January-February 2005
CORPORATE FINANCE
Lecture notes

Lecture 1. Introduction

The key responsibilities of the CFO

The two main parts of CF are **allocation** (investment) and **financing**, corresponding to the A&L sides of the balance sheet. The ultimate objective is to maximize the shareholders' wealth: achieve the highest return on the projects and ensure the cheapest financing. Evaluation of the investment projects is based on the discounted **cash flows**, which are different from the accounting profits (e.g. because of depreciation). The real options approach takes into account that managers can influence the CFs after the beginning of the project. Applying similar methods, one can value the company.

The company can be restructured from private to a public one via **IPO**, and vice versa. Another type of structural change comes from **M&A**s. The company's goal is to acquire the companies bringing synergy gains and those undervalued due to the inefficient management. The best defence from the acquisition is to maximize its own value.

The goal of **corporate governance** is to internalize the external effects, balance the economic interests of all stakeholders. In well-functioning financial markets, this maximizes shareholder value.

Typical CF questions:
- How to measure the project's worth for the company?
- Are companies' market prices justified? (e.g., dot-coms)
- How to choose among the projects given the budget constraint and external effects?
- How to account for risks associated with the project?
  - Systematic vs company-specific risks
- Are risks always bad?
- Is it good to have volatile oil prices?
  - Yes, if managers have flexibility in the future decisions.
- Should we invest now in a project, which seems unprofitable (has negative NPV)?
  - Probably, yes. It may yield high profit in certain future scenarios (oil pipeline)
- Should we invest now in a project, which is profitable (has positive NPV)?
  - Probably, not. It may be even more profitable next period (gold extraction)
- Should we give managers higher salaries or higher bonuses?
  - Bonuses encourage higher performance, but may also lead to the manipulations and excessive risk-taking.
- Does it matter how to finance the project: by debt or by equity?
- Would you like the company to have much debt?
  - Yes, to minimize taxes and to discipline the managers. Not too much, to avoid bankruptcy.
- Should the company borrow money from banks or issue bonds?
  - The company can renegotiate the terms of bank credit.
- Would you like the company to pay high dividends? (e.g., Microsoft)
  - Yes, if too much managerial discretion (Surgut). No, because of double taxation and signalling that the company has no valuable inv projects.
- How will the market react to the share buyback?
  - The company signals that its shares are undervalued.
- How will the market react to the new equity issue?

    - Usually negatively: either the company's shares are overvalued, or it needs to finance a new inv project.
- How should the company communicate with the market? Always provide precise info in time?
- What drives the company's decision to go public? Why are there hardly any IPOs in Russia?
- Would you like the company to grow via acquisitions?
    - Yes, if the main motivation comes form synergy gains, and not empire-building.

Specifics of corporation

Do not take the current form of corporations and stock markets as given, it is an endogenous outcome!
Advantages of **corporation** in comparison with **sole proprietorship** and **partnership**:

- Ltd liability: lesser risks for investors
    - Crucial for development of stock markets and diversification
- Easy transfer of ownership
    - Promotes liquidity
- Unlimited life
    - Makes it easier to attract financing

Disadvantages:

- Separation of ownership and control, the agency conflict
    - Solved in two ways: US vs Germany
- Double taxation

History: 1811, general act of incorporation in NY, specifying that *all* investors of NY corporations have limited liability

- Not so obvious that limiting the freedom of contracts is good
- Hot discussion at the time: could spur excessive risk taking
- California was the last to copy in 1931

The Objective in Corporate Finance

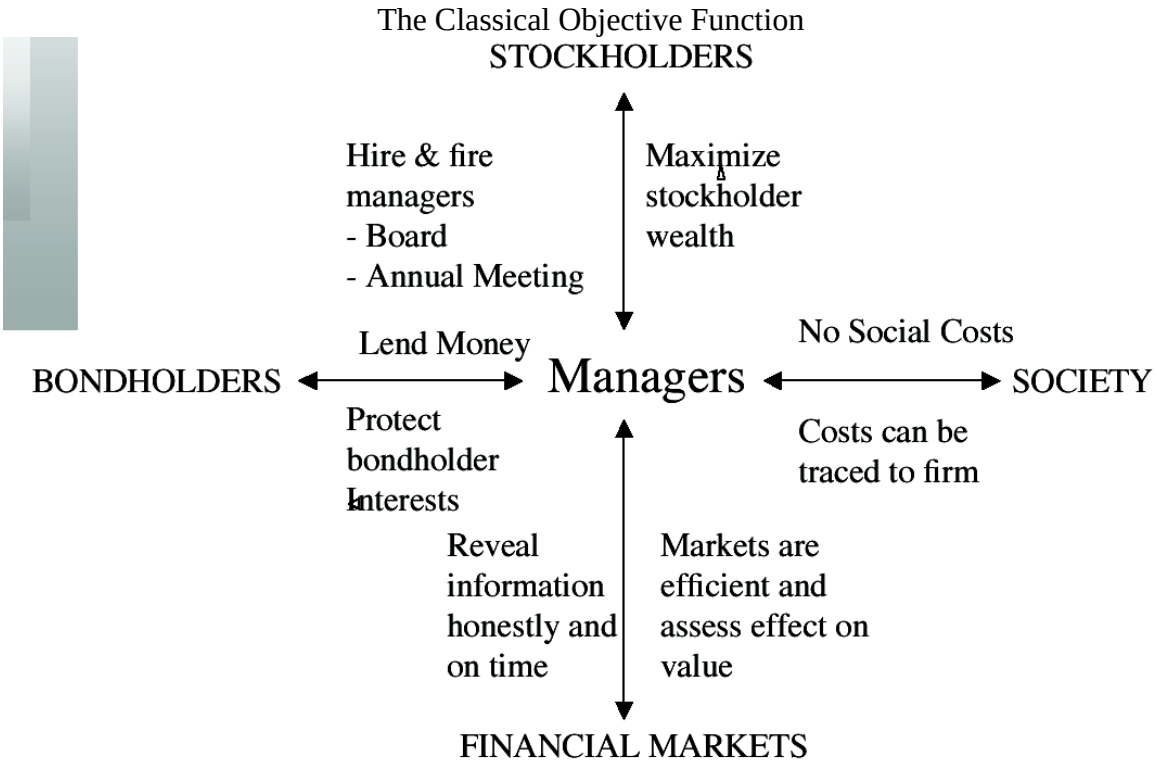
"If you don't know where you are going,
it does not matter how you get there"

