Comparaison Darcy vs invasion percolation

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Comparison between the different approaches of secondary and tertiary hydrocarbon migration modeling in basin simulators

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Abstract:

Two major techniques are commonly used to model secondary and tertiary hydrocarbon migration: Darcy flow and invasion percolation. These approaches differ from each other in many ways, most notably in the physical modeling, the methods of resolution, and the type of results obtained. The Darcy approach involves not only buoyancy, capillary pressures, and pressure gradient, but also transient physics, thanks to the viscous terms. Although it can be numerically difficult and therefore time consuming, it is appropriate for slow hydrocarbon movement and it is able to provide a good description of cap-rock leakage. The invasion percolation approach, at least in the context of the implementation used in our examples, does not consider either viscosity or permeability; only buoyancy and capillary pressures drive the hydrocarbon migration. This method is relatively quick and especially useful to simulate secondary migration. Nevertheless, the viscous terms cannot be universally neglected as they can impact the timing of trap filling.

Introduction

This paper addresses the modeling of the two main processes that occur at geological time-scales in sedimentary basins, namely secondary and tertiary hydrocarbon migration. Before elaborating on the scope of our study, let us briefly recall the nature of these phenomena in the context of the limited physical properties taken into account in basin modeling.

Secondary migration is the movement of hydrocarbons along a carrier bed from the source rock to the trap. As shown by Schowalter (1979) and England et al. (1987), it can be accounted for by three physical mechanisms.

The first and main driving process is buoyancy. "When two immiscible fluids (hydrocarbon and water) occur in a rock, a buoyant force is created owing to the density difference between the hydrocarbon phase and the water phase. The greater the density difference, the greater the buoyant force for a given length hydrocarbon column (always measured vertically)" (Schowalter, 1979, p. 10).

The second process is hydrodynamics. It adds a force that may be in any direction, depending on the nature of the flow involved (England et al., 1987). Indeed, the buoyant force can be reduced or increased when a hydrodynamic condition exists in the subsurface. However, the effects of hydrodynamics are not always of the utmost importance (Carruthers, 1998).

The third process is capillary pressure. This is in fact a resistance effect which controls the hydrocarbon trajectories. The factors that determine its magnitude are the radius of the pore throat of the rock, the hydrocarbon-water interfacial tension and wettability (Schowalter, 1979).

The combination of these three processes leads to the ascent of hydrocarbons through the carrier beds until the capillary pressure is sufficient to offset the effects of the difference of densities and hydrodynamics. Note that we have neglected compaction as a driving force for secondary migration as this is commonly assumed.

Tertiary migration is the leakage of hydrocarbons from traps. It is attributed to capillary leakage, hydraulic leakage and molecular diffusion (Sylta, 2004). Caprock leakage is possible when the driving processes (buoyancy, pressure gradients and molecular diffusion) exceed the resistant factors (capillary entry pressure or permeability) of the confining barrier (Thomas and Clouse, 1995; Burrus, 1997). In a normal pressure accumulation, a caprock reaches its maximum seal capacity when the pressure generated by the hydrocarbon column is equivalent to the capillary entry pressure of the barrier.

For an accumulation in overpressure (i.e., the difference between the fluid pressure and the hydrostatic pressure), the direction and the magnitude of fluid circulations are controlled by the global pressure field and the buoyancy generated by the hydrocarbon column is not the main force. The rate of leakage is then controlled by the permeability, the fluid viscosity and the pressure gradient (Watts, 1987; Schlomer and Krooss, 1997).

Two major techniques are commonly used to model secondary and tertiary hydrocarbon migration: Darcy flow and invasion percolation. These approaches differ from each other in many ways, most notably in the physical modeling, the methods of resolution, and the type of results obtained. This paper aims to summarize, compare and illustrate these two techniques through particular case studies. Its purpose is to highlight the capabilities of the different methods developed and to underline the advantages and drawbacks of each. Although it does not claim to add any insight into the physics and mechanics of hydrocarbon migration itself, we believe that such a comparison can help the practitioners who use migration modeling.

This paper is outlined as follows. First, we describe the Darcy approach, its physical principles, some standard numerical methods of resolution, and their limitations in Section 2. Section 3 is devoted to the invasion percolation approach, its algorithm and limitations. Then, we recapitulate the characteristics of each approach in Section 4. Finally, we illustrate their main differences through examples in Section 5.

