformation is summarized by Equation A3.7:
$R_{C_{K}}=f\left(R_{C_{\infty}}\right)$

## Eq A3.7: Light Matrix First Transformation

This is suggesting that the reality at the layer specified by the speed of light $C_{K}, R_{C_{K}}$ is a function of the reality at the layer specified by the speed of light $C_{\infty}$. This transformation translates the essential nature of a point into the sets described in Equation A3.4.

The second transformation is summarized by Equation A3.8:
$R_{C_{N}}=f\left(R_{C_{K}}\right)$

## Eq A3.8: Light Matrix Second Transformation

This is suggesting that the reality at the layer specified by the speed of light $C_{N}, R_{C_{N}}$ is a function of the reality at the layer specified by the speed of light $C_{K}$. This
transformation combines elements of the sets into unique seeds as suggested by Equation A3.5.

The third transformation is summarized by Equation A3.9:
$R_{C_{U}}=f\left(R_{C_{N}}\right)$

## Eq A3.9: Light Matrix Third Transformation

This is suggesting that the reality at the layer specified by the speed of light $C_{U}, R_{C_{U}}$ is a function of the reality at the layer specified by the speed of $\operatorname{light} C_{N}$. This transformation builds on the unique seeds suggested by Equation A3.5 to create the diversity of $U$ as specified by Equation A3.1.

In this framework the notion of wave-particle duality hence may become complementary block-field-wave-particle "quadrality" where block refers to phenomenon resident to $\infty$, field to phenomenon resident to N , wave to phenomenon resident to $K$, and particle to phenomenon resident to $U$.

Since each layer in the 'Space' and 'Time' components corresponds to the suggested reality of each layer in the Light Matrix, integrating the Light-Matrix in Equation A3.6 with the Unified Field Equation A1.4.1 yields Equation A3.10:


Eq A3.10: Light-Enhanced Unified Field Equation

## APPENDIX 4: LISTING OF DERIVED EQUATIONS

A. System Bias

Stagnation $=$ fn $\left(P_{L}^{-}\right)$
Stability $=$ fn $\left(P_{L}^{+}\right)$
Entropy $=f n\left(V_{L}^{-}\right)$
Energy $=$ fn $\left(V_{L}^{+}\right)$
Fragmentation $=f n\left(M_{L}^{-}\right)$
Sustainability $=f n\left(M_{L}^{+}\right)$
Outcome $_{\text {circumstance }}=\operatorname{Strongest}\left(P_{L}^{-}, P_{L}^{+}, V_{L}^{-}, V_{L}^{+}, M_{L}^{-}, M_{L}^{+}\right)$
B. Nature of a Point

System $_{P r} \equiv \operatorname{TS}_{0 \rightarrow N}\left[\begin{array}{c}\text { Presence } \\ \downarrow \\ \text { Opportunity }^{N}\end{array}\left[\begin{array}{ccc}P_{L} & \rightarrow & V_{L} \\ V_{L} & \rightarrow & M_{L}\end{array}\right]\right.$ \& Container $\left.\begin{array}{c}{\left[\begin{array}{l}\text { System }_{P} \\ \text { System }_{K} \\ \text { system }_{N}\end{array}\right]}\end{array}\right]$
System $_{P} \equiv$ power $>\sum_{T S=0}^{N} P_{R} * V_{R} * M_{R}$
System $_{K} \equiv T S_{0 \rightarrow N}\left[\begin{array}{c}\text { Knowledge } \\ \downarrow \\ \text { Leverage }\end{array}\left[\begin{array}{c}P_{I, C} \\ V_{I, C} \\ M_{I, C}\end{array}\right] \rightarrow\left[\begin{array}{lll}P_{L} & \rightarrow & V_{L} \\ V_{L} & \rightarrow & M_{L}\end{array}\right]\right]$
System $_{N} \equiv \operatorname{TS}_{0 \rightarrow N}\left(\int_{\text {Nurturing }}\left(\begin{array}{cc}P_{-} & M_{+} \\ V_{-} & V_{+} \\ M_{-} & P_{+}\end{array}\right) \bmod (r, \theta)\right)$
C. Architectural Forces
$S_{S y s t e m_{P r}} \ni$ [Service,Perfection,Diligence,Perseverance, ...]
$S_{\text {System }_{P}} \ni$ [Power, Courage, Adventure,Justice, ...]
$S_{\text {System }_{K}} \ni$ [Knowledge,Wisdom,Law Making,Spread of Knowledge ...]
$S_{\text {System }_{N}} \ni$ [Love,Compassion,Harmony,Relationship ...]
D. Uniqueness of Organizations

Sig $_{P}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Sig $_{V}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$\operatorname{Sig}_{M}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ a, b \text { are integers } ; ~\end{array}, b>b\right.\end{array}\right]$
Sig $_{I}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{\text {Pr }},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Sig $=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers; } a>b\end{array}\right]$
E. Emergence of Uniqueness
$\operatorname{Sig}_{E}=X\left[\begin{array}{c}C: \operatorname{Sig} * \bmod \left(\int=1\right) \\ F: \operatorname{Sig} \bmod (c) \\ I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\ M: \operatorname{Sig} * \bmod (G) \\ V: \operatorname{Sig} * \bmod (e) \\ P: \operatorname{Sig} * \bmod (\pi)\end{array}\right]$
F. Varying Culture of Organizations

Monoculture: elements $S_{\text {Set }_{(n)}}=$ elements $_{Y_{\text {set }_{(1)}}}$
Diverse Culture: elements ${Y_{\text {set }_{(n)}}} \propto(1+g)^{n}$
Integral

$$
=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{N} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow S_{\text {System }_{N}} \\
\left(\uparrow \text { Sig }_{\text {Sion }} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{I} \\
\left(\uparrow>P_{I)}\right. \\
U \rightarrow \text { Integral }_{U}
\end{array}\right] T C \rightarrow \text { Integral }_{T}, \text { where }\left[\begin{array}{l}
\text { Integral }_{U} \ni[\text { possession, usurpation, hidden agendas, } \ldots] \\
\text { Integral }_{T} \ni[\text { appreciation, shift POV, MPV, synthesis, } \ldots]
\end{array}\right]
$$

$$
\begin{aligned}
& \text { Physical }
\end{aligned}
$$

$$
\begin{aligned}
& T C \equiv\left(>P_{P}\right) \rightarrow \bmod \left(\sin , e^{x}, \pi\right) \\
& \text { Vital }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Mental }
\end{aligned}
$$