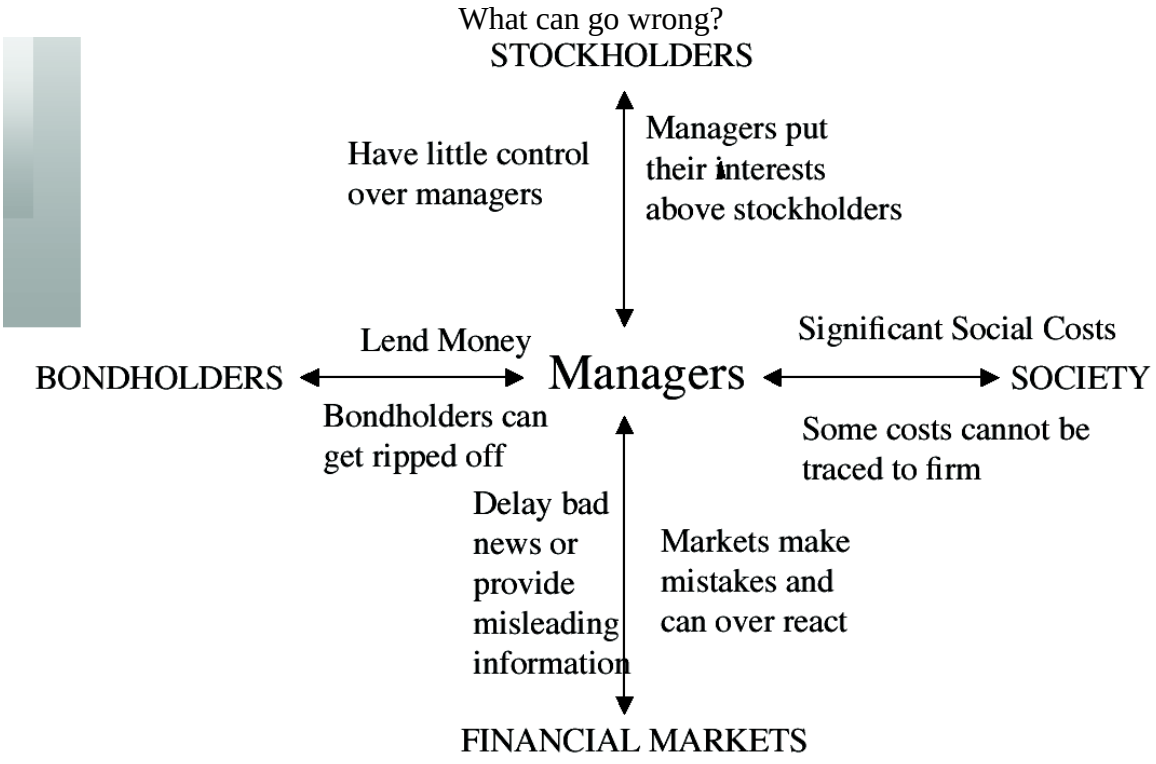
The Classical Viewpoint:
 Van Horne: "In this book, we assume that the objective of the firm is to maximize its value to its stockholders"
 Brealey & Myers: "Success is usually judged by value: Shareholders are made better off by any decision which increases the value of their stake in the firm... The secret of success in financial management is to increase value."
 Copeland & Weston: The most important theme is that the objective of the firm is to maximize the wealth of its stockholders."
 Brigham and Gapenski: Throughout this book we operate on the assumption that the management's primary goal is stockholder wealth maximization which translates into maximizing the price of the common stock.

Why focus on maximizing stockholder wealth?