Darcy approach

Darcy flow models assume that hydrocarbon displacement honors the Darcy law extended to multiphase fluids (Bear, 1972; Marle, 1972). Migration is driven by buoyancy, fluid pressure field and capillary pressure. Darcy migration is simulated by solving partial differential equations and the numerical treatment of the full set of equations is generally considered computationally costly and quite complicated (Schneider, 2003).

Physical principles

Based on the results of experiments on the water flow through beds of sand, Darcy (1856) formulated the law

$$\vec{U} = -\frac{K}{\mu}\nabla P$$

where,

U is the Darcy velocity (m/s), *P* is the pressure (Pa),

K is the permeability of the rock (m2), μ is the viscosity of the Newtonian fluid (Pa.s).

From the theoretical viewpoint, it has been proved that Darcy's law is not a constitutive law but a simplified form of the homogenized Navier-Stokes model (Hubbert, 1956; Irmay, 1958; Bear, 1972; Whitaker, 1986). The coefficient $\frac{K}{\mu}$ is a viscous term due to friction at the solid-fluid interface.

Moreover, in order to generalize Darcy's law to multiphase flow, the simplest approach is to assume that each fluid phase maintains a network of passages; the wetting fluid in the larger pores, with friction between fluid and solid (Bear, 1972).

In addition to the three main processes already mentioned (buoyancy, capillary forces and pressure gradient), the extension of Darcy's law to multiphase flow in porous media uses the concept of relative permeability. For two phases, this permeability correction term reflects the permeability reduction of a fluid flow caused by the presence of the second fluid in the porous medium (Guérillot and Kalaydjian, 1988). Then, the extended Darcy law reads

$$\begin{cases} \overrightarrow{U_{w}} = -\frac{KKr_{w}}{\mu_{w}} \left(\nabla P_{w} - \rho_{w} \overrightarrow{g} \right) \\ \overrightarrow{U_{h}} = -\frac{\overline{KKr_{h}}}{\mu_{h}} \left(\nabla (P_{w} + Pc) - \rho_{h} \overrightarrow{g} \right) \end{cases}$$

where,

 $\overrightarrow{U_{\alpha}}$ is the Darcy velocity of the phase α (m/s),

 μ_{α} is the viscosity of the phase α (Pa.s),

 \overline{K} is the intrinsic permeability tensor of the porous media (m2), Kr_{α} is the relative permeability,

Pc is the capillary pressure (Pa),

 P_{w} is the pore pressure in the water phase (Pa),

 \vec{g} is the gravitational acceleration vector (m/s2),

 ρ_{α} is the density of the phase α (kg/m3),

w refers to water phase and h to hydrocarbon phase.

The generalized Darcy law can be adequately applied to basin modeling if we accept that hydrocarbon migration occurs as a separate fluid flow, in a different phase from water, for both primary and secondary migration (England et al., 1987; Durand, 1988; Ungerer et al., 1990; Burrus, 1997).

Numerical modeling

Darcy model is classically coupled with a pressure-compaction model. This means not only that hydrocarbon migration depends on the pressure-compaction computation, but also that the pressure-compaction is influenced by the migration computation. Basin modeling simulators usually simultaneously solve the multiphase Darcy law, the mass-conservation equations for solid and fluids, and a compaction law. To solve this set of equations, finite difference, finite element or finite volume methods are used for the spatial discretization. Various time-schemes are also employed for the transport equations: the Impes with an implicit treatment for the pressure computation and an explicit one for all other unknowns; the Impims based on an implicit treatment for all the unknowns. These two time strategies solve sequentially in two separate stages the pressure-compaction problem and the hydrocarbon transport equations. On the contrary, with the Fully Implicit scheme, we have to solve a coupled system of non-linear equations for pressure and hydrocarbon saturation.

All of these schemes have distinct advantages and limitations (Wolf et al., 2011), but in all the cases, performing a simulation with a complete Darcy model is expensive in computing time. Indeed, to treat the non-linearity of the equations, a classical Newtonian scheme is used. The convergence of this scheme may be a delicate issue in some cases and may cause the time-step to decrease, particularly when a huge amount of hydrocarbon migrates rapidly. At each Newton iteration, solving the linear system represents a huge CPU-time-consuming part of the simulation (Willien et al., 2009). Furthermore, pressure dependent flow can significantly increase the computing time, especially in highly permeable layers. The management of computing time steps depends also on the strong heterogeneities of the fluid properties. Nevertheless, parallel techniques and specific preconditioners can improve the computing time for the Darcy approach (Requena et al., 2005).

