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$x_1(t)$	Stock de produits finis à l'instant t
$x_2(t)$	Stock de récupération à l'instant t
$u_1(t)$	Taux de production de la machine de fabrication à l'instant t (produit/UT)
$u_2(t)$	Taux de production de la machine de rectification/refabrication à l'instant t (produit/UT)
u_{max}^1	Taux maximal de production de la machine de fabrication (produit/UT)
u_{max}^2	Taux maximal de production de la machine de rectification/refabrication (produit/UT)
$\tilde{d}$	Taux de demande aléatoire (produit/UT)
r	Proportion de retours
$a_1(t)$	Age de la machine de fabrication à l'instant t
$\tilde{\beta}(.)$	Taux de rejets aléatoire de la machine de fabrication
p	Proportion de produits défectueux pour l'activité de rectification
n	Nombre de pannes
N	Nombre maximal de pannes
α	Mode des machines
B	Ensembles des modes des machines
$\xi(t)$	Processus aléatoire du système hybride à l'instant t
$\xi_1(t)$	Processus aléatoire de la machine de fabrication à l'instant t
$\xi_2(t)$	Processus aléatoire de la machine de rectification/refabrication à l'instant t
τ	Instant de saut du processus
$q_{\alpha\alpha'}(.)$	Taux de transition du mode α au mode α'
$Q(.)$	Matrice des taux de transition du système hybride
$\pi(.)$	Vecteur des probabilités limites
$\Gamma(.)$	Ensemble des commandes admissibles
$w_\alpha(.)$	Variable de contrôle pour le remplacement de la machine de fabrication en mode α
μ_D	Moyenne du taux de demande

$\mu_\beta(.)$	Moyenne dynamique du taux de rejets
σ_D	Écart type du taux de demande
$\sigma_\beta(.)$	Écart type du taux de rejets
$W_1(.)$	Mouvement Brownien standard pour le taux de demande
$W_2(.)$	Mouvement Brownien standard pour le taux de rejets
$G(.)$	Fonction coût instantané ($/UT)
$J(.)$	Fonction coût moyen actualisé
$\nu(.)$	Fonction valeur
$(\Omega, \mathcal{F}, P)$	Espace de probabilité
ρ	Taux d'actualisation
$h(.)$	Fonction coût de stockages et de pénuries
$c(.)$	Fonction coût de production
c^α	Fonction coût de réparation et de remplacement
c_1^+	Coût unitaire de stockage des produits finis ($/produit/UT)
c_2^+	Coût unitaire de stockage des produits récupérés ($/produit/UT)
c_1^-	Coût unitaire de pénuries des produits finis ($/produit manquant/UT)
c_{P_1}	Coût unitaire de fabrication ($/produit)
c_{P_2}	Coût unitaire de rectification/refabrication ($/produit)
c_{IP}	Coût unitaire d'inspection ($/produit)
c_d	Coût unitaire de rejet des produits défectueux ($/produit)
c_{r1}	Coût unitaire de réparation de la machine de fabrication ($/UT)
c_{r2}	Coût unitaire de réparation de la machine de rectification/refabrication ($/UT)
c_0	Coût de remplacement de la machine de fabrication ($)
UT	Unité de temps
FMSs	Flexible manufacturing systems
EMQ	Economic manufacturing quantity
EOQ	Economic ordering quantity
EOLP	End-of-life products
QVI	Quasi variational inequality
HJB	Hamilton-Jacobi-Bellman

HJBDD	Hamilton-Jacobi-Bellman equation in terms of directional derivatives
SMDP	Semi-Markov decision process
PDEs	Partial differential equations
$\tilde{E}$	Conditional expectation operator
HPP	Hedging point policy
$MTTF$	Mean time to failure
$MTTR$	Mean time to repair
$MTTRP$	Mean time to replacement
ABAO	As bad as old conditions
AGAN	As good as new conditions

INTRODUCTION GÉNÉRALE

Depuis plusieurs décennies, les entreprises manufacturières s'intéressent aux activités de logistique inverse en plus de leurs activités régulières au sein de leurs chaînes d'approvisionnement afin de traiter les retours de leurs produits. Diverses motivations les mènent à cela. L'économie de marché voudrait que l'entreprise offre un produit au meilleur coût avec une qualité supérieure et qu'il soit livré à temps. L'écologie et l'environnement quant à eux leurs imposent de réduire la consommation en ressources non renouvelables (matières premières, énergies) et de diminuer la production des déchets et des émissions de gaz à effet de serre. Du côté de la législation, avec la pression des gouvernements sur elles, les entreprises se voient désormais imposées de nouvelles contraintes et normes. En effet, elles sont maintenant responsables de la récupération et de la mise en valeur de leurs produits en fin de vie. En plus, les systèmes en production manufacturière sont souvent soumis à des phénomènes aléatoires tels que les pannes, les réparations et les dégradations des machines, les activités de maintenance, la détérioration de la qualité des pièces produites, les fluctuations de la demande des clients et du retour des produits à la fin de leur vie.

De nos jours, l'intérêt principal des entreprises manufacturières se porte souvent sur l'optimisation des performances des systèmes de production afin de satisfaire leurs clients tout en faisant un maximum de profits. Cela peut se faire notamment en minimisant les coûts liés à la gestion des diverses ressources de production ou tout autre élément qui interagit pour réaliser les activités de production; que ce soit dans le sens direct ou le sens inverse de la chaîne logistique. De ce fait, pour une gestion efficace, l'industriel doit posséder les outils nécessaires pour entreprendre et développer des stratégies afin d'atteindre ses objectifs; étant donné la complexité du problème d'optimisation dans l'environnement de production manufacturière causé par les phénomènes aléatoires qui perturbent le système. Néanmoins, dans certains cas, les questions de la prise en compte des aspects aléatoires dans les modèles mathématiques sont primordiales pour améliorer les performances des systèmes de production. Malheureusement, de nos jours, malgré les travaux récents dans le domaine de la commande des systèmes de production, il n'existe pas à notre connaissance de modèles qui explorent les problèmes de planification de la fabrication, de rectification ou refabrication des produits et de maintenance des systèmes manufacturiers en boucle fermée en présence d'incertitudes (demande, retour, détérioration de qualité, pannes et réparations des machines). Dans le but de relever les défis de recherche associés à cette problématique, nous proposons des modèles de contrôle innovants qui améliorent les performances des systèmes de production en tenant compte des effets des phénomènes aléatoires et d'autres concepts de détérioration sur la dynamique des systèmes étudiés.

La question qui se pose alors est la suivante. Quelle structure de loi de commande l'entreprise doit-elle adopter afin de déterminer une stratégie optimale hybride du problème de planification de la production (fabrication, rectification et refabrication) et de la maintenance ? Elle doit pour cela gérer efficacement ses ressources matérielles, maîtriser les phénomènes aléatoires qui se superposent au système manufacturier hybride et qui perturbent son fonctionnement dans les conditions normales d'utilisation. De plus, l'approche proposée doit aussi permettre la réduction des coûts totaux encourus reliés aux stockages, aux pénuries, à la production, à la maintenance et aussi à la détérioration de la qualité des pièces produites.

Notre contribution consiste à élaborer une structure de loi de commande en logistique inverse permettant de satisfaire une demande aléatoire avec la prise en compte des phénomènes aléatoires associés à la dynamique des machines, à la détérioration de la qualité des pièces produites et au retour des produits provenant du marché. Nous allons formuler le problème de planification hybride de fabrication, de rectification et de refabrication avec une possibilité de remplacement (pour acquérir une nouvelle machine) comme un problème de commande optimale stochastique et développer les conditions d'optimum décrites par des équations aux dérivées partielles de type HJB (Hamilton-Jacobi-Bellman) modifiées. En présence du caractère aléatoire au niveau de la demande, du retour et de la qualité, ces équations sont développées en second ordre, alors que dans le cas déterministe, elles sont de premier ordre. Généralement, on fait référence au second ordre dans des problèmes de mathématique financière concernant les cours en bourse. Cela reste toutefois un fait innovateur en système manufacturier hybride. Le terme du second ordre conduit à un résultat, connu dans la théorie du calcul stochastique sous le nom de « calcul d'Itô ». Le traitement analytique de ces équations aux dérivées partielles obtenues est beaucoup plus complexe. Nous allons aussi démontrer que ces équations d'HJB modifiées peuvent être résolues numériquement. L'approximation par différences finies et les méthodes itératives permettront de les résoudre. Les résultats obtenus seront confirmés par une analyse de sensibilité afin de vérifier la qualité de la solution proposée et sa robustesse en fonction de la variation de certains paramètres clés du modèle. Ces résultats, permettront aux industries non seulement d'économiser sur l'utilisation des ressources matérielles, mais aussi d'améliorer la disponibilité ou la fiabilité des équipements; ce qui aura un impact direct sur les coûts de production.

Dans le prochain chapitre, nous présentons la problématique de notre recherche et une revue de littérature dans le domaine de la commande optimale stochastique appliquée à la gestion des systèmes manufacturiers œuvrant dans des chaînes de production en boucle fermée.

CHAPITRE 1

PROBLÉMATIQUE ET REVUE DE LITTÉRATURE

1.1 Introduction

La première partie de ce chapitre présente la problématique de recherche de cette thèse. La deuxième partie du chapitre est une revue critique de la littérature sur la commande optimale stochastique des systèmes manufacturiers, de la logistique inverse avec ses différentes activités dans la chaîne logistique ainsi que de l'intégration de la ligne de retour en logistique inverse dans l'environnement de production de base. La troisième partie présente la méthodologie envisagée pour affronter notre problématique de recherche. Le chapitre se termine par les principales contributions et la structure de la thèse.

1.2 Problématique de recherche

Jusqu'à présent, un grand nombre de contributions intégrant la production, la maintenance et la logistique inverse d'un système manufacturier hybride en présence des aspects aléatoires ont été publiées. Toutefois, les efforts qui ont été accomplis pour maîtriser ces aspects ont donné lieu à des modèles incomplets qui traitent souvent les différents aspects de production séparément à cause de leur diversité. Malheureusement, dans les travaux disponibles à ce jour, nous n'avons pas trouvé des modèles qui tenaient en compte conjointement ces aspects afin de résoudre des situations industrielles. Rappelons que ces principaux aspects aléatoires, qui font du processus de prise de décision dans le domaine de commande des systèmes manufacturiers un processus très complexe, peuvent être les suivants :

- la dynamique des machines;
- la demande des clients;
- la nature du retour des produits en fin de cycle de vie en termes de quantité et qualité;
- la détérioration de la qualité des pièces produites;
- la détérioration de la disponibilité des machines.

Dans cette thèse, nous proposons d'amener une contribution vers cette direction. Pour mieux comprendre notre problème de planification de la production et de la maintenance d'un système manufacturier hybride fabrication/rectification/refabrication, nous allons faire un rappel de quelques concepts et mécanismes de base utilisés en contexte de détérioration.

1.2.1 Classification des détériorations en systèmes manufacturiers

La détérioration d'un système manufacturier en fonction des activités de réparation apparaît souvent lorsque les activités de maintenance sont imparfaites; que ce soit le niveau de détérioration de la machine avec l'âge, avec le nombre de pannes, avec la vitesse de production, *etc.* Cela nous amène à définir différents modèles de base de maintenance imparfaite comme c'est indiqué dans Love, Zhang, Zitron et Guo (2000), comme suit :

1. Modèle de réparation minimale (ABAO: As-Bad-As-Old). Dans chaque réparation, il y aura une détérioration cumulative. La machine est remise dans l'état où elle était, juste avant la défaillance. Le processus de panne correspond alors à un processus de poisson non-homogène.
2. Modèle de réparation parfaite (AGAN: As-Good-As-New). Il permet de restaurer l'état de la machine aux conditions initiales, et le processus de panne suit dans ce cas un processus de renouvellement.
3. Modèle de réparation imparfaite. Il permet de restaurer l'état de la machine à un niveau d'âge plus petit que son âge d'avant la défaillance, on parle alors d'âge virtuel. En réalité, il se trouve entre les réparations de type ABAO et AGAN qui sont les deux (2) cas extrêmes de modèles de réparation imparfaite.

Dans un contexte de production manufacturière, des réparations imparfaites sont effectuées pour plusieurs raisons. Une discussion sur ce sujet peut être trouvée dans Pham et Wang (1996) et Wang (2002). Comme raisons principales, on peut citer, les erreurs humaines durant l'intervention de maintenance, la réparation partielle des composants défectueux, la mauvaise qualité des pièces à remplacer et la complexité du problème de maintenance. En général, la maintenance pourrait concerner plusieurs activités comme par exemple, la réparation, le remplacement, la maintenance préventive ou la révision majeure (*major overhaul*).

Les différents mécanismes de détérioration répertoriés dans la littérature peuvent être détaillés comme suit :

1. Mécanisme de détérioration dépendant de l'âge de la machine. La dynamique de l'âge de la machine est une fonction du nombre d'unités de pièces produites ou du temps passé en opération (Dehayem-Nodem, Kenné et Gharbi, 2011). Elle peut être parfois une fonction combinée du taux de production et du taux de maintenance préventive (Boukas et Yang, 1996). Cette détérioration affecte de façon progressive la disponibilité de la machine et peut avoir aussi une influence négative sur la qualité des produits fabriqués. Elle se traduit par la probabilité de panne de la machine qui augmente avec son âge et aussi par un taux de rejets qui augmente avec son âge.
2. Mécanisme de détérioration dépendant du nombre de pannes de la machine. Cette détérioration affecte progressivement la disponibilité de la machine. Cette dépendance se traduit par une augmentation de la probabilité de panne ou par une diminution du taux de réparation (Kouedeu, Kenné, Dejax, Songmene et Polotski, 2015).
3. Mécanisme de détérioration dépendant de la vitesse de production de la machine. Cette détérioration affecte aussi la disponibilité et se traduit par une probabilité de pannes croissante avec le taux production de la machine. Autrement dit, plus la vitesse de production augmente plus souvent la machine tombera en panne (Kouedeu, Kenné, Dejax, Songmene et Polotski, 2014).
4. Mécanisme de détérioration dépendant de la qualité des pièces produites. Cette détérioration peut être représentée par un taux de rejets qui augmente progressivement avec l'âge de la machine ou avec le nombre de pannes (Rivera-Gomez, Gharbi et Kenné, 2013b; Rivera-Gomez, Gharbi, Kenné, Montano-Arango et Hernandez-Gress, 2016). Des exemples de ce type de détérioration se trouvent dans Huang, Chen et Fang (2013) pour le processus de rectification des produits défectueux et dans Jiang *et al.* (2016) pour le processus de refabrication des produits de nature hétérogène provenant du marché.

1.2.2 Structure du système étudié

Lorsque le produit récupéré retourne dans la chaîne initiale d'approvisionnement de la même entreprise manufacturière, la dynamique résultante est celle d'une chaîne d'approvisionnement en boucle fermée. La figure 1.1 adaptée de Govindan, Soleimani et Kannan (2015), illustre une chaîne d'approvisionnement générique en boucle fermée. Dans cette figure, nous nous intéressons aux composantes fondamentales qui sont en gras soit, les matières premières, la fabrication, la rectification/refabrication, le stock de service, le stock de récupération, la demande des clients, le retour des produits utilisés et défectueux, et la destruction.

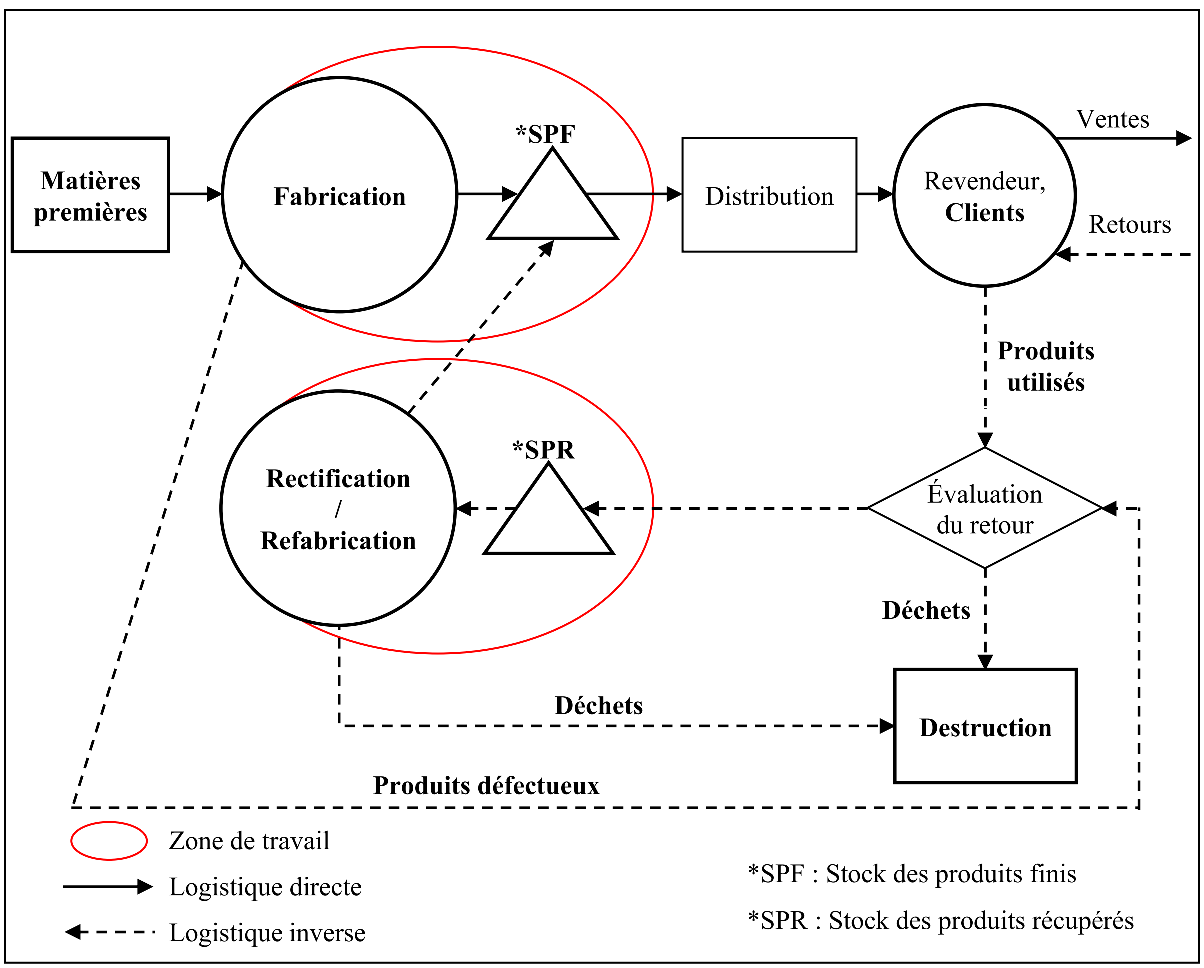

Figure 1.1 Structure générique d'une chaîne d'approvisionnement en boucle fermée adaptée de Govindan *et al.* (2015)

Dans notre zone de travail, nous allons considérer un système manufacturier hybride en boucle fermée composé de deux (2) machines produisant le même type de produit. Ces machines sont souvent sujettes à des pannes et réparations aléatoires. La première machine est utilisée dans le sens direct de la logistique pour réaliser les activités de fabrication. Elle se détériore avec son âge lorsqu'elle est en opération et le phénomène de détérioration de la machine affecte aléatoirement sa disponibilité et la qualité des produits qu'elle fabrique. Cela a pour effet d'augmenter progressivement les taux de panne et de rejets avec l'âge de la machine. La deuxième machine est placée dans le sens inverse de la chaîne logistique pour traiter les activités de rectification et de refabrication des produits. Nous définissons deux (2) stocks. Le premier dit de service contient les produits finis. Il permet de répondre à une demande aléatoire et peut être construit par la fabrication ou la rectification/refabrication. Le deuxième dit de récupération contient les produits en fin de vie récupérés du marché et les produits défectueux. Ces produits sont considérés comme matière première pour la deuxième machine. Ils peuvent être rectifiés, refabriqués ou tenus pour une rectification/refabrication future. Le processus de retour est considéré stochastique, il est représenté par un pourcentage du taux de la demande aléatoire. Les produits fabriqués sont examinés et évalués pour déterminer leurs états. Les produits parfaits sont tenus dans le stock de service, tandis que ceux étant défectueux sont rejetés et transférés dans le stock de récupération. Les produits récupérés en fin de vie sont utilisés pour le processus de refabrication, et ceux de mauvaise qualité peuvent également être réparés par le processus de rectification. Pour un système flexible, les activités de refabrication et de rectification peuvent être faites sur la même machine. Cependant, la détérioration de cette machine provient du traitement du flux de mauvaises pièces. Un tel flux de pièces défectueuses peut détériorer progressivement sa disponibilité. Finalement, les produits rectifiés sont inspectés, les produits de mauvaise qualité sont éliminés tandis que les autres sont considérés parfaits et seront stockés avec les produits fabriqués/refabriqués dans le stock de service pour répondre à la demande. Des activités de maintenance minimales ou imparfaites seront effectuées sur les deux (2) machines durant la panne. En raison des effets sévères de la détérioration sur la disponibilité des machines et la qualité des pièces produites, à long terme, le système manufacturier hybride atteint un certain niveau de détérioration où il devient incapable de satisfaire la demande des clients. Dans ce contexte de détérioration progressive, des activités de maintenance parfaite, aussi connues sous le nom de remplacement, peuvent être effectuées sur le système manufacturier hybride pour restaurer ses paramètres aux conditions initiales.

L'intégration de tous ces éléments dans la modélisation mathématique rend le problème d'optimisation très complexe et dans certains cas non faisable en raison de la complexité au niveau du caractère aléatoire et de la dimension du problème. La question qui se pose est de savoir comment planifier les opérations de fabrication, de rectification/refabrication et de maintenance dans un environnement de production manufacturière caractérisé par des incertitudes et des détériorations pour gérer efficacement les activités du système hybride, de manière à satisfaire la demande du client tout en faisant un maximum de profit ?

1.2.3 Hypothèses de travail

Les principales hypothèses de modélisation considérées et intégrées progressivement dans les chapitres suivants seront :

- les matières premières de même que les produits récupérés sont disponibles en tout temps;
- la demande du client est connue et représentée par un taux aléatoire au fil du temps (un processus stochastique);
- le taux de retour des produits est un pourcentage du taux de demande aléatoire;
- les pannes et réparations des machines sont aléatoires;
- la fonction de la détérioration de la qualité des pièces produites est connue et représentée par un taux de rejets aléatoire qui dépend de l'âge de la machine;
- la détérioration dépendant de l'âge de la machine est une fonction du nombre de pièces fabriquées;
- les activités de réparation minimale sur la machine restaurent son état à l'état d'avant la défaillance (*as-bad-as-old*);

- la détérioration dépendant de la nature hétérogène des produits à réusiner cause une réparation imparfaite dont le temps de réparation augmente progressivement avec le nombre de pannes;
- l'effet de détérioration sur la machine causé par son âge affecte progressivement et de façon aléatoire sa disponibilité et sa qualité des pièces produites;
- le traitement du flux de pièces récupérées de mauvaise qualité affecte progressivement et aussi de façon aléatoire la disponibilité de la machine de rectification;
- la qualité des produits réusinés et rectifiés n'est pas différente de celle des produits fabriqués;
- la refabrication et la rectification des produits peuvent être faites sur la même machine;
- le taux maximal de production de chaque machine est connu;
- le temps de traitement des activités de rectification et de refabrication est différent selon la nature des pièces traitées;
- une deuxième refabrication n'est pas autorisée pour les produits retournés du marché et qui ont déjà été réusinés. Cette option peut être vérifiée dans les centres de tri pour déterminer quels produits ne peuvent pas être réusinés;
- les produits rectifiés qui restent défectueux après inspection, seront éliminés et une deuxième rectification n'est pas permise;
- le coût de stockage des produits récupérés est inférieur au coût de stockage des produits finis.

1.2.4 Objectifs de recherche

Il est devenu primordial pour la survie d'une entreprise manufacturière d'être compétitive face à ses concurrents. Elle doit pour cela satisfaire les exigences du client qui sont de plus en plus pointilleuses. Les clients veulent des produits abordables, de bonne qualité et livrés dans les meilleurs délais. L'entreprise doit aussi répondre aux contraintes environnementales, écologiques et légales, et aux consommateurs qui désirent des produits sains pour l'environnement. Pour ces raisons, les entreprises doivent assurer et structurer la logistique de récupération de leurs produits. Afin de gérer efficacement les performances globales du système de production, il est nécessaire de contrôler le système manufacturier en boucle fermée. Ici, l'interaction de la logistique inverse dans l'environnement de production de base devient de plus en plus importante. Elle a pour but de réusiner les produits en fin de vie, de réparer les produits défectueux ou bien de les éliminer s'ils restent encore défectueux. Nous devons donc prendre en considération ce paramètre nouveau qui est en réalité aléatoire pour l'intégrer à la gestion de planification de sa production. En plus, nous allons tenir compte des caractères stochastiques du système manufacturier, imposés par les défaillances, les réparations et les effets de détérioration sur les machines en termes de disponibilité et qualité. Ces phénomènes aléatoires affectent le contrôle de la production et le processus de recherche de la décision optimale doit en tenir compte.

L'objectif général de cette thèse est la planification des activités de production et de maintenance d'un système manufacturier en boucle fermée en considérant les aspects de demande aléatoire, retour aléatoire, qualité aléatoire et les pannes et réparations des machines. L'objectif étant de permettre à l'entreprise de développer des stratégies optimales dans un contexte dynamique stochastique orienté vers la minimisation de ses coûts en récupérant les retours des produits usagés et défectueux à travers la logistique inverse. Pour ce faire, nous devons :

1. Intégrer progressivement dans la modélisation mathématique les pannes et les réparations aléatoires des machines, la demande et les retours aléatoires, les détériorations des machines et de la qualité des produits fabriqués ainsi que les activités de maintenance dans le but d'étudier leur impact sur la structure de la loi de commande;
2. Développer des méthodes de résolution de problèmes d'optimisation considérés;
3. Analyser les performances (exemple la minimisation des coûts liés à la production, aux stockages, aux ruptures de stocks et à la maintenance) des systèmes soumis aux stratégies obtenues;

4. Proposer des stratégies de contrôle de la production et de la maintenance dans le but de maîtriser les aléas afin de gérer efficacement les produits récupérés et les stocks en répondant aux exigences du client.

1.3 Revue critique de la littérature

Dans cette section, nous nous proposons de faire une revue critique de la littérature qui touche les différents sujets liés à la problématique de recherche abordée dans le paragraphe précédent. Pour ce faire, nous allons présenter les travaux sur :

- la commande optimale stochastique des systèmes manufacturiers;
- les demandes et qualités aléatoires;
- les chaînes d'approvisionnement en boucle fermée.

1.3.1 Commande optimale stochastique des systèmes manufacturiers

Dans le domaine de la planification de la production des systèmes manufacturiers, plusieurs politiques de commande optimale stochastique ont été développées afin d'optimiser selon un certain critère de performance le taux de production des machines perturbées par des phénomènes aléatoires. Les deux (2) outils principaux de la théorie de commande optimale traitant des processus évoluant dans un environnement stochastique sont le principe du maximum de Pontryagin (Pontryagin, Boltyanskii, Gamkrelidze et Mishchenko, 1962; Seierstad et Sydsaeter, 1987) et les équations de Hamilton-Jacobi-Bellman (HJB) obtenues en appliquant la programmation dynamique stochastique.

En se basant sur le dernier outil, la programmation dynamique stochastique, le formalisme de Rishel (1975) a permis de développer les conditions d'optimum (nécessaires et suffisantes) pour trouver la solution optimale du problème de commande stochastique des systèmes dont leur dynamique est modélisée par un processus Markovien homogène à états finis. Beaucoup de travaux de recherche considèrent que les phénomènes aléatoires qui perturbent le système de production suivent des processus Markoviens homogènes dont le temps de séjour dans un état suit une loi exponentielle pour le cas continu (ou une loi géométrique pour le cas discret). Par exemple, lorsqu'une machine tombe en panne, elle est remise à l'état neuf ce qui lui permet de produire à sa capacité maximale de production sans subir une certaine dégradation. Ces modèles ne représentent pas toujours la nature de la réalité industrielle. Certaines sources de phénomènes aléatoires telles que les pannes, les réparations et les activités de maintenance sont souvent modélisées par des processus semi-Markoviens en raison de la présence de détériorations. Ces détériorations affectent principalement et de façon progressive différents aspects de la machine, tels que la disponibilité, la qualité des pièces produites, *etc.* Le but des processus semi-Markoviens étant de modéliser une variété d'expériences dont le temps de séjour dans un état peut suivre une loi aléatoire quelconque. Dans un tel contexte, Boukas et Haurie (1990), Kenné et Gharbi (1999) et Boukas et Yang (1996) ont considéré que la probabilité des pannes des machines augmente avec son âge. Les deux (2) premières références citées disent que la dynamique de l'âge est une fonction du taux de production. Toutefois, Boukas et Yang (1996) considèrent que cette dynamique est liée à des taux de production et de maintenance préventive. Ils ont considéré qu'après chaque maintenance corrective ou préventive, l'âge de la machine est restauré à zéro (*as-good-as-new*). Ils ont pris en compte ce paramètre dans leur modélisation et ils ont développé la solution du problème de commande stochastique afin de déterminer la politique optimale de production. Boukas et Yang (1996) ont montré que lorsque la dynamique de l'âge dépend à la fois des taux de production et de maintenance préventive, la solution optimale est de type surface critique (*hedging surface*).

Les modèles mathématiques précédents ont été reformulés pour tenir compte de la détérioration progressive sur la machine de fabrication causée par une série d'événements tels que les pannes et réparations, l'usure des machines, les erreurs humaines, *etc.* Certains auteurs (Love, Zitron et Zhang, 1998) et (Love *et al.*, 2000) ont étudié des systèmes manufacturiers sujets à une détérioration progressive et ont déterminé les conditions optimales de réparation versus remplacement. Dans la même direction, Dehayem-Nodem *et al.* (2011) ont étendu le modèle de Love *et al.* (2000) pour intégrer la production. Ils ont proposé une politique hybride en fonction de l'âge et du nombre de pannes cumulées pour répondre à un taux de demande constant. Dans tous ces travaux, l'effet de détérioration est supposé affecter uniquement la disponibilité de la machine, tout en négligeant son effet sur la qualité des pièces produites. Pour rendre ces travaux plus réalistes, le modèle de détérioration de la machine doit également tenir compte de l'aspect de la qualité.

La prise en compte des aspects de qualité dans les problèmes de production n'a commencé à croître qu'avec la série de travaux de Kim et Gershwin (2005, 2008), qui ont développé des modèles mathématiques pour évaluer la performance des systèmes de production en considérant la qualité des pièces produites. Dans la même direction, Colledani et Tolio (2009, 2011) ont abordé l'évaluation de la performance des systèmes de production en utilisant des tableaux de contrôle statistique comme outils de contrôle de la qualité pour surveiller le comportement des machines. Ces travaux visent à étudier l'influence des aspects de qualité sur la dynamique des machines. Ils mettent l'accent sur l'analyse des mesures de performance. Une étude de l'impact des aspects de qualité sur la structure de la politique de production a été proposée dans plusieurs travaux. Radhoui, Rezg et Chelbi (2010) ont développé un modèle mathématique pour un système composé d'une machine non fiable produisant des lots de produits pour satisfaire un taux de demande constant. Chaque lot produit par la machine est soumis à un contrôle de qualité. Ils ont utilisé la proportion des pièces non conformes comme une variable décision pour définir le type d'action de maintenance à effectuer sur le système. Dans le même contexte, Bouslah, Gharbi et Pellerin (2016) ont proposé une nouvelle approche traitant le problème d'inspection de la production défectueuse pour un système de production dans un environnement dynamique stochastique soumis aux détériorations de la fiabilité et de la qualité. Les auteurs ont proposé un plan d'échantillonnage d'acceptation pour effectuer le contrôle de qualité. Une discussion plus détaillée sur l'effet combiné de la détérioration de la qualité et de la fiabilité peut être trouvée dans Rivera-Gomez *et al.* (2013b; 2016). Ils ont déterminé une politique conjointe de production et de contrôle de maintenance avec une option de faire une révision majeure et/ou faire appel à la sous-traitance afin d'augmenter la capacité de production d'un système manufacturier en présence de la détérioration. Dans leur modèle, le processus de vieillissement est défini à travers un ensemble d'états opérationnels. De plus, l'effet de détérioration sur la machine affectant la qualité des pièces produites et sa disponibilité augmente progressivement avec l'augmentation de l'usure et les interventions humaines. La prise de décision dans la plupart des travaux cités ci-dessus est principalement basée sur des variations de demande et de qualité déterministes. Une meilleure gestion devrait à cet effet intégrer le caractère aléatoire dans la modélisation afin d'obtenir des résultats plus réalistes.

1.3.2 Demandes et qualités aléatoires

Beaucoup de travaux dans la littérature ont souligné l'importance du caractère aléatoire qui a souvent été négligé sur la demande des clients. Il s'agit notamment du travail de Perkins et Srikant (2001), qui ont étudié un problème de planification de la production en système manufacturier d'une machine soumise à des pannes et réparations aléatoires avec une demande incertaine. Ils ont prouvé que la politique à seuil critique est optimale pour ce problème et ont fourni des expressions analytiques pour calculer le seuil critique. De même, Yin, Liu et Yin (2003) ont proposé des modèles mathématiques et des méthodes numériques applicables à la planification de la production des systèmes dans l'industrie papetière sous incertitudes. Utilisant des processus stochastiques pour décrire la dynamique du système, ils modélisent les processus de demande et de capacité aléatoire à l'aide des chaînes de Markov en temps continu et à espace d'états finis. Des politiques optimales de production sont obtenues tout au long de la durée de vie du processus pour différentes valeurs de demande et de capacité. Bensoussan, Liu et Sethi (2005) ont examiné un problème de la quantité économique à fabriquer (QÉF) des systèmes manufacturiers lorsque la demande suit un certain processus stochastique. À l'aide de la théorie du contrôle impulsionnel, ils ont réduit l'équation de Bellman du problème de la programmation dynamique à un ensemble d'inéquations quasi-variationnelles (IQV). Une étude analytique des IQV conduit à montrer l'existence d'une politique optimale. Plus tard, Presman et Sethi (2006) ont étendu le modèle de la QÉF avec une demande constante en ajoutant la portion aléatoire de cette demande.