- Stock price is easily observable and constantly updated (unlike other measures of performance, which may not be as easily observable, and certainly not updated as frequently).
- If investors are rational (are they?), stock prices reflect the wisdom of decisions, short term and long term, instantaneously.
- The objective of stock price performance provides some very elegant theory on:

- how to pick projects
- how to finance them
- how much to pay in dividends



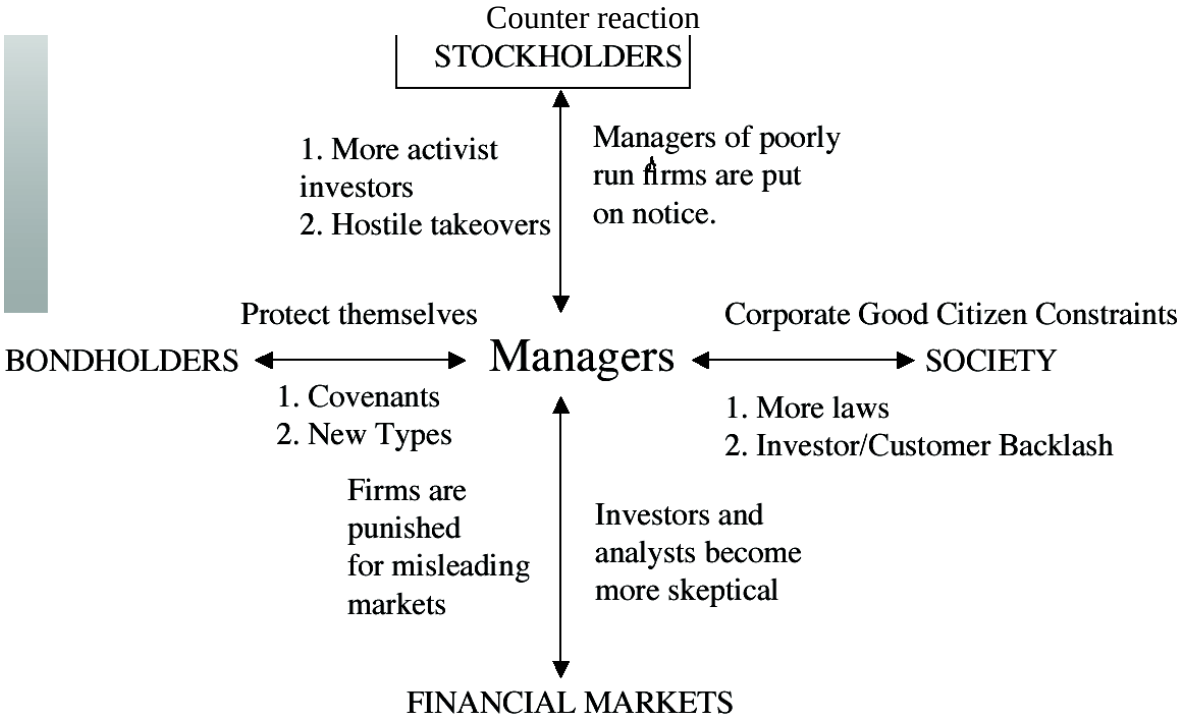


- Traditional corporate financial theory breaks down when the interests/objectives of the decision makers in the firm conflict with the interests of stockholders.
  - Bondholders (Lenders) are not protected against expropriation by stockholders.
  - Financial markets do not operate efficiently, and stock prices do not reflect the underlying value of the firm.

- Significant social costs can be created as a by-product of stock price maximization.

Solutions:

- Choose a different mechanism for corporate governance
- Choose a different objective:
  - Maximizing earnings / revenues / firm size / market share
  - The key thing to remember is that these are intermediate objective functions. To the degree that they are correlated with the long term health and value of the company, they work well. To the degree that they do not, the firm can end up with a disaster
- Maximize stock price, but reduce the potential for conflict and breakdown:
  - Making managers (decision makers) and employees into stockholders
  - Providing information honestly and promptly to financial markets



The strength of the stock price maximization objective function is its internal self correction mechanism. Excesses on any of the linkages lead, if unregulated, to counter actions which reduce or eliminate these excesses

- managers taking advantage of stockholders has lead to a much more active market for corporate control.
- stockholders taking advantage of bondholders has lead to bondholders protecting themselves at the time of the issue.
- firms revealing incorrect or delayed information to markets has lead to markets becoming more "skeptical" and "punitive"
- firms creating social costs has lead to more regulations, investor and customer backlashes.

The Modified Objective Function

- For publicly traded firms in reasonably efficient markets, where bondholders (lenders) are protected:
  - Maximize Stock Price: This will also maximize firm value
- For publicly traded firms in inefficient markets, where bondholders are protected:
  - Maximize stockholder wealth: This will also maximize firm value, but might not maximize the stock price

- For publicly traded firms in inefficient markets, where bondholders are not fully protected
    - Maximize firm value, though stockholder wealth and stock prices may not be maximized at the same point.
- For private firms, maximize stockholder wealth (if lenders are protected) or firm value (if they are not)

Relation to investment theory
- Use of CAPM to estimate the cost of capital
- Option pricing approach for valuing investment projects (real options), equity of the firm, and bonds' credit risk