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H. Stagnation \& Dynamic Growth

$$
\begin{aligned}
& \frac{d\left(T C_{x}\right)}{d t} \leq 0 \\
& \frac{d\left(T C_{x}\right)}{d t}>0
\end{aligned}
$$

IF $\frac{d\left(T C_{x}\right)}{d t} \leq 0$ THEN dS $>=0$, where $S$ is Entropy
I. Qualified Determinism in CAS
$D I_{V}\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow \text { S System }_{X} \\ \left(\uparrow \text { Sig }_{\text {Si }} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x}\right. \\ U \rightarrow x_{U}\end{array}\right] \Rightarrow$ x-state $\in\left(x_{U}\right.$, Sig $_{x}$, S $_{\text {System }_{X}}$ System $\left._{X}\right)$


Org_Dir =
 $x_{\_}$matrix $_{\text {strongest }} @$ level $_{\text {strongest }}$

J. Framing Organizational Transitions at Layer U

Leveraging Prigogine's Dissipative Structure inequality:
$\frac{d V_{O S}(x(t))}{d t} \leq u(t) \cdot y(t)$
Where:
$O S \epsilon\left[\begin{array}{c}\text { System }_{P r} \\ S_{\text {System }_{P r}} \\ \text { Sig }_{P} \\ \text { Physical }_{U m} \\ \text { Physical }_{U v} \\ \text { Physical }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{P} \\ \text { Stestem }_{P} \\ \text { Sig }_{V} \\ \text { Vital }_{U m} \\ \text { Vital }_{U v} \\ \text { Vital }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{K} \\ \text { Sistem }_{K} \\ \text { Sig }_{M} \\ \text { Sental }_{U m} \\ \text { Mental }_{U v} \\ \text { Mental }_{U p}\end{array}\right]\left[\begin{array}{c}\text { System }_{N} \\ \text { System }_{N} \\ \text { Sig }_{I} \\ \text { Integral }_{U m} \\ \text { Integral }_{U v} \\ \text { Integral }_{U p}\end{array}\right]$
$\frac{d V_{O S}\left(x_{U}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{T}(t)\right)}{d t}$

$$
\begin{aligned}
\frac{d V_{O S}\left(x_{U}(t)\right)}{d t} & \rightarrow \frac{d V_{O S}\left(x_{T}(t)\right)}{d t}: \\
\frac{d V_{O S}\left(x_{P}(t)\right)}{d t} & <\frac{d V_{O S}\left(x_{V}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{M}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{I}(t)\right)}{d t}<\frac{d V_{O S}\left(x_{F}(t)\right)}{d t} \\
& <\frac{d V_{O S}\left(x_{C}(t)\right)}{d t}
\end{aligned}
$$

K. Framing \& Modeling Shift in Innovation at Layer U

Modifying the original Turing activator-inhibitor equations:
Shift $_{\text {innovation }}=\left(\frac{\partial x_{T}}{\partial t}=f\left(x_{T}, x_{U}\right)+D_{x_{T}} \nabla^{2} x_{T}, \frac{\partial x_{U}}{\partial t}=g\left(x_{T}, x_{U}\right)+D_{x_{U}} \nabla^{2} x_{U}\right)$
L. Framing Complexity

Leveraging general measure of complexity:
Complexity $_{\text {System }}=f$ (Multiplicity, Interdependence, Diversity)

Complexity $_{\text {System }} \propto D I_{V}$

Emergence-Matrix $=\left|\begin{array}{c}C: \operatorname{Sig} * \bmod \left(\int=1\right) \\ F: \operatorname{Sig} \bmod (c) \\ I: \operatorname{Sig} \bmod \left(\int \overline{G, e, \pi}\right) \\ M: \operatorname{Sig} * \bmod (G) \\ V: \operatorname{Sig} * \bmod (e) \\ P: \operatorname{Sig} * \bmod (\pi)\end{array}\right|$

Complicated $_{\text {System }}=$ System $\left(\right.$ Sig $\left._{E}: X<I\right)$

Complex $_{\text {System }}=$ System $\left(\operatorname{Sig}_{E}: X \geq I\right)$
M. Molecular Plans at Cellular Level

Sig $_{\text {protein }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Sig $_{\text {nucleic acid }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Sig $_{\text {lipid }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\left.S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right]}^{a, b \text { are integers } ; a>b}\right.\end{array}\right]$

Sig $_{\text {poysaccharide }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Innovation orientation-x

$$
=\left(\left[\begin{array}{c}
\left.\left[\begin{array}{c}
\text { ientation-x } \\
M_{3} \rightarrow \text { System }_{X} \\
M_{2} \rightarrow \text { S System } \\
\left(\uparrow \text { Sig }_{X} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T} \text {, where }\left[\begin{array}{c}
x_{U} \ni[\ldots] \\
x_{T} \\
\exists[\ldots]
\end{array}\right]\right)_{\left\langle x_{U} \mid x_{T}\right\rangle}
\end{array}\right)\right.
$$

N. Space and Time Alteration as per Theory of Relativity

Leveraging Theory of Relativity equations for space and time alteration:
Length $_{\text {contraction }-U}=\operatorname{Length}_{U}\left(\sqrt{1-\frac{v_{U}^{2}}{c_{U}^{2}}}\right)$

Time $_{\text {elongation }-U}=\frac{\text { Time }_{U}}{\sqrt{1-\frac{v_{U}{ }^{2}}{c_{U}{ }^{2}}}}$