Kutzner et Kiesmüller (2013) ont étudié un problème de qualité dans la production d'une machine de fabrication produisant un seul type de produit pour satisfaire une demande aléatoire. La demande est modélisée comme une variable aléatoire discrète. Le processus de production est considéré imparfait et peut produire à la fois des pièces acceptables et défectueuses où leur pourcentage dépendent de l'état du processus. Dans l'état '*in-control*', la machine fabrique un pourcentage très élevé de pièces acceptables. Après un certain temps aléatoire, le processus démarre dans l'état '*out-of-control*' et la machine commence à fabriquer un pourcentage plus élevé de pièces défectueuses. Les auteurs ont montré comment la dynamique du système peut être décrite exactement avec une chaîne de Markov. Leurs résultats montrent qu'une optimisation conjointe des politiques de contrôle des stocks et de la maintenance conduit à une meilleure performance du système de production et à des coûts réduits.

Certains auteurs ont aussi abordé la question de l'aspect aléatoire de la qualité dans les systèmes manufacturiers. Papachristos et Konstantaras (2006) ont proposé un modèle de la quantité économique de commande (QEC) avec un processus d'approvisionnement non fiable dans le cas où le taux de pièces défectueuses est une variable aléatoire. Eroglu et Ozdemir (2007) ont étendu le modèle précédent de la QEC pour tenir compte de la rupture de stock. Ces modèles ont été développés dans un cadre très restreint en considérant les éléments de qualité imparfaite comme variable aléatoire. Bien qu'ils puissent être considérés approximativement comme tels, nous devons être conscients des problèmes qui peuvent survenir dans la pratique en raison de tant de facteurs inévitables. De toute évidence, si nous voulons une analyse plus exacte, il est nécessaire de généraliser la notion de variable aléatoire en traitant les modèles des éléments de qualité comme des processus stochastiques (une suite de variables aléatoires), ce qui rend les modèles proposés plus réalistes.

Comme les modèles précédents le montrent, dans l'environnement de production d'un système composé d'une machine et d'un produit, la demande aléatoire et les problèmes de qualité aléatoire sont abordés séparément et sans tenir compte de l'influence négative de la détérioration sur la disponibilité de la machine. Ces modèles doivent intégrer conjointement ces aspects aléatoires afin d'étudier leur impact combiné sur la structure de la loi de commande, en accordant une attention particulière au mécanisme de détérioration causé par le processus de production et les interventions humaines (spécialement reliées à l'usure de la machine et à sa réparation). De plus, dans les travaux développés dans les sections précédentes, la notion de retour des produits, ses contraintes et son intégration dans la chaîne initiale de production n'ont pas été abordées.

1.3.3 Chaînes d'approvisionnement en boucle fermée

Il est nécessaire d'aborder la fonction logistique; qu'elle soit directe ou inverse dans un concept dit de réseau ou de chaîne afin de supporter les différentes activités de l'entreprise manufacturière qui sont interdépendantes. Selon Fleischmann (2001), dans le cadre de la chaîne logistique directe (chaîne régulière d'approvisionnement), nous distinguons généralement six (6) activités primaires. Elles se présentent par les étapes suivantes :

1) processus de conception du produit;
2) logistique interne : réception et stockage de matières, de produits, transport et manutention;
3) logistique de production : les processus de transformation de matières premières en produits finis et en cours de production, l'entretien des machines et le contrôle de la qualité;
4) logistique externe : collecte, stockage et distribution des produits aux clients;
5) commercialisation et vente par les détaillants : comme le marketing, la promotion et la fixation des prix;
6) service de soutien auprès du consommateur pour maintenir le produit en valeur, telles que les réparations sous garantie, les pièces de rechanges.

Il existe plusieurs procédés dans la logistique inverse qui permettent aux produits retournés à l'entreprise d'avoir une seconde vie. Afin d'augmenter leur durée de vie, Fleischmann (2001) et Krikke, Bloemhof-Ruwaard et Wassenhove (2001) proposent quatre (4) étapes principales pour les activités primaires liées à la logistique inverse, soient :

1) collecte et acquisition pour récupérer les produits retournés;
2) évaluation et tri pour tester et examiner l'état des produits récupérés;
3) traitement des produits récupérés pour valorisation;
4) redistribution des produits valorisés sur le marché.

Thierry *et al.* (1995) classent les activités de traitement des produits récupérés dans le cadre de la logistique inverse en sept (7) catégories :

1) réutilisation directe,
2) réparation,
3) reconditionnement,
4) réassemblage,
5) récupération de composants,
6) recyclage des produits utilisés ou de leurs composants,
7) élimination des produits.

Dans cette classification, la réutilisation directe présentée par ces auteurs concerne les produits récupérés à l'état neuf retournés par les clients quelques jours après l'achat. Cette activité n'est pas considérée comme une activité de renouvellement des produits, car ces produits peuvent se réintégrer directement sur le marché après une opération mineure comme le nettoyage et la maintenance.

Une autre classification qui semble intéressante pour notre travail de recherche est celle de Lambert (2005) tirée de celle proposée par Giuntini et Andel (1995) et Rogers et Tibben-Lembke (1998). Les activités de traitement des produits dans un contexte de logistique inverse sont classées en deux (2) catégories :

1. Le renouvellement : il consiste soit à réparer le produit, à le réusiner ou alors à le remettre à neuf. Il s'agit d'une extension de la durée de vie du produit. On peut aussi recycler le produit en matière première, le reconfigurer ou encore récupérer certains ou la totalité des composants du produit.
2. L'élimination : il consiste à retirer le produit inutilisé de la circulation à des fins environnementales. Cette activité est réalisable seulement lorsqu'il est impossible de renouveler efficacement le produit (le réintégrer sur le marché).

1.3.3.1 Logistique inverse et types de retours

Au cours des deux (2) dernières décennies, les entreprises ont eu l'obligation de gérer efficacement le retour de leurs produits en fin de vie. Elles doivent pourvoir répondre aux questions et enjeux de la logistique inverse (De Brito, Dekker et Flapper, 2003). La logistique inverse n'était pas la priorité principale pour les entreprises. Ces entreprises accordaient peu d'attention à leurs retours de produits, mais en raison de la rentabilité qu'elle peut apporter, la logistique inverse est devenue de plus en plus considérée. Dans la littérature, on trouve plusieurs appellations utilisées pour représenter la logistique inverse. Selon le conseil exécutif américain de logistique inverse, la définition la plus couramment utilisée est celle donnée par Rogers et Tibben-Lembke (1998). Ils considèrent la logistique inverse comme étant « le processus de planification, de mise en œuvre et de contrôle, de manière rationnelle et avantageuse, des flux de matières premières, de produits semi-finis, de produits finis et d'informations y afférentes, du point de consommation jusqu'au point d'origine, dans le but de récupérer ou de créer de la valeur ou d'améliorer l'élimination des déchets ». Autrement dit, pour profiter du système des retours, une fois le produit récupéré, il faut choisir les meilleures activités liées à la logistique inverse qui permettront de valoriser le cycle de vie des matières premières, des en-cours de production, des produits finis ou de les retirer de la circulation de la façon la plus efficace pour diminuer l'impact négatif sur l'environnement. Beaulieu (2000) se base sur la définition donnée par Rogers et Tibben-Lembke (1998) pour redéfinir la logistique inverse. Il aborde la logistique inverse sous le nom de logistique à rebours, comme étant « un ensemble d'activités de gestion visant la réintroduction d'actifs secondaires dans des filières à valeur ajoutée ». Finalement, d'après Chouinard (2003), « la logistique inverse consiste à récupérer des biens du circuit commercial ou du consommateur même, de les orienter vers une nouvelle étape de leur existence et de les traiter dans le but d'en retirer le maximum de valeur en cherchant à les réintégrer sur le marché ou de les éliminer proprement ». Cette dernière définition semble intéressante, car elle permet d'assurer une utilisation efficace des produits récupérés pour les réintroduire sur le marché ou tout simplement de les éliminer proprement à des fins environnementales.

Dans la littérature (Thierry *et al.*, 1995), il est mentionné que les types des retours de produits ou services sont classés en quatre (4) catégories :

1) les produits réutilisables tels que les bouteilles en verre, les palettes, les cartouches d'imprimante à jet d'encre et laser rechargeables;
2) les services de retour ou de réparation de produits sous garantie se font suite à une panne ou un défaut de fabrication. Dans ce cas, le client s'attend à recevoir un produit identique après réparation ou un produit équivalent;
3) les produits en fin de vie peuvent être refabriqués pour prolonger la durée de vie du produit;
4) le recyclage des matières premières qui composent le produit et les déchets en fin de vie. Cela prolongera la durée de vie de la matière.

Selon cette classification, nous nous intéressons à la récupération des produits déjà utilisés et plus particulièrement aux produits qui présentent un défaut de fabrication pour les réintroduire dans le système de production. En effet, c'est dans cette catégorie de types de retours de produits que se situe notre problématique de recherche.

1.3.3.2 Planification de la production en logistique inverse

Ces dernières années, un nombre croissant de chercheurs ont commencé à s'intéresser à la planification de la production manufacturière en tenant compte des activités de la logistique inverse. C'est le cas de Fleischmann *et al.* (1997) qui ont proposé des modèles quantitatifs pour représenter les activités de refabrication et de recyclage dans l'environnement de la logistique inverse en se basant spécifiquement sur trois (3) types de problèmes, à savoir : 1) la planification de la distribution, 2) le contrôle des stocks et 3) la planification de la production. De Brito *et al.* (2003) ont effectué un recensement d'études de cas en logistique inverse publiés entre 1984 et 2002, où ils discutent des différentes structures de réseau ainsi que des différentes activités reliées à la récupération des produits en fin de vie. Nous pouvons citer par exemple : la refabrication, le recyclage et la redistribution dans plusieurs secteurs, plus particulièrement en planification et contrôle de la production et en gestion des stocks.

Dans la littérature, des modèles d'optimisation pour les chaînes d'approvisionnement en boucle fermée ont été proposés avec une attention particulière à la planification de la production et à la gestion des stocks dans des contextes déterministes ou stochastiques (en temps discret ou continu). L'impact de différents facteurs aléatoires tels que le taux de demande, le taux de retour et les délais de fabrication et refabrication sur les mesures de performance du système hybride en boucle fermée a été traité par Corum, Vayvay et Bayraktar (2014) en développant un modèle stochastique basé sur la simulation. Dans la même direction, Giri et Sharma (2016) examinent l'impact de diverses sources d'incertitudes sur la performance de la chaîne d'approvisionnement en boucle fermée et ses décisions optimales.

Kiesmüller et Scherer (2003) présentent une approche efficace pour déterminer la politique de commande optimale pour un système de récupération d'un produit, en tenant compte de l'incertitude de la demande des produits neufs et du retour des produits. Ils supposent que le retour est une variable aléatoire indépendante de la demande aléatoire. Ces auteurs traitent l'activité d'élimination des produits inutilisés. Ils considèrent aussi que les temps de fabrication et de refabrication sont égaux et déterministes. Dans leur étude, le produit récupéré peut être réusiné, éliminé ou resté en stock pour une refabrication future.

Inderfurth (2004) développe un modèle discret d'optimisation stochastique pour un système hybride dont les produits neufs de la fabrication sont différents des produits réusinés de la refabrication. Il suppose qu'il existe deux (2) marchés différents en parallèle, les demandes des deux (2) types de produits sont aléatoires et indépendantes, le retour des produits récupérés est aussi aléatoire. Ces produits peuvent être réusinés ou éliminés. Il considère que les temps de fabrication et de refabrication sont déterministes et il montre que la coordination entre les deux (2) activités (la fabrication et la refabrication) permet de maximiser la rentabilité du système. Dans le même contexte de logistique inverse, Nikoofal et Husseini (2010) développent un modèle de gestion de stock périodique pour un produit sur un horizon de planification fini afin de satisfaire la demande du même marché. Ils considèrent que le retour des produits récupérés dépend de la demande aléatoire, le produit réusiné sera alors comme neuf.

Hajej, Dellagi et Rezg (2010) traitent dans leurs travaux les activités de réutilisation directe des produits en logistique inverse. Quelques jours après l'achat du produit par le client, celui-ci a le droit de le retourner soit pour échange ou remboursement et le produit récupéré peut réintégrer directement le marché car il est intact. Ces auteurs ont développé des stratégies optimales de production et de maintenance d'un système de production composé d'une machine produisant un type de produit afin de satisfaire une demande aléatoire en tenant compte d'un retour des produits qui dépend de cette demande. Ces stratégies permettent de réduire aussi la fréquence des pannes de la machine ainsi que le coût moyen de production et de maintenance.

Oscar et Silva (2011) proposent un modèle d'optimisation stochastique à temps discret d'un système hybride fiable qui prend en compte; des activités de fabrication des produits neufs, de sous-traitance, de refabrication des produits retournés du marché et des produits de mauvaise qualité provenant de la ligne de fabrication ainsi que les activités d'élimination. Ils considèrent que la demande pour les produits est une variable aléatoire distribuée normalement et le retour des produits du marché dépend de cette demande. Les auteurs ont utilisé le principe de la programmation dynamique pour développer les politiques partielles optimales d'une série de sous-problèmes et déduire ensuite la politique optimale de l'ensemble.

Bien que les travaux développés dans les sections précédentes ne tiennent pas compte de la dynamique des machines dans les modèles d'optimisation stochastique à temps discret, ils donnent une idée claire des problèmes de coordination entre la fabrication et la refabrication en présence des aspects aléatoires au niveau de la demande et du retour. Aussi, l'aspect aléatoire de la détérioration progressive qui affecte la disponibilité des machines et la qualité des pièces produites n'a pas été abordé. Étant donné que ces aspects sont normalement observés dans des situations de production manufacturière, ils doivent donc être incorporés dans les modèles conjoints d'optimisation pour aider à fournir des décisions managériales plus appropriées.

Dobos (2003) propose un modèle continue pour l'optimisation de la fabrication, la refabrication et l'élimination dans un environnement dynamique déterministe en boucle fermée. Il suppose que la demande des clients est une fonction sinusoïdale et que le retour des produits récupérés est un pourcentage de cette demande. Il considère aussi que les machines fabrication/refabrication sont fiables. Les conditions nécessaires d'optimalité de la commande peuvent être obtenues analytiquement par application de la théorie de commande optimale en se basant sur le principe du maximum de Pontryagin (Pontryagin *et al.*, 1962; Seierstad et Sydsaeter, 1987). Ces conditions nécessaires permettent de minimiser le coût total du système hybride incluant les coûts de stockage, de fabrication, de refabrication et d'élimination. Leurs résultats sont limités à la demande et au processus de retour déterministes sans tenir compte de la dynamique des unités de production.

Dans un contexte dynamique stochastique, Kenné, Dejax et Gharbi (2012) ont développé un modèle de commande pour optimiser les performances globales de la chaîne d'approvisionnement en boucle fermée. Ils considèrent un système hybride de deux (2) machines montées en logistique inverse produisant le même type de produit. Les activités de refabrication sont intégrées dans la chaîne initiale de production. Les phénomènes aléatoires sont les pannes et les réparations des machines et ils supposent aussi que la demande des produits neufs est déterministe et connue, et que le retour des produits est un pourcentage connu de la demande. L'évolution de l'état des machines (pannes et réparations) est modélisée par un processus Markovien en temps continu et à état discret. Kenné *et al.* (2012) déterminent les conditions d'optimum qui permettent de résoudre le problème de commande optimale stochastique en se basant sur la programmation dynamique stochastique et les méthodes numériques. Dans la même direction, Polotski, Kenné et Gharbi (2017) ont suivi la méthodologie proposée dans Kenné *et al.* (2012) pour étudier un système de production hybride qui consiste en une seule machine et nécessite une stratégie de mise en course pour basculer entre les modes de fabrication et de refabrication. Cependant, dans les travaux de

Kenné *et al.* (2012) et Polotski *et al.* (2017), l'effet du phénomène de détérioration reflété sur la disponibilité des machines et la qualité des pièces produites, ainsi le problème de remplacement n'ont pas été abordés.

Huang *et al.* (2013) proposent un modèle d'optimisation déterministe continu pour un système de production hybride avec une logistique externe pour la refabrication des produits en fin de vie et une logistique interne pour la rectification des produits défectueux. Les produits réusinés et rectifiés sont utilisés pour satisfaire un taux de demande variable et le processus de fabrication de base peut être appelé pour compléter le reste s'il y a un manque dans la production. L'objectif de leur étude est de déterminer les stratégies optimales pour contrôler les taux de fabrication, de refabrication, de rectification et d'élimination. Les impacts des facteurs liés à la stratégie de production concernent l'évaluation des compromis entre la consommation de produits et la protection de l'environnement. Cette stratégie a été développée dans un cadre restreint d'un système de production hybride fiable. De plus, la détérioration de la qualité des produits a été limitée à un taux de rejets constant pendant toute la durée de vie du système.

Le problème de planification de la production avec détérioration de la disponibilité pour un système manufacturier hybride fabrication/refabrication en boucle fermée a été traité par Kouedeu *et al.* (2014). Ce système est composé de deux (2) machines non-identiques soumises à des pannes et réparations aléatoires produisant un seul type de produit pour répondre à un taux de demande constant. Le taux de retour des pièces récupérées sur le marché est considéré comme un pourcentage de ce taux de demande. Il est utilisé par la machine de refabrication pour combler la demande manquante. La machine de fabrication se dégrade sous l'hypothèse que son taux de pannes est fonction de son taux d'utilisation. Selon cette dégradation, la modélisation de la dynamique des machines a été faite par un processus Markovien non-homogène. La solution du problème est obtenue par la résolution numérique à travers des équations d'HJB. Les résultats de leur travail ont montré qu'il est possible de tenir compte de la fiabilité de la machine et de réduire le coût total du système de production lorsque le taux de production est réduit à une vitesse économique. Ces auteurs procèdent à des actions de maintenance destinées à restaurer le système de production dans l'état lui permettant de produire à sa pleine capacité.

Suite à l'analyse de cette classe de systèmes dans la section précédente, il ressort que la contribution de Kouedeu *et al.* (2014) représente la première tentative d'intégrer dans l'étude des systèmes hybrides de fabrication/refabrication en boucle fermée en plus des pannes et réparations aléatoires des machines, le mécanisme de détérioration sur la machine de fabrication. Cependant, l'option de maintenance préventive annule la considération de la détérioration progressive des machines et de ce fait ne traduit pas la vie réelle des systèmes manufacturiers. Dans le domaine de la détérioration des systèmes, il serait donc intéressant de fournir un cadre utile pour aborder davantage des contextes de production avec différentes sources de dégradations et d'aléas.

Au regard des travaux présentés ci-dessus, notre projet de recherche trouvera son originalité dans le fait de traiter en contexte de logistique inverse en plus des pannes, des réparations et des détériorations des machines, les aspects aléatoires associés à la demande, le retour et la qualité. Il est donc nécessaire d'intégrer progressivement ces aspects nouveaux dans le modèle de commande optimale stochastique pour rendre notre stratégie conjointe de production et de maintenance plus efficace et qui se rapproche le plus de la réalité. Le but étant de maîtriser ces aléas afin de bien gérer les stocks tout en répondant aux exigences des clients à moindre coût. Cependant, en intégrant simultanément tous ces aspects dans un même modèle, le problème d'optimisation devient de ce fait très complexe et nous proposons dans la section suivante la méthodologie détaillée pour résoudre ce problème.

1.4 Méthodologie proposée

La méthodologie que nous avons adoptée pour réaliser le travail est résumée dans la figure 1.2 en quatre (4) grandes étapes :

1. Modélisation : formuler les problèmes de planification de la production et de la maintenance correspondant à la dynamique des systèmes manufacturiers. Intégrer

progressivement des aspects aléatoires et d'autres concepts de détérioration afin d'atteindre l'objectif global de cette thèse.

2. Optimisation : faire appel aux principaux outils de la théorie de commande optimale stochastique. Se baser sur le principe de la programmation dynamique stochastique en temps continu pour développer les conditions d'optimum (nécessaires et suffisantes). Ces conditions sont décrites par des équations aux dérivées partielles modifiées et généralisées communément appelées les équations d'HJB (Hamilton-Jacobi-Bellman). Du fait que la demande des clients, que le retour des produits en fin de vie ainsi que la détérioration de la qualité des pièces produites soient aléatoires, nous développons des équations d'HJB du second ordre.

3. Résolution : en l'absence d'une solution analytique des équations d'HJB, nous optons pour une résolution numérique basée sur l'approximation par différences finies et les méthodes itératives pour déterminer les politiques de commande.

4. Simulation, validation et implémentation : nous appliquons les méthodes numériques développées à l'étape 3 sur des exemples tirés de la littérature. Des analyses de sensibilité sont faites suite aux variations des paramètres clés des modèles afin de confirmer les structures des politiques obtenues, de vérifier la qualité des solutions optimales trouvées et leurs robustesses avant de proposer des outils d'aide à la décision suite à l'implémentation des résultats sur les cas étudiés.

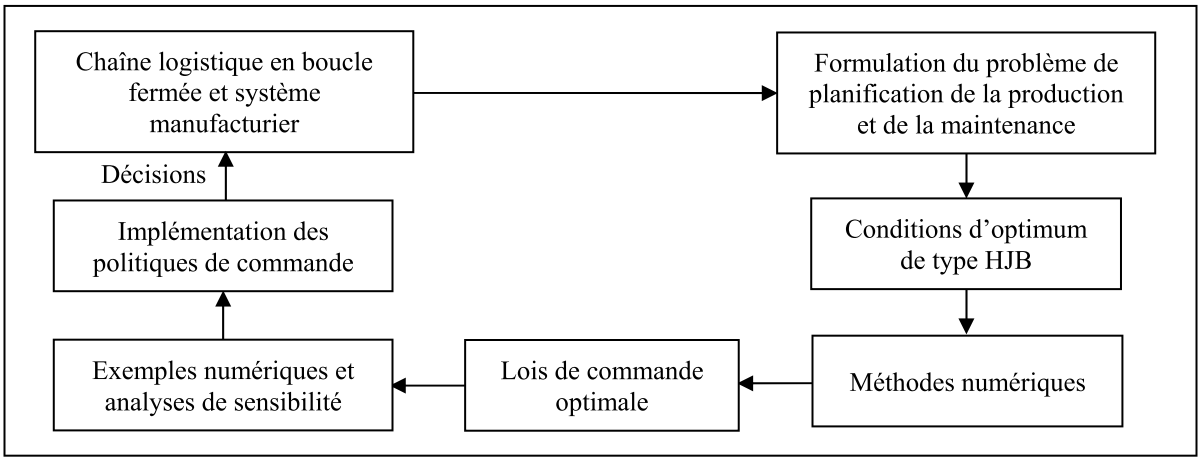

Figure 1.2 Méthodologie proposée

La prochaine section présente les contributions et la structure de la thèse.

1.5 Contributions et structure de la thèse

L'objectif principal de cette recherche réside dans le développement d'une nouvelle approche de modélisation mathématique pour résoudre une classe de problèmes de commande optimale stochastique en production manufacturière dans un contexte de chaîne d'approvisionnement en boucle fermée. Cette approche permet de fournir des lois de commande pour contrôler la production et la maintenance en présence de différents phénomènes aléatoires et les aspects de détérioration précédemment soulevés. Cet objectif principal est divisé en cinq (5) objectifs spécifiques (5 thèmes de recherche) :

1. Développer un modèle pour des stratégies de contrôle de la fabrication et du remplacement d'une machine soumise à des pannes, réparations et détériorations aléatoires.

2. Étendre le modèle précédent pour tenir compte des incertitudes au niveau de la demande des clients et de la qualité des pièces produites.

3. Faire une étude de planification de la production d'un système manufacturier hybride composé de machines de fabrication et de refabrication non fiables. La demande des clients et le retour des produits en fin de vie sont aléatoires.

4. Présenter un modèle d'optimisation conjointe de la production et du remplacement d'un système manufacturier hybride non fiable qui tient compte de la détérioration progressive générée par la refabrication des produits récupérés du marché qui sont souvent de nature hétérogène.

5. Étendre le modèle de détérioration pour tenir compte de l'effet simultané de la détérioration sur les deux (2) machines, de créer une relation entre panne et qualité aléatoires, et d'analyser son impact sur les politiques de contrôle de la production et du remplacement dans un système manufacturier hybride non fiable.

Les cinq (5) prochains chapitres sont constitués de cinq (5) articles de revue présentés dans l'ordre suivant :

L'article du chapitre deux (2) présente le problème de planification de la production et du contrôle de remplacement d'un système manufacturier dans un contexte de détérioration soumis à des pannes et réparations aléatoires. Le système est constitué d'une machine produisant un seul type de pièce. Nous avons introduit une détérioration proportionnelle à la production pour laquelle le vieillissement se traduit par l'âge que prend la machine chaque fois qu'une pièce est fabriquée. Cette détérioration affecte progressivement la disponibilité de la machine. Nous avons également tenu compte de la réparation minimale qui restaure l'âge de la machine aux conditions *as-bad-as-old*. Les variables de décision sont le taux de production et le taux de remplacement. Cet article est publié dans la revue Journal of Engineering Manufacture sous la référence :
S. Ouaret, J. P. Kenné, A. Gharbi, V. Polotski, (2015) "*Age-dependent production and replacement strategies in failure-prone manufacturing systems*". Proceedings of the

Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 231(3), pp. 540-554.

Dans l’article du chapitre trois (3), nous avons ajouté dans le système de base précédent les aspects aléatoires de la demande des clients et de la qualité des pièces défectueuses. L’effet du phénomène de détérioration causé par les processus de vieillissement et de réparation minimale est reflété sur la disponibilité de la machine et aussi sur la qualité des pièces produites. Nous avons analysé l’impact de l’introduction de la variabilité au niveau de la demande et la qualité sur la politique conjointe de production et de remplacement. Cet article est publié dans la revue European Journal of Operational Research sous la référence :
S. Ouaret, J. P. Kenné, A. Gharbi, (2018) “*Production and replacement policies for a deteriorating manufacturing system under random demand and quality*”. European Journal of Operational Research 264(2), pp. 623-636.

Dans l’article du chapitre quatre (4), une deuxième machine montée en logistique inverse est ajoutée à la première machine des chapitres 2 et 3 pour former un système hybride en boucle fermée. La première machine traite les activités de production de base alors que la deuxième machine traite le retour des produits en fin de vie. Les phénomènes aléatoires sont les pannes et réparations des machines, la demande des clients et le retour en fin de vie. La modélisation de la dynamique des machines a été faite en utilisant les chaînes de Markov homogènes. La demande des clients est modélisée par un processus stochastique et le retour étant un pourcentage de cette demande. Les variables de décision sont les taux de production des machines. Cet article est publié dans la revue Applied Mathematics sous la référence :
S. Ouaret, V. Polotski, J. P. Kenné, A. Gharbi, (2013) “*Optimal production control of hybrid manufacturing/remanufacturing failure-prone systems under diffusion-type demand*”. Applied Mathematics 4(3), pp. 550-559.

Dans l’article du chapitre cinq (5), nous avons introduit la notion de détérioration sur la machine de refabrication du chapitre 4 en raison de la qualité des produits en fin de vie qui sont de nature hétérogène. Ce processus de détérioration implique une réparation imparfaite sur cette machine dont le temps de réparation augmente progressivement d'une panne à l'autre. Nous faisons une étude en intégrant cette détérioration dans un système hybride fabrication/refabrication. Les machines sont sujettes aux pannes et aux réparations aléatoires. Dans ce cas les variables de décision seront les taux de production des machines ainsi que le taux de remplacement de la machine de refabrication. Cet article est en cours de révision. Il a été soumis dans la revue International Journal of Production Economics sous la référence :

S. Ouaret, J. P. Kenné, A. Gharbi, (2017) "*Production and replacement planning of a deteriorating remanufacturing system in closed-loop configuration*". Submitted on June 21th, 2017 (under revision). Submission Confirmation: IJPE-D-17-00830.

Dans l'article du chapitre six (6), nous avons étendu le concept de l'effet de détérioration du système hybride du chapitre 5 sur les deux (2) machines pour résoudre des problèmes plus proches de la réalité industrielle. La première machine traite les activités de production et la deuxième machine traite les activités de rectification des produits défectueux de la première machine et de refabrication des produits récupérés en fin de vie. L'effet de détérioration sur la machine de fabrication affecte progressivement et de façon aléatoire sa disponibilité et sa qualité des pièces produites. Tandis que cet effet sur la disponibilité de la machine de rectification/refabrication est généré par le flux des mauvaises pièces traitées. Les variables de décisions sont le taux de production de la première machine, le taux de production de la deuxième machine et le taux de remplacement de la première machine. Étant donné que la première machine cause de nombreuses pannes à la deuxième machine, son remplacement par une nouvelle machine permet de restaurer les paramètres du système hybride aux conditions initiales (*as-good-as-new*). Cet article a été soumis dans la revue Journal of Manufacturing Systems sous la référence :

S. Ouaret, J. P. Kenné, A. Gharbi, (2017) "*Stochastic optimal control of random quality deteriorating hybrid manufacturing/remanufacturing systems*". Submitted on November 20th, 2017. Submission Confirmation: SMEJMS-D-17-00517.

1.6 Conclusion

Dans ce chapitre, nous avons décrit la problématique de notre recherche. Nous avons également présenté une revue critique de la littérature qui touche les aspects d'ordre général associés à notre problématique. Par la suite, nous avons pu distinguer l'originalité de notre travail par rapport à l'ensemble des travaux déjà réalisés. Nous avons aussi présenté la méthodologie envisagée pour réaliser ce travail. En ce sens, nous avons présenté les contributions et la structure de la thèse.

Finalement, en guise de conclusion, nous résumons les principales contributions apportées et nous présentons les travaux futurs.

CHAPITRE 2

ARTICLE 1: AGE-DEPENDENT PRODUCTION AND REPLACEMENT STRATEGIES IN FAILURE-PRONE MANUFACTURING SYSTEMS

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Résumé

Un système manufacturier composé d'une machine produisant un type de produit est étudié. Les phénomènes aléatoires examinés sont les pannes et les réparations de la machine. Nous supposons que la machine subit une détérioration progressive pendant son fonctionnement et que le taux de panne de la machine est une fonction de son âge. Le vieillissement de la machine (la dynamique de l'âge de la machine) est supposé être une fonction croissante de son taux de production. Les activités de maintenance corrective sont imparfaites et restaurent l'âge de la machine dans son état juste avant panne. En cas de panne, la machine peut être réparée, et pendant la production, la machine peut être remplacée dépendamment de son âge. Lorsque l'action de remplacement est sélectionnée, la machine est remplacée par une nouvelle identique. Les variables de décision sont le taux de production et le taux de remplacement. L'objectif de cet article est de traiter le problème d'optimisation simultanée des politiques de production et de remplacement dans le contexte manufacturier avec détérioration et réparations minimales satisfaisant la demande du client et minimisant le coût total, comprenant les coûts de stockage, de pénurie, de production, de réparation et de remplacement sur un horizon infini de planification. Nous étudions en profondeur l'impact du vieillissement de la machine sur les politiques de production et de remplacement. Une attention particulière est accordée à la vérification des résultats mathématiques sous-jacents qui garantissent l'existence de solutions optimales et la convergence des méthodes numériques. En raison de réparations minimales, la dynamique du système est affectée par son historique et les processus semi-Markoviens doivent être utilisés pour la modélisation. Les conditions d'optimum sous la forme des équations de Hamilton-Jacobi-Bellman (HJB) sont développées et des méthodes numériques sont utilisées pour obtenir les politiques de commande optimale (politiques de production (taux) et de remplacement). Un exemple numérique est donné pour illustrer l'approche proposée et une analyse de sensibilité est présentée pour confirmer la structure des politiques de commande obtenue.

Abstract

A failure-prone manufacturing system that consists of one machine producing one type of product is studied. The random phenomena examined are machine breakdowns and repairs. We assume that the machine undergoes a progressive deterioration while in operation and that the machine failure rate is a function of its age. The aging of the machine (the dynamics of the machine age) is assumed to be an increasing function of its production rate. Corrective maintenance activities are imperfect and restore the age of the machine to as-bad-as-old conditions (ABAO). When a failure occurs, the machine can be repaired, and during production, the machine can be replaced, depending on its age. When the replacement action is selected, the machine is replaced by a new and identical one. The decision variables are the production rate and the replacement policy. The objective of this paper is to address the simultaneous production and replacement policy optimization problem in the context of manufacturing with deterioration and imperfect repairs satisfying the customer demand and minimizing the total cost, which includes costs associated with inventory, backlog, production, repair and replacement, over an infinite planning horizon. We thoroughly explore the impact of the machine aging on the production and replacement policies. Particular attention is paid to the verification of underlying mathematical results that guarantee the existence of optimal solutions and the convergence of numerical methods. Due to imperfect repairs, the dynamics of the system is affected by the system history and semi-Markov processes have to be used for modeling. Optimality conditions in the form of the Hamilton-

Jacobi-Bellman (HJB) equations are developed, and numerical methods are used to obtain the optimal control policies (production (rate) and replacement policies). A numerical example is given to illustrate the proposed approach, and an extensive sensitivity analysis is presented to confirm the structure of the obtained control policies.

Keywords: Manufacturing systems, Optimal control, Replacement policy, Corrective maintenance, Production planning, Numerical methods.

2.1 Introduction

In the manufacturing environment, the availability of the machine often decreases due to its age and also to imperfect maintenance activities. In general, corrective or preventive maintenance brings the state of the machine to a level which is not new, and it may not be able to meet the demand rate for the commodity produced. For this reason, the machine has to be replaced. We consider a machine that is subject to random breakdowns and repairs. It undergoes deterioration while in operation, and the failure rate increases with its age. The aging of the machine is an increasing function of its production rate. The corrective maintenance activities performed are imperfect and restore the machine to as-bad-as-old conditions. Replacement activities for their part renew the machine, which is similar to restoring the machine to as-good-as-new conditions (resetting its age to zero).

The first objective of this paper is to simultaneously determine production and replacement policies in a manufacturing environment under deterioration and imperfect repairs. We enhance existing mathematical models by including the production cost in the objective function. Given that the dynamics of the machine aging process depends on the production rate, penalizing the latter helps to amplify the impact of aging and push the system (optimal control policies) towards an appropriate solution. The proposed model appears to be better at addressing industrial reality, and is yet to be used in the literature in analyzing age-dependent production and replacement strategies. The obtained solution provides the simultaneous optimal control of production and replacement of the machine (assuming that one replacement is performed). The decision variables are the production rate and the replacement policy. The dynamics of the machine is described by a semi-Markov decision model due to the machine's deterioration and imperfect repairs (as-bad-as-old). The optimal control policies are determined in order to satisfy a deterministic customer demand and minimize inventory, backlog, production, repair and replacement costs over an infinite planning horizon.