Lecture 2. Analysis of financial statements

The Firm's Financial Statements
- Balance Sheet
- Income Statement
- Statement of Cash Flows

Functions: providing
- current status and past performance information to owners and creditors
- a convenient way for owners and creditors to set performance targets & to impose restrictions on the managers of the firm
- a convenient template for financial planning

Balance Sheet

Assets ≡ Liabilities + Shareholder's Equity

- Tabulates a company's assets and liabilities at a specific point in time
    - Info on value of the assets and the capital structure
- Sorting of
    - Assets by liquidity
    - Liabilities by maturity
- Assets and liabilities are represented by historical costs
    - The original cost adjusted for improvements and aging = Book Value
    - Avoid using market value, since is too volatile and easily manipulated
    - Preference for *under*estimating value
- Strict categorization into E or L: the liability must satisfy
    - The obligation will lead to CF at some specified or determinable date
    - The firm cannot avoid the obligation
    - The transaction behind the obligation has already happened
- However, important liabilities may be under-stated or omitted

Assets
- Current Assets (Оборотные средства): will convert into cash within a year
    - Cash
    - Accounts Receivable (Счета к получению)
        - Recognizing not collectible ones: reserves (danger of manipulation!)
    - Inventory (ТМЗ): valued by FIFO, LIFO, wdt-avg
        - LIFO increases costs and reduces taxes
        - LIFO reserve: difference between LIFO and FIFO valuations
- Investments and Marketable Securities (Рыночные цб)

    - Minority passive / active investment (<20% / 20-50% of the ownership): BV or MV
    - If majority active investment (>50%): include in the consolidated balance sheet
- Intangible Assets (Нематериальные активы): amortized over expected life (say, 40 years)
    - Patents and trademarks: valuation depends on whether generated internally or acquired
    - Goodwill: the difference between BV and MV of the acquired firm (purchase accounting)
- Fixed Assets (Основные средства; Land, Plant and Equipment): BV with adjustment for aging
    - Depreciation: straight line or accelerated (improves the earnings in the first years)

Liabilities and Stockholder's Equity

Liabilities

- Current Liabilities (Краткосрочные обязательства): valued as the amount due
    - Accounts Payable (Счета к оплате)
    - Short-term Borrowing
    - Other: Accrued Wages, Benefits, and Taxes
- Long-Term Debt: Bank Loans, Bonds
    - Valued as PV of future obligations at the time of borrowing (usually at par)
    - The premium or discount over the par is amortized over the bond's life
- Other Long-Term Liabilities
    - Leases
        - Capital lease (transfer of ownership): recognized as asset (depreciated) and liability
        - If operating lease: balance not affected, lease payments treated as operating expense
    - Employee Benefits: Pension Plans, Healthcare Benefits
        - DC: fixed contribution each year;
        - DB: contributions change depending on whether the plan is over- or underfunded
    - Deferred Taxes
        - Difference between the taxes on income reported in fin statements and actual taxes

Shareholder's Equity (Акционерный капитал) = Total Assets - Total Liabilities

- Preferred Stock
    - Hybrid: fixed (cumulative) dividend, but cannot result in bankruptcy
    - Valued at the original issue price + cumulated unpaid dividends
- Common Stock at Par
- Capital Surplus
    - Results from earnings on buying and selling stocks
    - Treasury Stock: repurchased shares, reduce BV of equity
- Retained Earnings (Нераспределенная прибыль)

Income Statement

Revenue – Expenses ≡ Income

- Summarizes the company's profitability *during a time period*
    - Records sales, expenses, taxes, and net income
- Matching principle of the accrual accounting:
    - Revenues and expenses are recognized when the good is sold
        - Becomes complicated for long-term contracts and buyers with credit risk
    - In contrast to the cash-based approach: recognizing revenues when received and expenses when paid
        - A company's accounting income and cash flow are two different things
- Categorization of expenses:

    - Operating: provide benefits only for the current period (cost of labor and materials)
        - Also included: depreciation (based on historical cost) and R&D
    - Financing: arising from non-equity financing (interest expenses)
    - Capital: generate benefits over multiple periods (buying land and buildings), written off as depreciation
- To improve forecasting, separately: nonrecurring items
    - Income from discontinued operations, extraordinary gains & losses, adjustments for changes in accounting principles
- Retained earnings are not added to the cash balance in the balance sheet, but are added to shareholder's equity
- Inflation distorts the measuring of income and the valuation of assets

Total operating revenues
- Cost of goods sold
- Selling, general, and administrative expenses
- Depreciation

Operating income
+ Other income
EBIT (Earnings before interest and taxes)
- Interest expense

Taxable income
- Taxes: Current + Deferred
Net income = Retained earnings + Dividends

The Statement of Cash Flows

CF(firm) ≡ CF(debt) + CF(equity)

- Reports how much cash is generated during a period.
    - Indicates where the cash comes from and what the firm did with that cash.
- Unlike the balance sheet and income statement, cash flow statements are independent of accounting methods
    - Accounting rules have a second-order effect on cash flows through taxes

Operating CF = EBIT + Depreciation - Taxes
- Capital Spending (net acquisitions of fixed assets)
- Additions to the Net Working Capital (current assets - current liabilities)