Inequality ${ }_{C}$ : $C_{U}<C_{M_{1}}<C_{M_{2}}<C_{M_{3}}$
O. Integration of $U$ and $M_{x}$
$I_{U}^{M} \rightarrow \Delta p X \Delta x \geq \frac{h}{4 \pi}$
P. Architecture of Quantum Particles

Sig $_{\text {Higgs-boson }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{S y s t e m}^{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Q. Elements of Periodic Table

Element ${ }_{S-\text { orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Alkali metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Element $_{\text {Alkali earth metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
As a representative element belonging to the Alkali Metal group the equation for Lithium (Li) would be:

Element $_{\text {Lithium }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {P-Orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Metal }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Metalloid }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {Sstem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$


As a representative element belonging to the Non-Metal group the equation for Carbon (C) would be:

Element $_{\text {Carbon }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Element $_{D-\text { Orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Transition Metal }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
As a representative element belonging to the Transition-Metal group the equation for Gold (Au) would be:

$$
\text { Element }_{\text {Gold }}=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{S_{\text {System }}^{P r}}\right] \\
Y \in\left[S_{S_{\text {Stem }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } s ; a>b]
\end{array}\right]
$$

Element $_{\text {F-Orbital }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {Lanthanide }}=$

Element ${ }_{\text {Actinide }}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$

As a representative element belonging to the Lanthanide group the equation for Lanthanum (La) would be:

Element $_{\text {Lanthanum }}=\begin{gathered}X \in\left[S_{\text {System }_{N}}\right] \\ X a+\overline{Y b_{0-n}} \text { where }\end{gathered}\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
And generally:

Element $_{\text {sig }_{P}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{\text {System }}^{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {sig }_{V}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Element $_{\text {sig }_{M}}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r} r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$


Equality: Point $_{A} \equiv$ Point $_{B}$

Uniqueness: Sig =
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{S_{\text {System }}^{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$

Alternative: Innovation orientation-x

$$
=\left[\begin{array}{c}
M_{3} \rightarrow \text { System }_{X} \\
(\uparrow F \rightarrow I) \\
M_{2} \rightarrow \text { S System }_{X} \\
\left(\uparrow \text { Sig }_{\text {S }} \rightarrow F\right) \\
M_{1} \rightarrow \text { Sig }_{x} \\
\left(\uparrow>P_{x)}\right. \\
U \rightarrow x_{U}
\end{array}\right] \text { TC } \rightarrow x_{T}, \text { where }\left[\begin{array}{lll}
x_{U} & \ni[\ldots] \\
x_{T} & \ni[\ldots]
\end{array}\right]
$$

Where:
$x \in($ physical, vital, mental, integral)
$X \in$ (Presence, Power, Knowledge, Nurturing)
Fractal: Orientation @ Complexity $_{n} \leftrightarrow$ Orientation $_{x}$ @ Complexity ${ }_{n+1}$
Intersection: Organization Orientation $\rightarrow$ Next Element $\left(S_{x}\right)$, where $S_{x}$ is the set (physical, vital, mental, integral).

Emergent: Org_Dir $=$ DI $\left(\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F r I) \\ M_{2} \rightarrow S_{\text {System }_{X}} \\ \left(\uparrow \text { Sig }^{\rightarrow} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{P)}\right. \\ U \rightarrow x_{U}\end{array}\right] x=p, v, m, i\right) \rightarrow$
$x_{-}$matrix $_{\text {strongest }} @$ level $_{\text {strongest }}$
Flowering: Possibility $_{T s_{n}}>$ Possibility $_{T s_{n-1}}$
Direction: Upward $\left[\begin{array}{c}M_{3} \rightarrow \text { System }_{X} \\ (\uparrow F \rightarrow I) \\ M_{2} \rightarrow \text { System }_{X} \\ \left(\uparrow \text { Sig }_{\text {Si }} \rightarrow F\right) \\ M_{1} \rightarrow \text { Sig }_{x} \\ \left(\uparrow>P_{x}\right. \\ U \rightarrow x_{U}\end{array}\right]$


Remember: Ubiquity $\left[\begin{array}{l}\stackrel{\leftarrow S=0}{\overrightarrow{T S>0}}\end{array}\right]$
Link: $\left[\begin{array}{rl}M_{3} & \rightarrow \text { System }_{X} \\ M_{2} & \uparrow \downarrow \\ & \uparrow S_{\text {System }_{X}} \\ & \uparrow \downarrow \\ M_{1} & \rightarrow \text { Sig }_{x} \\ & \uparrow \downarrow \\ U & \rightarrow x_{U}\end{array}\right]$


## T. Electromagnetic Spectrum Equations

$\left.\begin{array}{c}\text { EM_Spectrum }_{\text {Speed }} \\ X a+\overline{Y b_{0-n}} \text { where }\end{array}\right] \begin{gathered}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, \\ a, b \text { are integers } ; a>b\end{array}\right]\end{gathered}$
$[\operatorname{Nature}(U)]_{C_{U}}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$\left[\operatorname{Nature}\left(M_{1}\right)\right]_{C_{M_{1}}}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{N}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$\left[\operatorname{Nature}\left(M_{2}\right)\right]_{C_{M_{2}}}=$

$\left[\operatorname{Nature}\left(M_{3}\right)\right]_{C_{M_{3}}}$

$$
=X a+\overline{Y b_{0-n}} \text { where }\left[\begin{array}{c}
X \in\left[S_{\text {System }_{N}}\right] \\
Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\
a, b \text { are integers } ; a>b
\end{array}\right]
$$


What this also implies is that the significance or intent of the different types of waves that exist can also be expressed by this general equation where the X and Y elements will vary. What precisely these elements are will need to be worked out. Hence:

Gamma Rays ${ }_{\text {Intent }}=\begin{gathered}X \in\left[S_{\text {System }_{K}}\right] \\ X a+\overline{Y b_{0-n}} \text { where }\end{gathered}\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$