Limitations

It is a classical fact that Darcy's law can be considered as valid only for slow Newtonian flows, i.e., for Reynolds numbers between 1 and 10 (Bear, 1972; Burrus, 1997). It is also well-known that Darcy's law breaks down in extremely fine-grained clayey soils. The multiphase nature of the flow is likely to further restrict the validity of the Darcy model. Indeed, in two-phase systems, when the capillary term becomes important at "relatively low pressures, no continuous pathway through the rock is possible, and no flow will occur [...] this type of non-linear behaviour is obviously inconsistent with Darcy's law" (England et al., 1987, p. 335).

In an attempt to reduce the computation cost of Darcy approach, it is often suggested to use large cells. This requires permeability, relative permeability and capillary properties to be upscaled on a low-resolution numerical mesh. However, "the use of constant oil saturation in each computing cell results in too large average saturations being modelled when the vertical migration pathway has to overcome tight zones" (Sylta, 2004, chap. 10, p. 13). Due to this low resolution, Darcy approach tends to overestimate migration losses during secondary migration.

Invasion percolation approach

Percolation theory

The percolation method mathematically deals with disordered media, in which the disorder is defined by a random variation in the degree of connectivity. It can be used in different domains in physics, chemistry and materials science. Percolation theory is applied to porous media and deals with the description of interconnections of the porous and fractured network (Lenormand, 1981; Guéguen and Dienes, 1989). A regular network of "sites" or "bonds", that may or may not be occupied, represents physical properties (permeability, elastic properties...). Each site (or bond) contains the studied physical property which is characterized by a probability of occupation. A "cluster" is defined if several neighboring sites are occupied. These modes of representation of the fractured and porous medium can be used to compute a critical property of percolation (percolation threshold) (Sausse, 1998).

Wilkinson and Willemsen (1983) proposed a new form of percolation theory: invasion percolation. They looked at a wetting fluid (water), the invader, moving another non-wetting fluid (oil) in a porous medium under the action of capillary forces. Then, Wilkinson (1984) extended this model by adding the effects of buoyancy, which is very important for secondary migration modeling. Invasion percolation models assume that viscous effects can be neglected compared with those of capillary pressure and that the system is in a state of capillary equilibrium. Meakin et al. (2000) used this model in experiments and simulations for secondary migration modeling. Their model included the displacement between fluids in a fractured medium and the effects of wetting fluid flow under the influence of the gradient of hydraulic potential.

Invasion percolation algorithm adapted to basin scale

The traditional invasion percolation model assumes that the invading phase is in constant pressure communication and not only in the hydrocarbon accumulations. Carruthers (2003) states that "it is only applicable to small (submeter) systems", and not suitable for basin scale with several kilometers between the source rock and the reservoir zone. Moreover, "it assumes that the invading phase originates from a single point", which is not appropriate for a petroleum system containing several source points (Carruthers, 2003, p. 30).

Carruthers (1998) adapted the traditional invasion percolation algorithm for petroleum migration modeling. This approach of invasion percolation assumes that at basin time scale, hydrocarbons move only under the effects of buoyancy and capillary pressure, which opposes the movement. "Discontinuities within the oil phase are assumed to be ubiquitous except in accumulation zones. The buoyancy force generated as the result of the pressure head generated in an accumulation, is the only buoyancy force which will drive the oil through a carrier past its equilibrium phase pressure, and migration will always occur in a state of equilibrium" (Carruthers, 1998, p. 185).

For a vertical flow, the relationship governing the invasion percolation migration model is $(\rho_h - \rho_w)g\nabla z > \nabla Pc$

where z is the depth below sea level.

This model does not take into account viscous terms, which are negligible with respect to the capillary terms. Due to inviscid assumptions, the flow is steady-state, so transience is imposed by the rate of hydrocarbon generation from the source rock. Under these conditions, this invasion percolation model can be seen as a limit of the Darcy model under local equilibrium assumptions. In this modified form, petroleum migrates under buoyancy into an opposing network of cells populated with capillary pressures, pursuing the lowest entry pressure pathway. When this process proceeds, a migration backbone results, i.e., a set of pores through which an oil stringer is able to flow (Carruthers and Ringrose, 1998), and all subsequent migration occurs along the backbone. When a migration backbone reaches a barrier, a hydrocarbon column builds up. Then, the percolation finds a new pathway with the lowest entry pressure (Burley et al., 2000; Sylta, 2004). Figure 1 illustrates schematically hydrocarbon migration based on the invasion percolation model.