Cash Flow of the Firm

CF of debtholders = Interest – net long-term debt financing
CF of equityholders = Dividends– net equity financing

Financial Ratio Analysis

- Trend Analysis
- Cross-Sectional Analysis

Profitability Ratios
- Return on Assets (ROA) = EBIT(1-tax) / Total Assets
- Return on Equity (ROE) = Net Income / BV(equity)
- Gross Profit Margin = Gross Profit / TA
- Operating Profit Margin = EBIT / Sales
- Net Profit Margin = Net Income / Sales

Activity Ratios: measuring the efficiency of working capital management
- Accounts Receivable Turnover = Sales / Avg Accounts Receivable
- Inventory Turnover = Cost of Goods Sold / Avg Inventory
- Total Asset Turnover = Sales / Total Assets

Liquidity Ratios: measuring short-term liquidity
- Current Ratio = Current Assets / Current Liability
- Quick Ratio = (Current Assets – Inventory) / Current Liability

Financial Leverage Ratios
- Debt-to-Capital Ratio = Debt / (Debt + Equity)
- Debt-to-Equity Ratio = Debt / Equity
    - Can be based on BV or MV
    - Similarly: long-term debt ratios
- Interest Coverage Ratio = EBIT / Interest Expenses
- Cash Fixed Charges Coverage Ratio = EBITDA / Cash Fixed Charges

Market Value Ratios
- Price-to-Earnings Ratio = PS / EPS
    - Stock market price to earnings per share
- Dividend Yield = Div / PS
    - Latest dividend to current stock price
- Market-to-Book Value = MV / BV
    - Similarly: Market-to-Book Equity = ME / BE
- Tobin's Q = MV / Replacement Value

Links between the Ratios
- ROA = Profit Margin * Asset Turnover
    - Both for Net and Gross ROA and Profit Margin
    - Increasing ROA: trade-off between Profit Margin and Asset Turnover
- ROE = ROA * Equity Multiplier
    - where Equity Multiplier = Assets / Equity
    - Higher fin leverage magnifies ROE when ROA(gross) excess the interest on debt

Non-Financial Measures of Operating Effectiveness
- Innovation
- Customer Service
- Product Quality
- Reputation
- Good Employee Relations

Segmented Financial Statements
- Reports revenues, operating profits, and identifiable assets for each line of business
- Allows managers and shareholders to identify cross-subsidization

The DCF approach to bond and stock valuation

Computing Present Value
- Time value of money: discount rate R
- Single cash flow at T: CF_T
    - $PV_0 = CF_T/(1+R)^T$

- Perpetuity: $C_t = C$, $t>0$
  - $PV_0 = C / R$
- Growing perpetuity (with const rate g): $C_{t+1} = (1+g)C_t$
  - $PV_0 = C / (R - g)$
- Annuity: $C_t = C$, $t = 1,\ldots,T$
  - $PV_0 = (C/R) [1 – 1/(1+R)^T]$
- Growing annuity (with const rate g)
  - $PV_0 = (C/(R-g)) [1 – (1+g)^T/(1+R)^T]$

Computing Growth Rate of Dividends

Assume that the company does not grow unless a net investment is made. Then the company needs to retain part of its earnings to grow:

$$Earnings_{t+1} = Earnings_t + Retained\_ Earnings_t * R$$

where R is the return on the retained earnings, usually estimated by ROE

Divide by $Earnings_t$ to get Sustainable Growth Rate :

$$1 + g = 1 + Retention\ Ratio * ROE$$

where Retention Ratio = Retained Earnings / Earnings

Pricing Applications

- Bond with coupon C and face value F (at T)
  - $P_0 = (C/R) [1 – 1/(1+R)^T] + F_T/(1+R)^T$
- Stocks with dividends growing with const rate g
  - $PV_0 = Div_1/(R-g)$
- Project: $NPV = \Sigma_t CF_t/(1+R)^t$
- Value of the firm with Div=EPS:
  - *Discounted CF's*: $PV_0 = EPS/R + NPVGO$
    - EPS = earnings per share, GO = growth opportunities
  - *Multiples*: $P/E \equiv PV_0/EPS = 1/R + NPVGO/EPS$
    - P/E = price to earnings ratio

Lectures 3-6. Capital budgeting

Black-Scholes approach to the valuation of real options

Differences between real and financial options, which are crucial for the Black-Scholes approach:

1. The underlying asset is not traded

- Option pricing theory is built on the premise that a replicating portfolio can be created using the underlying asset and riskless lending and borrowing.

2. The price of the asset may not follow a continuous process

- If there are no price jumps, as it is with most real options, the model will underestimate the value of deep out-of-the-money options.
  - One solution is to use a higher variance estimate to value deep out-of-the-money options and lower variance estimates for at-the-money or in-the-money options.
  - Another is to use an option pricing model that explicitly allows for price jumps, though the inputs to these models are often difficult to estimate.

3. The variance may change over the life of the option

- The assumption that option pricing models make, that the variance is known and does not change over the option lifetime, is not unreasonable when applied to listed short-term options on traded stocks.