The second objective is to consider machine aging as a unique factor affecting the deterioration of the machine in order to get a better understanding of the phenomena specifically relating to machine aging without interference from other deterioration levels. Finally, the third objective is to provide a rigorous formulation of the underlying mathematical results and verification of all conditions in order to guarantee the existence of optimal solutions and convergence of numerical methods used to find these solutions.

The rest of the paper is organized as follows. We provide the overview of the existing body of works in the "Literature review" Section. In the "Assumptions and problem statement" Section, we describe the assumptions regarding the model and formulate the optimal control problem. The "Properties of the value function and optimality conditions" Section addresses optimality conditions in the form of HJB equations. A numerical example is presented in the "Numerical example" Section, followed in the "Sensitivity analysis" Section by the sensitivity analysis illustrating the robustness and effectiveness of the obtained control policies. Discussions of the results are presented in "Discussions" Section and finally, the paper is summarized in the "Conclusion" Section.

2.2 Literature review

To describe the behaviour of a repairable system subject to failures, various types of repairs are used in conventional models: the minimal repair, the perfect repair and the imperfect repair. The first one brings the system to its functioning condition just prior to a failure (e.g. as-bad-as-old); the second one is as-good-as-new, while the third one can be represented by its so-called virtual age, which is smaller than the real age. Pham and Wang (1996) suggest the faulty part being only partially repaired, human errors such as further damage during maintenance, replacement with faulty parts, etc., as reasons behind an imperfect repair. Chiu, Tseng, Liu and Ting (2009) for their part address system failures followed by imperfect rework using economic manufacturing quantity model with Poisson failure and abort/resume control policies. Many authors have studied the optimization problems of maintenance, repair and replacement in the context of a progressive deterioration, but without addressing production planning. Progressive deterioration is usually characterized either by the repair time increasing with the number of failures or by age accumulation when the machine is in operational mode. Phelps (1983) developed a semi-Markov decision model in which a minimal repair is performed on a system subject to random failures. He suggested three optimal replacement policies: in the first policy, the system is replaced at a fixed time T; in the second, the system is replaced after a fixed number of failures N; and in the third, the replacement takes place at the first failure after a fixed time T. Kijima, Morimura and Suzuki (1988) studied a problem of periodic replacement with a general repair: the system is replaced only at a regular time interval kT (k=0,1, …), and is repaired following a failure. They assumed that the costs of repair and replacement are constant. This general repair transfers the system state to a certain "better" state. A stochastic model describing this situation was proposed to find the optimal replacement period. Makis and Jardine (1993) described a system with deterioration subject to random failures modeled as a semi-Markov process, and demonstrated under appropriate conditions that a stationary optimal replacement policy exists. They assumed that the replacement cost is constant and that the repair cost depends on the number of failures and the real machine age. Love *et al.* (2000; 1998) proposed a discrete semi-Markov model: they determined the optimal policies of a machine subject to breakdowns, and which can be replaced or can undergo an imperfect repair. The virtual machine age has been introduced in the decision process, and it has been shown that the optimal repair/replacement policy is of threshold type. Zhang (1994) developed a bivariate replacement policy (T, N) for a repairable system. The replacement is performed at the first failure either after the cumulative age reaches T or the number of failures reaches N. It was assumed that the system after repair is not refurbished, and that the optimal solution can be obtained analytically or numerically under certain conditions.

Many researchers have studied the production planning problem for unreliable manufacturing systems. Based on the formalism of Rishel (1975), Olsder and Suri (1980) determined the optimum conditions of a planning problem in the production of a manufacturing system subject to random breakdowns and repairs, with its dynamics described by homogeneous Markov processes. The optimum conditions are described by the HJB equations. The pioneering work of Kimemia and Gershwin (1983) similarly showed that the obtained control policy for this problem is of threshold type (Hedging point policy). This allows the optimal production rate to be found in order to meet a constant demand rate and reduce the total cost, which is the sum of inventory and shortage costs. Boukas and Haurie (1990) determined a policy that combines the production and preventive maintenance for a manufacturing system consisting of two machines. They consider that the probability of failures of the machines increases with the age. Since the repair or preventive maintenance activities restore the cumulative age to zero, the dynamics of the system (machines) is modeled in Boukas and Haurie (1990) by a non-homogenous Markov process. The set of dynamic programming equations were solved numerically based on Kushner’s approach (Kushner and Dupuis, 1992), and the obtained control policy is of modified threshold type. Yan (2001, 2003) discussed a hierarchical stochastic production planning problem of flexible manufacturing systems (FMSs). Imposing some conditions on uncertain demand, the author formulated a stochastic non-linear programming problem and proposed the algorithms for its solution. By applying some approximations, a production plan with the lowest possible cost is obtained.

In the context of imperfect repairs, the optimal production and repair/replacement joint policy is of great importance for practitioners. It helps to better manage the manufacturing system performance, and to intervene with the acquisition of a new machine in a timely fashion. This situation was investigated in Dehayem-Nodem *et al.* (2011) where the authors proposed a hierarchical decision making approach to first determine the hybrid repair and replacement policy (T, N). Secondly, the optimal production rate is determined. The age and the number of failures are combined to make the decision about the repair/replacement of the machine. The repair activities are imperfect due to a number of failures affecting the system behaviour, and the replacement activities restore the age of the machine to initial conditions (AGAN). With respect to the modeling, the decision to repair or replace the machine is made when the failure occurs. In Rivera-Gomez, Gharbi and Kenné (2013a), the simultaneous production, repair/overhaul (equivalent to replacement) and preventive maintenance control policy is obtained under the effect of deteriorations resulting in product quality degradation: the authors combined the impact of two factors, namely, the wear of the machine and human interventions. When the machine is in operational mode, three types of actions can be taken: the machine produces, is sent to overhaul, or is sent for preventive maintenance. Upon failure, the machine has to be repaired. A semi-Markov process is used in both Dehayem-Nodem *et al.* (2011) and Rivera-Gomez *et al.* (2013a), due to the imperfect maintenance activities of the machine that depend on the history of breakdowns and repairs. The authors Dehayem-Nodem *et al.* (2011) and Rivera-Gomez *et al.* (2013a) developed the optimality conditions in the form of HJB equations, that allow the resolution of the problem of stochastic optimal control based on stochastic dynamic programming and numerical methods, in order to optimize the total cost (e.g. including the inventory, backlog, repair/replacement, preventive maintenance and defectives costs) and satisfy a demand assumed to be deterministic and known. However, due to the mixture of various deterioration mechanisms involved in the proposed models, it remained unclear which mechanism was the most responsible for the phenomena observed.

As mentioned in the introduction, a model that differs from the existing ones is developed in this paper. We explore the impact of deterioration mechanism related to the machine aging only, in order to eliminate interference from other sources of deterioration, as in Dehayem-

Nodem *et al.* (2011) and Rivera-Gomez *et al.* (2013a) and get the insight to age-induced deterioration phenomena. We also incorporate the production cost in the objective function. The rationale behind changing the cost function in such a way, given that the dynamics of the machine aging depends on the parts produced, penalizing the production rate with production cost will amplify the phenomenon of aging and push the system (optimal control) to enhance the obtained solution in manufacturing environment. The optimality conditions are developed in the form of HJB equations using the optimal control theory based on stochastic dynamic programming. The practical implication of the proposed model is verified through numerical simulations.

2.3 Assumptions and problem statement

This section defines the assumptions used throughout this article, as well as the optimal control problem statement.

2.3.1 Assumptions

The following assumptions are considered in this paper:

1. the raw materials are always available and unlimited;
2. the customer demand is known and described by a constant rate over time;
3. the maximum production rate of each machine is known;
4. the repair or replacement costs of the manufacturing machine are assumed constant (($/repair); ($/replacement));
5. the machine deteriorates in operational mode, in which the machine age is continuously growing;
6. the failure rate of the machine increases with its age;
7. the corrective maintenance activities are imperfect and associated with a minimal repair that restores the machine to as-bad-as-old conditions (ABAO);
8. the machine is replaced by a new identical one;
9. the production is more penalized when the machine increases its production rate.

2.3.2 Problem formulation

The manufacturing system under consideration consists of one machine producing one type of product. The machine is unreliable, and is subject to random phenomena, such as breakdowns and repairs. The machine deteriorates while in operation, and its failure rate increases with its age. In failure, the machine undergoes a minimal repair, and while it is in operation, a decision can be made to replace the machine with a new and identical one due to its deterioration. A service inventory is built at the end of the production process, which is used to satisfy a constant demand rate. This situation is illustrated in figure 2.1.

The system behaviour can be mathematically modeled by a stochastic control system in continuous time (not in multi-period time, as is normally used in the literature) characterized by a hybrid state, which is comprised of two continuous state variables (inventory level $x(t)$ and machine age $a(t)$) and one discrete state variable (mode of the machine $\xi(t)$) at time t.

The dynamics of the stock level is described by a one-dimensional ordinary differential equation:

$$\dot{x}(t) = u(t) - d \quad \text{with } t \in \mathbb{R}_+ \text{ and } x(0) = x^0 \tag{2.1}$$

where x^0 is the given initial inventory/backlog level. When $x(t) \geq 0$, the system has an inventory and a backlog, otherwise.

The cumulative age of the machine at time t, is the solution of the following differential equation:

$$\dot{a}(t) = f\big(u(t)\big)$$

where $f(.)$ is a positive real-valued function.

In the case considered herein, the machine age $a(t)$ is the number of produced parts, and the aging of the machine $\dot{a}(t)$ will always be an increasing function of the machine production rate. By referring to the literature, e.g. (see Boukas and Haurie (1990) and Kenné and Boukas (2003)), we can use the linear model that describes the relationship between the aging of the machine $\dot{a}(t)$ and its production rate $u(t)$. That is:

$$\dot{a}(t) = ku(t) \qquad \text{with } t \in \mathbb{R}_+ \text{ and } a(0) = a^0, a(T^+) = a(T^-) \text{ and } a(T) = 0 \tag{2.2}$$

where k is a given positive constant and a^0, T^+, T^- and T stand for the initial age, the last times of repair, operation and replacement of the machine, respectively. These values (T^+, T^-, T) imply that the repair is as-bad-as-old and the replacement is as-good-as-new.

The production rate $u(t)$ must satisfy the capacity constraint:

$$0 \leq u(.) \leq u_{max} \tag{2.3}$$

where u_{max} is the maximum production rate of the manufacturing machine.

The machine's mode can be classified as operational, denoted by $\xi(t) = 1$, under repair, denoted by $\xi(t) = 2$ and under replacement, denoted by $\xi(t) = 3$. The mode of the machine at time t is given by the random process $\{\xi(t) \geq 0\}$ with $\xi(t) \in B = \{1, 2, 3\}$ such that:

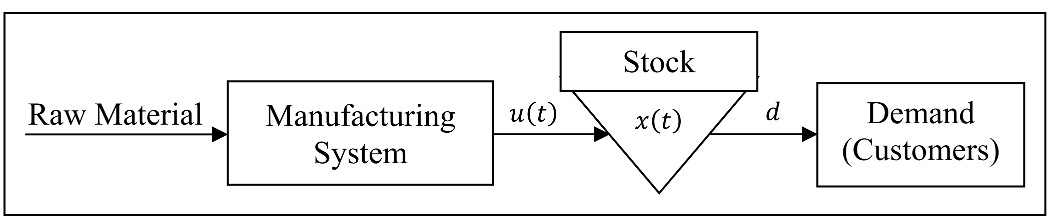

Figure 2.1 Structure of the production system

$$\xi(t) = \begin{cases} 1 & \text{operational} \\ 2 & \text{under failure} \\ 3 & \text{under replacement} \end{cases} \tag{2.4}$$

Given that the machine deteriorates with age and the fact that it is not new after repair activities (ABAO), its dynamics is modeled as a semi-Markov process in continuous time discrete state over an infinite horizon $\{\xi(t) \geq 0\}$. The transition diagram of such a process is illustrated in figure 2.2. We introduce a control variable $w(t) \in \{0, 1\}$ initially as a binary variable; it is set to a value 1 if the replacement of the machine is performed and to 0 otherwise. The transition rate $q_{13}(.)$ is defined as a linear function of $w(t)$: $q_{13}(.) = q.w(t)$ where q is a given constant.

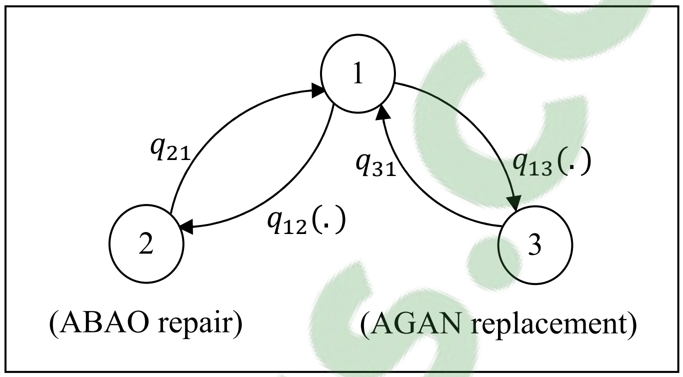

Figure 2.2 States transition diagram of the considered stochastic process

The failure rate $q_{12}(.)$ is an increasing function of a machine's age $a(t)$, and is given by:

$$q_{12}\big(a(t)\big) = A_0 + A_1^{\infty}\big(1 - e^{-(A_2 a(t)^3)}\big) \tag{2.5}$$

where the parameters A_0, A_1^{∞} and A_2 are given constants. The expression given by equation (2.5) describes the impact of a machine age on its dynamics, as in Kenné and Nkeungoue (2008). The inverse of the transition rate $q_{12}(a)$ represents the mean time to failure as a function of the age denoted by $MTTF(a)$. We use the notation $\mathrm{Ind}\{\Theta(.)\}$ for the indicator function of a condition $\Theta(.)$ defined as follows:

$$\mathrm{Ind}\{\Theta(.)\} = \begin{cases} 1 & \text{if } \Theta(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

The machine's mode switches from operational to replacement with a transition rate denoted by $q_{13}(.)$, the inverse of $q_{13}(.)$ represents the expected delay between the preparation of the order of the new machine, its installation kit and its arrival on the site. In order to allow this delay to vary continuously, we reintroduce a control variable $w(t)$ allowing it to take its values within the domain $]0,1]$. We also assume that the decision regarding the machine replacement can be taken only in operational mode. Thus, the delay corresponding to the inverse of $q_{13}(.) = q.w(t)$ is the transition time between the decision to perform the replacement and the effective switch from the operational mode to the replacement mode. The replacement is now allowed for all possible values of $w(t)$ within $]0,1]$. When the value of $w(t)$ is close to zero ($\mathrm{e.g.}\, 10^{-5}$), the delay before replacement is very large (but finite), thus describing the situation when replacements do not actually take place; as for machine repairs, they can occur at any failure instant. The transition rates q_{21} to repair the machine in mode 2 and q_{31} to replace the machine in mode 3 are assumed to be known constants (their inverses represent the mean time to repair and the mean time to replace, respectively). Other transition rates of the manufacturing system are equal to zero.

In the case of minimal repair, the repair rate is usually considered to be constant. In manufacturing systems, many repairs such as (as-bad-as-old) can be considered as minimal repairs. Such a repair brings the age of the machine in the state which is basically the same as it was just before the failure occurred. In other words, a minimal repair means that the age of the machine is not affected by failures: the machine age after each minimal repair is restored, and the failure rate gets the same value as it was before the failure. Thus, after minimal repair, both failure rate and repair rate are restored to their values before the last failure; $q_{12}^{new} = q_{12}^{old}$, and $q_{21}^{new} = q_{21}^{old}$. The ratio of repair duration τ_r and the mean time to failure $1/q_{12}$ are then kept constant in the case of minimal repair and then:

$$\tau_r^{new} = \tau_r^{old}.\frac{q_{12}^{old}}{q_{12}^{new}} = \tau_r^{old}.\frac{q_{12}^{old}}{q_{12}^{old}} \Rightarrow \tau_r^{new} = \tau_r^{old}$$

As a result, the repair duration τ_r is represented by the constant repair rate q_{21}. More detailed description of the minimal repair models can be found in Zio (2009). For example: once the machine is repaired after the first failure, the age continues to accumulate when the machine begins to operate again and the failure rate continues also to increase with the age. When the next failure occurs, and a minimal repair is executed, the failure rate gets the same value as before the last failure and according to the equality above ($\tau_r^{new} = \tau_r^{old}$), the repair duration does not change; it is the same as before the failure.

Figure 2.3 shows the jump times occurring within our model in the different modes e_i. It should be recalled that the mode of the manufacturing system at time t is given by the finite-state semi-Markov process $\xi(t) \in B = \{1, 2, 3\}$, and can be characterized by the matrix $Q(w) = [q_{\alpha\beta}(.)]$. Its entries $q_{\alpha\beta}$ are real numbers, and depend on the decision variable $w(.)$, with $\alpha, \beta \in B$. If $\alpha \neq \beta$ we have

$$q_{\alpha\beta}(x, a, u, w) \geq 0 \tag{2.6}$$

$$\sum_{\beta} q_{\alpha\beta}(x, a, u, w) = 0 \tag{2.7}$$

and if $\alpha = \beta$, we have

$$q_{\alpha\alpha}(x, a, u, w) = -\sum_{\alpha \neq \beta} q_{\alpha\beta}(x, a, u, w) \tag{2.8}$$

The transition probabilities of the manufacturing system from mode α to mode β after time t are given by:

$$\begin{aligned} &P[\xi(t+\delta t) = \beta | \xi(t) = \alpha, a(t) = a, u(t) = u, w(t) = w] \\ &\quad = q_{\alpha\beta}(x, a, u, w)\delta t + o(x, a, \delta t) \end{aligned} \tag{2.9}$$

$$\begin{aligned} &P[\xi(t+\delta t) = \alpha | \xi(t) = \alpha, a(t) = a, u(t) = u, w(t) = w] \\ &\quad = 1 + q_{\alpha\alpha}(x, a, u, w)\delta t + o(x, a, \delta t) \end{aligned} \tag{2.10}$$

where $o(x, a, \delta t)$ is a quantity such that:

$$\lim_{\delta t \to 0} \frac{o(x,a,\delta t)}{\delta t} = 0 \quad \text{for all } \alpha, \beta \in B$$

The corresponding (3×3) transition matrix of the semi-Markov chain $\xi(t)$ for the considered system is given by:

$$Q(w) = \begin{bmatrix} q_{11} & q_{12}(a(t)) & q_{13}(.) \\ q_{21} & q_{22} & 0 \\ q_{31} & 0 & q_{33} \end{bmatrix} \tag{2.11}$$

To address the feasibility of the manufacturing system, we introduce as usual, the limiting probabilities π_i of mode i, $i \in \{1, 2, 3\}$ that are known to be steady state solutions of the forward Kolmogorov equations $\dot{\pi}(t) = \pi(t)Q(t)$. When $t \to \infty$, $\dot{\pi}(t) = 0$, as in Ross (2003). Therefore, we have:

$$\pi(.)Q(.) = 0 \tag{2.12}$$

with the normalizing condition:

$$\sum_{i=1}^{3} \pi_i = 1 \tag{2.13}$$

where $\pi(.) = (\pi_1, \pi_2, \pi_3)$ and $Q(.)$ is a (3×3) transition rates matrix given by equation (2.11). The manufacturing system is considered feasible if:

$$\pi_1 u_{max} \geq d \tag{2.14}$$

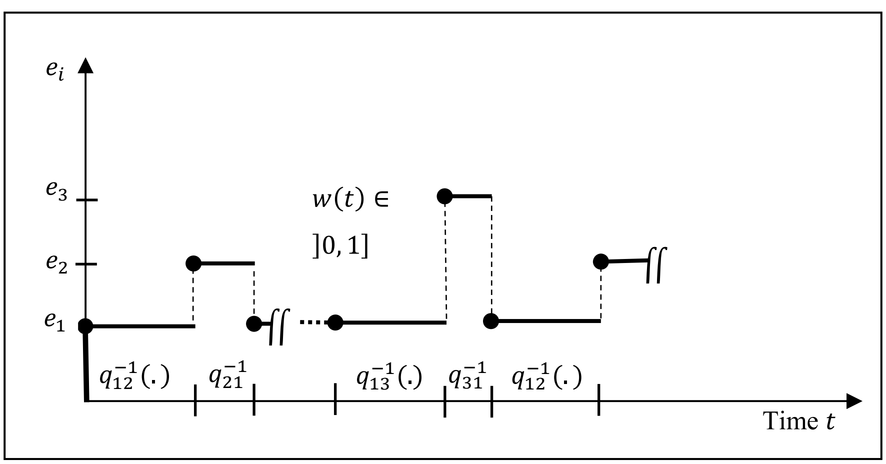

Figure 2.3 Jump times of different modes e_i ($i = 1, 2$ and 3)

Let $G(.)$ be the cost rate defined as follows:

$$G(\alpha, x, a, u, w) = c^+x^+ + c^-x^- + c^\alpha + c(u) \tag{2.15}$$

The constants c^+ and c^- are costs incurred per unit of produced parts and per unit time, and are used to penalize parts inventory and backlog, respectively, with $x^+ = \max(0, x)$, $x^- = \max(0, -x)$ and c^α being a constant defined as follows:

$$c^\alpha = c_r.\,\mathrm{Ind}\{\xi(t) = 2\} + c_0.\,q_{31}.\,\mathrm{Ind}\{\xi(t) = 3\}$$

where c_r is the cost rate (positive constant) for repair activity on the machine and $c_0.\,q_{31}$ is the replacement cost (positive constant) depends on the duration of the replacement activity of the machine, respectively. Let $c(u)$ be a linear production cost function given by $c(u) = c_u u$. The constant c_u is the cost incurred per unit of produced parts, and used here to penalize the production rate of the machine.

The objective of this research is to find the two decision variables, namely, the production rate $u(.)$ and the replacement variable $w(.)$, that would minimize the expected discounted cost $J(.)$ given by:

$$J(\alpha, x, a, u, w) = \tag{2.16}$$

$$E\left\{\int_0^\infty e^{-\rho t} G(.)dt \,|\, \xi(0) = \alpha, x(0) = x, a(0) = a\right\}, \forall\, (u(.), w(.)) \in \Gamma(\alpha)$$

where ρ is the discount rate, and (α, x, a) are the initial values of the state variables. The set of admissible decisions $\Gamma(\alpha)$ that define the feasible plan $(u(a,.), w(a,.))$ depends on the stochastic process $\xi(t)$, and is given by:

$$\Gamma(\alpha) = \tag{2.17}$$

$$\left\{\left(u(a,.), w(a,.)\right) \in \mathbb{R}^2, 0 \le u(a,.) \le u_{max}\mathrm{Ind}\{\xi(t) = 1)\}, 0 < w(a,.) \le 1\right\}$$

The value function of such a problem is given by:

$$v(\alpha, x, a) = \inf_{(u(\alpha,.),w(\alpha,.))\in\Gamma(\alpha)} J(\alpha, x, a, u, w), \forall \alpha \in B, x \in \mathbb{R}, a \in \mathbb{R}_+ \quad (2.18)$$

The properties of the value function leading to HJB equations of the stochastic optimal control problem are presented in the next section.

2.4 Properties of the value function and optimality conditions

In this section, we develop the optimality conditions using the optimal control theory based on stochastic dynamic programming. The optimal control policy $(u^*(.), w^*(.))$ denotes a minimizer over $\Gamma(\alpha)$ on the right hand side of equation (2.16). This policy shows that the value function $v(.)$ given by equation (2.18) satisfies the set of partial differential equations known as the Hamilton-Jacobi-Bellman equations (HJB). In Annex II, we develop the derivation of the HJB equations. However, the differentiability and viscosity properties of the value function can be found in Kenné, Boukas and Gharbi (2003). From equation (2.15), $G(\alpha, a, x, u, w)$ can be written as follows: $G(\alpha, z, U) = h(x) + c^\alpha + c(u)$, where $z = (x, a)$ and $U = (u, w)$. We extend the domain of the decision variable w serving to describe the controlled replacement to a continuous domain $]0, 1]$, in order to use the notions of continuity and convexity. This however will not affect the optimization result since for the type of problem we are solving, the optimum (over w) always occurs on the boundary of $]0, 1]$. We should recall that in our case, the replacement is allowed for all possible values of the variable w. Below, we will use the following assumption, definition and Lemmas.

Assumption A2.1. $Q(w)$ is a continuous function in w.
Lemma 2.1.

1. $G(\alpha, z, U)$ is jointly convex if, for each $\alpha \in B$ and for every z_1, z_2, U_1 and U_2 and $\delta \in [0, 1]$, e.g.

$$G(\alpha, \delta z_1 + (1-\delta)z_2, \delta U_1 + (1-\delta)U_2) \leq \delta G(\alpha, z_1, U_1) + (1-\delta)G(\alpha, z_2, U_2)$$

Then $v(\alpha, z)$ is also convex if, for each $\alpha \in B$ and for every z_1 and z_2 and $\delta \in [0, 1]$, e.g.

$$\nu(\alpha, \delta z_1 + (1-\delta)z_2) \leq \delta\nu(\alpha, z_1) + (1-\delta)\nu(\alpha, z_2)$$

Moreover, $G(\alpha, z, U)$ is strictly jointly convex in z if the inequality above of G holding as an equality for some $0 < \delta < 1$ implies $z_1 = z_2$ and $U_1 \neq U_2$. Then $\nu(\alpha, z)$ is also strictly convex in z if the inequality above of ν is strict whenever $z_1 \neq z_2$ and $0 < \delta < 1$.

2. For some constants C_g and $K_g > 0$, if $G(\alpha, z, U)$ is locally Lipschitz in z, e.g.

$$|G(\alpha, z_1, U) - G(\alpha, z_2, U)| \leq C_g(1 + |z_1|^{K_g} + |z_2|^{K_g})|z_1 - z_2|$$

Then $\nu(\alpha, z)$ is also locally Lipschitz in z, e.g.

$$|\nu(\alpha, z_1) - \nu(\alpha, z_2)| \leq C_g(1 + |z_1|^{K_g} + |z_2|^{K_g})|z_1 - z_2|$$

Proof. Any function which is convex and locally Lipschitz is continuously differentiable. To prove this property for the value function $\nu(\alpha, z)$ with respect to z and for each $\alpha \in B$, we can refer the reader to the book by Sethi, Zhang and Zhang (2005), and the proof is similar to their proof of Lemma E.1. ■

The value function $\nu(\alpha, \mathrm{z})$ is continuously differentiable in z if and only if $D^+\nu(\alpha, z)$ and $D^-\nu(\alpha, z)$ are both singletons. In this case:

$$D^+\nu(\alpha, z) = D^-\nu(\alpha, z) = \{\nu_z(\alpha, z)\}$$

where $z = (x, a)$. The HJB equations corresponding to the optimal control problem are written as follows:

$$\rho\nu(\alpha, x, a) = \min_{(u,w)\in\Gamma(\alpha)} \begin{bmatrix} G(\alpha, x, a, u, w) + (u-d)\dfrac{\partial}{\partial x}\nu(\alpha, x, a) \\ +ku\dfrac{\partial}{\partial a}\nu(\alpha, x, a) \\ +\displaystyle\sum_{\beta} q_{\alpha\beta}(.)\nu\big(\beta, x, \varphi_a(\xi, a)\big) \end{bmatrix} \tag{2.19}$$

where $\varphi_a(\xi, a)$ is the function that restores the age of the machine to as-good-as-new after the machine is replaced, and to as-bad-as-old after imperfect repair activities. Therefore, at a jump time τ for the process ξ one can write this function as follows:

$$\varphi_a(\xi, a) = \begin{cases} a(\tau^-) & \text{if } \xi(\tau^+) = 1 \text{ and } \xi(\tau^-) = 2 \\ 0 & \text{if } \xi(\tau^+) = 1 \text{ and } \xi(\tau^-) = 3 \\ a(\tau^-) & \text{otherwise} \end{cases} \tag{2.20}$$

where $\xi(t) = \alpha \in B$, and $\frac{\partial v(\alpha,x,a)}{\partial x}$ and $\frac{\partial v(\alpha,x,a)}{\partial a}$ are the first-order partial derivatives of the value function $v(\alpha, x, a)$ with respect to x and to a, in that order. The following definition will be used to prove the Lemma given below.

Definition 2.1. The superdifferentiability $D^+f(x)$ and subdifferentiability $D^-f(x)$ of any function $f(x)$ with respect to x are defined as follows:

$$D^+f(x) = \left\{ s \in \mathbb{R}^n \colon \lim_{h\to 0} \sup \frac{f(x+h) - f(x) - h.s}{|h|} \leq 0 \right\}$$

$$D^-f(x) = \left\{ s \in \mathbb{R}^n \colon \lim_{h\to 0} \inf \frac{f(x+h) - f(x) - h.s}{|h|} \geq 0 \right\}$$

Lemma 2.2. The value function $v(.)$ defined in equation (2.18) is the unique viscosity solution to the HJB equations (2.19).

Proof. To prove the *Lemma 2.2*, we use the concept given by *Definition 2.1* to extend Theorem 2 presented by Yan and Zhang (1997). ■

When the value function is available, an optimal control policy can be obtained as a solution of the HJB equations (2.19). Since an analytical solution is impossible to obtain in general, it is a common practice to develop numerical methods for solving the HJB equations. Boukas and Haurie (1990) implemented a numerical method initially introduced by Kushner and Dupuis (1992) to solve such a problem in the context of production planning. Kushner's method used to solve the proposed optimality conditions is presented in Annex II. In the subsequent section, we provide a numerical example to illustrate the structure of the control policies.

2.5 Numerical example

The computational domain of the state variables is defined as $G_{xa} = G_x^h \times G_a^h$ such that:

$$G_x^h = \{x: -10 \le x \le 30\},\ G_a^h = \{a: 0 \le a \le 100\} \tag{2.21}$$

with $h_x = 0.5$ and $h_a = 2$. The imperfect repair is characterized by a minimal repair which brings the machine to as-bad-as-old conditions. The machine failure rate is assumed to be age-dependent. Using the model of the failure rate given by equation (2.5), we can obtain its trajectory according to the machine age, as illustrated in figure 2.4 with values of $A_0 = 10^{-4}, A_1^{\infty} = 0.01$ and $A_2 = 5 \times 10^{-6}$.

Other parameters needed in the numerical example are presented in Table 2.1. These parameters have been chosen such as to ensure that the system is feasible. Using the formula of equation (2.14), the feasibility of the system was verified and respected on the all computational domains of the age, where $(q_{21}^{-1}, q_{31}^{-1}, q_{13}^{-1})$ are the mean time to repair, the mean time to replace and the mean time between replace of the machine, respectively.

The production rate and replacement policy are presented in figures 2.5 and 2.6, respectively. We can see that the obtained optimal production control policies can be viewed as an extension of the hedging point policy presented in Akella and Kumar (1986). In our case, however, we take into account the effect of the deterioration of the machine along the production phase, as well as the imperfect repairs. The optimal production control policy presented in figure 2.5, and defined by $Z(a)$, which shows the numbers of parts to hold in inventory in order to hedge against failures of the machine with the age a. This control policy consists of the following rule:

- if the current stock level $x(t)$ is under the threshold level value $Z(a)$, the production rate is to be set to its maximum value;
- if it is exactly at the threshold level, the production rate is to be set to the demand rate, and
- if it is above the threshold level, the production rate is to be set to zero (produce nothing).

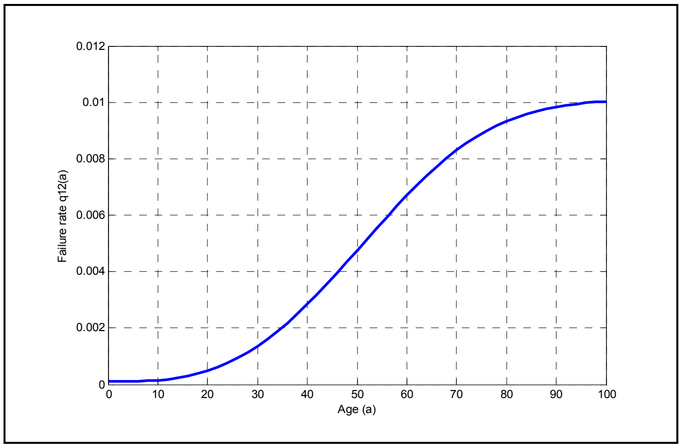

Figure 2.4 Age-dependent failure rate of the machine

Table 2.1 Parameters of the numerical example

Parameter	c^+	c^-	c_r	c_0	c_u
Unit	(\$/product/time unit)	(\$/missing product/time unit)	(\$/time unit)	(\$/replacement)	(\$/product)
Value	10	150	25	3000	100
Parameter	u_{max}	d	ρ	k	w
Unit	(products/time unit)	(products/time unit)			
Value	0.55	0.4	0.01	0.8	]0,1]
Parameter	q_{21}^{-1}	q_{13}^{-1}	q_{31}^{-1}		
Value	20	45	14		

Thus, the production control policy satisfies:

$$u^*(1,x,a) = \begin{cases} u_{max} & \text{if } x(t) < Z(a) \\ d & \text{if } x(t) = Z(a) \\ 0 & \text{if } x(t) > Z(a) \end{cases} \tag{2.22}$$

where $Z(a)$ is the machine-age dependent function that gives the optimal production threshold in operational mode for each value of age of the machine, as illustrated in figure 2.5.

We could not prevent the machine from getting older because only minimal repair (ABAO) is possible. Therefore, the machine deteriorated further, and after a certain age, could no longer satisfy the demand. Thus, a replacement policy defines when to replace the machine, taking into account the required stock level allowing the demand rate to be met when the machine is sent for replacement. In other words, the problem to be solved is determining the levels (x^*, a^*) at which the machine should be replaced with a new identical one in order to optimize the manufacturing system.

Figure 2.6 shows that the optimal control for the replacement policy is a bang-bang solution. The reason for this is that the optimal control switches from the upper bound to the lower bound, and is restricted somewhere between the two. Let $w(1, x, a)$ denotes a switching function based on the stock level $x(t)$ and the age of the machine $a(t)$, with the maximum value $w_{max} = 1$ if the machine must be sent for a replacement activity, and the minimum value (lower value) $w_{min} = 10^{-5}$ if the option to replace the machine is not recommended. We should recall that we only consider the decision process until the first replacement. Thus, the replacement policy can be written as follows:

$$w^*(1, x, a) = \begin{cases} w_{max} & \text{if } \big(x(t), a(t)\big) \in \text{zone B} \\ w_{min} & \text{otherwise} \end{cases} \tag{2.23}$$

where states variables $\big(x(t), a(t)\big)$ are the parameters of zone B. We can also see that figure 2.6 shows the behaviour of the decision variable responsible for replacement, based on the machine age and inventory level. To better understand the obtained policy, we divide the plane (x, a) into two zones, A and B.