Figure 1: Schematic representation of the hydrocarbon migration route in the case of invasion percolation.

During the search of a new migration path, the invasion percolation approach is akin to graph exploration techniques. It is a sequential computation and does not use an iterative algorithm. This method requires a precise distribution of capillary pressures in the basin and of their evolution through time. It can be very efficient in term of CPU-time and the cost of the simulation depends only on the size of the area swept by the migration paths and not on the total amount of cells in the model. There is only a small set of input parameters in the model so they can be easily modified to perform a sensitivity analysis.

Limitations

As secondary hydrocarbon migration in permeable areas occurs along thin stringers, the modified invasion percolation approach, which results in a migration backbone, provides a good description of this process. Nevertheless, Sylta (2004) explains that for a low caprock permeability (less than 10-2 mD), a typical leakage flow rate is high. The leakage in a such situation occurs on a wide area and not through a narrow migration backbone (Hantschel and Kauerauf, 2009). That is why "the method of percolation modeling does not provide a valid description of the caprock leakage process if the caprock permeability is very low" (Sylta, 2004, chap. 10, p. 11). On the contrary, with Darcy flow, when the rate of hydrocarbon migration into a trap is high compared to the leak capacity of the caprock, the hydrocarbon column increases in the accumulation and the filling of the trap reaches its maximum level. This induces a leak in a wider area in order to have an equilibrium in the hydrocarbon accumulation. "The Darcy method will distribute hydrocarbons at low-saturation within a relatively broad migration "chimney" above the trap, while the percolation will only saturate a very thin migration backbone" (Sylta, 2004, chap. 10, p. 5). This narrow migration backbone through a caprock sequence can, in some cases, miss small sand lens. In systems where filling rates are high, it can be problematic because hydrocarbon losses could be overestimated. Figure 2 shows that with Darcy approach, as the caprock leakage covers a wide area, oil can reach zones beyond the top-point of the structure, and in particular, isolated sand lenses. On the contrary, invasion percolation modeling can bypass sand units because they are not on its migration backbone. This limitation is due to the inviscid assumption carried in the modified invasion percolation approach. Under this assumption, the Darcy model should also degenerate and result in a thin migration backbone through caprock.



Figure 2: Hydrocarbon leakage in caprock containing sand lenses (after Sylta, 2004).

Summary of the comparison between the different approaches

Based on the description of models and our practical experience in basin modeling, we have compared the two approaches. For the invasion percolation migration model, we consider the modified form introduced by Carruthers (1998), which does not take into account viscosity and permeability. This section is a summary of the previous ones in order to highlight the advantages and the drawbacks of each of them. For this, we focused on their differences in three aspects: the modeling, the methods of resolution and the outputs obtained after a standard simulation with each kind of model.

Modeling

Darcy migration is a non-stationary model contrary to the invasion percolation approach, which assumes that the viscous terms are negligible with respect to the capillary terms so that the petroleum system can be considered at each time in a quasi-static equilibrium state. The Darcy model is well suited to follow transient flow in low permeable areas, which is not the case for the invasion percolation. Darcy approach is able to give a good description of hydrocarbon leakage through mudrock sequences, whereas the invasion percolation sometimes gives an incorrect description of hydrocarbon leakage out of traps and misses small accumulations, but gives a good simulation of secondary processes.

Methods of resolution

Performing a simulation with a multi-component and multi-phase Darcy model is expensive in computing time due to the complex system of non-linear partial differential equations that must be solved, but parallel techniques can help to improve the performance. On the contrary, invasion percolation is a sequential computation that does not use any iterative algorithm and has a fast computing time.

Results

After a simulation using the Darcy approach, values of pressure, saturation and hydrocarbon composition are obtained in each cell of a 3D block, but post-processing is needed to identify areas of hydrocarbon accumulation. On the contrary, at the end of an invasion percolation simulation we obtain areas of pathway and hydrocarbon accumulations that are well identified. Moreover, due to computing time with the Darcy model, only a limited number of migration scenarios is usually tested. Unlike this approach, invasion percolation allows testing many different migration scenarios by changing the input parameters.

Examples

The objective of the following examples is to focus on the impact of the above-mentioned differences in the migration methods through 2D synthetic cases and 2D sections from a real case study. The reason why we have chosen well controlled 2D cases rather than 3D blocks is that the former are easier to analyze and to understand.