White Light Intent $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$\left.\begin{array}{c}\text { Violet Light }_{\text {Intent }}= \\ X a+\overline{Y b_{0-n}} \text { where }\end{array} \begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Indigo $_{\text {Light }}^{\text {Intent }}$ $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{\text {Pr }}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Blue Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Green Light Intent $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Yellow $^{\text {Light }}{ }_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{K}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ a, b \text { are integers } ;\end{array}, a>b\right.\end{array}\right]$

Orange Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Red Light $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Infrared $_{\text {Intent }}=X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{y s t e m_{K}}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},} S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Millimeter Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Microwaves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

FM Radio Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Short Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

AM Radio Waves $_{\text {Intent }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{K}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
EM_Spectrum $_{\text {Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Gamma Rays Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$X-$ Rays $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Ultraviolet $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

White Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Violet Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Indigo Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Blue Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Green Light Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Yellow Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Orange Light Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Red Light ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Infrared $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Millimeter Waves ${ }_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Microwaves $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

FM Radio Waves $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Short Waves $_{\text {Nature of Energy }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$
AM Radio Waves Nature of Energy $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

EM_Spectrum $_{\text {Mass_Possibility }=}$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Gamma Rays Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
$X-$ Rays $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S y s t e m_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Ultraviolet Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

White Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$

Violet Light Mass_Possibility $=$


Indigo Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r} r}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Blue Light $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Green Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Yellow Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r} r}, \\ a, b \text { are integers } ;\end{array}, a>b\right.\end{array}\right]$

Orange Light Mass_Possibility
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Red Light Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[\begin{array}{c}S_{\text {System }_{P r}}, \\ S_{\text {System }_{P}}, S_{\text {System }_{K}}, \\ a, b \text { are integers } ; a>b\end{array}\right]\end{array}\right]$

Millimeter Waves Mass_Possibility $=$
$X \in\left[\begin{array}{c}\left.\text { M }_{\text {System }_{P r}}\right]\end{array}\right.$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c} \\ Y \in\left[S_{S_{\text {System }}^{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}},\right. \\ a, b \text { are integers } ; a>b\end{array}\right]$
Microwaves $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
FM Radio Waves Mass_Possibility $=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{\text {System }_{P r}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
Short Waves $_{\text {Mass_Possibility }}=$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{\text {System }_{P r}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}},}, S_{\text {System }_{P}}, S_{\text {System }_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
AM Radio Waves $_{\text {Mass_Possibility }=}$
$X a+\overline{Y b_{0-n}}$ where $\left[\begin{array}{c}X \in\left[S_{S_{\text {System }}}\right] \\ Y \in\left[S_{S_{\text {System }_{P r}}}, S_{\text {System }_{P}}, S_{S y s t e m_{K}}, S_{\text {System }_{N}}\right] \\ a, b \text { are integers } ; a>b\end{array}\right]$
U. Wave Equations

Leveraging Schrodinger's equation:
$\widehat{\mathrm{H}}_{Q}<\widehat{\mathrm{H}}_{A}<\widehat{\mathrm{H}}_{C}$
Where:
$\widehat{\mathrm{H}}_{X} \propto$ Energy $_{\text {Sig }_{X}}$

Such that, and as specified by the number of components of $\operatorname{Sig}_{X}$ :
Energy $_{\text {Sig }_{Q}}<$ Energy $_{\text {Sig }_{A}}<$ Energy $_{\text {Sig }_{C}}$

Wave equations for quantum, atomic, and cellular respectively:
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{Q}=\widehat{H}_{Q} \psi_{Q}$
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{A}=\widehat{H}_{A} \psi_{A}$
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi_{C}=\widehat{H}_{C} \psi_{C}$
V. Uncertainty Principle

Leveraging Heisenberg's Uncertainty Principle:
$@_{\mathrm{U}}: \Delta p X \Delta x \geq \frac{h}{4 \pi}$
$@ \mathrm{C}_{\mathrm{Q}}: \Delta p X \Delta x \rightarrow 0$
$@ C_{C_{M_{3}}}: \Delta p X \Delta x=0$
W. Dark Matter, Dark Energy

Matter $_{\text {Complexity }} \propto\left|M S_{\text {System }_{P r}} \cup M S_{\text {System }_{P}} \cup M S_{\text {System }_{K}} \cup M S_{\text {System }_{N}}\right|$

Matter $_{\text {Complexity }} \propto[$ Visible Matter + Dark Matter $]$
$\left|M S_{\text {System }_{P r}} \cup M S_{\text {System }_{P}} \cup M S_{\text {System }_{K}} \cup M S_{\text {System }_{N}}\right| \propto$ Matter $_{\text {Complexity }}$

MS_Growth $_{\text {Threshold }}:$ Acceleration $_{\text {Galaxies }}>$ Gravity $_{\text {Universe }}$
!MS_Growth Threshold : Acceleration Galaxies $^{<}$Gravity $_{\text {Universe }}$

MS_Growth Threshold $\propto$ Dark Energy
X. Emergence of CAS in Universe

Emergence space-time

Y. Qubits and Quantum Computing
$\left\lvert\, \psi>=D I\left[\begin{array}{c}M_{3} \\ M_{2} \\ M_{1} \\ U\end{array}\right]\right.$
Z. Nature of Light
$C_{U}:[$ Physical, Vital, Mental, Connection]
$C_{N}>C_{U}$
$C_{\infty}:$ [Presence, Power, Knowledge, Harmony]
$C_{K}:\left[S_{P r}, S_{P o}, S_{K}, S_{H}\right]$
$C_{N:} f\left(S_{P r} \times S_{P o} \times S_{K} \times S_{H}\right)$

$$
\begin{aligned}
& \text { Light }_{\text {Matrix }}=\left|\begin{array}{c}
C_{\infty}:[P r, P o, K, H] \\
\left(\downarrow R_{C_{K}}=f\left(R_{C_{\infty}}\right)\right) \\
C_{K}:\left[S_{P r}, S_{P o}, S_{K}, S_{H}\right] \\
\left(\downarrow R_{C_{N}}=f\left(R_{C_{K}}\right)\right) \\
C_{N:}: f\left(S_{P r} \times S_{P o} \times S_{K} \times S_{H}\right) \\
\left(\downarrow R_{C_{U}}=f\left(R_{C_{N}}\right)\right) \\
C_{U}:[P, V, M, C]
\end{array}\right| \\
& R_{C_{K}}=f\left(R_{C_{\infty}}\right) \\
& R_{C_{N}}=f\left(R_{C_{K}}\right) \\
& R_{C_{U}}=f\left(R_{C_{N}}\right)
\end{aligned}
$$