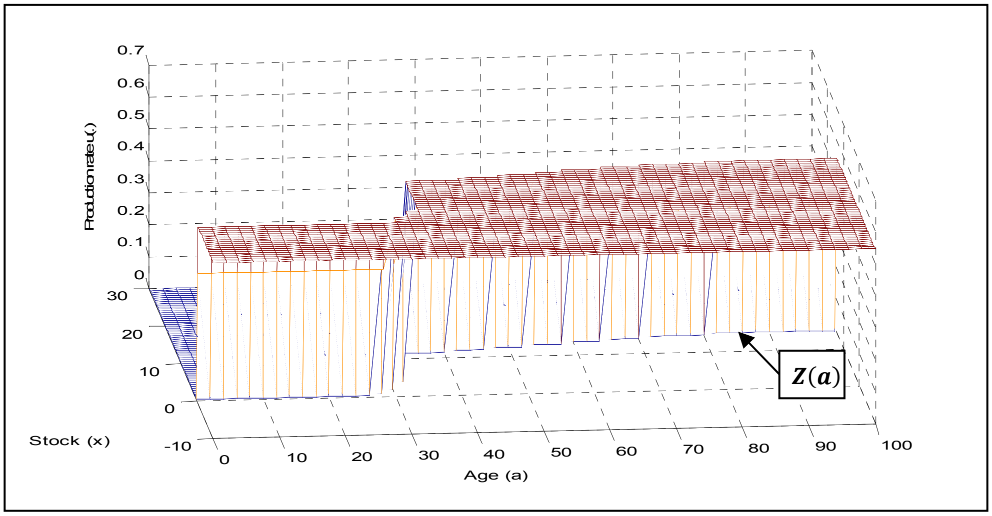

Figure 2.5 Production policy of the manufacturing system

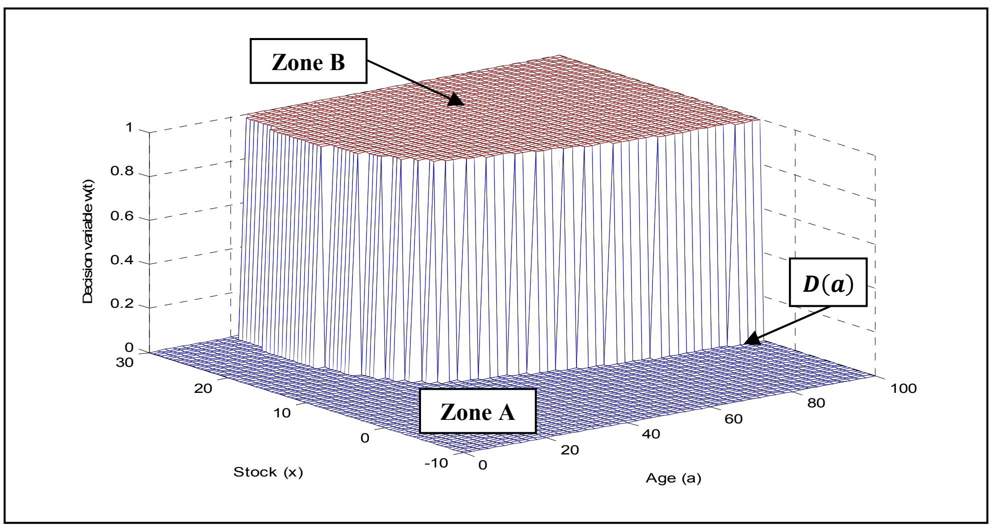

Figure 2.6 Replacement policy

Zone A: the replacement policy does not recommend sending the machine for replacement and the inventory level is low. The machine is still new in this zone, and able to satisfy the demand with a rare fear of failure. Thus, it is not necessary to build a significant inventory level. This result is in a good agreement with the "zero inventory condition" of Bielecki and Kumar (1988), which asserts that a zero-inventory policy can be exactly optimal even in the presence of uncertainty. Hence, the decision variable $w(.)$ is set to its minimum value, $(w(.) = 10^{-5} \approx 0)$.

Zone B: the machine is aging and the failure rate increases. Here, the replacement cost is justified, and performing a replacement becomes necessary. However, before that is done, the manufacturing system must ensure a certain inventory level to satisfy the demand, and to hedge against possible failures and a non-productive replacement time. In this case, the decision variable $w(.)$ is set to its maximum value, ($w(.) = 1$).

To better illustrate the optimal production and replacement policies, we can use their boundaries defined by $Z(a)$ in figure 2.5 and $D(a)$ in figure 2.6, respectively. The control policy defined by figure 2.7 is known as the switching curve policy, and is characterize by a vector z of two threshold parameters $z = (Z(a), D(a))$, the optimal production and replacement switching levels. For every fixed a, when $Z(a) \equiv Z$, and $D(a) \equiv D$, the switching curve policy obtained becomes an extension of the so-called hedging point policy, and Z and D are the hedging points. Exploring the impact of aging, we are interested in the functions $Z(a)$ and $D(a)$, which are the optimal threshold levels of production and replacement for each age a. In the context of deterioration, we note that the production threshold $Z(a)$ presented in figure 2.7 increases progressively. Indeed, when the machine is in its early life period, it is still relatively new and failures are rare. Thus, the inventory level should be maintained almost to a value closer to zero. When the machine is aging and the failure rate increases, building a certain inventory level becomes important, and this level increases with the age of the machine. We note that all values of $w(.)$ chosen between $(10^{-5} \approx 0)$ and 1 are permitted in order to extend the domain of $w(.)$ to a continuous domain, and can apply the theory of convexity, but we know that the optimal solution is reached at the boundary value 10^{-5} or 1, confirming what was mentioned in the section, "Properties of the value function and optimality conditions".

In figure 2.7, the intersection between the production threshold $Z(a)$ and the replacement trace $D(a)$ defines the feasible zone C, where the manufacturing system resides. This zone recommends optimally, the age of the machine and the necessary stock level when the machine should be replaced. Thus, the replacement policy can be rewritten as follows:

$$w^*(1,x,a) = \begin{cases} w_{max} & \text{if } \big(x(t),a(t)\big) \in \text{zone C} \\ w_{min} & \text{otherwise} \end{cases}, \text{ with C} = \text{A} \cap \text{B} \tag{2.24}$$

where states variables $\big(x(t),a(t)\big)$ are the parameters of zone C. We note that the point s highlighted on figure 2.7 is a point where the production threshold $Z(a)$ intersects with the replacement trace $D(a)$, and this is a recommended point for machine replacement.

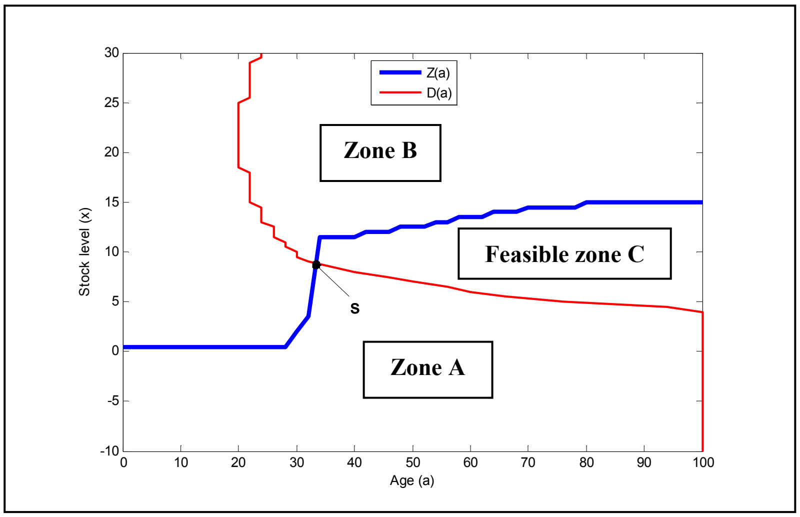

Figure 2.7 Production threshold and replacement trace

In the next section, we will confirm the obtained structure of the control policies through a sensitivity analysis.

2.6 Sensitivity analysis

The obtained control policies are validated through a sensitivity analysis by varying some parameters of the model. We analyze the behaviour of the production threshold $Z(.)$ and the replacement trace $D(.)$ by varying the following parameters: the costs of backlog, surplus, production and replacement, the mean time to repair and the mean time to replace. In the following cases of the variation of parameters, we can note that the symbols s_1, s_2 and s_3 are highlighted on the following figures, in order to illustrate the intersection points of the trace

$D(.)$ and the production threshold $Z(.)$ when the replacement of the machine is recommended for first time. The three intersection points s_1, s_2 and s_3 correspond respectively to the small, middle, and high values of the parameter which will be varied in the sensitivity analysis.

2.6.1 Backlog cost variation

The results presented in figure 2.8 for three different backlog cost values $c^- = 100, 150$ and 200 show that when the backlog cost increases, the replacement of the machine is more highly recommended. That is because in this situation, the production threshold $Z(.)$ must be increased as c^- increases in order to avoid shortages during periods of future failures of the machine, and to reach this level, the machine must produce more parts at its maximum production rate u_{max}. In other words, the machine takes more time to produce at u_{max}. Thus, the machine will deteriorate more rapidly, and for this reason, we replace it earlier: $a(s_3) < a(s_2) < a(s_1)$. Consequently, this increases the feasible zone C, and we can observe that the variation of the backlog cost is directly linked to the size of the replacement zone.

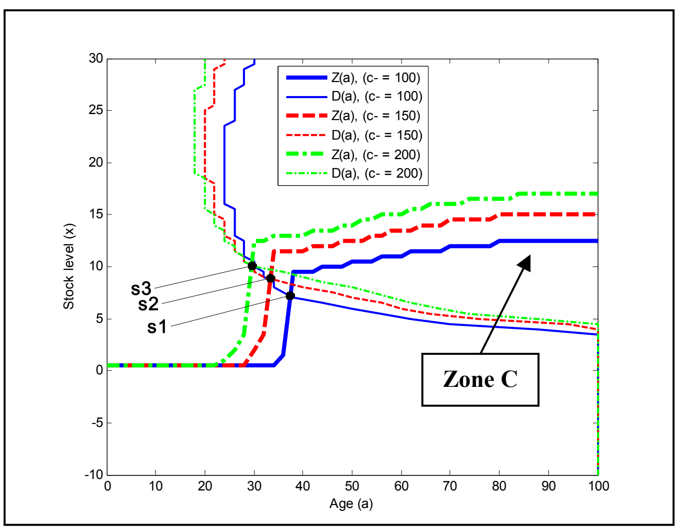

Figure 2.8 Variation of the backlog cost and its effect on the production and replacement policies

2.6.2 Inventory cost variation

As we can see in figure 2.9, the next step in the sensitivity analysis is to examine the inventory cost parameter. The values used are $c^+ = 5, 10$ and 15. When the inventory cost c^+ increases, the replacement of the machine is less recommended. The reason for this is that the production threshold $Z(.)$ must be decreased as c^+ increases, and in this case, the machine must produce for a shorter period at its maximum production rate u_{max}. Thus, the machine will deteriorate more slowly, and for this reason, we replace it later: $a(s_1) < a(s_2) < a(s_3)$; this means the feasible zone C is decreased as c^+ increases.

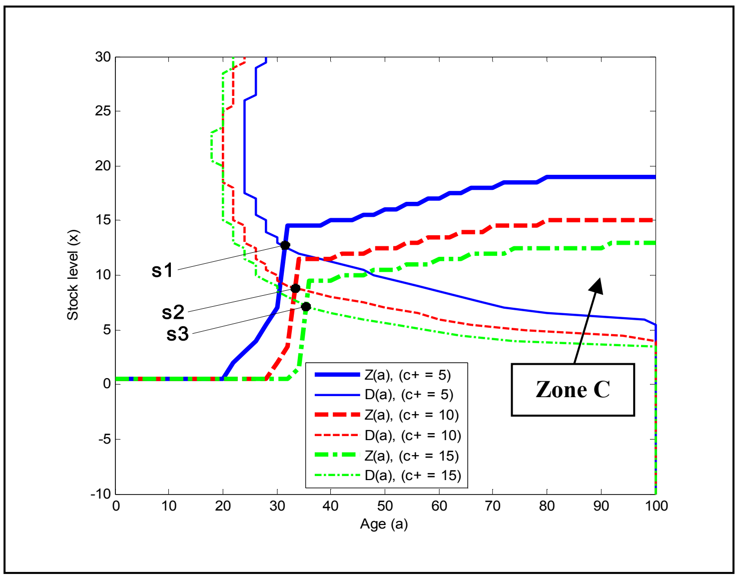

Figure 2.9 Variation of the inventory cost and its effect on the production and replacement policies

We can conclude from figures 2.8 and 2.9 that the effects of the backlog cost on the production and replacement policies are the inverse of what is seen with the inventory cost. We know that the machine deteriorates with age when it produces the parts. When the threshold increases (in the case of c^- increases or c^+ decreases), the machine produces more parts at its maximum production rate u_{max} and therefore it is aging rapidly, and it is recommended that it be replaced, and for this reason, the feasible zone C increases. Conversely, when the threshold decreases (in the case of c^- decreases or c^+ increases), the machine produces fewer parts, and ages slowly, and for this reason, the feasible zone C decreases.

2.6.3 Production cost rate variation

We will now analyze the variation of the production cost rate c_u for three values $c_u = 100, 500$ and 1000. The results presented in figure 2.10 show that by increasing the production cost rate, the replacement is less recommended, since when the production is more penalized, the production threshold $Z(.)$ must be decreased, and to reach this level, the machine must take less time to produce parts at its maximum production rate u_{max}. Thus, the machine will deteriorate more slowly, and we have to replace it later: $a(s_1) < a(s_2) < a(s_3)$ and the feasible zone C will decrease.

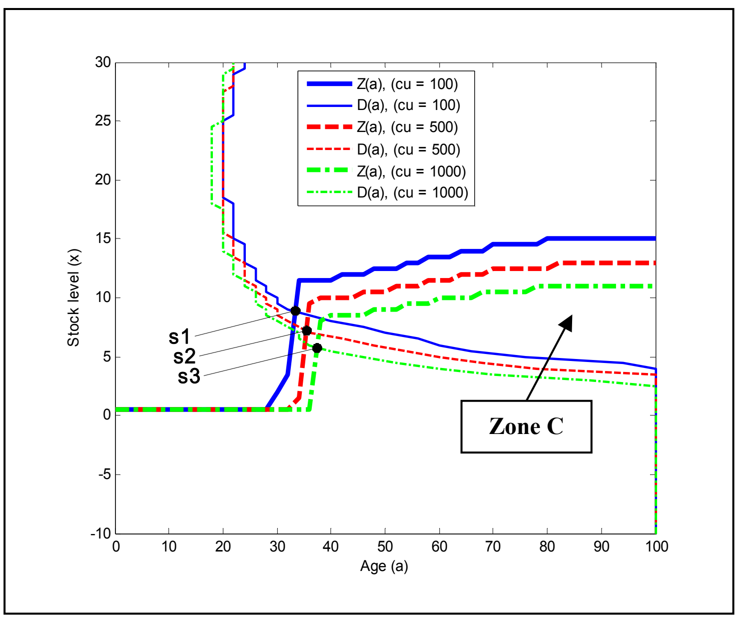

Figure 2.10 Variation of the production cost rate and its effect on the production and replacement policies

2.6.4 Replacement cost variation

Now, we illustrate the effect of the variation of the replacement cost c_0 on the production and replacement policies. As we can see from figure 2.11, with three different cases, as $c_0 = 1500, 3000$ and 4500, when the replacement cost increases, replacement is less recommended. The reason is that the cost to replace the machine is higher, and it is preferable to keep it longer. That is why we replace the machine later: $a(s_1) < a(s_2) < a(s_3)$. However, with respect to the production policy, the replacement costs have not reported any influence on the production threshold $Z(.)$ when the machine is at its early stage of life. The change will occur only before entering in the replacement zone, and the system will build a certain stock level to hedge against shortages and to continue to meet customer demand. Once the decision to send the machine for replacement is made, it seems logical that the behaviour of the production threshold $Z(.)$ over age remains the same (as observed in all three cases from figure 2.11), because the mean time to replacement is the same in all three cases. Finally, it can be seen that the feasible zone C decreases as the replacement cost increases.

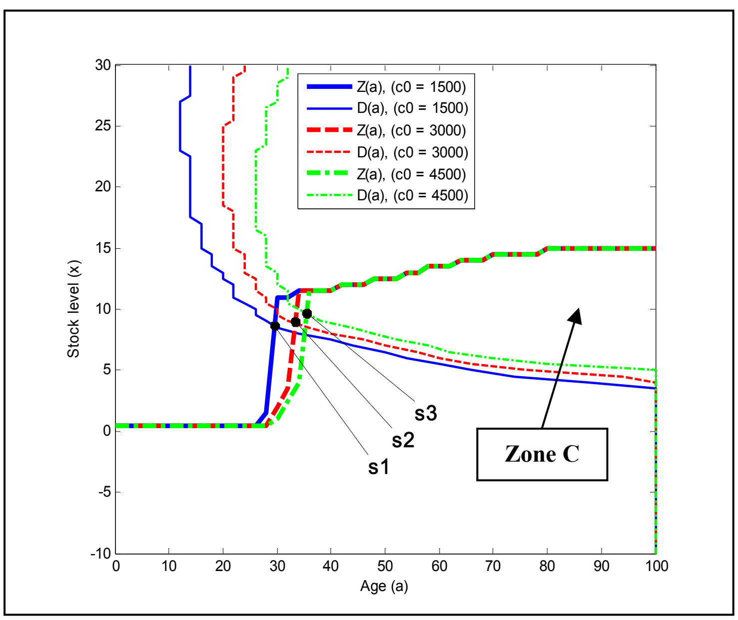

Figure 2.11 Variation of the replacement cost and its effect on the production and replacement policies

2.6.5 Mean time to repair variation

We now discuss the variation of the mean time to repair $MTTR$ for three values, $MTTR = 15, 20$ and 25. From figure 2.12, we can observe that when the $MTTR$ increases, replacement is more highly recommended. The reason is that the machine becomes less available, and to continue to meet customer demand, the production threshold $Z(.)$ must be increased; however, to be able to do that, the machine must produce more parts at its maximum production rate u_{max}. Thus, we will have a more rapid deterioration of the machine, which is why we replace the machine earlier. We conclude that the age at the intersection point is $a(s_3) < a(s_2) < a(s_1)$, and the feasible zone C will be increased.

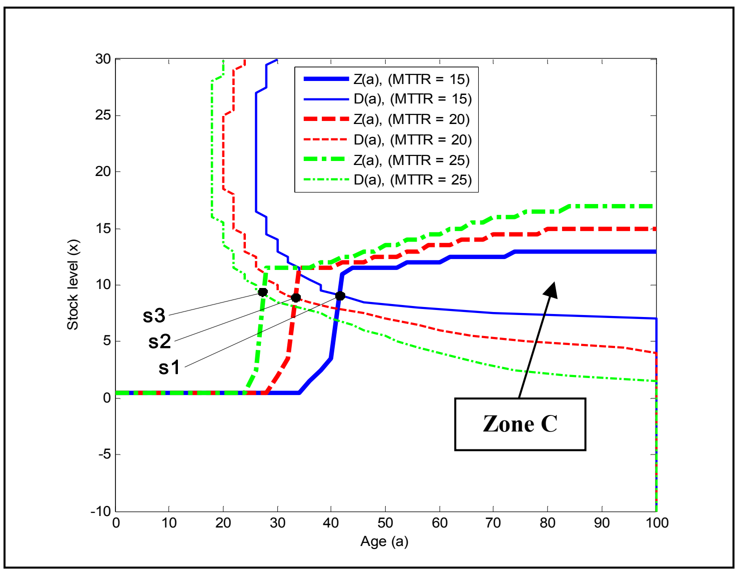

Figure 2.12 Variation of the mean time to repair and its effect on the production and replacement policies

2.6.6 Mean time to replacement variation

To complete the sensitivity analysis, we study the variation of the mean time to replacement $MTTRP$, for the values $10, 14$ and 18. We know that the availability of the machine decreases when more time is needed to replace it. As we observe in figure 2.13, when the

$MTTRP$ increases, replacement is less recommended. The reason is that when it takes time to replace the machine, for the optimal control policy, it is preferable to keep it longer. Thus, we replace the machine later with the age at the intersection point $a(s_1) < a(s_2) < a(s_3)$. Once the decision to send the machine for replacement is made, the machine must first produce more parts at its maximum production rate u_{max} to hedge against periods of non-production, when the $MTTRP$ increases. Thus, the replacement will be performed with a higher age, and with a higher stock level. In this case, the feasible zone C will be decreased. We note that the $MTTRP$ has the same effect on the replacement policy as the replacement cost c_0.

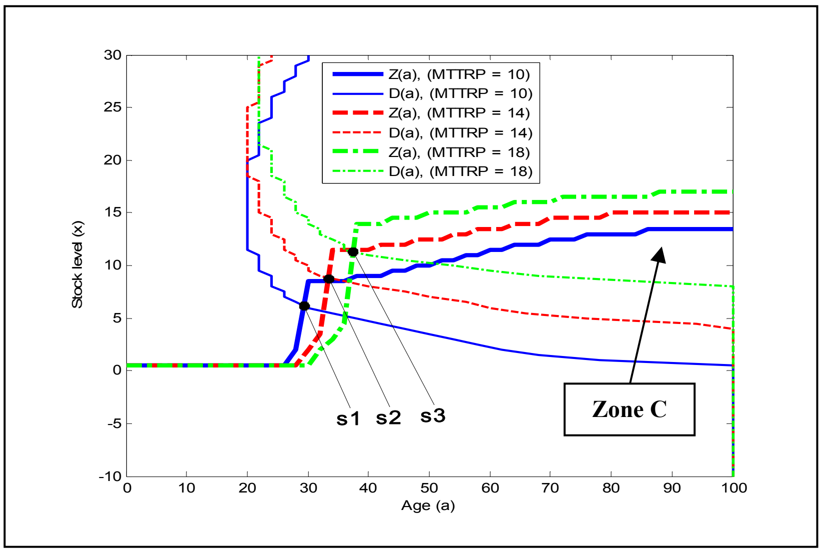

Figure 2.13 Variation of the mean time to replace the machine and its effect on the production and replacement policies

2.7 Discussions

The numerical results presented in this paper show that the control policy for the considered manufacturing system is not a traditional hedging point policy, but rather, is a modified one when the machine deteriorates with age, and the repair activities are as-bad-as-old. This policy is of hedging point type in the sense that it is fully characterized by a curve within a domain of two parameters - the stock level x and the age a. We can conclude from the sensitivity analysis presented that the structure of the control policy obtained by our proposed approach is maintained when the parameters of the system vary. Incorporating the new production cost factor in the cost function leads to a reduction of the inventory level while continuing to meet the customer demand, and keeps the machine longer within a low cost level, because it is aging more slowly. The results presented were obtained with the semi-Markovian model because the simpler (Markovian) model is not appropriate for systems subject to deterioration caused by machine aging. A careful analysis of applicability of optimization techniques based on a numerical solution of HJB equations was performed. The production and replacement policies are defined simultaneously by the function $Z(a)$ and $D(a)$, the production threshold trace and the replacement trace, respectively. These two traces, and their intersection point in particular, determine the feasible zone C and the recommended stock and age levels, at which the decision regarding the machine replacement has to be made.

2.8 Conclusion

In this paper, we determine simultaneously the optimal production and replacement strategies for the case of one machine and one product subject to random failures and repairs. A stochastic optimization model in continuous time has been developed. The considered manufacturing system is under machine's age deterioration with the combined effect of imperfect repairs and AGAN-replacement. By penalizing the production rate, we can explore and observe the impact of the machine aging on the optimal control strategies (production (rate) and replacement policies). From our results, it seems reasonable to incorporate the production parameter into the process of finding optimal control policies in order to get a better solution. This work is strictly related to the age phenomenon, and provides a mathematically accurate analysis of the problem at hand in order to ensure the convergence of numerical methods based on the convexity of two-dimensional (u, w) controls. Since the machine is not new after repair activities, a semi-Markov decision process has to be used in describing its dynamics, which seems more realistic in practice. The optimality conditions were developed in the form of the HJB equations using the stochastic optimal control and dynamic programming approach. The solution of the stochastic control problem is obtained using numerical methods. An extensive sensitivity analysis is performed to validate the structure of the obtained control policies. Finally, an extension of this model to the case of several machines remains to be done, and is a part of our ongoing research.

CHAPITRE 3

ARTICLE 2: PRODUCTION AND REPLACEMENT POLICIES FOR A DETERIORATING MANUFACTURING SYSTEM UNDER RANDOM DEMAND AND QUALITY

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Résumé

Ce travail étudie la planification de la production d’un système manufacturier non fiable dans un contexte de détérioration et de présence d’incertitudes. L’effet du phénomène de détérioration sur la machine est principalement observé dans sa disponibilité et dans la qualité des pièces produites, où le taux de défaillance et le taux de rejets augmentent avec l’âge de la machine. L’option de remplacer la machine devrait être considérée pour atténuer l’effet de la détérioration afin d’assurer une satisfaction à long terme de la demande. L’objectif de cet article est de trouver le taux de production et la politique de remplacement qui minimisent le coût total actualisé, incluant les coûts de stockage, de pénurie, de production, de réparation et de remplacement, sur un horizon de planification infini. Nous formulons le problème de commande stochastique dans le cadre d’un processus de décision semi-Markov pour considérer l’historique de la machine. L’intégration des comportements aléatoires de la demande et de la qualité nous a amené à proposer une nouvelle approche de modélisation en développant les conditions d’optimum en termes d’approximation de second ordre des équations de Hamilton-Jacobi-Bellman (HJB). Des méthodes numériques sont utilisées pour obtenir les politiques de commande optimale. Finalement, un exemple numérique et une analyse de sensibilité sont présentés afin d'illustrer et de confirmer la structure de la solution optimale obtenue.

Abstract

This work investigates the production planning of an unreliable deteriorating manufacturing system under uncertainties. The effect of the deterioration phenomenon on the machine is mainly observed in its availability and the quality of the parts produced, with the rates of failure and defectives increasing with the age of the machine. The option to replace the machine should be considered to mitigate the effect of deterioration in order to ensure long-term satisfaction of demand. The objective of this paper is to find the production rate and the replacement policy that minimize the total discounted cost, which includes inventory, backlog, production, repair and replacement costs, over an infinite planning horizon. We formulate the stochastic control problem in the framework of a semi-Markov decision process to consider the machine's history. The integration of random demand and quality behaviour led us to propose a new modeling approach by developing optimality conditions in terms of a second-order approximation of Hamilton-Jacobi-Bellman (HJB) equations. Numerical methods are used to obtain the optimal control policies. Finally, a numerical example and a sensitivity analysis are presented in order to illustrate and confirm the structure of the optimal solution obtained.

Keywords: Flexible manufacturing systems, Random process, Production planning, Random quality, Random demand, Minimal repairs, Replacement policy, Numerical methods.

3.1 Introduction

During the past few decades, the problem of random phenomena in the control of manufacturing systems has come to represent a fertile area that has been widely investigated in order to maintain the company's market position. Many stochastic factors inherent in manufacturing systems can make it more difficult for managers to plan and control the production process. Among these factors are machine breakdowns and repairs, customer demand, and the quality of the produced parts. Each of these factors must be managed appropriately in order to develop control strategies for a more accurate industrial practice. Additionally, the manufacturing system is subject to deterioration, which has a somewhat negative influence, not only on the normal system operation, but also on the quality of the parts produced. Therefore, the manufacturing system is no longer capable of fulfilling customer demand after a certain level of deterioration. One of attractive options is to reduce the effects of deterioration in order to improve the system performance through the use of major maintenance activities, also known as replacement. However, in the context of this deterioration, including such random elements in mathematical models may take into consideration a certain amount of complexity in terms of modeling and analysis, and render the tasks of production planning and optimization very challenging. Furthermore, further research is required in order to have a better understanding of the behaviour of the production system.

The importance of production planning problem under uncertainties has been widely recognized. A one-machine, one-part type production system was formulated as a stochastic optimal control problem in Akella and Kumar (1986). Moreover, the demand rate was considered constant and the dynamics of failures and repairs of the machine was modeled by a two-state continuous-time Markov chain. There, the structure of the optimal control policy was obtained analytically, and was given by a threshold policy to minimize the discounted costs of inventory and backlog over an infinite horizon. Later on, for a similar production system, the counterpart of long-run-average cost was studied by Bielecki and Kumar (1988) and the optimal solution can be regarded as the so-called hedging point policy, which is a special type of threshold policy. Many important features of real life systems, not taken into account by these authors, are the random aspects of demand, the influence of quality issues on the control policy and the possibility to do replacement activity in order to reduce the effects of deterioration.

In Dehayem-Nodem, Kenné and Gharbi (2009), the authors extended the work of Love *et al.* (2000) to a manufacturing system facing a constant demand rate under imperfect repairs to simultaneously determine the production and repair/replacement policies. Their results are limited to deterministic demand with no consideration of its randomness. Considering repairs or other maintenance activities as imperfect, they assumed that the machine was aging, and that this deterioration affected only its availability, while disregarding its effect on quality. To make this assumption more realistic, the deterioration process of the machine also had to incorporate the quality of parts produced.

Given this context, we found some authors that linked the deterioration of the machine to the quality of the parts produced. For instance, Kim and Gershwin (2005, 2008) developed mathematical models integrating production and quality issues to analyze the performance of small and larger manufacturing systems. In the same direction, Colledani and Tolio (2011) presented a new approximate analytical method to evaluate the performance of production systems by simultaneous tackling quality and production issues. Panagiotidou and Tagaras (2007), Xiang (2013) and Panagiotidou (2014) presented an economic model for the optimization of preventive maintenance in a production process with two possible quality states characterized by different failure rates. Furthermore, Radhoui *et al.* (2010) developed a mathematical model for one machine system producing lots of products to satisfy a constant demand rate. Each lot produced by the machine is subject to a quality control. They proposed a joint strategy of production, quality control and preventive maintenance for systems with random failure characteristics. In the same context, Bouslah *et al.* (2016) proposed a new holistic approach addressing the problem of inspection of defective production for a stochastic production system subject to both reliability and quality deteriorations, in which the quality control is performed by a simple acceptance sampling plan. Additionally, a more detailed discussion about the combined effect of deterioration on quality and reliability can be found in Rivera-Gomez *et al.* (2016). The authors proposed a joint production and major maintenance control policy with subcontracting options available to supplement the limited production capacity for a deteriorating manufacturing system. Similarly, Ouaret, Kenné, Gharbi and Polotski (2015) analyzed the simultaneous production planning and quality control problem for an unreliable single-product-single-machine system responding to a constant demand rate. The authors considered that the deterioration caused by the age of the machine progressively affects its availability and the rate of defectives (defectives rate is considered variable). We note that although mathematical models provided in this set of works did not incorporate random aspects of demand and quality, they give a clear idea of industrial benefits addressing the relationship problem among quality and productivity in manufacturing systems.

Most of the above cases are mainly based on deterministic demand and deterministic quality issues. In industrial contexts, omit the randomness does not give realistic results, and consequently, not to consider them is very restrictive (Sethi and Thompson, 2000). In the stochastic optimal control theory, the state of the system is represented by a controlled stochastic process. We will introduce in this paper the possibility of controlling a system governed by a stochastic differential equation of a type known as Itô equation. This equation arises when the state equation is perturbed by stochastic diffusion-type processes (Davis, 1979; El-Gohary, Tadj and Al-Rahmah, 2007; Yin *et al.*, 2003). We know that the randomness of customer demand is an external parameter which is a function of the market, while the randomness of quality usually depends on system parameters, such as imperfect or minimal repairs and deterioration caused by operating age. Pham and Wang (1996) and Wang (2002) suggested some reasons responsible of minimal or imperfect repairs, including that the faulty part is often repaired only partially and that human interventions often lead to further damage during maintenance. The operating age as a factor is due to the fact that after a certain age, the machine loses its perfect adjustability. This leads to production errors, and increases the quantity of products that are discarded. From a practical point of view, it is clear that manufacturing systems experience breakdowns, repairs, fatigue, wear, human interventions and corrosion, all leading to the random deterioration of different aspects of the machine, such as availability, reliability and quality. However, when we consider that both demand and quality processes are stochastic, the optimization problem becomes more complex, and thus making it crucial to require the use of appropriate models for its analysis.

Some papers have highlighted the importance of the randomness that has often been neglected on the quality and demand; these include the work by Perkins and Srikant (2001), which provided the hedging point policy for a class of failure-prone single machine production systems with a compound Poisson process demand. Likewise, Yin *et al.* (2003) obtained the optimal production policies of a paper manufacturing machine by applying the dynamic programming principle. They considered two types of uncertainty, demand and production capacity, and formulated them using two finite-state continuous-time Markov chains. Bensoussan *et al.* (2005) reviewed the stochastic inventory problem with a demand model consisting of a mixture of diffusion and Poisson processes, and another with a constant demand and a Poisson process. Using the theory of impulse control, they reduced the Bellman equation of the dynamic programming problem to a quasi-variational inequality (QVI) set in order to obtain the optimal policy. Later, Presman and Sethi (2006) addressed a stochastic inventory problem in continuous time with both average and discounted criteria. The authors extended the inventory model of the economic manufacturing quantity (EMQ) with constant demand, adding the Poisson-distributed portion to it. Kutzner and Kiesmüller (2013) introduced a mathematical model to cover a random demand and the influence of deterioration on part quality for the case of degrading system. The demand is modeled as a discrete random variable, and the production process is considered imperfect and may produce both, acceptable and defective parts depending on the process state. In other words, produce a very high percentage of acceptable goods with the in-control state and a higher percentage of defective products with the out-of-control state. According to the above discussion, we can observe that the demand was the only source of random behaviour and quality deterioration of the parts produced (due to operating age) and replacement option were completely neglected. Further, some authors have incorporated random quality issues in the inventory system. For instance, Papachristos and Konstantaras (2006) investigated the economic ordering quantity (EOQ) model with unreliable supply process for deteriorating items characterized by a proportional imperfect quality, which is a random variable. Eroglu and Ozdemir (2007) extended the EOQ model to the case that both defective and scrap rates are random variables. This idea of randomness, which has been addressed by Papachristos and Konstantaras (2006) and Eroglu and Ozdemir (2007), is developed in a very restricted context by considering the assumption that the quality is modeled as a random variable. But due to so many unavoidable factors this assumption may not always be true in practice. Obviously, to make a model for quality more realistic, it is necessary to generalize the notion of the random variable by using a stochastic process (a collection of random variables). Moreover, the defective items are due to ordered lots where usually the quantity received is deteriorating in nature. However, they have not included the effects of deterioration on the machine (its availability and its quality of the parts produced) and the replacement in their model.