As the models share common physical principles, we obtain similar results for several petroleum system simulations, but in some special cases differences are magnified. For that purpose, we developed a prototype which ensures the same input data for the computational domain, the initial and the boundary conditions. It also guarantees the same computation of geometry, thermal history and hydrocarbon generation.

For the Darcy model, the prototype simultaneously solves the mass-conservation equations for solid and fluids, a compaction law and the generalized Darcy equations for two-phase flow. It uses finite volume methods and a fully implicit scheme for the transport equations. Pressures and hydrocarbon saturations are strongly coupled.

For the invasion percolation model, the prototype computes water pressures and porosities using Darcy's law for the single water phase and afterwards an invasion percolation algorithm is performed for hydrocarbon migration. The computation of pressures is decoupled from the hydrocarbon migration computation. As a consequence, this methodology allows us to observe the effects of the coupling between the pressure-compaction model and the hydrocarbon migration.

In the upcoming sections, we are not interested in the computation times, but prefer to qualitatively compare the results for the locations of accumulations and pathways taken by the hydrocarbons. The capillary pressure model and the permeability, viscosity computations are detailed in Appendix 1. The actual parameters used in each example are enumerated in Appendix 2.

Secondary migration

Synthetic case: a carrier bed with a low slope

This first synthetic example is a geological section consisting of a carrier bed with a low slope and containing 200 grid cells (Figure 3). At -5 Ma, the trap is not yet formed, but the source rock is mature (the transformation ratio is close to 1) and the hydrocarbons begin migration. Then, at -2.5 Ma, the structural trap is formed.



Figure 3: First synthetic case, a carrier bed with low slope. Description of lithology, structural evolution and source rock transformation ratio as a function of time (ages: -5 Ma, -2.5 Ma, 0 Ma).

We carried out two Darcy simulations by changing the permeabilities in the carrier bed to see their sensitivity on the time of trap filling. Using standard permeabilities for a carrier bed, the migration is not very fast. Hydrocarbons go to the surface but not the total amount expelled from the source rock. Hydrocarbons remaining in the carrier bed can fill the trap. At present day, we observe an accumulation of hydrocarbons (Figure 4).

Using higher permeabilities, even though the slope is low, all of the hydrocarbons go to the surface before trap formation. There is no present day hydrocarbon accumulation (Figure 5).

With invasion percolation, the permeabilities have no effect because only the capillary pressures are able to provide resistance to hydrocarbon migration. Because hydrocarbon generation takes place before the trap formation, all of the hydrocarbons go to the surface. No present-day accumulation is observed (Figure 6).



Figure 4: First synthetic case, a carrier bed with low slope. Evolution of hydrocarbon saturation obtained with Darcy migration model (ages: -5 Ma, -2.5 Ma, 0 Ma).







Figure 6: First synthetic case, a carrier bed with low slope. Evolution of hydrocarbon saturation obtained with invasion percolation migration model (ages: -5 Ma, -2.5 Ma, 0 Ma).

With Darcy approach, which is transient because it solves the viscous term, the filling history is controlled by permeabilities and the slope of the migration pathways, whereas with invasion percolation only the rate of hydrocarbon generation has an impact on trap filling. In conclusion, the viscous effect cannot be universally neglected.

Real case study: long distance pathways

This first real case study comes from Africa. It corresponds to a Palaeozoic and Early Mesozoic depression with a thick sedimentary series. This intracratonic basin is characterized by a major Late Palaeozoic unconformity with most of the known hydrocarbon accumulations located in the overlying Triassic reservoirs. The source rocks are Palaeozoic in age and range from thermally mature at the border of the basin to overmature in the central part. Maturation is controlled by the source rock thermal histories, characterized by a first phase of deepening, followed by an important uplift and a restart of the sedimentation, which lead the source rocks to their today maturity levels. Therefore, two phases of expulsion and migration took place associated with long-distance migration along the unconformity.

Figure 7 shows an enlarged 2D section of the studied area. The lower left part of this section is composed of two Upper Palaeozoic source rock layers with a thin overlying carrier bed covered by a caprock. Its upper right part contains three layers of Middle Palaeozoic source rocks. High permeable layers are located above and below these sources. The traps, located on the right, are not represented.



Figure 7: First real case, long distance pathways. Description of lithology, capillary pressures and vertical permeabilities.