- When option pricing theory is applied to long-term real options, there are problems with this assumption, since the variance is unlikely to remain constant over extended periods of time and may in fact be difficult to estimate in the first place.

4. Exercise is not instantaneous

- The option pricing models are based upon the premise that the exercise of an option is instantaneous. This assumption may be difficult to justify with real options, where exercise may require the building of a plant or the construction of an oil rig, actions which are unlikely to happen in an instant.
- The fact that exercise takes time also implies that the true life of a real option is often less than the stated life.

Valuing Natural Resource Options/ Firms

Input	Estimation Process
1. Value of Available Reserves of the Resource	• Expert estimates (Geologists for oil..); The present value of the after-tax cash flows from the resource are then estimated.
2. Cost of Developing Reserve (Strike Price)	• Past costs and the specifics of the investment
3. Time to Expiration	• Relinqushment Period: if asset has to be relinquished at a point in time. • Time to exhaust inventory - based upon inventory and capacity output.
4. Variance in value of underlying asset	• based upon variability of the price of the resources and variability of available reserves.
5. Net Production Revenue (Dividend Yield)	• Net production revenue every year as percent of market value.
6. Development Lag	• Calculate PV of reserve based upon the lag.

Example: A gold mine

- Consider a gold mine with an estimated inventory of 1 million ounces, and a capacity output rate of 50,000 ounces per year.
- The price of gold is expected to grow 3% a year.
- The firm owns the rights to this mine for the next twenty years.
- The present value of the cost of opening the mine is $40 million, and the average production cost of $250 per ounce. This production cost, once initiated, is expected to grow 4% a year.
- The standard deviation in gold prices is 20%, and the current price of gold is $350 per ounce. The riskless rate is 9%, and the cost of capital for operating the mine is 10%. The inputs to the model are as follows:

Inputs for the Option Pricing Model

- Value of the underlying asset = Present Value of expected gold sales (@ 50,000 ounces a year) = (50,000 * 350) * (1- ($1.03^{20}/1.10^{20}$))/(.10-.03) - (50,000*250)* (1- ($1.04^{20}/1.10^{20}$))/(.10-.04) = $ 42.40 million
- Exercise price = PV of Cost of opening mine = $40 million
  Variance in ln(gold price) = 0.04
- Time to expiration on the option = 20 years
- Riskless interest rate = 9%
- Dividend Yield = Loss in production for each year of delay = 1 / 20 = 5%
  - Note: It will take twenty years to empty the mine, and the firm owns the rights for twenty years. Every year of delay implies a loss of one year of production.

Valuing the Option

Based upon these inputs, the Black-Scholes model provides the following value for the call:
d1 = 1.4069, N(d1) = 0.9202
d2 = 0.5124, N(d2) = 0.6958
Call Value= 42.40 exp(-0.05)(20) (0.9202) -40 (exp(-0.09)(20) (0.6958)= $ 9.75 million

The value of the mine as an option is $ 9.75 million, in contrast to the static capital budgeting analysis which would have yielded a net present value of $ 2.40 million ($42.40 million - $ 40 million). The additional value accrues directly from the mine's option characteristics.

Example: Valuing an oil reserve

- Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil, where the present value of the development cost is $12 per barrel and the development lag is two years.
- The firm has the rights to exploit this reserve for the next twenty years and the marginal value per barrel of oil is $12 per barrel currently (Price per barrel - marginal cost per barrel).
- Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8% and the variance in ln(oil prices) is 0.03.

Inputs to the Black-Scholes Model

- Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $12 * 50 /(1.05)2 = $ 544.22
  - If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period. Hence, the discounting of the reserve back at the dividend yield
- Exercise Price = Present Value of development cost = $12 * 50 = $600 million
- Time to expiration on the option = 20 years
- Variance in the value of the underlying asset = 0.03
- Riskless rate =8%
- Dividend Yield = Net production revenue / Value of reserve = 5%

Valuing the Option

Based upon these inputs, the Black-Scholes model provides the following value for the call:
d1 = 1.0359, N(d1) = 0.8498
d2 = 0.2613, N(d2) = 0.6030
Call Value= 544 .22 exp(-0.05)(20) (0.8498) -600 (exp(-0.08)(20) (0.6030)= $ 97.08 million

This oil reserve, though not viable at current prices, still is a valuable property because of its potential to create value if oil prices go up.

Valuing product patents as options

Input	Estimation Process
1. Value of the Underlying Asset	• PV of Cash Inflows from taking project now • This will be noisy, but that adds value.
2. Variance in value of underlying asset	• Variance in cash flows of similar assets or firms • Variance in PF from capital budgeting simulation.
3. Exercise Price on Option	• Option is exercised when investment is made. • Cost of making investment on the project; assumed to be constant in present value dollars.
4. Expiration of the Option	• Life of the patent

5. Dividend Yield	• Cost of delay • Each year of delay translates into one less year of value-creating cashflows

Valuing Equity as an option

A simple example

- Assume that you have a firm whose assets are currently valued at $100 million and that the standard deviation in this asset value is 40%.
- Further, assume that the face value of debt is $80 million (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%, how much is the equity worth? What should the interest rate on debt be?