Emergence light-space-time


## APPENDIX 5: VENSIM EQUATIONS AND VARIABLES

A. Time Variables
(01)

- INITIAL TIME $=0$
- Units: Month
- The initial time for the simulation.
(02)
- FINAL TIME $=120$
- Units: Month
- The final time for the simulation.
(03)
- TIME STEP $=0.25$
- The time step for the simulation.


## B. Overall System Innovation

(04)

- System Innovation= "Physical (-)"+"Vital (-)"+"Mental (-)"+"Integral ( )"+"Physical (+)"+"Vital (+)" +"Mental (+)"+"Integral (+)"
- Units: Innovation Contribution
- System Innovation is sum of initial values, $\mathrm{P}(-), \mathrm{V}(-), \mathrm{M}(-), \mathrm{I}(-)$, and built-up positive values, $\mathrm{P}(+), \mathrm{V}(+), \mathrm{M}(+), \mathrm{I}(+)$
C. Initial States at Untransformed Layer, U
(05)
- "Physical (-)"= RANDOM UNIFORM(-2,5,0)
- Units: Innovation Contribution
- A random 'Innovation Contribution' from -2 to 5
(06)
- "Vital $(-)$ "= RANDOM UNIFORM $(-2,5,0)$
- Units: Innovation Contribution
- A random 'Innovation Contribution' from -2 to 5
(07)
- "Mental (-)"= RANDOM UNIFORM(-2,5,0)
- Units: Innovation Contribution
- A random 'Innovation Contribution' from -2 to 5
(08)
- "Integral (-)"= RANDOM UNIFORM(-2,5,0)
- Units: Innovation Contribution/Month
- A random 'Innovation Contribution' from -2 to 5
D. Meta-Layer Influence at Layer-U
(09)
- "Physical (+)"= "Physical (sig)"+"Force (presence)"+"Point (presence)"
- Units: Innovation Contribution
- $P(+)$ is sum of $P(s i g), F(p r), P t(p r)$
(10)
- "Vital (+)"= "Vital (sig)"+"Point (power)"+"Force (power)"
- Units: Innovation Contribution
- $\mathrm{P}(+)$ is sum of $\mathrm{V}($ sig $), \mathrm{F}($ pow $), \mathrm{Pt}($ pow $)$
(11)
- "Mental (+)"= "Mental (sig)"+"Force (knowledge)"+"Point (knowledge)"
- Units: Innovation Contribution
- $M(+)$ is sum of $M(s i g), F($ know $), \operatorname{Pt}($ know $)$
(12)
- "Integral (+)"= "Integral (sig)"+"Force (nurture)"+"Point (nurture)"
- Units: Innovation Contribution
- $\mathrm{I}(+)$ is sum of $\mathrm{I}($ sig $), \mathrm{F}$ (nurture), $\mathrm{Pt}($ nurture)


## E. Activating Meta-Layer 1

- "Overcome (physical patterns)" $=1$
- Units: Innovation Contribution/Month
- Overcome patterns (or contribute to innovation) at the rate of $1 /$ month
(14)
- "Overcome (vital patterns)"= 1
- Units: Innovation Contribution/Month
- Overcome patterns (or contribute to innovation) at the rate of $1 /$ month
(15)
- "Overcome (mental patterns)"= 1
- Units: Innovation Contribution/Month
- Overcome patterns (or contribute to innovation) at the rate of $1 /$ month
- "Overcome (integral patterns)"= 1
- Units: Innovation Contribution/Month
- Overcome patterns (or contribute to innovation) at the rate of $1 /$ month


## F. Signature-Based Dynamics at Meta-Layer 1

(17)

- "Physical (sig)"= INTEG ("Overcome (physical patterns)",0)
- Units: Innovation Contribution
- If 'Overcome (physical patterns)' then contribute to 'System Innovation'
(18)
- "Vital (sig)"= INTEG ("Overcome (vital patterns)",0)
- Units: Innovation Contribution
- If 'Overcome (vital patterns)' then contribute to 'System Innovation'
(19)
- "Mental (sig)"= INTEG ("Overcome (mental patterns)",0)
- Units: Innovation Contribution
- If 'Overcome (mental patterns)' then contribute to 'System Innovation'
(20)
- "Integral (sig)"= INTEG ("Overcome (integral patterns)",0)
- Units: Innovation Contribution
- If 'Overcome (integral patterns)' then contribute to 'System Innovation'


## G. Activating Meta-Layer 2

(21)