Hydrocarbons expelled from the left lower part of Figure 7 go into the carrier bed. With the Darcy model, all of the hydrocarbons do not go instantaneously into the structural traps, but they are distributed along the carrier bed because of low permeabilities. With the invasion percolation model, the hydrocarbons go directly into the traps as soon as they are generated (Figure 8).



Figure 8: First real case, long distance pathways. Comparison between the hydrocarbon saturation obtained with Darcy migration model and those using invasion percolation.

We focus on the history of a cell located in the carrier bed. With Darcy, the hydrocarbon saturation of this cell fluctuates over time. These changes in time depend on the fluid expulsion rate from the source rock and the geometry variation due to compaction or tectonic movement. With invasion percolation, this cell is identified as a migration pathway, so as soon as the hydrocarbon saturation of the cell has reached the critical hydrocarbon saturation, it does not vary anymore.

In the upper right part of the section, hydrocarbons, expelled from the Middle Palaeozoic source rocks, go above and below these layers with Darcy and accumulate under the source rocks. However, with the invasion percolation model, hydrocarbon migration is mainly driven by buoyancy so preferentially upward and there is no accumulation under the source rocks.

Caprock leakage

Synthetic case: sand lens

This synthetic example is a geological section composed of a structural trap. Figure 9 depicts this trap under a caprock that contains a sand lens. Hydrocarbons are expelled from the source rocks and then migrate vertically until they reach the first caprock. They form an accumulation under this barrier before caprock leakage. We focussed on the behaviour of this leakage. For that purpose, we use three different grid resolutions and for each we compare the results obtained using the Darcy and the invasion percolation models. The parameters of capillary pressures, permeabilities and the other data for the geological section are identical for all of the grid resolutions. The first grid resolution is coarse and contains 320 cells. As explained in Section 3.3, with the Darcy model, we have caprock leakage through a wide "chimney" and hydrocarbons can migrate into the sand lens and form an accumulation. With invasion percolation, the leakage follows a pathway starting from the highest point of the structure. Hydrocarbons do not migrate into the sand unit because the lens is not on this pathway (Figure 9).

The second and the third grid resolutions are finer and contain 1280 and 2560 cells respectively (Figure 10 and Figure 11). We observe the same phenomenon as that obtained using the coarse grid. The invasion percolation pathway is increasingly narrow with the resolution and misses the sand unit, contrary to the broad leakage for the Darcy model, which leads to hydrocarbon accumulation in the sand lens.



Figure 9: Second synthetic case, a sand lens, low grid resolution. Comparison between the hydrocarbon saturation obtained with Darcy migration model and those using invasion percolation.



Figure 10: Second synthetic case, a sand lens, medium grid resolution. Comparison between the hydrocarbon saturation obtained with Darcy migration model and those using invasion percolation.



Figure 11: Second synthetic case, a sand lens, high grid resolution. Comparison between the hydrocarbon saturation obtained with Darcy migration model and those using invasion percolation.

In conclusion, the Darcy model better described the process of caprock leakage as a whole, whatever the grid resolution. Furthermore, with the invasion percolation model, bypassing of the lens at all three grid resolutions results in a shallower accumulation under the second caprock and an underestimate of hydrocarbon losses.

Real case study: pressure-migration coupling

This second 2D section comes from the real case study described in the section on long-distance pathways and is also enlarged. It contains five Middle Palaeozoic source rock layers. Our zone of interest is an anticline located above the source rocks and under a salt layer. It is composed of three groups of layers: the first layer contains three argillaceous sand units (the main reservoir area), the second layer is composed of shale and the third layer contains sandstones and shales (Figure 12).



Figure 12: Second real case, pressure-migration coupling. Description of lithology, capillary pressures and vertical permeabilities.

We want to study the sensitivity of the coupling between pressure and oil saturation. To this end, Figure 13 compares the pressures obtained using the Fully Implicit Darcy method, which couples the pressure porosity computation with the oil saturation computation, with the invasion percolation method. Figure 14 displays the results of hydrocarbon saturation obtained using these two models.

The pressure in the lower accumulation, localized in the three argillaceous sand units, is higher with the Darcy model than with the invasion percolation model (Figure 13). This pressure difference of 3 MPa leads to leakage for the Darcy model, but not for the invasion percolation model (Figure 14).



Figure 13: Second real case, pressure-migration coupling. Comparison between the pressures obtained with a pressure-migration coupled model and without coupling. In the reservoir zone, the pressure is equal to 42 MPa with the coupled model and to 39 MPa without coupling.