Model Parameters

Value of the underlying asset = S = Value of the firm = $ 100 million
Exercise price = K = Face Value of outstanding debt = $ 80 million
Life of the option = t = Life of zero-coupon debt = 10 years
Variance in the value of the underlying asset = $\sigma 2$ = Variance in firm value = 0.16
Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Valuing Equity as a Call Option

Based upon these inputs, the Black-Scholes model provides the following value for the call:
d1 = 1.5994, N(d1) = 0.9451
d2 = 0.3345, N(d2) = 0.6310
Value of the call = 100 (0.9451) - 80 exp(-0.10)(10) (0.6310) = $75.94 million
Value of the outstanding debt = $100 - $75.94 = $24.06 million
Interest rate on debt = ($ 80 / $24.06)1/10 -1 = 12.77%

Input	*Applicability in valuation* Estimation Process
Value of the Firm	• Cumulate market values of equity and debt (or) • Value the firm using FCFF and WACC (or) • Use cumulated market value of assets, if traded.
Variance in Firm Value	• If stocks and bonds are traded, $\sigma 2firm = we2\ \sigma e2 + wd2\ \sigma d2 + 2\ we\ wd\ \rho ed\ \sigma e\sigma d$ where $\sigma e2$ = variance in the stock price we = MV weight of Equity $\sigma d2$ = the variance in the bond price wd = MV weight of debt • If not traded, use variances of similarly rated bonds. • Use average firm value variance from the industry in which company operates.
Maturity of the Debt	• Face value weighted duration of bonds outstanding (or) • If not available, use weighted maturity

Monte-Carlo approach to the valuation of real options

Stochastic Processes for Oil Prices

Geometric Brownian Motion Simulation

- The real simulation of a GBM uses the real drift a. The price at future time t is given by:

$$P_t = P_0 \exp\{ (a - 0.5\, s^2)\, D_t + s\, N(0, 1) \}$$

  - **s is the volatility of P**

  With real drift use a risk-adjusted (to P) discount rate
- The risk-neutral simulation of a GBM uses the risk-neutral drift a' = r - d . The price at t is:

$$P_t = P_0 \exp\{ (r - d - 0.5\, s^2)\, D_t + s\, N(0, 1) \}$$

  - **d is the convenience yield of P**
  - With risk-neutral drift, the correct discount rate is the risk-free interest rate.

Mean Reversion Process

- Consider the arithmetic mean reversion process

$$dx = \eta\,(\bar{x} - x)\,dt + \sigma\,dz$$

- The solution is given by the equation with stochastic integral:

$$x(T) = x(0)\, e^{-\eta T} + \bar{x}\,(1 - e^{-\eta T}) + \sigma\, e^{-\eta T} \int_0^T e^{-\eta t}\, dz(t)$$

- Where h is the reversion speed. The variable x(t) has normal distribution with mean and variance given by:

$$E[x(T)] = x(0)\, e^{-\eta T} + \bar{x}\,(1 - e^{-\eta T})$$

$$Var[x(T)] = [1 - e^{-2\eta T}]\,\sigma^2/(2\eta)$$

- We want a mean reversion process for the oil prices P with lognormal distribution with mean E[P(T)] = exp{E[x(T)]}

Risk-Neutral Mean Reversion Process for P

- The risk-neutral process for the variable x(t), considering the AR(1) exact discretization (valid even for large Δt) is:

$$x_t = x_{t-1}\, e^{-\eta \Delta t} + [\,\bar{x} - ((\rho - r)/\eta)]\,(1 - e^{-\eta \Delta t}) + \sigma \sqrt{(1 - \exp(-2\eta \Delta t))/(2\eta)}\; N(0, 1)$$

- The variable x(t) reverts to a long run mean
- Prices reverts to a long run equilibrium level, say $20/bbl
- In order to get the desirable mean is necessary to subtract from x the half of variance Var[x(t)], which is a deterministic function of the time: $P(t) = \exp\{ x(t) - (0.5 * Var[x(t)])\}$
  - This is necessary due the log-normal properties
- Using the previous equation relating P(t) with x(t), we get the risk-neutral mean-reversion sample paths for the oil prices.

Risk-Neutral Simulation vs Real Simulation

- For the underlying asset, you get the same value:
  - Simulating with real drift and discounting with *risk-adjusted discount rate r* = a + d
  - Or simulating with risk-neutral drift (r - d) but discounting with the risk-free rate (r)
- For an option/derivative, the same is not true:
  - Risk-neutral simulation gives the correct option result (discounting with r) but the real simulation does not gives the correct value (discounting with r)
  - Why? Because the risk-adjusted discount rate is "adjusted" to the underlying asset, not to the option
- Risk-neutral valuation is based on the absence of arbitrage, portfolio replication (complete market)

Excerpt from Dias-Rocha (2001)