- "Physical (sig) becomes Force"= 2
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $2 /$ month
(22)
- "Vital (sig) becomes Force"= 2
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $2 /$ month
(23)
- "Mental (sig) becomes Force"= 2
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $2 /$ month
- "Integral (sig) becomes Force"= 2
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $2 /$ month
H. Architectural force-based dynamics at Meta-Layer 2
(25)
- "Force (presence)" = INTEG (IF THEN ELSE("Physical (sig)">0, "Physical (sig) becomes Force", 0), 0)
- Units: Innovation Contribution
- If $\mathrm{P}(\mathrm{sig})>0$ then contribute to System Innovation
(26)
- "Force (power)"= INTEG (IF THEN ELSE("Vital (sig)">0,"Vital (sig) becomes Force",0), 0)
- Units: Innovation Contribution
- If V(sig) > 0 then contribute "Vital (sig) becomes Force" to System Innovation
- "Force (knowledge)"= INTEG (IF THEN ELSE("Mental (sig)">0,"Mental (sig) becomes Force",0), 0)
- Units: Innovation Contribution
- If $\mathrm{M}($ sig $)>0$ then contribute "Mental (sig) becomes Force" to System Innovation
(28)
- "Force (nurture)" $=$ INTEG (IF THEN ELSE("Integral (sig)">0,"Integral (sig) becomes Force",0), 0)
- Units: Innovation Contribution
- If I(sig) > 0 then contribute "Integral (sig) becomes Force" to System Innovation


## I. Activating Meta-Layer 3

- Presence becomes impersonal= 3
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $3 /$ month
(30)
- Power becomes impersonal= 3
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $3 /$ month
(31)
- Knowledge becomes impersonal=3
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $3 /$ month
(32)
- Nurture becomes impersonal= 3
- Units: Innovation Contribution/Month
- Contribute to innovation at the rate of $3 /$ month


## J. Point-based Dynamics at Meta-Layer 3

- "Point (presence)"= INTEG (IF THEN ELSE("Force (presence)">0:AND:"Force (power)">0:AND:"Force (knowledge)">0:AND:"Force (nurture)">0, Presence becomes impersonal,0),0)
- Units: Innovation Contribution
- If "Force (presence) > 0" and "Force (power) > 0" and "Force (knowledge) > 0" and "Force (nurture) >0" and "Presence becomes impersonal", then contribute to System Innovation
(34)
- "Point (power)"= INTEG (IF THEN ELSE("Force (power)">0:AND:"Force (presence)">0:AND:"Force (knowledge)">0:AND:"Force (nurture)">0,Power becomes impersonal,0),0)
- Units: Innovation Contribution
- If "Force (power) > 0" and "Force (presence) >0" and "Force (knowledge) >0" and "Force (nurture) > 0" and "Power becomes impersonal", then contribute to System Innovation
- "Point (knowledge)"= INTEG (IF THEN ELSE("Force (knowledge)">0:AND:"Force (power)" $>0: A N D: " F o r c e ~(p r e s e n c e) ">0: A N D: " F o r c e ~(n u r t u r e) ">0, K n o w l e d g e ~$ becomes impersonal,0),0)
- Units: Innovation Contribution
- If "Force (presence) >0" and "Force (power) >0" and "Force (knowledge) >0" and "Force (nurture)>0" and "Knowledge becomes impersonal", then contribute to System Innovation
- "Point (nurture)"= INTEG (IF THEN ELSE("Force (presence)">0:AND:"Force (power)">0:AND:"Force (knowledge)">0:AND:"Force (nurture)">0,Nurture becomes impersonal,0), 0)
- Units: Innovation Contribution
- If "Force (presence) >0" and "Force (power) >0" and "Force (knowledge) >0" and "Force (nurture)>0" and "Nurture becomes impersonal", then contribute to System Innovation


## APPENDIX 6: STANFORD UNIVERSITY MEDICAL CENTER CASE-RELATED COMMUNICATION AND ENDORSEMENTS

The following are exhibits referred to in Chapter 7 and also include parts of communications and endorsements from the Leadership of Stanford University Medical Center related to the case described in Chapter 7.

## A. Communication Regarding Use of Stanford University Medical Center as Case in this Dissertation with Director of Department

From: pravir malik [mailto:pravir.malik@deepordertechnologies.com]
Sent: Saturday, March 04, 2017 6:51 PM
To: Prigge, Todd
Subject: Re: Collaboration - The Fractal Organization
Hello Todd,

Wanted to let you know that i am just completing a PhD (part-time) in Mathematics of Innovation in Complex Adaptive Systems and ex-post-facto have leveraged the work we did together, culminating in the joint field guide in The Fractal Organization. So you and the various experiments we attempted at Stanford are immortalized in my dissertation.

Best Always, Pravir

On Mon, Mar 6, 2017 at 8:05 AM, Prigge, Todd wrote:
(personal info deleted)
Congrats on your doctorate and your advancements in how people view complex adaptive systems.
(personal info deleted)
Regards,
Todd Prigge
Administrative Director, Training and Organization Development Stanford Health Care
1850 Embarcadero Rd., Suite B • Palo Alto, CA 94303
you have received this communication in error and that any review, disclosure, dissemination, distribution or copying of it or the attachments is strictly prohibited. If you have received this communication in error, please contact me and destroy all copies of the communication and attachments. Thank you.

## B. Acknowledgement By Author of Dissertation in The Fractal Organization, for Stanford University Medical Center and the Director of Organizational Development

As reproduced from the Acknowledgement Section of The Fractal Organization:
"Part 2 of this book - Exercises - was developed entirely while I was at Stanford University Medical Center. I worked closely with Todd Prigge, the Director of Organizational Development, and we created Part 2 togeher. Todd has a wealth of experience in translating concept into practical exercises and with his help we crafted a number of practical exercises so that the concepts in Part 1 - Theory, could be more viscerally felt by practitioners, consultants, and decision-makers alike. I am grateful to Stanford University Medical Center for allowing this collaboration to happen, and especially grateful to Todd for agreeing to collaborate with me on this."
C. Sample Courses Taught by Researcher of this Dissertation at Stanford Center of Education and Professional Development


# The Flower Chronicles: A Way of Seeing \& Being 



Stanford Center of Education and Professional Development

Dec 06, 2012

Pravir Malik \& Lee Glickstein

Copyright Aurosoorya


## D. Representative Emotional Intelligence Builder Context-Sensitive Help Illustrating Capturing of Soft-Side

ENTRY allows observations relating to states of being to be entered into the diary. One state is entered at a time, and this can be repeated until all significant observations are recorded.