Figure 14: Second real case, pressure-migration coupling. Comparison between the hydrocarbon saturation obtained with Darcy migration model and invasion percolation.

In conclusion, the pressure-migration coupling in the Darcy model induces leakage which is not captured at all by the invasion percolation model.

Conclusion

As stated in the introduction, the aims of this paper were (1) to compare the capabilities of the Darcy and invasion percolation methods to model secondary and tertiary hydrocarbon migration, and (2) to illustrate the main differences through examples. From our investigation of selected case studies, the following features emerge.

The Darcy approach takes into account all of the relevant physical processes because it involves not only buoyancy, capillary forces and pressure gradient, but also transient physics thanks to the viscous terms. Although it can be numerically difficult and therefore time-consuming, especially in case of fast movement of fluids, it is appropriate for slow hydrocarbon movement through, for instance, mudrock sequences or a low angle slope where buoyancy is not very strong. Moreover, it is able to provide a good description of caprock leakage due to, among other things, pressure-migration coupling, which is present in our studied Darcy model.

The invasion percolation approach, at least in the context of the implementation used for this paper, does not take into account either viscosity or permeability; only buoyancy and capillary pressures drive the hydrocarbon migration. These two processes allow us to manage the pathways and the accumulation areas, but not the timing of trap filling, which is only imposed by the rate of hydrocarbon generation from the source rocks. The invasion percolation method is relatively quick and especially useful to simulate secondary migration in continuous, high permeability migration pathways. Nonetheless, in certain cases, it may be inappropriate for modeling hydrocarbon leakage out of traps. Furthermore, we must keep in mind that the viscous terms cannot be universally neglected from the equations as they can impact hydrocarbon system dynamics and the saturated bulk rock volume in fine-grained rocks.

These different approaches may be amended by improving some of the observed limitations, such as inclusion of viscosity in the invasion percolation method (Carruthers and de Lind van Wijngaarden, 2000).

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APPENDIX 1

Capillary pressure model

For all of the case studies, we used the following capillary pressure model depending only on porosity φ .

$$Pc(\varphi) = Pc_0 + (Pc_{\lim} - Pc_0) \left[\frac{\varphi_0 - \varphi}{\varphi_0 - \varphi_{\lim}} \right]^{\varphi_0}$$

where,

Pc0 is the capillary pressure at surface or maximum porosity (Pa), Pclim is the capillary entry pressure at maximum burial or minimal porosity (Pa), $\varphi 0$ is the initial porosity when effective stress is equal to zero, φ lim is the minimum porosity when effective stress is infinite, φ PcEx is the curvature of the capillary entry pressure/porosity function.

Permeability computation

The intrinsic permeability tensor \overline{K} is the product of an anisotropy tensor and the intrinsic permeability K

$$\overline{\overline{K}} = K(\varphi) \bullet \begin{bmatrix} Kx & 0 & 0 \\ 0 & Ky & 0 \\ 0 & 0 & Kz \end{bmatrix}$$

The intrinsic permeability is computed using the modified Kozeny-Carman formula:

$$K(\varphi) = \frac{0.2\varphi^3}{S^2(1-\varphi)^2} \quad \text{if } \varphi \ge 0.1$$
$$K(\varphi) = \frac{20\varphi^5}{S^2(1-\varphi)^2} \quad \text{if } \varphi < 0.1$$

where,

S is the specific surface area of the porous medium (m2/m3), Kx is the anisotropy coefficient for the horizontal direction, Kz is the anisotropy coefficient for the vertical direction. Viscosity computation

The water viscosity μ_w is a function of temperature T according to the Bingham formula.

The hydrocarbon viscosity μ_h is a function of temperature T: $\mu_h = \mu_0 \exp\left(\frac{Ak_0}{T}\right)$

where,

 μ_0 is a reference viscosity (Pa.s),

Ak0 is a temperature-dependant viscosity parameter (K).

APPENDIX 2: Detailed data for each example

This appendix uses the notations described in Appendix 1.

First synthetic case: a carrier bed with a low slope

The capillary pressures follow the law: $Pc(\varphi) = Pc_{\lim} \left[\frac{\varphi_0 - \varphi}{\varphi_0 - \varphi_{\lim}} \right]$

For each lithology, we used the parameters given in Table 1.