As it can be seen from these models, the random demand and the random quality issues facing finished products are addressed separately, and without any consideration of the deterioration effect on the machine. Hence, our proposal is the joint integration of random behaviour of demand and quality issues, with special attention paid to deterioration mechanisms caused by the production process and human interventions (especially related to wear and minimal repairs, respectively). To the best of our knowledge, there is no available work in the literature covering jointly these issues, and our work is the first one that aims to investigate the combined impact of the random components of demand and quality in a context of aging, and show its influence on the joint control of production and replacement. This approach leads us to review our production planning strategies, and to develop an integrated mathematical model in which the random aspects of demand and quality deterioration are considered, and their interaction examined. In our work, the manufacturing system under consideration consists of one machine and one produced part type. The random phenomena considered are machine breakdowns and repairs, the quality of the parts produced, and customer demand. The machine deteriorates while in operation; corrective maintenance activities are considered minimal, and restore the age of the machine to as-bad-as-old conditions. Thus, if the machine undergoes deterioration with its age, and only minimal repairs are conducted, this leads to a memory problem. In this situation, the semi-Markov model is more appropriate for describing the dynamics of the machine than the Markov model. Since the age is not reset to zero, the machine should be replaced in the long term in order to continue to satisfy the random customer demand.

The optimization problem consists in the joint determination of the production and replacement policies in a stochastic manufacturing environment with deteriorations and minimal repairs. In this paper, we intend to develop a new mathematical model of quality deterioration and demand including their random components into the system dynamics. Our focus is related to the random aspects of quality deterioration, coupled with random demand, and their integration into a production planning and replacement problem. The decision variables are the production and replacement rates, are determined in order to minimize the total discounted cost, including inventory, backlog, production, repair and replacement costs, over an infinite planning horizon. With random demand and random quality, the stochastic control problem becomes very complex and has not been studied in the literature. We will demonstrate that when uncertainties of demand and quality are introduced, it becomes inevitable to extend the classical stochastic model into an appropriate version in order to capture these random phenomena. We formulate the stochastic control problem as a dynamic programming problem and develop optimality conditions in the form of a second-order approximation of HJB equations. The variability in the demand and quality deterioration rates leads to the second-order terms, contrary to the case of constant demand rate and defectives rate, which results in first-order equations. However, an analytical solution for such a production planning problem is impossible to obtain. Therefore, the use of numerical methods is then necessary to approximate the optimal solution. A sensitivity analysis is presented in order to confirm the structure of different scenarios studied.

The rest of the paper is structured as follows. The industrial context of the paper is presented in Section 3.2. In Section 3.3, we describe the notations and the assumptions used in the model. The stochastic control problem is also formulated in detail in Section 3.3. A numerical example is illustrated in Section 3.4. Section 3.5 discusses some managerial implications for the obtained results. A sensitivity analysis is given in Section 3.6 to demonstrate the performance of the obtained control policies. A comparison of different scenarios of random demand and defectives rates and discussions are presented in Section 3.7, while Section 3.8 concludes the paper.

3.2 Industrial context

The model proposed in this paper addresses a production control problem for a deteriorating manufacturing system. The machine concerned undergoes a progressive deterioration that has the effect of aging it with parts produced. It can also be characterized by the fact that minimal repairs conducted on it decrease its availability, and it must be replaced after a certain level of aging. Our model can be applied to many industries, in situations where the production system may deteriorate over time and can be subject to random breakdowns and repairs, random demands, as well as to random quality deterioration of parts produced. The phenomena of failures and repairs of such systems have already been experienced in aircraft engines, machine tools and paper manufacturing plants, as mentioned in Kouedeu *et al.* (2014). Many practitioners and researchers have looked at production problems with deterioration, where customer demand and/or quality factors are considered constant. Examples of such systems can be found in Dehayem-Nodem *et al.* (2009) and Rivera-Gomez *et al.* (2016). A typical example covering random demand is presented in Yin *et al.* (2003), in the context of a large paper manufacturing process. Other examples examining the random effect of deterioration of machines on part quality are described in Kim and Gershwin (2005) for the automotive sector, where the machine starts producing bad parts when the defect takes place due to common causes of variations (i.e. due to a failure, such as sudden tool damage).

This work is thus motivated by the need for better production planning in many industrial applications such as those mentioned in this section. The primary tools (formulation, approaches and procedures) in this work could possibly be generalized to production planning problems in other industries characterized by deterioration (such as the pharmaceutical, semiconductor, computer and telecommunications industries), when random demand and random quality are present, machines are unreliable, and their production rates can be controlled. The applicability of our obtained decisions can improve the performance of the manufacturing system by determining the necessary stock and the appropriate moment for machine replacement. Basing decisions on random demand and random quality with deterioration is very interesting due to the fact that it has not previously been discussed in the literature, whereas the industry is indeed facing this kind of problem. Below, we formulate a novel stochastic optimization model and apply appropriate techniques for its solution.

3.3 Formulation of the control problem

This section defines the notations and assumptions used throughout this paper, as well as the problem statement.

3.3.1 Notations

$x(t)$ stock level at time t

$a(t)$ age of the machine at time t

$u(t)$ production rate of the manufacturing system at time t

$w(.)$ control variable for the replacement of the manufacturing system

u_{max} maximum production rate of the manufacturing system

$D(.)$ random demand rate of customers

$\beta(.)$ random rate of defectives

$\xi(t)$ stochastic process of the system at time t

$Q(.)$ transition rate matrix

$q_{\alpha\alpha'}(.)$ transition rate from mode α to mode α'

ρ	discount rate
$G(.)$	instantaneous cost function
$J(.)$	expected discounted cost function
$v(.)$	value function
$c(.)$	production cost function
c_r	repair cost rate
c_u	production cost rate
c_0	replacement cost
c^+	inventory cost
c^-	backlog cost

3.3.2 Assumptions

The following is a summary of the main assumptions considered in this paper:

1. The customer demand is known and subject to a compound diffusion process rate (a mixture of a constant demand rate and a diffusion process) over time (products/time unit);
2. The rate of defectives is assumed to be a compound diffusion process of the machine age (a mixture of a dynamic average rate and a diffusion process);
3. The machine deteriorates with its operating age, and the failure rate of the machine increases with its age;
4. The machine repair activities are minimal, and restore its age to as-bad-as-old conditions;
5. The machine is replaced by a new identical one when it undergoes a serious deterioration;
6. The maximum production rate of the machine is known, and
7. The production cost includes the manufacturing, inspection and handling costs related to good quality and defective products.

3.3.3 Problem formulation

In this section, we study the problem of joint optimization of production and replacement in a continuous-time stochastic mathematical model, with a production process subject to random machine breakdowns and repairs, quality deterioration and customer demand. It is assumed that upon breakdown, the repair brings the state of the failed machine to as-bad-as-old conditions. The machine deteriorates with its age while in operation, and this process has a random negative influence on its availability and on the quality of the parts produced. Because the age is not reset to zero, the impact of deterioration pushes us to replace the machine in order to mitigate its effects. Meanwhile, with the replacement activity, the machine is replaced by a new one, and restores the machine age, its availability, and the quality of the parts produced to initial conditions. The system under consideration consists of one machine which produces one part type. After inspection, perfect products are stored in the serviceable inventory and used to meet demand, while the remainder is defective, and is rejected. The situation is schematically illustrated in figure 3.1.

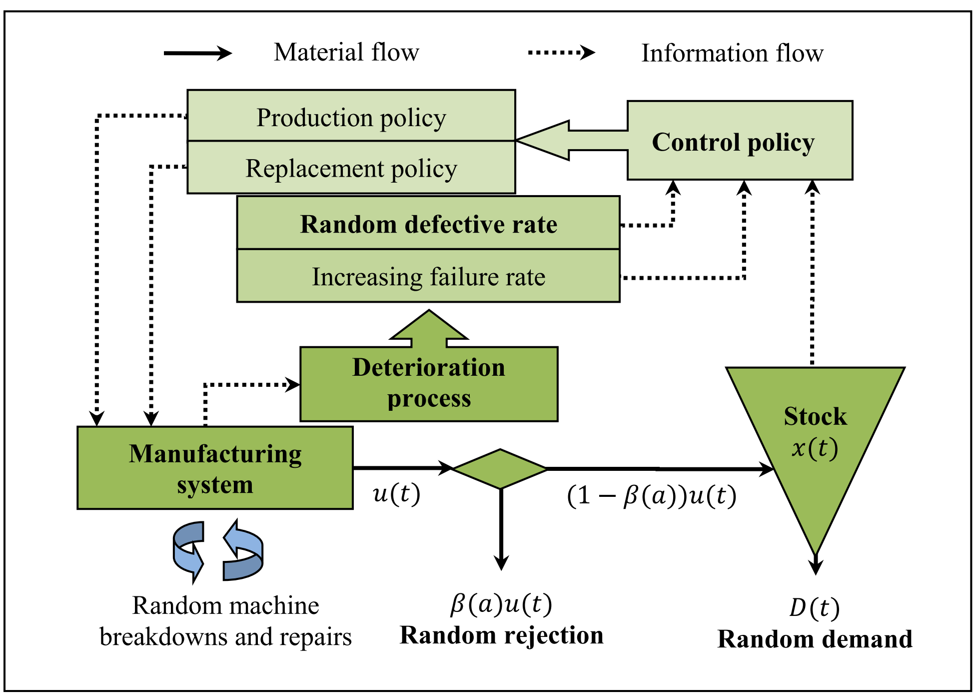

Figure 3.1 Block diagram of the manufacturing system

Let us consider the state equation, which specifies that the dynamics of the stock level $x(t) \in \mathbb{R}$ at time t increases with the production rate, and decreases with demand and defectives rates. This dynamics can be written as:

$$\dot{x}(t) = \Big(1 - \beta\big(a(t)\big)\Big)u(t) - D(t), \quad \text{with } t \in \mathbb{R}_+ \text{ and } x(0) = x^0 \in \mathbb{R} \tag{3.1}$$

where x^0 and $u(t) \in \mathbb{R}_+$ are the given initial values of inventory or backlog level and production rate of the machine at time t, respectively, $D(t) \in \mathbb{R}_+$ is the demand rate at time t, and $\beta(a(t)) \in \mathbb{R}_+$ is the defectives rate at a given age $a(t) \in \mathbb{R}_+$ of the machine. Since the demand rate $D(t)$ and defectives rate $\beta(a(t))$ are assumed to be random functions, the more rigorous Itô form (Chiarella, He and Nikitopoulos, 2015) of equation (3.1) will be used. We consider the case of a demand process $\{D(t), t \geq 0\}$ described by the Ornstein-Uhlenbeck diffusion-type process which is constructed as an output of the shaping filter excited by the white noise, and can be verified by the following Itô stochastic differential equation:

$$dZ_D(t) = -b_1 Z_D(t)dt + \sigma_1 dW_1(t), \;\; Z_D(0) = Z_D^0, \;\; Z_D^0 > 0 \tag{3.2}$$

where $Z_D(t)$ is the random varying portion of the demand rate at time t, b_1 is the non-negative constant (linear drift coefficient), σ_1 is the non-negative diffusion coefficient, $W_1(t)$ is the standard Brownian motion (Wiener process) at time t, and Z_D^0 is a given random variable. Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $Z_D(t)$ be an adapted to the filtration $\{\mathcal{F}_t\}$ for all $t \geq 0$ with respect to the σ-algebra generated by the random one-dimensional variable Z_D^0 and the history of one-dimensional standard Brownian motion $W_1(t)$ until time t, denoted as $\sigma\{Z_D^0, W_1(t): 0 \leq s \leq t\}$. It is known that equation (3.2) has a limiting variance $\sigma_D^2 = \frac{\sigma_1^2}{2b_1}$ when $t \to \infty$ (Chiarella *et al.*, 2015). For fast numerical convergence, we set the initial condition for equation (3.2) as $Z_D^0 \sim \mathcal{N}(0, \frac{\sigma_1^2}{2b_1})$, where Z_D^0 is a normally distributed initial value with zero mean and $\frac{\sigma_1^2}{2b_1}$ variance, and is assumed to be independent of W_1.

While $\beta(a(t))$ represents the rate of defectives, it is also described by a diffusion-type process of the age of the machine $\{\beta(a(t)),\ a(t) \geq 0\}$. The impact of the defective products is to decrease the production rate when the quality of the parts produced deteriorates. We define the random varying portion of the defectives rate process $\{\beta(a(t)),\ a(t) \geq 0\}$ as an output of the shaping filter excited by the white noise, and satisfies an equation in the Itô form, considering the age as a time-type variable:

$$dZ_\beta(a(t)) = -b_2 Z_\beta\big(a(t)\big)da(t) + \sigma_2(a)dW_2(a(t)),\ \ Z_\beta(0) = Z_\beta^0,\ \ Z_\beta^0 = 0 \tag{3.3}$$

where $Z_\beta(a(t))$ is the random varying portion of the defectives rate at age a. When the age tends to infinity ($a \to \infty$), equation (3.3) has a limiting variance $\sigma_\beta(a)^2 = \frac{\sigma_2(a)^2}{2b_2}$ (Chiarella *et al.*, 2015). Z_β^0 is a given random variable in initial conditions, and a zero value is imposed on it since we end up with a negative value that has no meaning, and the random portion of the quality cannot be allowed to become negative when the machine is new. b_2 is a drift coefficient, and is a given non-negative constant, while $\sigma_2(a)$ is a non-negative diffusion coefficient, and is assumed to be a continuous bounded and increasing function of the age of the machine a. The value of $\sigma_2(a)$ is closer to zero when the machine is still new, with hardly any fear of failure, and it begins to increase when the machine is aging and failures become significant. W_2 is a one-dimensional standard Brownian motion (or Wiener process), which is independent of both Z_β^0 and W_1. We note that $Z_\beta(a(t))$ is adapted to the filtration $\{\mathcal{F}_t\}$ for all $t \geq 0$ generated by $\sigma\{Z_\beta^0, W_2(a(s)) \colon 0 \leq s \leq t\}$. For more details on stochastic processes, we refer the reader to Ross (2003) and Chiarella *et al.* (2015).

Finally, we consider the following models for the demand and defectives rates, with a mixture of a deterministic component (constant or variable) and a (stochastic) diffusion-type process:

$$D(t) = \mu_D + Z_D(t) \tag{3.4}$$

$$\beta(a(t)) = \mu_\beta(a(t)) + Z_\beta(a(t)) \tag{3.5}$$

where μ_D is a constant and known average demand rate, $\mu_\beta(a(t))$ is a dynamic and known average defectives rate, which is also considered as a continuous bounded and increasing function of the machine age a and is of the same shape as the diffusion coefficient $\sigma_2(a)$.

Since the quality and demand characteristics of our system over an infinite horizon are stochastic with finite variance, the only process to model them more appropriately is the Ornstein-Uhlenbeck process. This choice is justified by the fact that the Ornstein-Uhlenbeck diffusion process is the only nontrivial process that is stationary, Gaussian and Markovian, up to allowing linear transformations of the space and time variables. Even though the initial distributions of Z_D^0 and Z_β^0 are not Gaussian distributed with mean 0, the processes $Z_D(t)$ and $Z_\beta(a(t))$ are not anymore stationary. However, over time, they will tend to move back towards the stationary processes (see Wong and Hajek (1985) and Chiarella *et al.* (2015) for more details). In the portions $Z_D(t) = \sigma_D v_D(t)$ and $Z_\beta(a(t)) = \sigma_\beta(a)\ \ v_\beta\big(a(t)\big)$, the stochastic processes $v_D(t)$ and $v_\beta(a(t))$ are stationary and normally distributed with zero mean and unity variance: $v_D(t) \sim \mathcal{N}(0,1)$ and $v_\beta\big(a(t)\big) \sim \mathcal{N}(0,1)$, and σ_D and $\sigma_\beta(a)$ are the standard deviations of the demand and defectives rates diffusion processes, respectively. Thus, whatever the initial distributions of Z_D^0 and Z_β^0, the portions $Z_D(t)$ and $Z_\beta(a(t))$ for $(t \to \infty)$ and $(a \to \infty)$ are approximately normally distributed with zero mean and $\frac{\sigma_1^2}{2b_1}$ and $\frac{\sigma_2(a)^2}{2b_2}$ variances: $Z_D(t) \sim \mathcal{N}(0, \frac{\sigma_1^2}{2b_1})$ and $Z_\beta(a(t)) \sim \mathcal{N}(0, \frac{\sigma_2(a)^2}{2b_2})$, and their correlation functions are: $R_D(s,t) = \frac{\sigma_1^2}{2b_1} e^{-b_1|t-s|}$ and $R_\beta(a_s, a_t) = \frac{\sigma_2(a_t-a_s)^2}{2b_2} e^{-b_2|a_t-a_s|}$ for all $0 < s < t$. Generally among the characteristics of Ornstein-Uhlenbeck process, it is considered as a good noise because it is able to generate random variations around the average of demand rate and the dynamic average of defectives rate without changing their main trajectories. Otherwise we are completely moving away from the original problem by dealing another type of problem. One of the practical advantages of the models given by equations (3.4) and (3.5) is that they allow us to obtain realistic random demand and defectives rates trends by varying the parameters of shaping filter (b_1 and/or σ_1) and (b_2 and/or $\sigma_2(.)$) to adjust the trajectories corresponding to a desired customer market, and for a specific machine. Thus, the role of these parameters is to reduce the variation rate excited by the white noise (represented by the differential forms of a Wiener process $dW_1(t)$ and $dW_2(a(t))$). So, the influence of the hypothesis in which demand and defectives rates are described by a diffusion-type can be quantified by the three main models:

1) Model of Wiener process when ($b_1 = 0$ and $\sigma_1 \neq 0$) or ($b_2 = 0$ and $\sigma_2(.) \neq 0$), the variances $\frac{\sigma_1^2}{2b_1} \to \infty$ and $\frac{\sigma_2(a)^2}{2b_2} \to \infty$.
2) Model of deterministic process when ($b_1 = \infty$ or $\sigma_1 = 0$) or ($b_2 = \infty$ or $\sigma_2(.) = 0$) , the variances $\frac{\sigma_1^2}{2b_1} \to 0$ and $\frac{\sigma_2(a)^2}{2b_2} \to 0$.
3) Model of Ornstein-Uhlenbeck process when ($0 < b_1 < \infty$ and $\sigma_1 \neq 0$) or ($0 < b_2 < \infty$ and $\sigma_2(.) \neq 0$), the variances $0 < \frac{\sigma_1^2}{2b_1} < \infty$ and $0 < \frac{\sigma_2(a)^2}{2b_2} < \infty$.

Let us consider the aging of the machine at time t as an increasing function of its production rate defined as:

$$\dot{a}(t) = ku(t), \text{ with } t \in \mathbb{R}_+ \text{ and } a(0) = a^0 \in \mathbb{R}_+,\ a(T^+) = a(T^-) \text{ and } a(T) = 0 \tag{3.6}$$

where k and a^0 are the given positive constant and initial age. The random variables T^+, T^- and T are the last times of repair, operation and replacement of the machine, respectively.

The production rate constraint is given by:

$$0 \leq u(t) \leq u_{max} \tag{3.7}$$

where u_{max} is the maximum production rate of the machine, and $u(t)$ is Markov-modulated; in other words, $0 \leq u(t) \leq u_{max}$ when the machine is up and $u(t) = 0$ when the machine is down or under replacement.

Using equations (3.4)-(3.5) for the demand and defectives rates models, the stochastic state differential equations (3.1) and (3.6) during short intervals $(\delta t, \delta W_1, \delta W_2)$ can be rewritten as:

$$\begin{bmatrix} \delta x(t) \\ \delta a(t) \end{bmatrix} = \begin{bmatrix} \left(1-\mu_\beta(a)\right)u-\mu_D & -\sigma_D & -\sigma_\beta(a)k^{-1} \\ ku & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta t \\ \delta W_1 \\ \delta W_2 \end{bmatrix} \tag{3.8}$$

with $\delta W_1 = v_D(t)\delta t$ and $\delta W_2 = v_\beta(a(t))\delta a(t)$.

Equations (3.8) will be also used in the following generic form:

$$\begin{bmatrix} \delta x(t) \\ \delta a(t) \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \delta t + \begin{bmatrix} g_1 & g_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta W_1 \\ \delta W_2 \end{bmatrix} \tag{3.9}$$

The machine's modes can be classified as: i) operational $(\xi(t)=1)$, producing a combination of good products $((1-\beta(a))u)$ and defective products $(\beta(a)u)$; ii) under repair $(\xi(t)=2)$, where the maintenance activity is minimal and restores the machine to as-bad-as old conditions, and iii) under replacement $(\xi(t)=3)$, where the maintenance activity is perfect and restores the machine to as-good-as-new conditions (resetting its age to zero). Because the repair activities are minimal, the dynamics of the machine should be modeled as a semi-Markov process in continuous time and discrete state over an infinite horizon $\{\xi(t) \geq 0\}$, with $\xi(t) \in B = \{1, 2, 3\}$. The transition diagram describing the considered system is illustrated in figure 3.2.

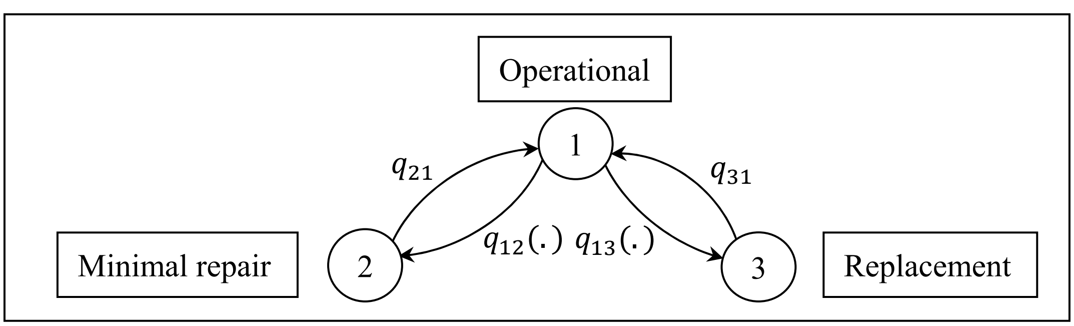

Figure 3.2 States transition diagram of the considered system

We assume that the failure rate $q_{12}(.)$ is a continuous bounded and increasing function of the machine's age $a(t)$, and is given by:

$$q_{12}\big(a(t)\big) = A_0 + A_1^{\infty}(1 - e^{-(A_2 a(t)^3)}) \tag{3.10}$$

where the parameters A_0, A_1^{∞} and A_2 are given constants. The model given by equation (3.10) describes the deterioration caused by the machine age on its dynamics, as in Rivera-Gomez *et al.* (2016). The inverse of $q_{12}(a)$ represents the mean time during which the machine remains operational before failure. The transition rate q_{21} from mode 2 to mode 1 is assumed constant. We introduce a continuous control variable $w(t)$, and the decision to send the machine for replacement is taken only in operational mode for all its possible values within $]0, 1]$. We assume that the transition rate $q_{13}(.)$ is defined as a linear function of $w(t)$:

$$q_{13}(.) = q.w(t) \tag{3.11}$$

where q is a given constant such that if the value of $w(t)$ is close to zero ($\text{i.e.}\, 10^{-7}$), the delay before replacement is very big (but finite), and no replacement takes place (see Ouaret *et al.* (2015) for more details). This situation describes the machine when it remains operational, and at any failure instant, a repair action is considered. The inverse of $q_{13}(.)$ represents the mean delay-time between the decision to perform the replacement and the effective switch from operational mode to replacement mode. The transition rate q_{31} from mode 3 to mode 1 is assumed constant. The other transition rates of the considered system are equal to zero.

The stochastic semi-Markov process $\xi(t)$ involves a generator matrix $Q(.)$, such that $Q(.) = [q_{\alpha\alpha'}(.)]$, where $q_{\alpha\alpha'}(.)$ indicates the transition rate from mode α to α' (with $\alpha, \alpha' \in \{1, 2, 3\}$), and verifies some properties. For more details about these properties, we refer the reader to the work of Ouaret et al. (2015).

Let $\Gamma(\xi(t))$ denote the following set of the feasible control policies, depends on the process $\xi(t)$, and is given by:

$$\Gamma(\alpha) = \{(u(\alpha,.), w(\alpha,.)) | 0 \le u(\alpha,.) \le u_{max} \text{Ind}\{\xi(t) = 1\}, w(\alpha,.) \in]0,1]\} \quad (3.12)$$

where $\xi(t) = \alpha$, $u(.)$ and $w(.)$ are production and replacement controls, respectively. The indicator function Ind$\{.\}$ used in equation (3.11) is defined as follows:

$$\text{Ind}\{\Theta(.)\} = \begin{cases} 1 & \text{if } \Theta(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

Let $G(.)$ be the cost rate function to be defined as follows:

$$G(\alpha, x, a, u, w) = c^+x^+ + c^-x^- + c^\alpha + c(u) \quad (3.13)$$

where c^+ and c^- are the inventory and backlog costs, respectively, with $x^+ = \max(0, x), x^- = \max(0, -x)$ and c^α being a constant defined as follows:

$$c^\alpha = c_r.\text{Ind}\{\xi(t) = 2\} + c_0.q_{31}.\text{Ind}\{\xi(t) = 3\} \quad (3.14)$$

where c_r is the cost rate for repair on the machine, and $(c_0.q_{31})$ is the replacement cost, which depends on the duration of replacement of the machine. Let $c(u)$ be a non-negative production cost function, $c(0) = 0$, and is twice differentiable. Moreover, $c(u)$ is either strictly convex or linear. In this paper, it is assumed to be a linear cost function, and is given by $c(u) = c_u u$. The constant c_u is the cost incurred per unit of produced parts, including the manufacturing, inspection and handling costs related to both confirming products $((1 - \beta(a))u)$ and defective products $(\beta(a)u)$.

Our objective is to determine the control policies of the production rate $u(.)$ and the replacement variable $w(.)$ so as to minimize the expected discounted cost given by:

$$J(\alpha, x, a, u, w) = E\{\int_0^\infty e^{-\rho t} G(.)dt | \xi(0) = \alpha, x(0) = x, a(0) = a\}, \quad \forall (u(.), w(.)) \in \Gamma(\xi(t)) \quad (3.15)$$

where ρ is the discount rate. The value function of the planning problem is defined as follows:

$$v(\alpha, x, a) \underset{(u(.),w(.))\in\Gamma(\alpha)}{=\inf} J(\alpha, x, a, u, w), \quad \forall\alpha \in B, x \in \mathbb{R}, a \in \mathbb{R}_+ \tag{3.16}$$

The properties of the value function $v(.)$ leading to the first-order Hamilton-Jacobi-Bellman (HJB) equations can be found in Kenné *et al.* (2003). Such equations describe the optimality conditions for stochastic control. The derivation of the optimality conditions in the form of HJB equations associated with the optimal control problem under study is detailed below:

After the formulation of the stochastic control problem, we now develop the optimality conditions in the form of second-order HJB equations using the optimal theory control and Itô's stochastic calculus. According to the Bellman optimality principle for the cost function given by equation (3.14), we derive the value function between t and $t + \delta t$ as follows:

$$v(\alpha(t), x(t), a(t), t) = \min_{\substack{(u,w)\\ t\le s\le t+\delta t}} E\left\{\begin{matrix}\int_t^{t+\delta t} G(.,s)ds + \\ e^{-\rho\delta t} v(.,t+\delta t)\end{matrix}\middle| \alpha(t), x(t), a(t)\right\} \tag{3.17}$$

Using the conditional expectation operation $\tilde{E}$ (i.e. for any function $H(\alpha)$, $\tilde{E}\{H(\alpha(t + \delta t))\} = E\{H(\alpha(t + \delta t))|\alpha(t)\}$), we obtain the following equation:

$$v(\alpha(t), x(t), a(t), t) = \min_{(u,w)} \tilde{E}\left\{\begin{matrix} G(.,t)\delta t + (1 - \rho\delta t) \\ [v(\alpha(\,t + \delta t), ., t + \delta t)] + o(\delta t)\end{matrix}\right\} \tag{3.18}$$

Applying the Taylor series expansion in the second order, we extend the usually differential form for the first order, in order to capture the stochastic aspects of the random demand and defectives rates. So, equation (3.18) becomes:

$$v(\alpha(t), x(t), a(t), t) =$$

$$\min_{(u,w)} \tilde{E}\left\{\begin{matrix} G(.,t)\delta t + (1 - \rho\delta t) \\ \begin{bmatrix} v(\alpha(\,t + \delta t), x(t), a(t), t) + v_x\delta x + v_a\delta a + v_t\delta t \\ +\frac{1}{2}v_{xx}\delta x^2 + \frac{1}{2}v_{aa}\delta a^2 + \frac{1}{2}v_{tt}\delta t^2 \\ +v_{xa}\delta x\delta a + v_{xt}\delta x\delta t + v_{at}\delta a\delta t + o(\delta x^2, \delta a^2, \delta t^2)\end{bmatrix}\end{matrix}\right\} \tag{3.19}$$

A technical step consists in computing the value function $\nu(\alpha(t+\delta t), x(t), a(t), t)$ using the dynamics of the machine. To that end, we expand the conditional expectation $\tilde{E}\{H(\alpha(t+\delta t))\} = H(\alpha(t)) + \sum_j H(j) q_{j\alpha(t)}(.)\,\delta t + o(\delta t)$, where the term $o(\delta t)$ is negligible as compared to δt. Second-order terms over δx are kept for further analysis because diffusion-type processes affect system dynamics (caused by random demand and defectives rates). Averaging over random realizations of the demand and defectives rates driven by the Brownian inputs, $\delta W_1(t)$ and $\delta W_2(a)$, using equations (3.9) and applying the rules of stochastic calculus introduced by Itô, we get:

$$E[(g_1\delta W_1 + g_2\delta W_2)\nu_x] = g_1\nu_x E[\delta W_1] + g_2\nu_x E[\delta W_2] = 0,\; E\left[\delta {W_1}^2\right] = \delta t,\; E\left[\delta {W_2}^2\right] = \delta a$$
$$E[\delta a^2] = E[\delta t^2] = 0,\; E[(\delta x\delta t)\nu_{xt}] = E[(\delta x\delta a)\nu_{xa}] = E[(\delta a\delta t)\nu_{at}] = 0$$
$$E[\delta W_1\delta W_2] = E[\delta W_1].E[\delta W_2] = 0,\; E[\delta x^2] = E[g_1^2\delta W_1^2 + g_2^2\delta W_2^2] = g_1^2\delta t + g_2^2\delta a.$$

Now, neglecting all terms of order higher than 1 over δt, taking the limit in equation (3.19), and considering the stationary regime $\nu(\alpha, x, a, t) \to \nu(\alpha, x, a)$ as $t \to \infty$ and $\frac{\partial \nu}{\partial t} \to 0$, the HJB equations in the second-order Itô from can be further simplified to:

$$\rho\nu(\alpha, x, a) = \min_{(u,w)\in\Gamma(\alpha)} \left\{ \begin{matrix} G(.) + \sum_{\alpha'} q_{\alpha\alpha'}(.)\nu(\alpha', x, \varphi_a(\xi, a)) \\ + f_1\frac{\partial \nu}{\partial x} + f_2\frac{\partial \nu}{\partial a} + \frac{1}{2}(g_1^2 + g_2^2 f_2)\frac{\partial^2 \nu}{\partial x^2} \end{matrix} \right\} \tag{3.20}$$

where $f_1 = (1-\mu_\beta(a))u - \mu_D$, $f_2 = ku$, $g_1 = -\sigma_D$ and $g_2 = -\sigma_\beta(a)k^{-1}$. Stochastic calculus differs from standard calculus because $(\delta W_1(t))^2$ and $(\delta W_2(a))^2$ are in order of δt and δa, respectively. In terms of which, in computing the HJB equation, one need to go to second order terms to capture the combined effect of variability on demand and quality. The term $\varphi_a(\xi, a)$ is the reset function that brings the age of the machine to zero after the machine is replaced, and to as-bad-as-old after a minimal repair activity. Therefore, let τ denote a jump time for the process ξ. Then we can write the function $\varphi_a(\xi, a)$ as follows:

$$\varphi_a(\xi, a) = \begin{cases} a(\tau^-) & \text{if } \xi(\tau^+) = 1 \text{ and } \xi(\tau^-) = 2 \\ 0 & \text{if } \xi(\tau^+) = 1 \text{ and } \xi(\tau^-) = 3 \\ a(\tau^-) & \text{otherwise} \end{cases} \tag{3.21}$$

where $\xi(t) = \alpha \in B$, $\frac{\partial v(\alpha,x,a)}{\partial x}$ and $\frac{\partial v(\alpha,x,a)}{\partial a}$ are the first-order partial derivatives of the value function $v(\alpha, x, a)$, and $\frac{\partial^2 v(\alpha,x,a)}{\partial x^2}$ is the second-order partial derivative of the value function $v(\alpha, x, a)$. The numerical solution of equations (3.20) is developed in Annex III.

In the next section, we provide a numerical example to illustrate the structure of the joint control policy.

3.4 Numerical example

In this section, a numerical example is conducted to illustrate the manufacturing system presented in section 3.3. The computational domain $G_{xa} = G_x^h \times G_a^h$ is such that:

$$G_x^h = \{x: -10 \le x \le 30\}, \quad G_a^h = \{a: 0 \le a \le 100\} \tag{3.22}$$

with $h_x = 0.5$ and $h_a = 2$. Let us define the dynamic averages of the defectives rate and diffusion coefficient as functions of the age of the machine.

$$\mu_\beta(a(t)) = \beta_0 + \beta_1^\infty(1 - e^{-(\beta_2\theta_d a(t)^3)}) \tag{3.23}$$

$$\sigma_2(a(t)) = \sigma_{20} + \sigma_{21}^\infty(1 - e^{-(\sigma_{22}\theta_s a(t)^3)}) \tag{3.24}$$

with $0 \le (\theta_d, \theta_s) \le 1$ being the adjustment parameters to obtain the desired trajectory of the rate of defectives and diffusion coefficient, respectively; β_0 and σ_{20} are the rate of defectives and diffusion coefficient at age zero (very low values), respectively. β_1^∞, β_2, σ_{21}^∞ and σ_{22} are given constants, and can be estimated from an analysis of historical data of the machine, obtained during the production process (Lam, Zhu, Chan and Liu, 2004). Additionally, as mentioned in Colledani and Tolio (2011), the behaviour of the machine can be monitored by control charts, which are used to provide information about its state. Table 3.1 summarizes the parameters used in this paper.