The state to be entered is selected from the appropriate column. If logging entries as part of a team, please keep the following core questions in mind. Note that these questions do not need to be exhaustively answered for all meetings. However, these questions serve as joggers, and the most significant ones can be answered. The determination of what to answer is up to you and will depend on the time you have available. Each team member should individually and independently enter information. Any resulting duplication of information from multiple perspectives will help to establish underlying and more significant patterns that the team may be experiencing:

1. How do team members respond to pressure?
2. How do members get along with one another? What is the level of tolerance between team members?
3. Are there serious issues and feelings that have been avoided? Are real issues being addressed by the team?
4. What is the level of confrontation? How does this show up on the team?
5. What leadership patterns have emerged?
6. Does the team agree on a single goal and a mutual plan?
7. What is the nature and result of conflict? How is conflict managed?
8. How are decisions made?

Note, that it is also possible, once you have established a sense for 'states of being' that arise in a meeting, to enter these without reference to the questions.

The INTENSITY bar allows the appropriate intensity of the state to be recorded. The range is from 1 to 10 , with 1 being 'low' and 10 being 'high'. You wil be enforced to choose a suitable value.

The DURATION bar allows the duration the state was active to be recorded. The range is from 5 minutes to 300 minutes. You wil be enforced to choose a suitable value.

The EVENT box allows the associated event accompanying the state to be recorded. You wil be enforced to enter text. You can get as detailed as you want in these entries. You can cross-reference these entries to the questions listed above. Please get as nuanced as you need to, to describe what happened. Note that when entering data as a member of a team, these entries will remain hidden from normal view in the REVIEW module until such time as the team elects to have all entries be displayed. the team will then inform the system administrator to switch this viewing capability on.

The ADD ENTRY button actually records all the data on the form into the diary. This button should be selected only after an observation has been completely entered.

The RESET button clears all entered values.
The LEARN button allows the user to learn more about the specific state being entered. This is done by opening the LEARN module in a separate window. After the user has read the related information he or she can exit from that module and return to the ENTRY module.

The HELP button in the Master Menu on top, displays this help file.
The Date field defaults to the date of entry. It is not necessary to change the date field as it is understood that entries entered on the date will refer to past meetings till the current date. You could reference individual meetings in the EVENT box if you want to.

## E. Illustrative States Recorded in Emotional Intelligence Builder Software by PICU Leadership

| Duration | Intensity | State | Event |
| :---: | :---: | :---: | :---: |
| 300 | 9 | Fatigue | re: their is really a discrepancy in what some people feel they are giving and what others perceive they are giving |
| 300 | 8 | Fragmentary | Unclear about ultimate goals. Each member might have a different view of a central and single goal that will have greatest impact. |
|  |  |  | Because the demands placed on the team are great, the members avoid exposing their true feelings for one 's sense of safety and to avoid |
| 300 | 8 | Sensitiveness | misunderstandings. |
| 190 | 5 | Repeatability | leadership quality |
| 180 | 8 | Sensitiveness | Some decisions are reached by consensus which at times backfires. |
| 180 | 5 | Joy | It is a joy to work with each other. However, under extreme duress the team's functionality suffers and we treat each other with less tolerance. |
| 145 | 8 | Fear | fear of confronation |
| 145 | 7 | Inertia | some team members feel inertia when dealing with serious issues. |
| 120 | 6 | Energy | Most team members are very energetic dedicating an enormous amount of effort to produce positive results. |
| 100 | 8 | Anxiety | Is there enough trust in the group to be really honest with each other to change the team dynamics. |
| 100 | 8 | Haste | Sometimes too much haste before there is a real vision of the goal. re: working under pressure |
| 100 | 8 | Perseverance | working under pressure: persistance |
| 100 | 6 | Determination | We all want to be a highly functioning team. <br> working under pressure: sometimes don't have all the pieces. Some people |
| 100 | 6 | Fragmentary | seem to know whats going on and others don't have the information |
| 100 | 5 | Short-sighted | re; dealing with serious issues |

## F. Foreword to The Fractal Organization by Dean Dipak Jain

"This is a different kind of business leadership book.
It is, in some sense, a book about archeology and architecture on a cosmic and commercial scale. It's informed by the author's extensive experience as a global consultant, but it is equally inspired by his profound desire to examine the metaphysical subtleties of the world-and, indeed, the universe. This is a book that explores what might be called "the wholeness of the parts" in an effort to demonstrate the significance of patterns-specifically, fractal patterns-in our lives.

Pravir Malik adopts a fresh and daring perspective in an attempt to bridge the science of fractals with the larger world comprised of these smaller pieces. From individual to institution, there are profound connections, he says. Malik explores these relationships at the micro level and endeavors to highlight their accumulated power at the macro level too.

By helping us become more aware of this elegant latticework that runs through nature and human life, from physics and biotechnology to business, Malik hopes to point the way to a dramatically new kind of leader and organization. He aspires to share a model that encourages the growth of progressive, sustainable organizations.

Like the Ancient Greeks and their notion of the atom-then considered the smallest particle of matter-Malik invites the reader on a journey into the fractal realm to show how patterns at this level are reproduced and reverberate throughout our social and cultural institutions, including our organizations. He is a firm believer in cultivating greater awareness of these patterns so that individuals can best contribute to human progress at all levels: in the organization, the community, the nation, region and world.

By combining theory with examples from his professional life, as well as with exercises designed to enhance understanding of his model-one with roots in his previous book, Connecting Inner Power with Global Change: The Fractal Ladder-Malik provides readers with an ambitious and intriguing framework unlike any other.

Open-minded practitioners from all walks of life should find plenty of food for thought and reflection within these pages.
— Dipak C. Jain, Director, Sasin Graduate Institute of Business Administration"

## G. TOC of The Fractal Organization Highlighting Part II - Exercises (aka the "field guide")

The "field guide" is highlighted as Part II - Exercises in the TOC.