Lithology	Pc _{lim} (Pa)	φ0	Φlim
shale	1.5×10^{8}	0.702	0.03
overburden rock	1.5×10^{8}	0.434	0.05
fault - carrier	$1.0 \mathrm{x} 10^5$	0.702	0.03
source rock	1.0×10^{6}	0.702	0.03

Table 1: Capillary pressure parameters for the first synthetic case study. Pclim : capillary entry pressure at maximum burial or minimal porosity (Pa), $\varphi 0$: initial porosity when effective stress is equal to zero,

 φ lim : minimum porosity when effective stress is infinite (see Appendix 1).

The water density ρ_w is equal to 1030 kg.m⁻³ and the hydrocarbon density ρ_h is equal to 140 kg.m⁻³.

For all the lithologies, the connate water saturation Swc is equal to 0.9 and the critical hydrocarbon Soc is equal to 0.02.

To compute the hydrocarbon viscosity, we used the parameters: $\mu_0 = 1.45 \times 10^{-5}$ Pa.s and $Ak_0 = 1533.15$ K.

We used a source rock containing type I kerogen.

To compute the permeabilities, for the first kind of simulations and also for the second kind of simulation with Darcy and high permeabilities, we used the parameters given in Table 2.

Lithology	First kind of	Second kind	Kx	Kz
	simulation	of simulation		
	$S(m^{2}/m^{3})$	$S(m^{2}/m^{3})$		
shale –	5×10^{7}	5×10^{7}	1	0.5
overburden rock				
fault - carrier	$2x10^{6}$	1×10^{5}	1	0.001
source rock	5×10^{7}	$5x10^{5}$	1	1

 Table 2: Permeability parameters for the first synthetic case study.

S : specific surface of the porous medium (m2/m3),

Kx : anisotropy coefficient for the horizontal direction,

Kz : anisotropy coefficient for the vertical direction(see Appendix 1).

Real case study

The capillary pressures follow the law: $Pc(\varphi) = Pc_{\lim} \left[\frac{\varphi_0 - \varphi}{\varphi_0 - \varphi_{\lim}} \right]^{\varphi_{PCEx}}$

For each lithology, we used the parameters given in Table 3 and to compute the permeabilities, the parameters are detailed in Table 4. For the salt we used an extremely high capillary pressure and a permeability equal to zero.

Lithology	Pc _{lim} .(Pa)	φPcEx	φ ₀	φlim
marl	3.0×10^{6}	0.5	0.5	0.02
limestone and salt	$1.0 \mathrm{x} 10^{6}$	0.5	0.35	0.02
salt and carbonate	3.0×10^{6}	0.5	0.1556	0.008
limestone	3.0×10^{6}	0.5	0.35	0.02
silt	7.5×10^5	0.5	0.4186	0.02
silty shale	3.0×10^{6}	0.5	0.4639	0.02
sandstones and shale	5.0×10^5	0.5	0.3814	0.02
marl and silt	2.0×10^{6}	0.5	0.4186	0.02
argillaceous sand	$1.0 \mathrm{x} 10^{6}$	0.5	0.4186	0.02
shales	5.0×10^{6}	0.5	0.5	0.02
sand	$5.0 \mathrm{x} 10^5$	1.0	0.36	0.02
sandy shale	3.0×10^{6}	0.5	0.4478	0.02

Table 3: Capillary pressure parameters for the real case study.

Pclim : capillary entry pressure at maximum burial or minimal porosity (Pa),

 ϕ PcEx : curvature of the capillary entry pressure/porosity function,

 $\phi 0$: initial porosity when effective stress is equal to zero,

φlim : minimum porosity when effective stress is infinite (see Appendix 1).

Lithology	$S(m^2/m^3)$	Kx	Kz
marl	5.0×10^7	1	1
limestone and salt	$1.0 \mathrm{x} 10^7$	1	1
salt and carbonate	2.0×10^7	1	1
limestone	$1.0 \mathrm{x} 10^{6}$	1	1
silt	1.0×10^{6}	1	0.2
silty shale	$1.0 \mathrm{x} 10^7$	1	0.5
sandstones and shale	1.0×10^{6}	50	25
marl and silt	5.0×10^7	1	1
argillaceous sand	1.0×10^{6}	5	2
shales	$1.0 \mathrm{x} 10^8$	10	1
sand	$1.0 \mathrm{x} 10^{6}$	200	100
sandy shale	1.0×10^8	500	100

Table 4: Permeability parameters for the real case study.

S : specific surface of the porous medium (m2/m3) (see Appendix 1).