Table 3.1 Parameters of the numerical example

Parameter	c^+	c^-	c_r	c_0	c_u	u_{max}	ρ	k	q_{21}	q
Value	10	350	25	3000	100	1	0.02	0.8	1/20	100
Parameter	q_{31}	A_0	A_1^∞	A_2	Z_D^0	σ_1	b_1	Z_β^0	σ_{20}	σ_{21}^∞
Value	1/15	10^{-4}	0.01	5.10^{-6}	0.02	0.2	2	0	10^{-4}	0.23
Parameter	σ_{22}	θ_s	b_2	β_0	β_1^∞	β_2	θ_d	μ_D		
Value	15.10^{-6}	0.6	0.4	10^{-4}	0.3	15.10^{-6}	0.6	0.43		

For the chosen parameters presented in Table 3.1, the manufacturing system is initially considered to be able to satisfy the demand rate if the feasibility condition given by the following equation (3.25) is verified. In other words, the average inventory level should be positive to guaranty the demand satisfaction of products. However, due to the deterioration effect on failure and defectives rates, the manufacturing system may not be able to ensure the long-term satisfaction of demand. In this case, a maintenance plan to replace the machine should be put in place to fulfill the product demand.

$$\pi_1(.)u_{max} \geq \frac{E[D]}{(1-E[\beta])} \tag{3.25}$$

with $E[D] = \mu_D$, $E[\beta] = \mu_\beta$ and $\pi_1(.)$ is the limiting probability of the manufacturing system in operational mode. Note that the limiting probabilities of modes 1, 2 and 3, are computed as follows:

$$\pi(.)Q(.) = 0 \text{ and } \textstyle\sum_{i=1}^{3} \pi_i(.) = 1 \tag{3.26}$$

where $\pi(.) = (\pi_1(.), \pi_2(.), \pi_3(.))$ is the vector of limiting probabilities. The values of the parameters of equations (3.2)-(3.5) can be obtained from industrial market customer service data and historical maintenance data service, respectively. The simulation of trajectories of demand and defectives rates processes is obtained by a numerical procedure by solving stochastic equations (3.2) and (3.3) using an approximation of the Milstein numerical scheme. A second-order approximation by iteration used to obtain the solutions (variability) has a high order of convergence equal to 1. More details about the numerical scheme can be seen in Annex IV. The trajectories of demand rate are shown for different initial conditions. As can be seen in figures 3.3 and 3.4, the parameters of shaping filter affect the variances of realizations of random demand and defectives rates σ_D^2 and $\sigma_\beta(a)^2$, respectively. In other words, the perturbations are less severe by increasing (b_1 or b_2) and more severe by increasing (σ_1 or $\sigma_2(.)$). As compared to the Wiener process when $b_1 = 0$ or $b_2 = 0$, the variation rate is very high and it is much greater than in the diffusion-type process case.

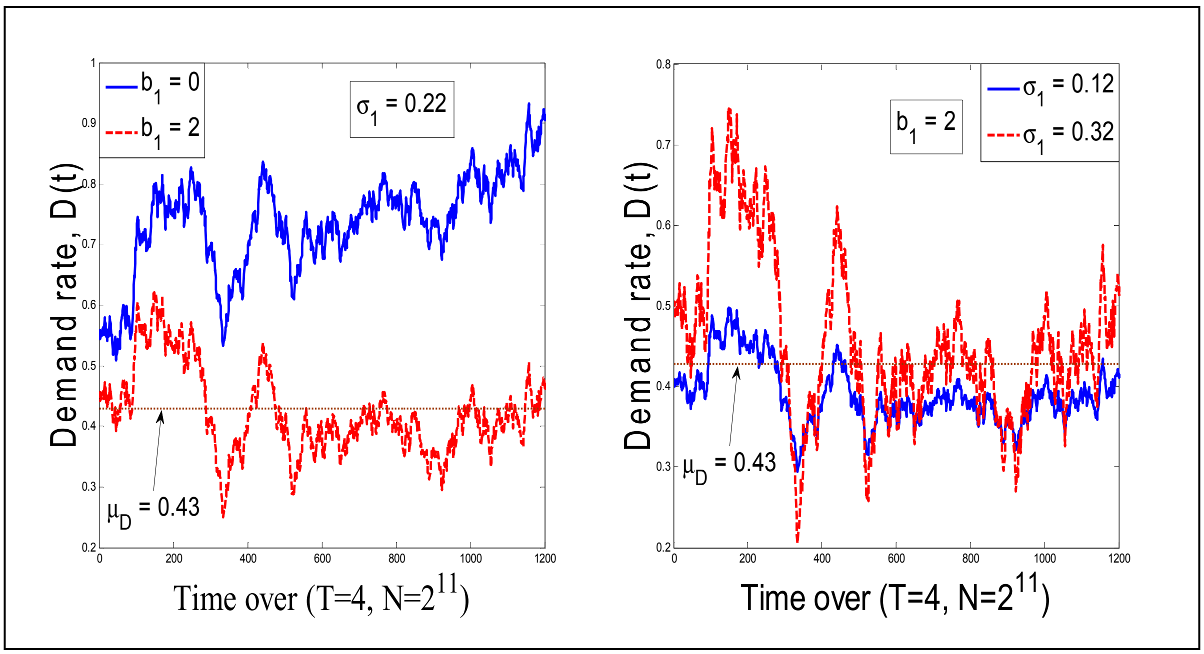

Figure 3.3 Trajectories of demand rate process over time with variation of b_1 and σ_1

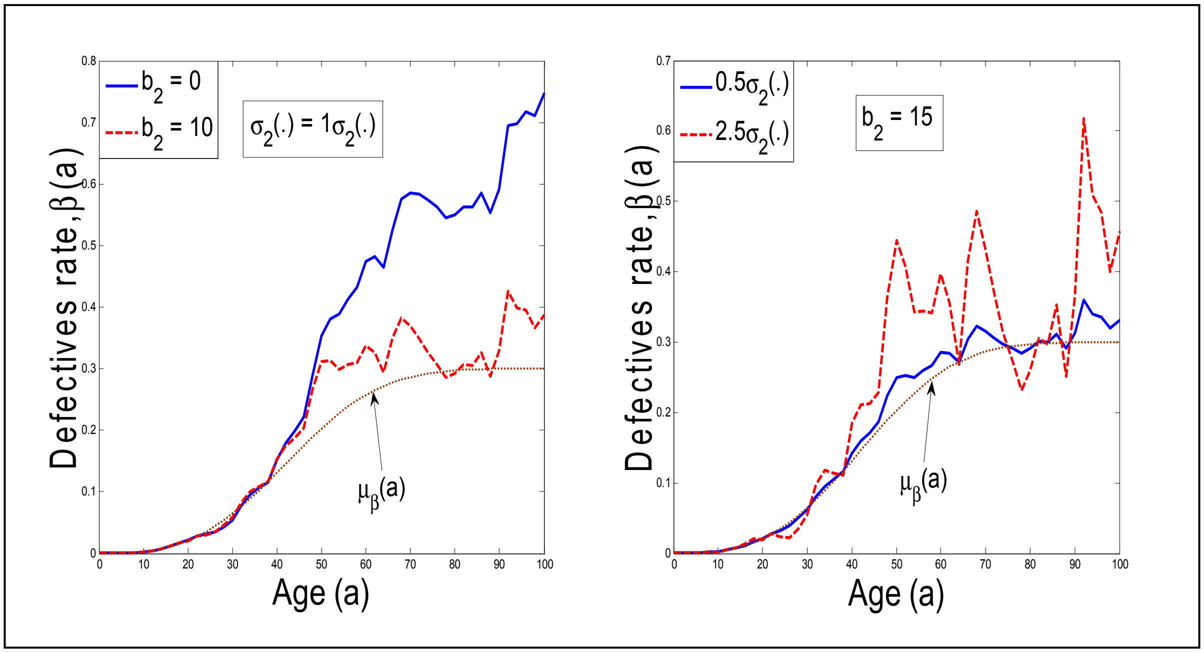

Figure 3.4 Trajectories of defectives rate process over age with variation of b_2 and $\sigma_2(.)$

The optimal production control policy obtained in figure 3.5 can be viewed as an extension of the so-called threshold policy presented in Akella and Kumar (1986). However, in our case, this extension is due to the fact that the deterioration caused by the age of the machine and minimal repairs affects the availability of the machine and the quality of the parts produced randomly. We also include random customer demand in our model. The production control policy is of the production threshold form, i.e. we have the optimal number of parts $Z(a)$ as a function of the age of the machine a, which provides protection against breakdowns and allows customer demand to be met when there is quality deterioration. It has been shown in Yin *et al.* (2003) that the optimal structure of the production control policy (production rate) $u^*(1, x, a)$ is when the second derivative of $c(u)$ is strictly positive. This policy can be generalized to our stochastic control problem by minimizing the following quantity: $f_1 v_x(\alpha, x, a) + f_2 v_a(\alpha, x, a) + \frac{1}{2} g_2^2 f_2 v_{xx}(\alpha, x, a) + c(u)$. Thus:

$$u^*(1,x,a) = \begin{cases} u_{max} & \text{if } x(t) < Z(a) \\ \dfrac{\mu_D}{\left(1-\mu_\beta(a)\right)} & \text{if } x(t) = Z(a) \\ 0 & \text{if } x(t) > Z(a) \end{cases} \quad (3.27)$$

when $c(u) = c_u u$ for some constant $c_u > 0$.

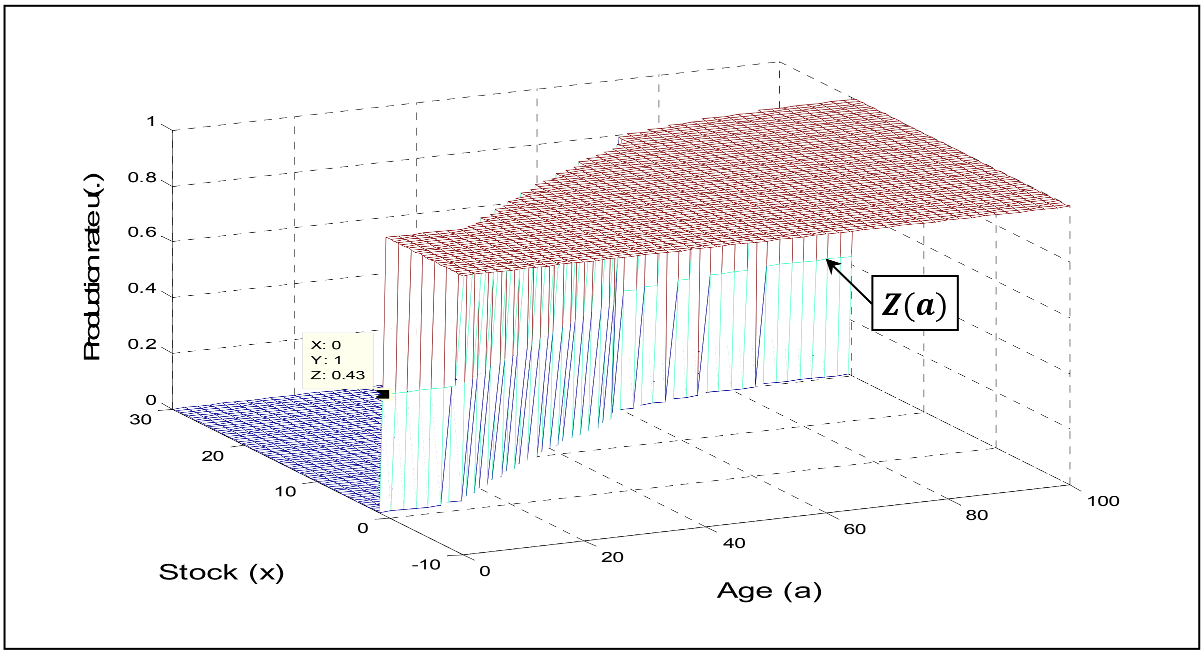

Figure 3.5 Production policy of the manufacturing system

In figure 3.6, we present the replacement policy $w(.)$, which determines the optimal conditions for the machine to be replaced or to be repaired upon failure. We can divide the plane (x, a) into two zones, A and B. We note that in zone A, the replacement policy does not recommend sending the machine for replacement, because the machine is still new. In this case, the replacement decision variable is set to its minimum value ($w_{min} = 10^{-7} \approx 0$). However, in zone B, because the machine is aging, with deteriorations in quality and availability, replacement becomes necessary, and its cost is economically justified. Hence, the decision variable is set to its maximum value ($w_{max} = 1$). The replacement activity should restore the age of the machine to zero. In other words, the failure and defectives rates are restored to initial conditions. We recall that the replacement is assumed to be continually allowed for all possible values of the decision variable w within $]0,1]$. However, the structure of the control replacement policy switches between the maximum and minimum values of w.

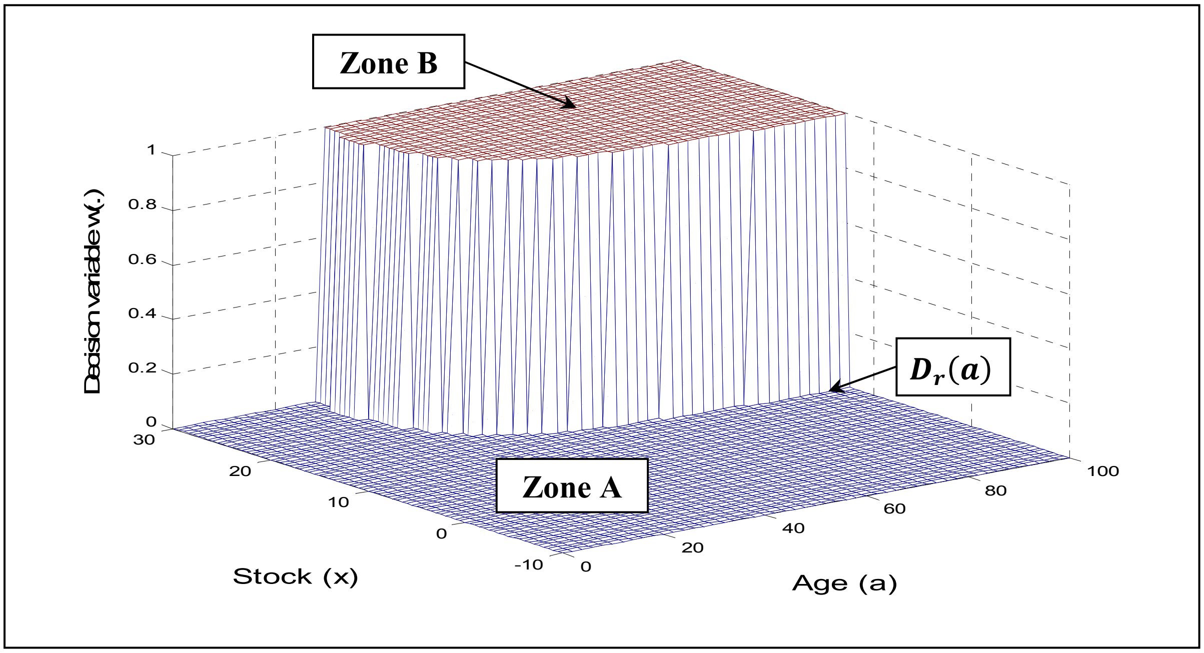

Figure 3.6 Replacement policy

Let $Z(.)$ and $D_r(.)$ define the production threshold and replacement trace, respectively, for the production and replacement policies, and let the intersection between them give the feasible zone C. We define the intersection point s as the point where the replacement is first recommended. As presented in figure 3.7, the property of zone C determines the optimal levels (x^*, a^*) when the machine should be replaced. Thus, the optimal replacement policy can be defined as follows:

$$w^*(1, x, a) = \begin{cases} 1 & \text{if } \big(x(t), a(t)\big) \in \text{zone C} \\ 10^{-7} & \text{otherwise} \end{cases} \tag{3.28}$$

We can observe from figure 3.7 that usually, the manufacturing system resides below the production threshold $Z(.)$, but it can still reside on the hedging point. Only in the feasible zone C which is described by the area below the production threshold $Z(.)$ and the area above the trace $D_r(.)$ is replacement more recommended, and here, the necessary stock is available to support the replacement process. Regarding the replacement, it is not recommended in the following cases:

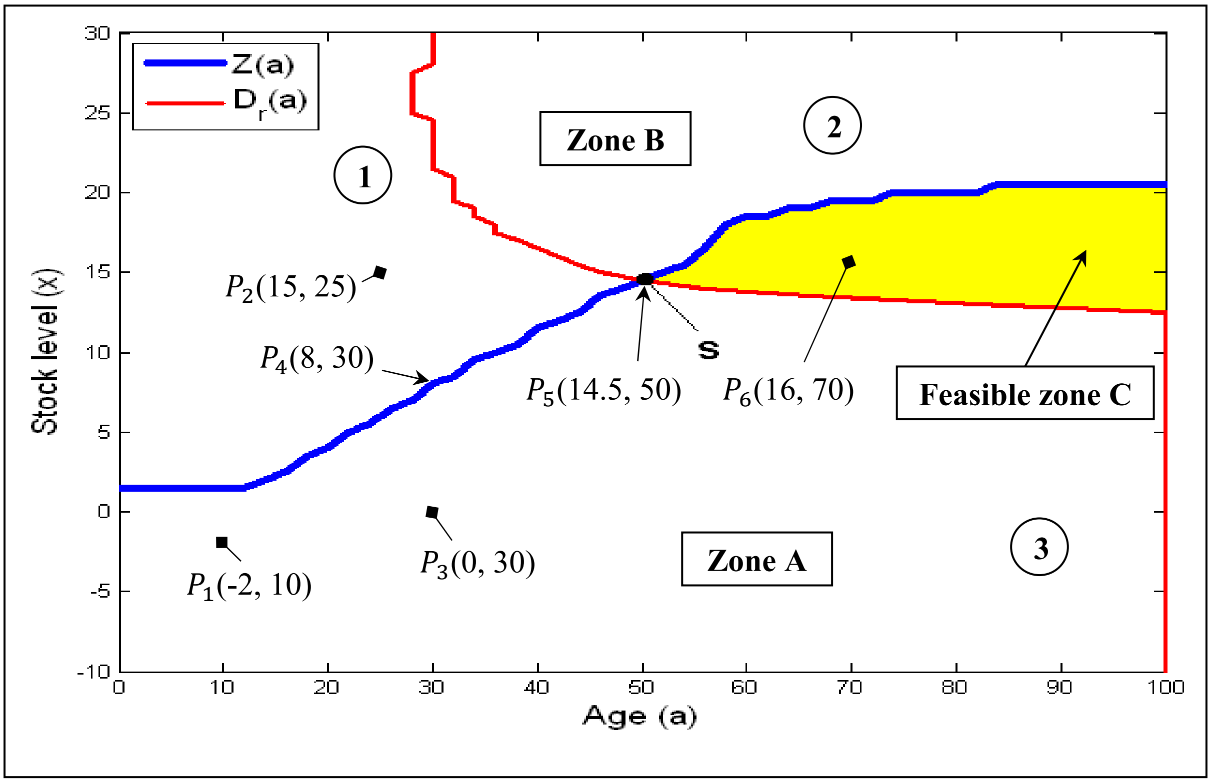

Figure 3.7 Production threshold and replacement trace

(1) In zone A, above the production threshold $Z(.)$, and below the trace $D_r(.)$, because the machine is still new.

(2) In zone B, above the production threshold $Z(.)$, and above the trace $D_r(.)$, we obtain an old machine with a high stock level, but the dynamic of the system resides below the production threshold $Z(.)$. That is why it is not recommended to replace the machine in this zone.

(3) In zone A, below the production threshold $Z(.)$, and below the trace $D_r(.)$, the machine is aging. Here, we can recommend repairs over a replacement, because we do not have the necessary stock to support the backlog when the machine is not available, and in real

life systems, the replacement process is much more expensive than repairs in terms of time and cost.

In the next section, we illustrate the implementation of the results obtained in order to facilitate the control of the production system.

3.5 Managerial implications

Although the numerical computation needed in solving HJB equations increases with its second-order, we get to solve of such a problem and obtain the optimal production and replacement rates. Therefore, this computation is done off-line and after that an implementation of this policy in the form of a decision tool is proposed in order to facilitate the task to the manager. Figures 3.5 to 3.7 are used in the implementation of our joint control policy. Figure 3.8 shows an implementation control flowchart for decisions that should be taken by the manager when the machine is operational (mode 1), under repair (mode 2), and under replacement (mode 3). Our joint policy can be applied to control the manufacturing system by knowing the stock level (inventory and backlog) and the age of the machine. Based on the diagram of figure 3.8, our policy proposes two levels, one to make decisions concerning the stock level and the other to indicate when to perform the replacement. However, minimal repair activities are performed whenever the machine fails. For control actions, the information about the stock level and the age of the machine must be updated continually. The results obtained will allow an easier optimization of the production process as they have a direct impact on the management of the manufacturing system control parameters. We give an example to illustrate the joint control policy, which is monitored by the block diagram presented in figure 3.8.

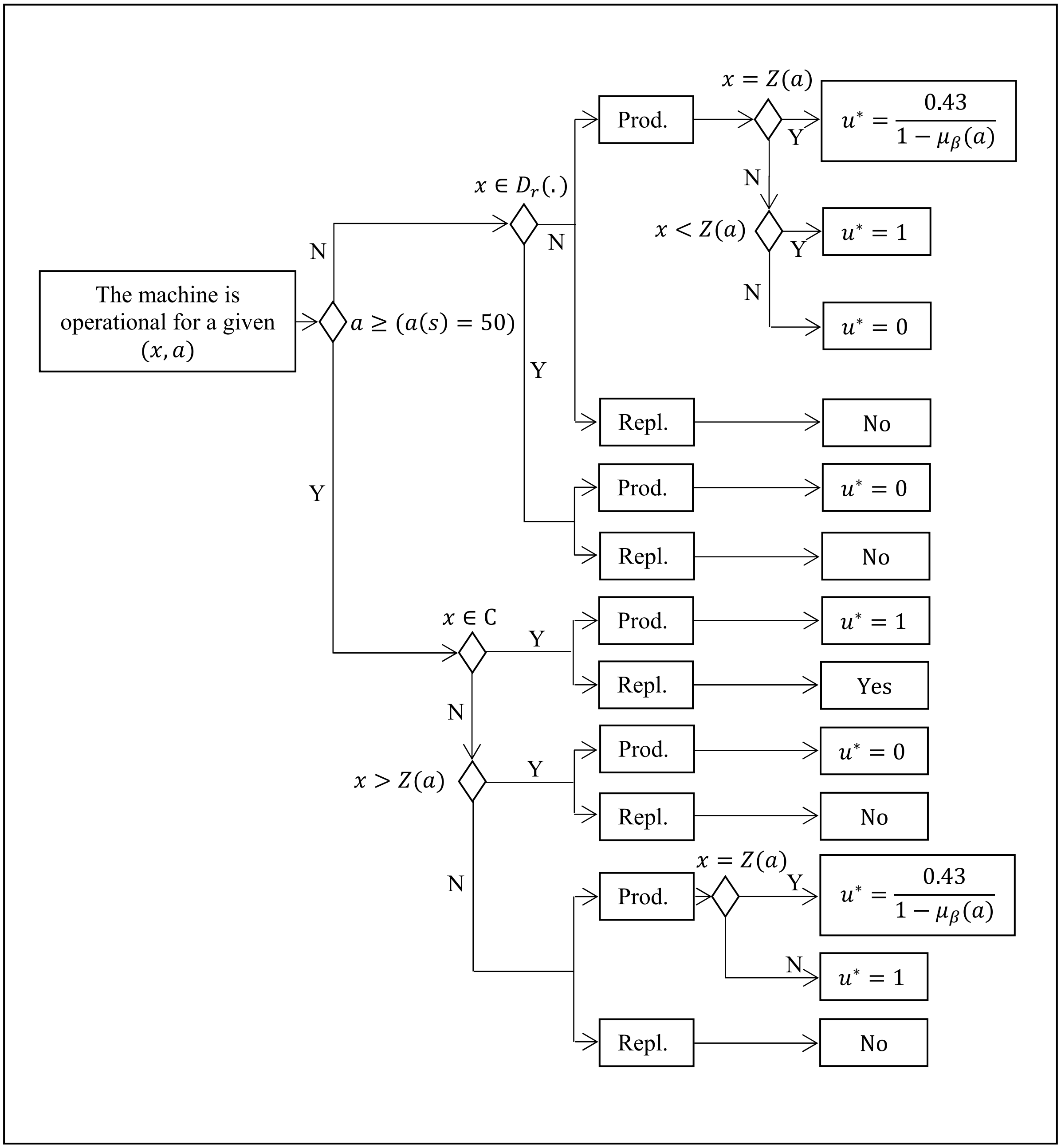

Figure 3.8 Implementation of the joint control policy

Assuming that the machine is operational and waiting for its next failure, for the selected points (P_1 to P_6) on the grid (x, a) in figure 3.7, the joint control policy is more straightforward. The control variables for production and replacement are shown in Table 3.2.

Table 3.2 Comparison data

Point	(x, a)	Z	$u^*(.)$	$w^*(.)$
P_1	$(-2, 10)$	1.5	$u_{max} = 1$	$w_{min} = 10^{-7}$
P_2	$(15, 25)$	-	0	$w_{min} = 10^{-7}$
P_3	$(0, 30)$	8	$u_{max} = 1$	$w_{min} = 10^{-7}$
P_4	$(8, 30)$	8	$0.43/(1 - 0.065) = 0.46$	$w_{min} = 10^{-7}$
P_5	$(14.5, 50)$	14.5	0	$w_{max} = 1$
P_6	$(16, 70)$	19.5	0	$w_{max} = 1$

In the next section, we will analyze the sensitivity of the above results to observe the dependence of the threshold according to state variables (x, a). Additionally, we will concentrate our efforts on the replacement feasible zone C judged to be the most appropriate in order to facilitate the analysis of this policy.

3.6 Sensitivity analysis

We will now consider the scenario for both demand and defectives rates defined as diffusion-type processes. We can choose different parameter model values to analyze the variation of the effects of demand and quality on the control policies, and confirm their obtained structure. A sensitivity analysis was carried out by exploring a stochastic demand and quality deterioration, and by varying the parameters of their models. We can compute the optimal control policies for different values of the drift coefficients (b_1, b_2) and diffusion coefficients $(\sigma_1, \sigma_2(a))$ for demand and defectives rates. Generally, an increase or decrease in the value of one of these parameters is reflected in the variability of demand or defectives rate. The models of $D(t)$ and $\beta(a(t))$ are integrated into the HJB equations model using equations (3.4)-(3.5), with the standard deviations σ_D and $\sigma_\beta(a)$ of the random varying portions. In other words, it is somewhat like a means of conversion, implemented to overcome some existing mathematical difficulties in the literature (i.e. no dynamic model exists covering demand and defectives rates, as is the case of equation (3.6) for machine aging), and to be able to formulate it in the HJB equations while keeping its characteristics. This scenario analyzes the sensitivity of the control policy according to only three different values of diffusion coefficient of defectives rate. We have: ($\sigma_2(.) = 0.5\sigma_2(.), 1\sigma_2(.)$ and $1.5\sigma_2(.)$). Other parameters of random demand and quality models, such as b_1, σ_1 and b_2, can be used in the same way to represent the corresponding joint control policy. In the following analysis, we can note that the intersection points s_1, s_2 and s_3 are highlighted in figure 3.9, and correspond respectively to the small, middle and high values of $\sigma_2(.)$ when the replacement of the machine is recommended for the first time.

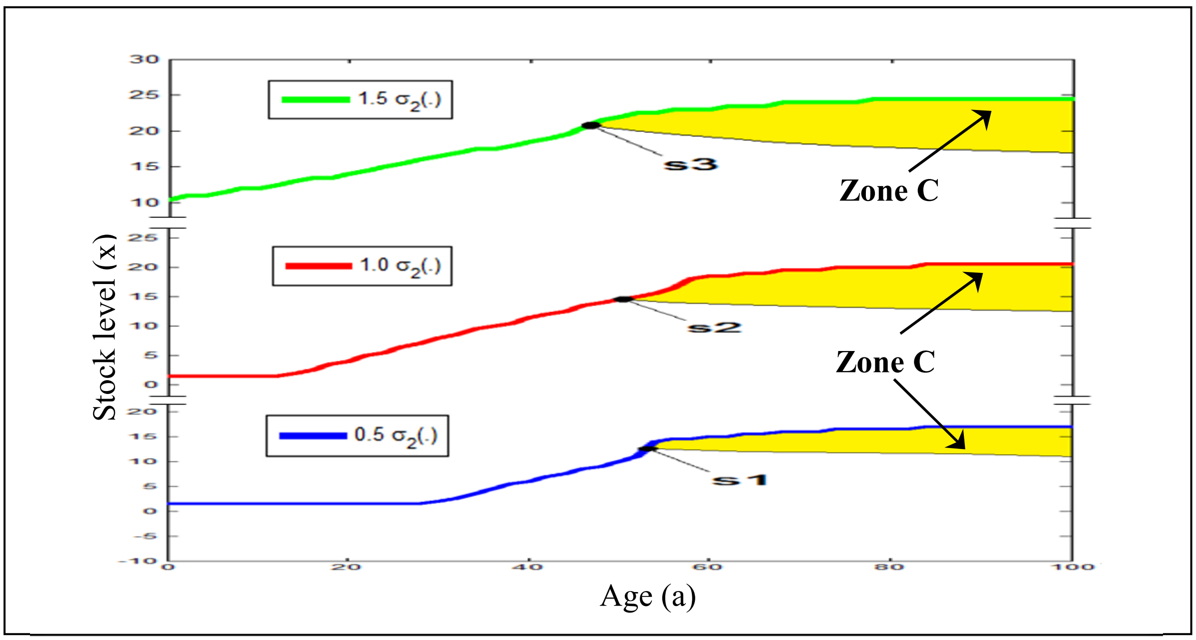

Figure 3.9 Production and replacement policies with variation of diffusion coefficient $\sigma_2(.)$

From the results presented in figure 3.9, for the scenario where both demand and the defectives rates are random, the following is observed:

When the diffusion coefficient $\sigma_2(.)$ increases, we know that the variability of the demand and defectives rates increases, and perturbations will become more difficult to predict, and then more severe. In this situation, as the risk of shortages should be increased, the production threshold $Z(.)$ must also be increased to hedge against shortages during periods of non-production and compensate for variability in demand and quality. To reach this level, the machine takes more time to produce at its maximum production rate u_{max}. Thus, it should deteriorate more rapidly (with age deterioration being dependent on the production rate), and as a result, we replace it earlier, and at a smaller age value: $a(s_3) < a(s_2) < a(s_1)$, and the feasible zone C increases. A sensitivity analysis was conducted with respect to other parameters of the system (i.e. σ_1, b_1, b_2). We noticed that the effect of the diffusion coefficient (σ_1) on the production and replacement control policies is the same as that of the diffusion coefficient ($\sigma_2(.)$). Moreover, we observed that the effect of the drift coefficients (b_1, b_2) on the control policy is the inverse of the effect of the diffusion coefficients ($\sigma_1, \sigma_2(.)$).

Following that sensitivity analysis of the proposed scenario, we will now carry out a comparison between the various scenarios which we can find in the literature or in the industrial reality to better understand the impact of random phenomena of demand and quality on production and replacement policies.

3.7 Comparison of scenarios and discussions

The objective of this analysis is to compare the control policies which can be derived from a basic case. Further, the effect of the variability in the trajectory of the demand and defectives rates on the control parameters (stock level, recommended replacement point and the feasible zone C) is analyzed. The following Table 3.3 summarizes the different demand and defectives rates scenarios which we are going to analyze.

Table 3.3 Types of demand and quality

Scenario	Demand (demand rate)	Quality (defectives rate)
1 (basic case)	Random	Random
2	Constant	Random
3	Random	Constant/dynamic
4 (classical case)	Constant	Constant/dynamic

<u>Scenario 1</u>: We start the analysis with the same scenario as the basic example, where the models of random portions of demand and defectives rates are assumed to follow diffusion-type processes:

$$dZ_D(t) = -b_1 Z_D(t)dt + \sigma_1 dW_1(t)$$

$$dZ_\beta(a(t)) = -b_2 Z_\beta\big(a(t)\big)da(t) + \sigma_2(a)dW_2(a(t))$$

<u>Scenario 2</u>: The random portion of the defectives rate is assumed to be a diffusion-type process, and the demand rate is considered constant, and can be written as follows:

$$D(t) = \mu_D$$

$$dZ_\beta(a(t)) = -b_2 Z_\beta\big(a(t)\big)da(t) + \sigma_2(a)dW_2(a(t))$$

Similarly to the previous scenario, some terms in the HJB equations (3.20) should be eliminated. Thus, the random terms of demand are equal to zero, and the generic terms of the HJB model become: $f_1 = \big(1 - \mu_\beta(a)\big)u - \mu_D$, $f_2 = ku$, $g_1 = 0$ and $g_2 = -\sigma_\beta(a)k^{-1}$.

<u>Scenario 3</u>: The random portion of demand rate is assumed to be a diffusion-type process, while the defectives rate is a dynamic average function which increases with the age of the machine, and can be written as follows:

$$dZ_D(t) = -b_1 Z_D(t)dt + \sigma_1 dW_1(t)$$

$$\beta(a(t)) = \mu_\beta(a(t)) = \beta_0 + \beta_1^\infty(1 - e^{-(\beta_2 \theta_d a(t)^3)})$$

Thus, the HJB equations (3.20) should be changed by setting the random terms of quality to zero, and the generic terms of the HJB model become: $f_1 = \big(1 - \mu_\beta(a)\big)u - \mu_D$, $f_2 = ku$, $g_1 = -\sigma_D$ and $g_2 = 0$ (with $\mu_\beta(a)$ being the mean value of $\beta(a(t))$.

<u>Scenario 4</u>: The demand rate is assumed to be constant and the defectives rate as a dynamic average function of the age of machine, and can be written as follows:

$$D(t) = \mu_D$$

$$\beta(a(t)) = \mu_\beta(a(t)) = \beta_0 + \beta_1^\infty(1 - e^{-(\beta_2 \theta_d a(t)^3)})$$

In the classical model, the random terms of demand and quality are set to zero. Thus, the second-order terms will be eliminated from the HJB model when $g_1 = g_2 = 0$.