## Contents

- Acknowledgment
- Introduction


## Part I - Theory

- Chapter 1 - The Pattern
- Chapter 2 - The Person Pattern
- Chapter 3 - The Business Pattern
- Chapter 4 - The Economy Pattern
- Chapter 5 - The System Pattern
- Chapter 6 - The Evolution Pattern
- Chapter 7 - The Fractal Ladder
- Chapter 8 - Fractal Properties
- Chapter 9 - The Nature of Progress
- Chapter 10 - Remaking the Business World
- Chapter 11 - Creating Enterprises of Tomorrow


## Part II - Exercises

- Exercises for Chapter 1 - The Pattern
- Exercises for Chapter 2 - The Person Pattern
- Exercises for Chapter 3 - The Business Pattern
- Exercises for Chapter 4 - The Economy Pattern
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- Bonus - Exercises for Creating Your Personal Manifesto

Part III - Reflections

- A Personal Experience of Fractals
- Reflections at the Individual Level
- Reflections on Organizational Development
- Reflections on Organizational Design
- Reflections on Industry Development
- Reflections on Financial Crises
- Reflection From The 2012 US Presidential Elections
- Reflections on Various Global Political Development
H. Endorsements by Stanford University Medical Center Leadership


## Greg Souza, Chief Human Resources Office, Stanford Children's Health

I've been impressed with Pravir's work using Fractal thinking specifically as it relates to managing change. The methodology in The Fractal Organization: Creating Enterprises of Tomorrow opens up perspectives not normally considered, that can lead to highly accelerated adoption of the change process.

Laura Gottlieb, Director - Enterprise Learning \& Development, Lucille Packard Children's Hospital at Stanford

By calling attention to the fractal patterns that reside within and all around us, Pravir Malik has given us a set of keys to unlock our inner potential, enabling us to more fully partner with "Progress" and ascend the fractal ladder. This concept is both evolutionary and revolutionary and has been proposed by some of our greatest thinkers, including Mahatma Gandhi who once said, "You must be the change you wish to see in the world." With great depth and heart, Malik goes beyond merely referencing this powerful quote, he offers readers a simple formula (via the physical, vital and mental fractal) to BE the change. This book makes the connection between the individual and the world we each helped create, while at the same time elevating readers' consciousness to envision and co-create an even brighter future."

## Dale Spartz, Ph.D., VP - Human Resources, Stanford Hospitals \& Clinics

We live in a world of constant change and there seems to be a new book out every day. Unfortunately, many of the latest books could be categorized as "fads" or rehashing similar ideas. "The Fractal Organization: Creating Enterprises of Tomorrow" is quite different. It builds on the previous work of the author (Connecting Inner Power with Global Change: The Fractal Ladder (Malik, 2009) and is based on a solid theoretical foundation from the fields of psychology, economics, anthropology, and behavioral science. It offers a unique view of change and system patterns. This book reviews the theory behind the Fractal Ladder, provides exercises at an individual or group level for reflection and learning, and provides reflections \& analysis for individual, group, and organizational levels (I enjoyed the Dark Knight and the Fractal for Progress section as a great example behind the theory). The book is not intended to be an "easy read." One needs to spend time understanding the theory and the "fractal architecture" of physical orientation, vital orientation, and mental orientation. However, if you spend the time and study carefully the flow of ideas, you can gain a new perspective of systems and change.

## APPENDIX 7: EXISTING EQUATIONS LEVERAGED IN DERIVED MATHEMATICS

Prigogine's Dissipative Structure inequality:
$\frac{d V_{O S}(x(t))}{d t} \leq u(t) \cdot y(t)$
Turing Activator-Inhibitor Equations:

$$
\begin{aligned}
& \frac{\partial u}{\partial t}=f(u, v)+D_{u} \nabla^{2} u \\
& \frac{\partial v}{\partial t}=g(u, v)+D_{v} \nabla^{2} v
\end{aligned}
$$

Framing Complexity (Sargut \& McGrath, 2011)
Complexity $_{\text {System }}=f$ (Multiplicity, Interdependence, Diversity)
Einstein's Length Contraction
Length $_{\text {contraction }}=$ Length $\left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)$
Schrodinger's Wave Equation
$i \frac{h}{2 \pi} \frac{\partial}{\partial x} \psi=\widehat{H} \psi$
Heisenberg's Uncertainty Principle
$\Delta p X \Delta x \geq \frac{h}{4 \pi}$

## APPENDIX 8: AUTHOR'S PUBLISHED ARTICLES AND AWARDS RELATED TO THIS DISSERTATION

## Published Journal Articles

1. Malik, P. 2003. Business Transformation Through the Creation of a Complex Adaptive System. Journal of Human Values 9:2. New Delhi: Sage Publications
2. Malik, P. 2004. Introduction to Fractal Dynamics. Journal of Human Values 10:2. New Delhi: Sage Publications
3. Malik, P. 2013. Current Economic Challenges: Making Sense of Market Crises. Integral Review. 9(3), 50-63

Published /To-Be Published Conference Proceedings

1. Malik, P et al., 2015. A Mathematical Basis of Innovation. Conference Proceedings IAMOT 2015
2. Malik, P. et al., 2016. Leveraging a Mathematics of Innovation to Create Complex Adaptive Systems. Conference Proceedings IAMOT 2016
3. Malik, P., et al., 2017. Qualified Determinism in Emergent-Technology Complex Adaptive Systems. IEEE TEMSCON 2017

Published Books

1. Malik, P. 2009. Connecting Inner Power with Global Change: The Fractal Ladder. New Delhi: Sage Publications
2. Malik, P. 2011. Redesigning the Stock Market: A Fractal Approach. New Delhi: Sage Publications
3. Malik, P. 2015. The Fractal Organization. New Delhi: Sage Publications

## Awards

1. Award for Best Presentation at a Doctoral Research Colloquium. "Application of a Generalized Equation for Innovation at the Cellular Level. IAMOT 2015.
2. Best Book Award. "The Fractal Organization". IAMOT 2016

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