One practical "market-way" to estimate ρ is taking the net convenience yield (δ) time series (calculated by using futures market data from longest maturity contract with liquidity)[1], together with spot prices series, estimating ρ by using the equation: $\boldsymbol{\rho(t) = \delta(t) + \eta(\overline{P} - P(t))}$. Here δ is just the difference between the discount rate (total required return) and the expected capital gain E(dP/P), like a dividend. The parameter δ is endogenous in our model and, from a market point of view, is used in the sense of Schwartz's (1997b, p.2) description: "*In practice, the convenience yield is the adjustment needed in the drift of the spot price process to properly price existing futures prices*". High oil prices P in general mean high convenience yield δ (positive correlation), and for very low P the net convenience yield can even be negative. There is an offsetting effect in the equation (even though not perfect), so we claim as reasonable the approximation of ρ constant. As compensation, we do not need to assume constant interest rate (because it does not appear in our model) or constant convenience yield (this implicitly changes with P). The time series (P, r, δ) generate the ρ time series. In this way, the value of ρ depends of the assumed values for $\overline{P}$ and η. Using 10-year oil futures data (from July/89 to June/99) and 12-month T-Bond interest rates, we found the time series for both ρ and δ, and the standard deviation of ρ was about the half of the δ, confirming our intuition of more stability for ρ. The simple regression P x δ permits us to estimate "market" values for ρ and η. We found ρ = 9.3% and η = 0.03 (used in the base case). We get the same value ρ = 9.3% p.a. at the equilibrium level \$20/bbl (for P = $\overline{P}$, ρ = δ) using the equation of regression.

[1] The known formula for a commodity futures prices is $F(t) = e^{(r-\delta)t} P$. This equation is deduced by arbitrage and assumes that δ is deterministic, so it looks contradictory with our assumption of systematic jump and with our model that implies that δ is as uncertain as P. But we want an *implicit* value for δ and so for ρ, to get a *market reference* for ρ. It is only a practical "market evaluation" for the discount rate that is assumed constant in our model.

Lectures 7-9. Capital structure

Treatment of Warrants and Convertibles

Warrants and conversion options (in convertible bonds, for instance) are long term call options, but standard option pricing models are based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for warrants and convertibles, since their exercise increases the number of shares outstanding and brings in fresh cash into the firm, both of which will affect the stock price. The expected negative impact (dilution) of exercise will make warrants less valuable than otherwise similar call options. The adjustment for dilution in the Black-Scholes to the stock price involves three steps:

Step 1: The stock price is adjusted for the expected dilution from warrant exercise.
Dilution-adjusted $S = (S\ n_s + W\ n_w) / n_s$
where,
S = Current value of the stock
n_w = Number of warrants outstanding
W = Market value of warrants outstanding
n_s = Number of shares outstanding
When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding.
Step 2: The variance used in the option pricing formula is the variance in the value of the equity in the company (i.e., the value of stocks plus warrants, not just the stocks).
Step 3: The call is valued with these inputs.
Dilution-adjusted value = Call Value from model

Lecture 10. Payout (dividend) policy
Lecture 11. IPOs
Lecture 12. Mergers and acquisitions

Example on the accounting for acquisitions
Assume that there are firms A and B with the current value and number of shares given in the first columns of the table. Together, firms A and B are worth 700. The shareholders of B require half of the synergy gain, i.e., at least 150 for their shares.

	A	B	A* after the merger	A+B after the merger
Market value, $	500	100	550	700
# shares	25	10	25	31.818
Share price, $	20	10	22	22

1. The *purchase method*: A makes a cash offer of 150 for B's shares.
In this case, the combined firm A* is worth 700-150=550 or $22 per share after the merger.

2. *Pooling of interest*: A issues n new shares in exchange for 10 shares of B, such that n shares of the combined firm are worth 150.
Solving the equation 150/700 = n/(25+n), we find n=6.818. Naturally, the share price is the same as in the previous case: $22, as A's shareholders must be indifferent between the two methods given a fixed premium for B's shareholders.

Given the probability of merger q, say, equal to 0.6, one can compute the pre-merger price of A:
Pre-merger $P(A) = 0.6P^*(A) + 0.4P_0(A) = 0.6*20 + 0.4*22 = \$20.8$.

Lecture 13. Corporate governance

Enron & WorldCom

- Энергокомпания Enron завысила прибыль более чем на $1 млрд и скрывала долги на $8 млрд на счетах офшорных структур. В декабре 2001 г. Enron стала крупнейшим банкротом США с активами $63,4 млрд.
- В июле 2002 г. обанкротился телекоммуникационный гигант WorldCom с активами $107 млрд. Масштабы махинаций с бухгалтерской отчетностью в WorldCom достигли $11 млрд.
- В конце прошлой недели было объявлено о двух громких соглашениях, в рамках которых члены советов директоров поступились личными деньгами, чтобы отвести от себя обвинения акционеров. Десять независимых директоров WorldCom согласились в рамках внесудебного соглашения выплатить $18 млн, а 10 директоров Enron — $13 млн.
- С 2002 г. введена персональная ответственность генерального и финансового директоров публичных компаний за достоверность бухгалтерской отчетности. Если они сознательно завизируют поддельную отчетность, им грозит до 20 лет тюрьмы и штраф в $5 млн; неумышленная подпись под недостоверным документом грозит 10 годами и $1 млн.