The results of the control policies for different scenarios are illustrated in figure 3.10, and are obtained by varying the standard deviation values of demand and defectives rates σ_D and $\sigma_\beta(.)$, which can be represented by σ_1 and $\sigma_2(.)$. The comparison method is then applied with the data presented in Table 3.4. We note that the intersection points s_1, s_2, s_3 and s_4 which are highlighted in figure 3.10 correspond to the recommended replacement of the machine for scenarios 1, 2, 3 and 4, respectively.

Table 3.4 Comparison data

Scenario	σ_1	$\sigma_2(.)$
1	0.2	$1.5\sigma_2(.)$
2	0	$1.5\sigma_2(.)$
3	0.2	0
4	0	0

We start the comparison between scenario 4 ($\sigma_D = 0$ and $\sigma_\beta(.) = 0$) and scenario 3 ($\sigma_D \neq 0$). As we can see in figure 3.10, given that we have more variability in scenario 3 than in scenario 4 due to random demand, the system increases its production to hedge against backlogs (the stock level increases). To reach this objective, the machine should produce more parts at its maximum production rate u_{max}. Hence, the machine ages and the replacement will occur earlier: $(a(s_3) = 68) < (a(s_4) = 86)$. As a result, the feasible zone C increases in the presence of randomness at the demand level.

Next, we compare scenario 4 ($\sigma_D = 0$ and $\sigma_\beta(.) = 0$) with scenario 2 ($\sigma_\beta(.) \neq 0$). The same remark can be made as in the previous case. In figure 3.10, we have more variability with scenario 2 than with scenario 4 due to the increase in the random portion of the defectives rate. Hence, the stock level represented by the production threshold increases, the machine ages with the number of the produced parts, and replacement takes place earlier: $(a(s_2) = 56) < (a(s_4) = 86)$. Consequently, the feasible zone C should be increased.

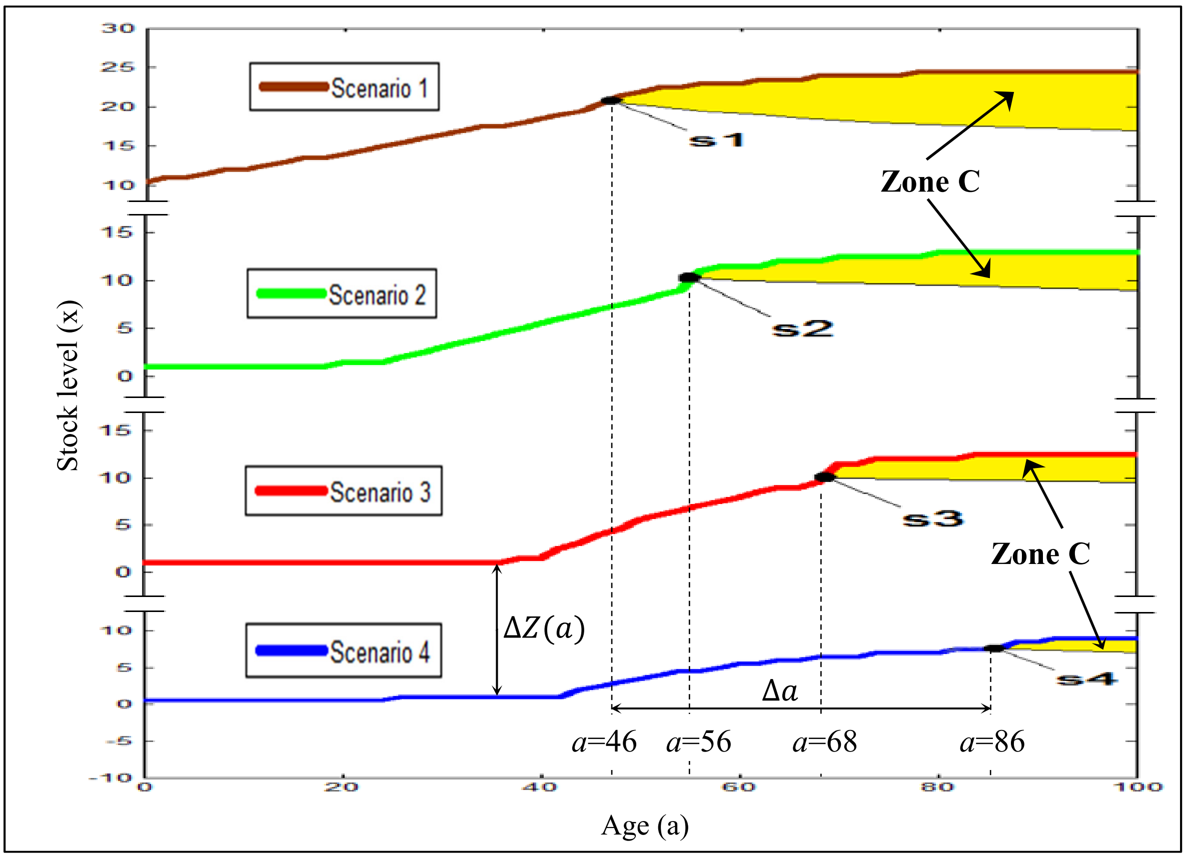

Figure 3.10 Comparison of production and replacement policies for scenarios 1, 2, 3 and 4

Finally, the last comparison is between scenario 4 ($\sigma_D = 0$ and $\sigma_\beta(.) = 0$) and scenario 1 ($\sigma_D \neq 0$ and $\sigma_\beta(.) \neq 0$). As we can see in figure 3.10, due to both random demand and quality, the production threshold in scenario 1 is higher than that in scenario 4. This is logical because when more random phenomena are present, the system faces a greater degree of disturbance, and more parts should be produced to manage the necessary stock level as protection against random failures. Thus, the production threshold increases, and the replacement should be more frequent. In this situation, the recommended replacement point $(a(s_1) = 46) < (a(s_4) = 86)$ and the feasible zone C should be increased.

From figure 3.10, we can observe that scenario 4 (classical case) and scenario 1 (basic case) are two extreme cases of these demand and quality models. In other words, the right side holds both deterministic demand and quality models (scenario 4) when the replacement is recommended for later, and the left side holds both stochastic demand and quality models (scenario 1) when the replacement is recommended earlier. However, between the left and right extreme cases, we have the other models representing the times when only the demand (scenario 3) or quality (scenario 2) is stochastic. From this comparison, we notice that the degree of variability increases as the number of random phenomena integrated in the model increases, which requires that a greater stock level be built in the inventory before entering in the replacement zone. To reach this level, the machine must produce parts at its maximum production rate u_{max} for long time. Given the machine aging as a result of this number of parts produced, the machine deteriorates more rapidity, and hence the risk of shortages will become more important. For this reason, the replacement will occur earlier, and consequently, the feasible zone C increases. Thus, we can conclude that in the existing models of random demand or random quality, the recommended age for replacement should be somewhere between $a(s_1) < (a(s_2);\ a(s_3)) < a(s_4)$. When random phenomena in mathematical models are ignored, the errors are important. As, we can see in figure 3.10, the difference between scenario 1 and scenario 4 is very significant, in terms of threshold level and recommended replacement age. The error $\Delta Z(a) = Z_{s_1}(a) - Z_{s_4}(a) = 12.5$ and the error $\Delta a = |a(s_1) - a(s_4)| = 40$. It is therefore crucial to use appropriate mathematical models of demand and quality in the presence of randomness in order to provide better appropriate control policies. From the above analysis, it is clearly appears that the results of our computations for the four different scenarios make sense, and that the structure of the joint control policy for the considered manufacturing system under deterioration is maintained. Depending on the industrial situation, the model presented in this paper in the form of second-order HJB equations allows us to consider that both demand and defectives rates are stochastic.

3.8 Conclusion

In this research, a simultaneous production planning and replacement control problem for a manufacturing system under deterioration is considered. We integrate random demand and quality issues in the mathematical model in order to determine the control policies. The effects of deterioration on the machine are reflected in its availability and the quality of the parts produced. The failure rate increases progressively with the machine age, while the rate of defectives is a random process. This deterioration is often due to the concept of the aging process and repair actions. The problem of minimization of the total discounted cost, including, inventory, backlog, production, repair and replacement costs, over an infinite planning horizon was addressed. We formulated a stochastic control problem using dynamic programming in the framework of a semi-Markov decision. We developed optimality conditions in the form of HJB equations, and showed that due to the (stochastic) diffusion components of the demand and defectives rates, the HJB equations are extended to the second-order Itô form. Numerical methods were used to obtain the optimal production and replacement policies. The results obtained are interesting in practice when more naturally addressing the growing number of industrial applications. Regardless of the situation, where demand or quality or both demand and quality are random processes, our model is able to provide the optimal solution, in addition to providing control policies that are better suited to industrial realities. As a subject of future research, due to the competition, we will recover defective manufactured products and used products collected from the market by integrating reverse logistics to the manufacturing process in order to improve the productivity and profitability of a company.

CHAPITRE 4

ARTICLE 3: OPTIMAL PRODUCTION CONTROL OF HYBRID MANUFACTURING/REMANUFACTURING FAILURE-PRONE SYSTEMS UNDER DIFFUSION-TYPE DEMAND

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Résumé

Le problème de contrôle de la production d'un système hybride fabrication/refabrication sous incertitude est analysé. Deux (2) sources d'incertitude sont considérées : les machines sont sujettes à des pannes et réparations aléatoires, et le niveau de la demande est modélisé en tant que processus stochastique de type diffusion. Contrairement à la plupart des études où le niveau de demande est considéré comme constant ou constant par morceaux, la demande est modélisée ici comme un processus de type diffusion. En particulier, les processus de Wiener et Ornstein-Uhlenbeck pour les demandes cumulatives sont analysés. Nous formulons le problème de commande stochastique et nous développons les conditions d'optimum pour ce problème sous la forme d'équations aux dérivées partielles (EDPs) de Hamilton-Jacobi-Bellman (HJB). Nous démontrons que les équations d'HJB sont du second ordre contrairement au cas du taux de demande constant (correspondant à la demande moyenne dans notre cas), où les équations d'HJB sont des EDPs linéaires. Nous appliquons l'approche de Kushner et la procédure d'amélioration de la politique pour résoudre les équations d'HJB numériquement et montrons que la politique de production optimale est de type seuil critique pour les deux (2) modèles de demande que nous avons introduits, comme dans le cas connu d'un taux de demande constant. Les résultats obtenus permettent de calculer numériquement la politique de production optimale pour les systèmes hybrides fabrication/refabrication en tenant compte de la variabilité de la demande et montrent également que le schéma discret de type Kushner peut être appliqué avec succès pour résoudre les équations sous-jacentes d'HJB de second ordre.

Abstract

The problem of production control for a hybrid manufacturing/remanufacturing system under uncertainty is analyzed. Two sources of uncertainty are considered: machines are subject to random breakdowns and repairs, and demand level is modeled as a diffusion type stochastic process. Contrary to most of studies where the demand level is considered constant and fewer results where the demand is modeled as a Poisson process with few discrete levels and exponentially distributed switching time, the demand is modeled here as a diffusion type process. In particular Wiener and Ornstein-Uhlenbeck processes for cumulative demands are analyzed. We formulate the stochastic control problem and develop optimality conditions for it in the form of Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDEs). We demonstrate that HJB equations are of the second order contrary to the case of constant demand rate (corresponding to the average demand in our case), where HJB equations are linear PDEs. We apply the Kushner-type finite difference scheme and the policy improvement procedure to solve HJB equations numerically and show that the optimal production policy is of hedging-point type for both demand models we have introduced, similarly to the known case of a constant demand. Obtained results allow to compute numerically the optimal production policy in hybrid manufacturing/remanufacturing systems taking into account the demand variability, and also show that Kushner-type discrete scheme can be successfully applied for solving underlying second order HJB equations.

Keywords: Stochastic control, Manufacturing systems, Optimization, Failure, Random process.

4.1 Introduction

In recent years, the reverse logistics framework allowing the unified analysis of manufacturing planning and the inventory management has gained a substantial interest among the researchers working in the field. In Fleischmann (2001), the author described the quantitative models to represent the activities of remanufacturing and recycling in the context of reverse logistics emphasizing three issues: namely: distribution planning, inventory management and production planning. In the De Brito *et al.* (2003) survey, authors analyzed more than sixty case studies in reverse logistics published between 1984 and 2002 and discussed network structures and activities related to the recovery of products up to the end of life. Various optimization models for supply chains with a recovery of returned products have been proposed with special attention to the production control and inventory management using both, deterministic and stochastic approaches. In the majority of previous studies discrete time (as opposed to continuous time) settings is used. Kiesmüller and Scherer (2003) present an effective approach to determine the discrete policy of the optimal control for a system with product recovery, taking into account the uncertainty in the demand of new and returned products. They model the demand and the return as discrete independent random variables. In Inderfurth (2004), a new discrete stochastic inventory model for a hybrid system is proposed: new and returned products are manufactured separately, the demands are independent but production policies are synchronized. Nikoofal and Husseini (2010) develop a periodic inventory model on a finite planning horizon with consideration of production, remanufacturing and disposal activities. In Oscar and Silva (2011), authors propose a model of discrete time stochastic optimization for a hybrid system taking into account, the production, subcontracting, remanufacturing of returned products, return market of poor quality products the production line and disposal activities. The demand is a random variable normally distributed and the return of products depends on the demand. A continuous time optimization model is considered in Dobos (2003) for the production, remanufacturing and disposal in a dynamic deterministic settings.

Manufacturing systems subject to random breakdowns and repairs were systematically analyzed in Fleming, Sethi and Soner (1987), Boukas and Haurie (1990) and Kenné and Boukas (2003) in continuous time using stochastic optimization technique. Recently in Kenné *et al.* (2012), this methodology has been extended to address the global performances of the manufacturing system with the supply chain in closed loop. A stochastic dynamic system consisted of two machines dedicated respectively to manufacturing and remanufacturing; the random phenomena are breakdowns and repairs of the machines, the demand of new products was considered deterministic and known, the returned product was a portion of this demand.

The constant demand is a prevailing assumption in the large body of the research devoted to stochastic continuous time optimization of production management in failure-prone systems. Some papers develop optimality conditions and use them for searching numerical solutions (Yan and Zhang, 1997); others present analytical solutions as the recent article (Khemlnitsky, Presman and Sethi, 2011). In much fewer studies where the random demand is analyzed – it is most often modeled as a Poisson process. This approach allows to keep the usual framework of random discrete events changing the state of the system for both machine breakdowns and demand jumps (Perkins and Srikant, 2001). Poisson-type demand is used more systematically in inventory optimization problems (Presman and Sethi, 2006). A combined model: Poisson process coupled with the diffusion process has been recently proposed in Bensoussan *et al.* (2005) for modeling the demand in inventory problem. In fact diffusion-type processes were used for modeling the demand in the classical paper of Fleming *et al.* (1987) were optimality conditions have been obtained, however it was the only source of random behavior since the machine breakdowns were not considered.

The system considered in this paper contains reverse logistics loop with manufacturing and remanufacturing branches revisiting the model proposed in Perkins and Srikant (2001). We use continuous time stochastic control approach and adopt the diffusion-type component into the demand model merging this source of random behavior with random machine breakdowns described by Poisson process as in Boukas and Haurie (1990) and Kenné and

Boukas (2003). As a direct consequence of an adopted demand model the optimality conditions lead to the Hamilton-Jacobi-Bellman (HJB) equation of the second order. Second order HJB is often met in option price modeling, but for stochastic control in manufacturing systems the HJB is usually of the first order (Boukas and Haurie, 1990; Kenné and Boukas, 2003; Perkins and Srikant, 2001). Analyzing the second order HJB we use the Kushner finite difference approximations and the policy improvement algorithm (Kushner and Dupuis, 1992).

The paper is structured as follows. In Section 4.2 we describe the model of the hybrid system consisting of 2 machines. The first machine uses primary product, and the second - returned product; both are subject to breakdowns and repairs constituting the first source of uncertainty. We describe in details our demand model using diffusion type random processes constructed as an output of shaping filter excited by the white noise. We study 2 versions of such model simple Brownian motion and first order Markovian process. Latter version seems more realistically fit the real world situations. In Section 4.3 we derive optimality conditions in the form of Hamilton-Jacobi-Bellman (HJB) equations which are second order partial differential equations (PDEs) for the chosen demand model. In Section 4.4 we describe the numerical method based on finite difference approximations and policy improvement approach following the methodology proposed in Kushner and Dupuis (1992) and also in Yan and Zhang (1997). In Section 4.5 we apply the developed methodology to the manufacturing system described in Section 4.2, compute the optimal production policy and show that it is of classical hedging point type. In conclusion we discuss the proposed methodology and obtained results, and outline the possible directions for future works.

4.2 Model of a hybrid manufacturing system suitable for stochastic control

We consider a hybrid manufacturing/remanufacturing system consisting of two parallel machines denoted M_1 and M_2 respectively, producing the same type of product. Stochastic phenomena are demand level and machine breakdowns/repairs. We take into account the activity of production in forward direction and the activity of reutilization of returned products in reverse logistics. The demand must be satisfied by inventory for serviceable items. This inventory will be built by the products manufactured or reused. The returned products will be in the second inventory namely recovery, they can be remanufactured, or be hold on stock for future remanufacturing. In our problem, we assume that the maximal production rates for each machine are known and the machine M_2 is producing at average supply for return rate, which is also its maximal rate. This situation is illustrated in figure 4.1.

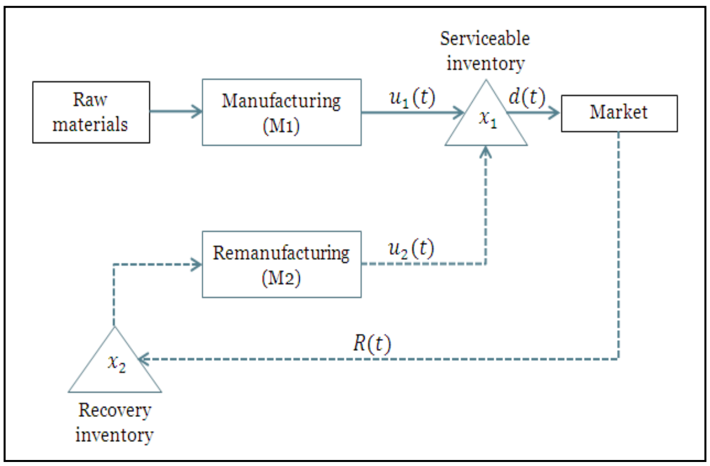

Figure 4.1 System structure

State of the machine M_i with $i = \{1, 2\}$ is modeled as a Markov process in continuous time with discrete state $\{\xi_i(t) \geq 0\}$, with $\xi_i(t) \in B_i = \{0, 1\}$ ($B_i = 1$ - machine is operational, $B_i = 0$ - machine is out of order). We may the define $\xi(t) = \xi_i(t) \times \xi_j(t) \in B = B_1 \times B_2 = \{(1,1), (1,0), (0,1), (0,0)\} \equiv \{1, 2, 3, 4\}$. State transition diagram is shown in figure 4.2. Hybrid system is in production while in modes 1, 2 and 3. Transition probabilities from state α to state β for machine M_i.

$$P[\xi(t+\delta t) = \beta | \xi(t) = \alpha] = \begin{cases} q_{\alpha\beta}(.)\delta t + o(\delta t) & \text{if } \alpha \neq \beta \\ 1 + q_{\alpha\alpha}(.)\delta t + o(\delta t) & \text{if } \alpha = \beta \end{cases} \quad \alpha, \beta \in B \tag{4.1}$$

with $q_{\alpha\beta} \geq 0, q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}, \lim_{\delta t \to 0} \frac{o(\delta t)}{\delta t} = 0.$

State transition (4×4) matrix $Q = [q_{\alpha\beta}]$ is therefore given by:

$$Q = \begin{bmatrix} -(q_{12}+q_{13}) & q_{12} & q_{13} & 0 \\ q_{21} & -(q_{21}+q_{24}) & 0 & q_{24} \\ q_{31} & 0 & -(q_{31}+q_{34}) & q_{34} \\ 0 & q_{42} & q_{43} & -(q_{42}+q_{43}) \end{bmatrix} \quad (4.2)$$

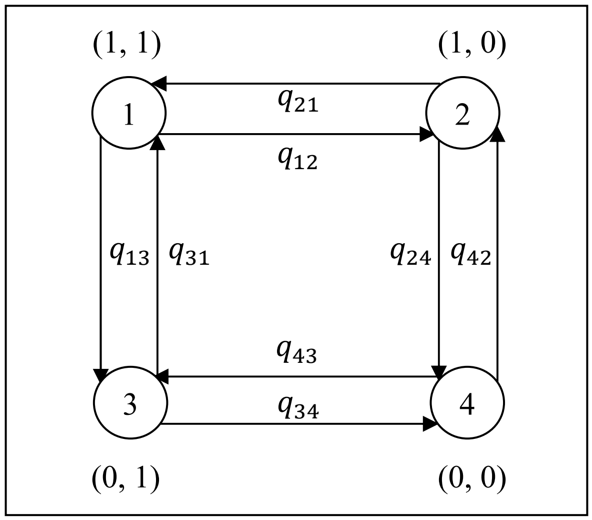

Figure 4.2 State transition diagram

State equations can be written in the simplified form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} -d(t) \\ R(t) \end{bmatrix} \quad (4.3)$$

Since the demand $d(t)$ and return $R(t)$ rates are considered as stochastic processes the more rigorous Itô form of equations (4.3) will be used later. Namely let $d(t)$ be a stationary Gaussian process with the constant mean and variance $d(t) \sim \mathcal{N}(\mu_D, {\sigma_D}^2) \equiv \mu_D + \sigma_D \varepsilon(t)$, where $\varepsilon(t) \sim \mathcal{N}(0,1)$. Below we further specify $\varepsilon(t)$ in one of two ways: either an increment of a standard Brownian motion, or an increment of the first order Markov process defined later using the shaping filter.

For the return (remanufacturing) rate, an assumption is made that it is proportional to the customer demand rate $R(t) = r * d(t) \equiv \mu_R + \sigma_R \varepsilon(t)$ with r is a percentage of return.

Stochastic state differential equations (4.3) can be rewritten in Itô form using notation $\varepsilon(t)\delta t = \delta z$:

$$\begin{bmatrix}\delta x_1(t)\\ \delta x_2(t)\end{bmatrix} = \left\{\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix} + \begin{bmatrix}-\mu_D\\ \mu_R\end{bmatrix}\right\}\delta t + \begin{bmatrix}-\sigma_D\\ \sigma_R\end{bmatrix}\delta z \tag{4.4}$$

Equations (4.4) will be also used in the following generic form:

$$\delta x(t) = \begin{bmatrix}\delta x_1(t)\\ \delta x_2(t)\end{bmatrix} = f(x,\alpha,u,w,t)\delta t = \begin{bmatrix}f_1(x,\alpha,u,t)\\ f_2(x,\alpha,u,t)\end{bmatrix}\delta t + \begin{bmatrix}g_1\\ g_2\end{bmatrix}\delta z \tag{4.5}$$

For the **case A** the input δz to equations (4.4) is specified as a standard Brownian motion increment $\delta z = \delta W$.

For the **case B** the input δz to equations (4.4) is specified as an increment of the shaping filter output (Ornstein-Uhlenbeck process):

$$\delta z = -a.z\,\delta t + b\delta W \quad \text{where } a > 0, b > 0 \tag{4.6}$$

Process $z(t)$ is a first order Markovian, its correlation function is $k(t) = (\frac{b^2}{2a}).exp(-a|t|)$. Additional insight to the proposed demand model can be given by considering the cumulative demand: $D(t) = \mu_D t + V_D(.)$ where μ_D is a constant demand ramp, $V_D(.)$ is a randomly varying portion of the demand. For the case A: $V_D = W$ (Wiener process), for the case B: $V_D = z$ (Ornstein-Uhlenbeck process). Also, case A can be obtained from B setting $a = 0, b = 1$.

Following constraints have to be added to equations (4.5)-(4.6):

$$\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix} \geq \begin{bmatrix}0\\ 0\end{bmatrix};\ x_2(t) \geq 0 \tag{4.7}$$

Let the cost rate function to be defined as follows:

$$G_\alpha(x_1,x_2,u) = c_1^+x_1^+ + c_1^-x_1^- + c_2^+x_2^+ + c_pu_1 + c_ru_2 + c^\alpha \quad \text{with } \alpha \in B \tag{4.8}$$

Here $x_{1,2}^{+} = \max\{0, x_{1,2}\}$, $x_1^{-} = \max\{-x_1, 0\}$; c_1^{+}, c_1^{-}: inventory holding and backlog costs for manufactured product (per time unit); c_2^{+}: inventory holding cost for remanufactured product (per time unit); c_p, c_r: production costs for manufacturing and remanufacturing processes (per unit); c^{α}: maintenance cost for nonoperational state of the machines:

$$c^{\alpha} = c_r^2 \mathrm{Ind}\{\xi(t) = 2\} + c_r^1 \mathrm{Ind}\{\xi(t) = 3\} + (c_r^1 + c_r^2)\mathrm{Ind}\{\xi(t) = 4\}$$

where $\mathrm{Ind}\{P(.)\} = \begin{cases} 1 & \text{if } P(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$

The objective is to determine the production rates $u_1(.)$ and $u_2(.)$ in order to minimize the expected discounted cost (ρ is the discount rate):

$$J(\alpha, x_1, x_2, u) = E\left\{\int_0^{\infty} e^{-\rho t} G_{\alpha}(x_1, x_2, u)\, dt | x_1(0) = x_1, x_2(0) = x_2, \xi(0) = \alpha\right\}, \quad \forall\big(u_1(.), u_2(.)\big) \in \Gamma(\alpha) \tag{4.9}$$

The domain $\Gamma(\alpha)$ of admissible controls is defined as:

$$\Gamma(\alpha) = \left\{ \begin{matrix} (u_1(.), u_2(.)) \in \Re^2 | 0 \le u_1(.) \le u_{max}^1 \mathrm{Ind}\{\xi(t) = \alpha\}, \\ 0 \le u_2(.) \le u_{max}^2 \mathrm{Ind}\{\xi(t) = \alpha\} \end{matrix} \right\}, \forall\big(u_1(.), u_2(.)\big) \in \Gamma(\alpha) \tag{4.10}$$

Defined hybrid system is said to be meeting feasibility condition if:

$$\sum_{i=1}^{4} \pi_i u_{max}^i = \pi_1(u_{max}^1 + u_{max}^2) + \pi_2 u_{max}^1 + \pi_3 u_{max}^2 > E\big(d(t)\big) = \mu_D \tag{4.11}$$

where π_i et u_{max}^i are limiting probabilities and maximal production rates. We recall that the vector of limiting probabilities is defined as an eigenvector of the transition matrix $Q(.)$:

$$\sum_{i=1}^{4} \pi_i = 1 \text{ and } \pi(.)Q(.) = 0 \tag{4.12}$$

4.3 Optimality conditions for stochastic control problem

Let us define the value function as a minimum (infimum) of expression (4.9) over all possible control inputs:

$$v(\alpha, x_1, x_2) = \inf_{(u_1(.), u_2(.)) \in \Gamma(\alpha)} J(\alpha, x_1, x_2, u), \quad \forall \alpha \in B \tag{4.13}$$

Let us briefly recall the guidelines for obtaining optimality conditions. Introducing time-dependant α-dependant cost function and value function we have:

$$J(., t) = E\left\{\int_0^t e^{-\rho s} G_\alpha\big(x_1(s), x_2(s), u(s)\big)\, ds | x_1(0) = x_1, x_2(0) = x_2, \xi(0) = \alpha\right\} \tag{4.14}$$

According to Bellman optimality principle for cost function at $t + \delta t$ we can write:

$$v(\alpha, x_1, x_2, t) = \min_u E \left\{ \begin{matrix} \displaystyle\int_t^{t+\delta t} e^{-\rho s} G_\alpha\big(x_1(s), x_2(s), u(s)\big)\, ds \\ +e^{-\rho\delta t} v(\alpha(t+\delta t), x_1(t+\delta t), x_2(t+\delta t), t+\delta t) \end{matrix} \right\} \tag{4.15}$$

$$= \min_u E\left\{G_\alpha(x_1, x_2, u)\delta t + e^{-\rho\delta t} v(\alpha(t+\delta t), x_1(t+\delta t), x_2(t+\delta t), t+\delta t)\right\}$$

Using Taylor expansion for the term $e^{-\rho\delta t}$ and the value function $v(\alpha(t+\delta t), x_1(t+\delta t), x_2(t+\delta t), t+\delta t)$ over last 3 arguments, and keeping linear terms over δt and up to second order terms over δx we get:

$$e^{-\rho\delta t} v(\alpha(t+\delta t), x_1(t+\delta t), x_2(t+\delta t), t+\delta t) = \tag{4.16}$$

$$\left(\begin{matrix} (1-\rho\delta t)(v(\alpha(t+\delta t), x_1, x_2, t) + v_t\delta t + v_{x_1}\delta x_1 \\ +v_{x_2}\delta x_2 + \frac{1}{2} v_{tt}(\delta t)^2 + \frac{1}{2} v_{x_1 x_1}(\delta x_1)^2 + \frac{1}{2} v_{x_2 x_2}(\delta x_2)^2 \\ +v_{x_1 t}(\delta x_1 \delta t) + v_{x_2 t}(\delta x_2 \delta t) + v_{x_1 x_2}(\delta x_1 \delta x_2)) + o(\delta t^2) \end{matrix} \right)$$

Second order terms over δx are kept for further analysis because of diffusion-type processes affecting system dynamics. One more technical step consists of computing the value function $v(\alpha(t+\delta t), x_1, x_2, t)$ using Markov chain-type machine dynamics (4.2) defined through transition probabilities:

$$v(\alpha(t+\delta t), x_1, x_2, t) = v(\alpha, x_1, x_2, t) + \sum_{\beta} q_{\alpha\beta}\, v(\beta, x_1, x_2, t)\delta t \tag{4.17}$$

Merging equations (4.16) and (4.17) we get:

$$e^{-\rho\delta t} v(\alpha(t+\delta t), x_1(t+\delta t), x_2(t+\delta t), t+\delta t) = \tag{4.18}$$

$$\left((1-\rho\delta t)\begin{pmatrix} v(\alpha, x_1, x_2, t) + \sum_{\beta} q_{\alpha\beta}\, v(\beta, x_1, x_2, t)\delta t \\ +v_t\delta t + v_{x_1}\delta x_1 + v_{x_2}\delta x_2 + \frac{1}{2} v_{tt}(\delta t)^2 + \frac{1}{2} v_{x_1x_1}(\delta x_1)^2 \\ +\frac{1}{2}\, v_{x_2x_2}(\delta x_2)^2 + v_{x_1t}(\delta x_1\delta t) + v_{x_2t}(\delta x_2\delta t) + v_{x_1x_2}(\delta x_1\delta x_2) \end{pmatrix} \atop +o(\delta t^2) \right)$$

Averaging over random realizations of the demand driven by the Brownian input δw, using equations (4.5) and applying Itô's lemma we get:

$$E\left[(g_1 v_{x_1} + g_2 v_{x_2})\delta w\right] = g_1 v_{x_1} E\delta w + g_2 v_{x_2} E\delta w = 0,$$

$$E\left[v_{x_1t}(\delta x_1\delta t)\right] = E\left[v_{x_2t}(\delta x_2\delta t)\right] = 0, E(\delta w)^2 = \delta t,$$

$$E(\delta x_1)^2 = g_1^2, E(\delta x_2)^2 = g_2^2, E(\delta x_1\delta x_2) = g_1 g_2$$

Now neglecting all terms of order higher than 1 over (δt), taking $\lim_{\delta t\to 0}$, and considering the stationary regime $\frac{\partial v}{\partial t} = 0 \Rightarrow v(\alpha, x_1, x_2, t) = v(\alpha, x_1, x_2)$, we finally get HJB equations in the following form:

$$\rho v(\alpha, x_1, x_2) = \tag{4.19}$$

$$\min_{(u_1(.),u_2(.))\in\Gamma(\alpha)} \begin{Bmatrix} G_\alpha(x_1, x_2, u) + \sum_\beta q_{\alpha\beta}\, v(\beta, x_1, x_2) + f_1 \frac{\partial v}{\partial x_1} \\ +f_2 \frac{\partial v}{\partial x_2} + \frac{1}{2} g_1^2 \frac{\partial^2 v}{\partial x_1^2} + \frac{1}{2} g_2^2 \frac{\partial^2 v}{\partial x_2^2} + g_1 g_2 \frac{\partial^2 v}{\partial x_1 \partial x_2} \end{Bmatrix}, \alpha, \beta \in B$$

4.4 Numerical method – policy improvement

A numerical approach proposed by Kushner and Dupuis (1992) and successfully used in the series of works of Boukas and Haurie (1990) and Yan and Zhang (1997) consists of introducing the grid in the state space (x_1, x_2) for approximating the value function $v(\alpha, x_1, x_2)$ approximating the first derivatives by "up wind" finite differences, then use policy improvement – discrete analog of a gradient descent in policy (control) space. Use of "up-wind" derivatives results in conditional computations but greatly improve convergence of the numerical.

4.4.1 Computations of first derivatives

To describe conditional computations of the derivatives let us introduce the following notation:

$$K_\alpha^+ = \mathrm{Ind}\{u_{1\alpha} + u_{2\alpha} - \mu_D \geq 0\}, K_\alpha^- = \mathrm{Ind}\{u_{1\alpha} + u_{2\alpha} - \mu_D < 0\}$$

with $\mathrm{Ind}\big(P(.)\big) = \begin{cases} 1 & \text{if } P(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$

The first derivatives of the value function with respect to x_1 and x_2 are:

$$v_{x_1}(\alpha, x_1, x_2) = \begin{cases} \dfrac{1}{hx_1}\big(v(\alpha, x_1 + hx_1, x_2) - v(\alpha, x_1, x_2)\big). K_\alpha^+ \\ \dfrac{1}{hx_1}\big(v(\alpha, x_1, x_2) - v(\alpha, x_1 - hx_1, x_2)\big). K_\alpha^- \end{cases} \tag{4.20}$$

$$v_{x_2}(\alpha, x_1, x_2) = \frac{1}{hx_2}\big(v(\alpha, x_1, x_2 + hx_2) - v(\alpha, x_1, x_2)\big) \tag{4.21}$$

It worth emphasizing that there is no conditional computation for v_{x_2} since $(\mu_R - u_{2\alpha}) \geq 0$ all the time due to assumption described in section 4.2.

4.4.2 Computations of second derivatives

For $v_{x_1 x_1}(\alpha, x_1, x_2)$ and for $v_{x_2 x_2}(\alpha, x_1, x_2)$ we have:

$$v_{x_1x_1}(\alpha, x_1, x_2) = \frac{1}{{hx_1}^2}\big(v(\alpha, x_1 + hx_1, x_2) + v(\alpha, x_1 - hx_1, x_2) - 2v(\alpha, x_1, x_2)\big) \tag{4.22}$$

$$v_{x_2x_2}(\alpha, x_1, x_2) = \frac{1}{{hx_2}^2}\big(v(\alpha, x_1, x_2 + hx_2) + v(\alpha, x_1, x_2 - hx_2) - 2v(\alpha, x_1, x_2)\big) \tag{4.23}$$

Both expressions above do not need conditional computations, contrary to the cross derivative $v_{x_1x_2}(\alpha, x_1, x_2)$ which might need up to four different schemes. Since the return inventory is always positive we will use just two schemes (main inventory can be positive or negative).

- If $(u_{1\alpha} + u_{2\alpha} - \mu_D) \geq 0$ and $(u_R - u_{2\alpha}) \geq 0$ we have:

$$v_{x_1x_2}(\alpha, x_1, x_2) = \frac{1}{hx_1 hx_2}\begin{pmatrix} v(\alpha, x_1 + hx_1, x_2 + hx_2) - v(\alpha, x_1 + hx_1, x_2) \\ -v(\alpha, x_1, x_2 + hx_2) + v(\alpha, x_1, x_2) \end{pmatrix} \tag{4.24}$$

- If $(u_{1\alpha} + u_{2\alpha} - \mu_D) < 0$ and $(u_R - u_{2\alpha}) \geq 0$ we have:

$$v_{x_1x_2}(\alpha, x_1, x_2) = \frac{1}{hx_1 hx_2}\begin{pmatrix} v(\alpha, x_1, x_2 + hx_2) - v(\alpha, x_1 - hx_1, x_2 + hx_2) \\ +v(\alpha, x_1 - hx_1, x_2) - v(\alpha, x_1, x_2) \end{pmatrix} \tag{4.25}$$

As a result, we obtain for the case of Brownian motion the following discrete HJB equations:

- In mode 1: $\alpha = 1$; we obtain:

$v(1, x_1, x_2) =$

$$\min_{(u_1(.),u_2(.))\in\Gamma(1)} \frac{\left\{\begin{array}{c} G_1(x_1, x_2, u) + q_{12}v(2, x_1, x_2) + q_{13}v(3, x_1, x_2) \\ +\dfrac{|u_{11} + u_{21} - \mu_D|}{hx_1}\begin{bmatrix} v(1, x_1 + hx_1, x_2)K_1^+ \\ + \\ v(1, x_1 - hx_1, x_2)K_1^- \end{bmatrix} + \dfrac{(\mu_R - u_{21})}{hx_2}\big(v(1, x_1, x_2 + hx_2)\big) \\ +\dfrac{1}{2}\dfrac{{\sigma_D}^2}{{hx_1}^2}\big(v(1, x_1 + hx_1, x_2) + v(1, x_1 - hx_1, x_2)\big) \\ +\dfrac{1}{2}\dfrac{{\sigma_R}^2}{{hx_2}^2}\big(v(1, x_1, x_2 + hx_2) + v(1, x_1, x_2 - hx_2)\big) \\ -\dfrac{\sigma_D\sigma_R}{hx_1 hx_2}\begin{bmatrix} \big(v(1, x_1 + hx_1, x_2 + hx_2) - v(1, x_1 + hx_1, x_2) - v(1, x_1, x_2 + hx_2)\big)K_1^+ \\ + \\ \big(v(1, x_1, x_2 + hx_2) - v(1, x_1 - hx_1, x_2 + hx_2) + v(1, x_1 - hx_1, x_2)\big)K_1^- \end{bmatrix} \end{array}\right\}}{Q_{11}} \tag{4.26}$$

- Mode 2: $\alpha = 2$; $u_2 = 0$ and

$$\nu(2,x_1,x_2) =$$

$$\min_{(u_1(.),u_2(.))\in\Gamma(2)} \frac{\left\{\begin{array}{c} G_2(x_1,x_2,u) + q_{21}\nu(1,x_1,x_2) + q_{24}\nu(4,x_1,x_2) \\ +\frac{|u_{12}-\mu_D|}{hx_1}\begin{bmatrix}\nu(2,x_1+hx_1,x_2)K_2^+ \\ + \\ \nu(2,x_1-hx_1,x_2)K_2^-\end{bmatrix} + \frac{\mu_R}{hx_2}(\nu(2,x_1,x_2+hx_2) \\ +\frac{1}{2}\frac{\sigma_D{}^2}{hx_1{}^2}\big(\nu(2,x_1+hx_1,x_2)+\nu(2,x_1-hx_1,x_2)\big) \\ +\frac{1}{2}\frac{\sigma_R{}^2}{hx_2{}^2}\big(\nu(2,x_1,x_2+hx_2)+\nu(2,x_1,x_2-hx_2)\big) \\ -\frac{\sigma_D\sigma_R}{hx_1hx_2}\begin{bmatrix}\big(\nu(2,x_1+hx_1,x_2+hx_2)-\nu(2,x_1+hx_1,x_2)-\nu(2,x_1,x_2+hx_2)\big)K_2^+ \\ + \\ \big(\nu(2,x_1,x_2+hx_2)-\nu(2,x_1-hx_1,x_2+hx_2)+\nu(2,x_1-hx_1,x_2)\big)K_2^-\end{bmatrix}\end{array}\right\}}{Q_{21}} \tag{4.27}$$

- Mode 3: $\alpha = 3$; $u_1 = 0$ and

$$\nu(3,x_1,x_2) =$$

$$\min_{(u_1(.),u_2(.))\in\Gamma(3)} \frac{\left\{\begin{array}{c} G_3(x_1,x_2,u) + q_{31}\nu(1,x_1,x_2) + q_{34}\nu(4,x_1,x_2) \\ +\frac{|u_{23}-\mu_D|}{hx_1}\begin{bmatrix}\nu(3,x_1+hx_1,x_2)K_3^+ \\ + \\ \nu(3,x_1-hx_1,x_2)K_3^-\end{bmatrix} + \frac{(\mu_R-u_{23})}{hx_2}(\nu(3,x_1,x_2+hx_2) \\ +\frac{1}{2}\frac{\sigma_D{}^2}{hx_1{}^2}\big(\nu(3,x_1+hx_1,x_2)+\nu(3,x_1-hx_1,x_2)\big) \\ +\frac{1}{2}\frac{\sigma_R{}^2}{hx_2{}^2}\big(\nu(3,x_1,x_2+hx_2)+\nu(3,x_1,x_2-hx_2)\big) \\ -\frac{\sigma_D\sigma_R}{hx_1hx_2}\begin{bmatrix}\big(\nu(3,x_1+hx_1,x_2+hx_2)-\nu(3,x_1+hx_1,x_2)-\nu(3,x_1,x_2+hx_2)\big)K_3^+ \\ + \\ \big(\nu(3,x_1,x_2+hx_2)-\nu(3,x_1-hx_1,x_2+hx_2)+\nu(3,x_1-hx_1,x_2)\big)K_3^-\end{bmatrix}\end{array}\right\}}{Q_{31}} \tag{4.28}$$

- Mode 4: $\alpha = 4$; $u_1 = 0$ and $u_2 = 0$, $-\mu_D < 0\ \forall\ t$ and the value function

$$\nu(4,x_1,x_2) =$$

$$\min_{(u_1(.),u_2(.))\in\Gamma(4)} \frac{\left\{\begin{array}{c} G_4(x_1,x_2,u) + q_{42}\nu(2,x_1,x_2) + q_{43}\nu(3,x_1,x_2) \\ +\frac{\mu_D}{hx_1}\nu(4,x_1-hx_1,x_2) + \frac{\mu_R}{hx_2}(\nu(4,x_1,x_2+hx_2) \\ +\frac{1}{2}\frac{\sigma_D{}^2}{hx_1{}^2}\big(\nu(4,x_1+hx_1,x_2)+\nu(4,x_1-hx_1,x_2)\big) \\ +\frac{1}{2}\frac{\sigma_R{}^2}{hx_2{}^2}\big(\nu(4,x_1,x_2+hx_2)+\nu(4,x_1,x_2-hx_2)\big) \\ -\frac{\sigma_D\sigma_R}{hx_1hx_2}\big(\nu(4,x_1,x_2+hx_2)-\nu(4,x_1-hx_1,x_2+hx_2)+\nu(4,x_1-hx_1,x_2)\big)\end{array}\right\}}{Q_{41}} \tag{4.29}$$

where:

$$Q_{11} = \rho + q_{12} + q_{13} + \frac{|u_{11} + u_{21} - \mu_D|}{hx_1} + \frac{(\mu_R - u_{21})}{hx_2} + \frac{{\sigma_D}^2}{{hx_1}^2} + \frac{{\sigma_R}^2}{{hx_2}^2} + \frac{\sigma_D\sigma_R}{hx_1hx_2}(K_1^+ - K_1^-) \tag{4.30}$$

$$Q_{21} = \rho + q_{21} + q_{24} + \frac{|u_{12} - \mu_D|}{hx_1} + \frac{\mu_R}{hx_2} + \frac{{\sigma_D}^2}{{hx_1}^2} + \frac{{\sigma_R}^2}{{hx_2}^2} + \frac{\sigma_D\sigma_R}{hx_1hx_2}(K_2^+ - K_2^-) \tag{4.31}$$

$$Q_{31} = \rho + q_{31} + q_{34} + \frac{|u_{23} - \mu_D|}{hx_1} + \frac{(\mu_R - u_{23})}{hx_2} + \frac{{\sigma_D}^2}{{hx_1}^2} + \frac{{\sigma_R}^2}{{hx_2}^2} + \frac{\sigma_D\sigma_R}{hx_1hx_2}(K_3^+ - K_3^-) \tag{4.32}$$

$$Q_{41} = \rho + q_{42} + q_{43} + \frac{\mu_D}{hx_1} + \frac{\mu_R}{hx_2} + \frac{{\sigma_D}^2}{{hx_1}^2} + \frac{{\sigma_R}^2}{{hx_2}^2} - \frac{\sigma_D\sigma_R}{hx_1hx_2} \tag{4.33}$$

In the second case (filter demand) we have similar HJB equations in four modes, but with slightly different parameters in the first derivative of the value function namely: $f_{1\alpha} = u_{1\alpha} + u_{2\alpha} + a.D$ and $f_{2\alpha} = -(u_{2\alpha} + raD)$.

4.5 Optimal production policy for hybrid system–simulation results

The first case we have analyzed corresponds to the hybrid system with manufacturing costs set relatively high in order to enforce production in remanufacturing loop. The (cumulative) demand is modeled as a Brownian process. The results are shown in figures 4.3-4.6. Figure 4.3 illustrates the shape of the value function $v(\alpha = 1, x_1, x_2)$ depending on the stock levels of manufactured (x_1) and remanufactured (x_2) products in mode 1. Value functions in other modes have similar shapes and are not shown. Figures 4.4 and 4.5 illustrate the optimal policy for the machine 1 (manufacturing) in mode 1 (both machines in operation) and mode 2 (remanufacturing machine 2 in failure) respectively. The optimal policies for the machine 1 are of hedging-point type, namely: maximal production if the stock level (x_1) is below the threshold, zero production above the threshold and production “on demand’’ at the threshold-level. Comparing figures 4.3 and 4.4 one can observe that the threshold level in mode 2 when machine 2 is in failure is higher than in mode 1.

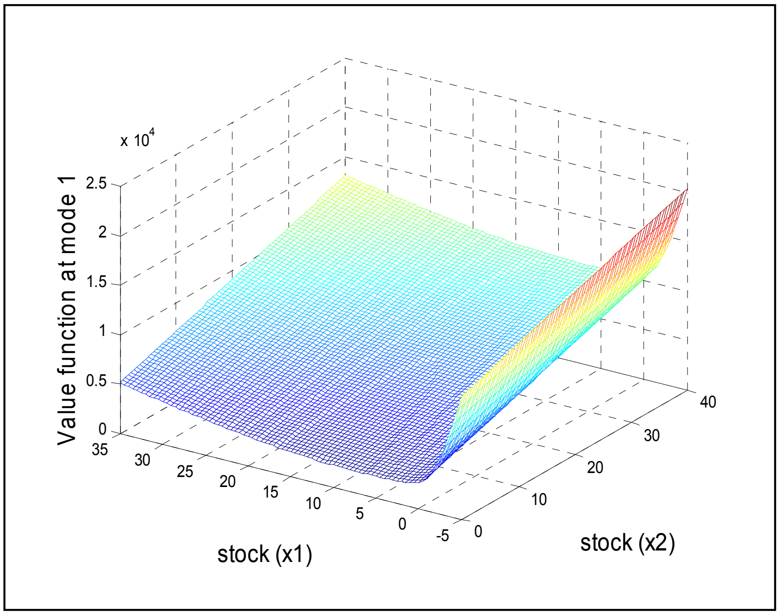

Figure 4.3 Value function in mode 1

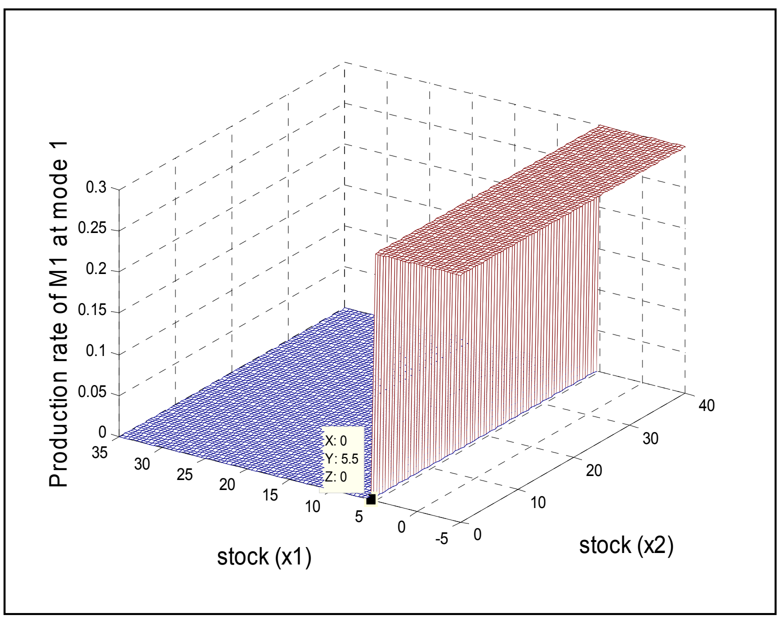

Figure 4.4 Production policy: machine 1, mode 1

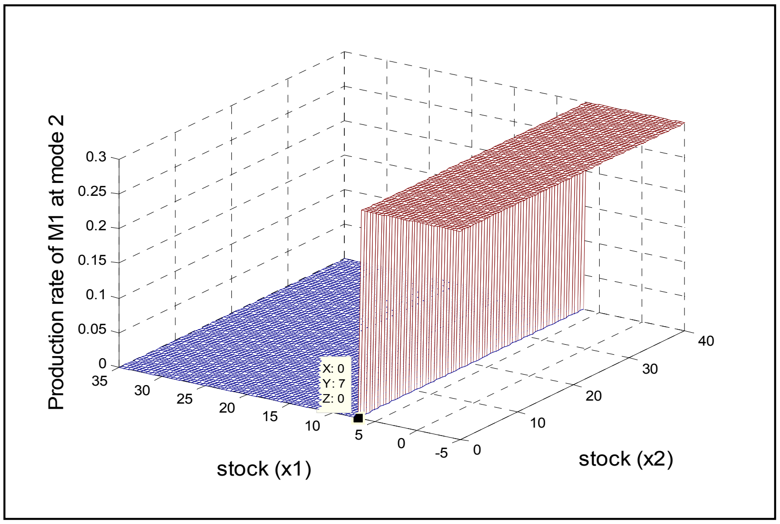

Figure 4.5 Production policy: machine 1, mode 2

Figure 4.6 illustrates the optimal policy for the machine 2 (remanufacturing) in modes 1 when both machines are in operation (optimal policy of the machine 2 in mode 3 is identical). Machine 2 must produce at average supply (proportional to demand) rate – which is also its maximal rate as explained in the section 4.2.

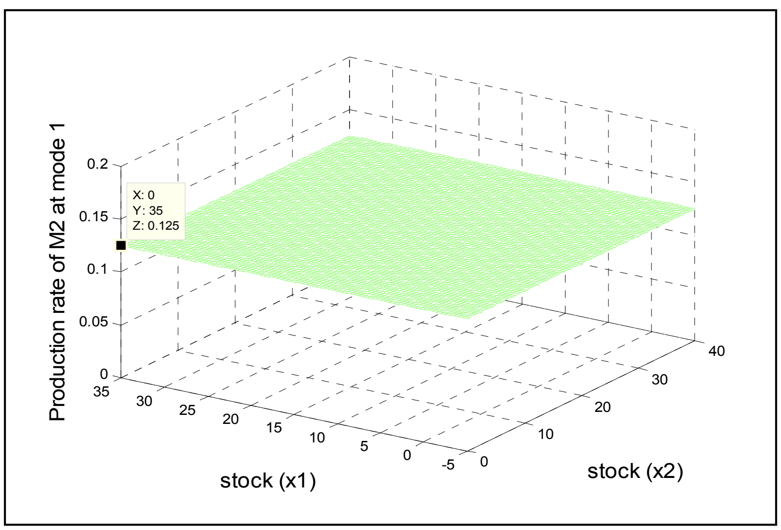

Figure 4.6 Production policy: machine 2, mode 1

Figures 4.7 and 4.8 show the realizations of Brownian and Markov-type (filtered) demand rates respectively (the variance σ_D is set to the same value). Comparing two graphs one can see that in Brownian case (figure 4.7) the variation rate is much faster than in Markov case (figure 4.8).

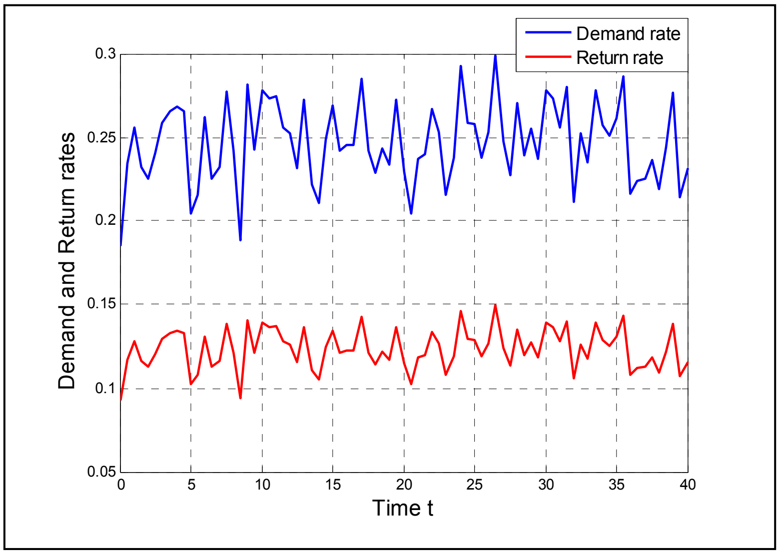

Figure 4.7 Brownian demand and return rates

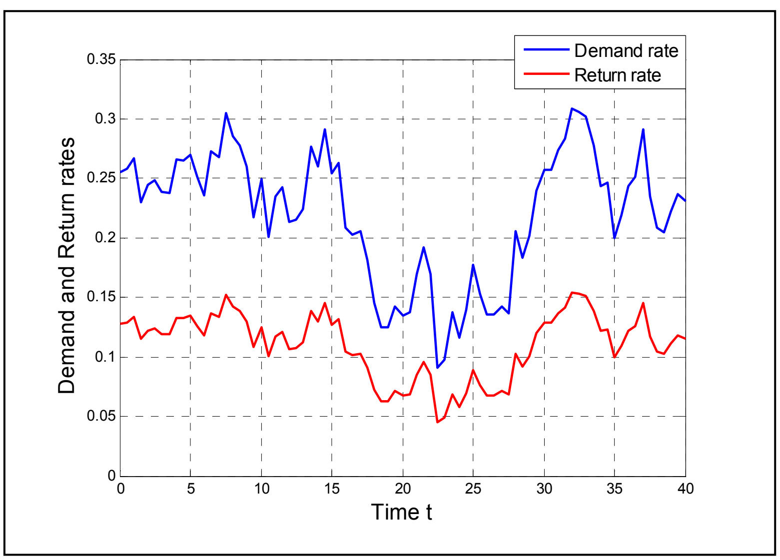

Figure 4.8 Filtered demand and return rate

Figures 4.9 and 4.10 illustrate the optimal policy for the machine 1 (manufacturing) in mode 1 (both machines in operation) and mode 2 (remanufacturing machine 2 in failure) respectively. The results are to be compared with those shown in figures 4.4 and 4.5. One can observe that the threshold values for the case of slower varying Markov demand are lower as compared to the case of Brownian demand. Parameters used for simulations are summarized in Table 4.1.

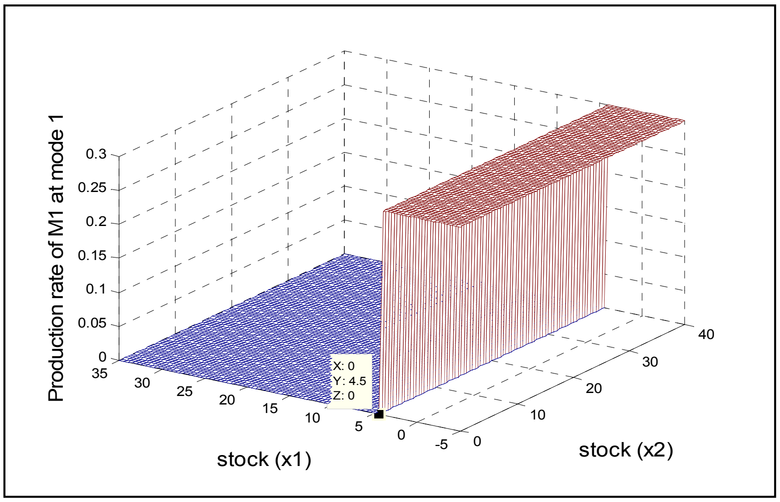

Figure 4.9 Production policy: machine 1, mode 1

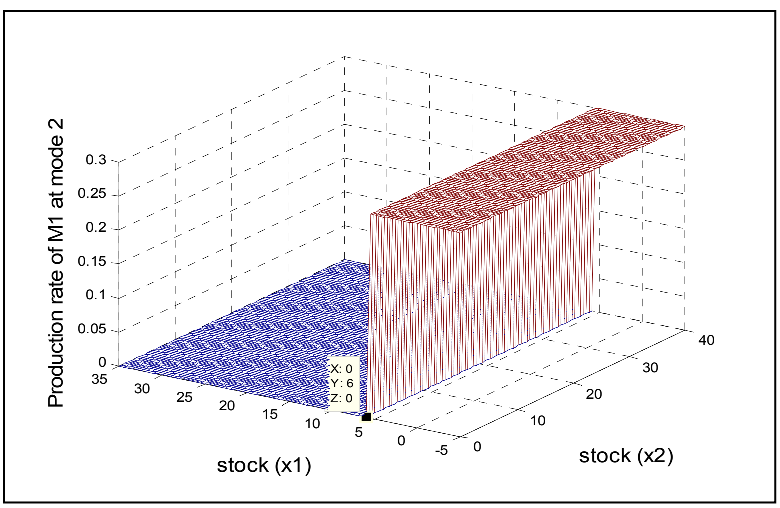

Figure 4.10 Production policy: machine 1, mode 2

Table 4.1 Parameters of the numerical example

c_1^+	c_2^+	c_1^-	c_p	c_r	c^α	ρ	u_{max}^1
2	2	100	10	8	50	0.01	0.3
u_{max}^2	q_{13}, q_{24}	q_{12}, q_{34}	$q_{31}, q_{42}, q_{21}, q_{43}$	r	a	b	D_0
0.125	0.01	0.02	0.067	0.5	0.01	0.035	0.25

In the second case of Markov-type (filtered) demand the parameters of the filter may be used to fit the model to the characteristics observed in the real life applications. A classical assumption of the constant demand in this context means that the variability of the demand is ignored and only its average rate is taken into account. A second order terms in HJB equations reflecting demand variability are in that case neglected and the optimal policy is found using the first order approximation of HJB equation. Optimal policy for the main (manufacturing) machine is of hedging point in both studied (Brownian and Markov) cases - as it is for the constant demand. According to figures 4.4, 4.5, 4.9 and 4.10 one can see that more the demand varies rapidly in time, more the hedging stock level increases in order to respond to the demand variability. In addition, the average total cost also increases from 1939 (Markov) to 2236.5 (Brownian) as the demand variability increases.

4.6 Conclusion and future work

We have shown that the problem of stochastic control corresponding to optimization of production planning in failure prone hybrid manufacturing/remanufacturing systems with random demand can be successfully analyzed for diffusion-type demands. We investigate this problem in continuous time which seems to be the most natural setting. We develop optimality conditions in the form of HJB equations and show that due to the Brownian component in the demand the HJB equations are the second order PDEs, contrary to the case of a constant demand where they are of the first order. We use finite difference approximations for HJB equations reducing a continuous time optimization problem to the discrete time, discrete state, infinite horizon dynamic programming problem, and use policy improvement technique of Kushner and Dupuis (1992) for solving it. Value functions of the stochastic optimization problems are usually non-smooth and corresponding HJB equations have to be addressed using generalized approaches such as viscosity solutions (Fleming and Soner, 2005). Theoretical studies of the convergence of discrete approximations to an exact (viscosity-type) solution of HJB equations when the size of the grid tends to zero is addressed in Barles and Jakobsen (2002) and Krylov (2000). Such theoretical analysis is out of the scope of this paper where we propose a numerical approach targeting the new model for the uncertain demand that allows addressing more naturally the growing number of industrial applications. Considering possible extensions of the study presented in this paper we count to explore a compound demand model of Poisson and diffusion-type process thus allowing both the jumps and continuous random variation of the demand.

CHAPITRE 5

ARTICLE 4: PRODUCTION AND REPLACEMENT PLANNING OF A DETERIORATING REMANUFACTURING SYSTEM IN CLOSED-LOOP CONFIGURATION

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Résumé

Le présent article traite le mécanisme de détérioration d'un système hybride fabrication/refabrication répondant à une seule demande de type de produit. Les machines impliquées sont sujettes à des pannes et réparations aléatoires. Compte tenu de la nature hétérogène des produits retournés, la machine de refabrication se détériore avec le temps en raison de réparations imparfaites et doit être remplacée, tandis que la machine de fabrication reçoit des matières premières homogènes et n'est pas affectée par ce genre de détérioration. Les variables de décision pour le problème de commande sont les taux de production des deux (2) machines et le taux de remplacement de la machine de refabrication. L'objectif principal de cet article est de trouver les variables de décision permettant d'avoir un coût total minimal, incluant les coûts de production, les coûts de stockage, les coûts de pénurie, les coûts de réparation et les coûts de remplacement, sur un horizon de planification infini. Une nouvelle approche de modélisation mathématique est proposée pour traiter une classe de problèmes reliés à l'historique de pannes et de réparations des machines. Cette nouvelle approche est basée sur l'extension de l'espace d'état et conduit à un modèle de décision de Markov, ce qui nous permet d'appliquer les techniques puissantes développées pour l'optimisation stochastique de ces modèles. Les conditions d'optimum sous la forme d'équations de Hamilton-Jacobi-Bellman (HJB) sont développées. Nous montrons que malgré la complexité au niveau de la dimensionnalité de l'espace d'état, le problème reste traitable et les solutions des équations d'HJB sont obtenues numériquement. Un exemple numérique est donné et une analyse de sensibilité est réalisée pour illustrer cette nouvelle approche proposée et assurer sa robustesse en montrant l'impact de différents paramètres du système sur les politiques de commande obtenue.

Abstract

The present paper deals with the deterioration mechanism for a hybrid manufacturing/remanufacturing system responding to a single product type demand. The machines involved are subject to random breakdowns and repairs. Given the heterogeneous nature of returned products, the remanufacturing machine deteriorates with time as a result of imperfect repairs, and needs to be replaced, while the manufacturing machine receives homogeneous raw materials, and is not affected by this type of deterioration. The decision variables for the control problem are the production rates of both machines and the replacement rate of the remanufacturing machine. The main objective of this paper is to find the decision variables minimizing the total cost, including production, inventory holding, backlog, repair and replacement costs, over an infinite planning horizon. A new mathematical modeling approach is proposed for the underlying class of problems related to the history of breakdowns and repairs. This new approach is based on the extension of the state space, and leads to a Markov decision model, which in turn allows us to apply the powerful techniques developed for the stochastic optimization of such models. Optimality conditions in the form of Hamilton-Jacobi-Bellman (HJB) equations are developed. We show that despite the increased state space dimension, the problem remains tractable, and the solutions of HJB equations are obtained numerically. Finally, an illustrative example and a sensitivity analysis are provided to ensure the robustness of the control policies obtained.

Keywords: Manufacturing/remanufacturing systems, Reverse logistics, Heterogeneous products, Imperfect repairs, Production planning, Replacement policy, Numerical methods.

5.1 Introduction

Faced with environmental, ecological, legal and economic factors, as well as market globalization, manufacturing companies are forced to become innovative in dealing with their end-of-life products. They must handle performance optimization problems in their global supply chain management. In this framework, most companies pay particular attention to the development of reverse logistics activities and the benefits they can bring. This attention is aimed at changing the structure of internal policies in order to integrate recovery activities into existing forward logistics and to obtain better synchronization. In such contexts, even a good supply chain management system will remain incomplete if unaccompanied by a concurrent effort to effectively manage returned products. Moreover, acting holistically can increase the productivity, profitability and competitiveness of a company and reduce the negative environmental impacts caused by the extraction of raw materials. One of the definitions of reverse logistics most commonly encountered in the literature is that given by Rogers and Tibben-Lembke (1998), who describe the concept as "The process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal". Indeed, reverse logistics processes start with end-of-life products collected from clients, and take different forms, including the recycling, remanufacture, repair, and finally, disposal of some used parts. Many works have been dedicated to closed-loop logistics systems, without taking into account the combined effect of stochastic aspects related to the dynamics of machines, the limit on the quantity of products to be remanufactured and the effects of deterioration caused by the return of heterogeneous used products. Since production planning and replacement problems are more complex in a hybrid context involving manufacturing and remanufacturing activities, especially in the presence of these aspects, we aim to establish a new optimization approach with an extension of the states that can be applied on this class of problems. Hence the objective of this study is to develop the optimal control of production and replacement rates for the hybrid manufacturing/remanufacturing system in a stochastic context subject to deterioration and reverse logistics constraints.

The rest of the paper is organized as follows. A literature review is presented in Section 5.2. In Section 5.3, we present the notations and the assumptions used in this study. The problem statement is also described in detail in Section 5.3. Optimality conditions in the form of HJB equations and a numerical example are addressed in Section 5.4. The structure of the optimal policies is presented in Section 5.5, followed in Section 5.6 by a sensitivity analysis illustrating the robustness of the obtained policies. Discussions of the results and implementation of the joint control policy are presented in Section 5.7. Finally, the paper is concluded in Section 5.8.

5.2 Literature review

In an industrial context, manufacturing systems may be affected by deteriorations due to a combination of a number of factors, including machine breakdowns and repairs, wear, human interventions, etc. This deterioration is usually progressive, and may have a severe effect on the reliability of the production system. In the long term, the intensity of failures will increase and the machine becomes less available. Thus, it becomes economically justifiable to replace it, but the problem is to determine the appropriate moment at which to carry out the replacement or to know for how long it should continue to be used (Dehayem-Nodem *et al.*, 2011). In industrial practice, it is important to incorporate repair/replacement and reverse logistics activities into the production environment, and to deal with the corresponding complexity in order to determine more realistic control policies.

In the context of imperfect repairs, the optimal production and repair/replacement control problem has become a very significant one for practitioners. It can be shown that the application of a joint policy for this problem allows for a better management of the manufacturing system performance, and permits a timely intervention, through the acquisition of a new machine. The first comprehensive work on this interaction was presented by Dehayem-Nodem *et al.* (2011), using a semi-Markov decision model. The authors studied a manufacturing system which deteriorates with age and imperfect repair. Rivera-Gomez *et al.* (2013b) proposed a quality deterioration model, and used a combination of two factors: the wear of the machine due to an aging process, and human interventions tied to imperfect repairs. In the same vein, Kouedeu, Kenné, Songmene and Polotski (2015) developed a stochastic optimization model for a single-machine, single-product failure-prone manufacturing system under imperfect repairs, when the failure rate increases with the number of failures. Following a preventive maintenance activity, the machine becomes as good as new. As we can notice in the research works of Dehayem-Nodem *et al.* (2011), Rivera-Gomez *et al.* (2013b) and Kouedeu *et al.* (2015), the stochastic models are limited to the production and maintenance control problem with the deterioration on a manufacturing system. Nevertheless, a key observation is that none of them have considered the effect of deterioration on remanufacturing systems. Since this aspect is widely observed in practical situations, it will be very useful to be taken into account in our model.

Remanufacturing has both economic and environmental benefits (Mitra, 2007; Sasikumar and Kannan, 2008). Remanufacturing, which is an important part of a closed-loop supply chain, is one of the most crucial processes in product recovery (Jena and Sarmah, 2014; Zhou, Tao and Chao, 2011). Kumar and Putnam (2008) considered that remanufacturing is gaining significance in the area of product recovery, and find it to be singularly different from the repair activity, since in remanufacturing, products are completely disassembled, cleaned and inspected; following that, all parts are restored to “like new” conditions by reuse, reconditioning and replacement operations. Bela G. and Halit (2012) for their part defined remanufacturing as an industrial process in which a new product is reassembled from an old one, except that many of the components used are taken from used parts, and worn out components are replaced with new or rebuilt ones.

Recently, Jiang *et al.* (2016) studied a remanufacturing process planning in a situation where the failure rate of remanufacturing operations is influenced by the quality of used products returned. The authors showed that depending on the type of damage suffered by the part (wear, corrosion, fatigue, etc.), and the level involved (slight, medium, serious), a variety of remanufacturing operations can be employed. Despite the relevance of the paper, their results are limited to the open-loop reverse supply chain with no consideration on the basic production environment.

Traditional areas where remanufacturing is commonly performed include the automotive and aeronautic sectors and those involving machinery and mechanical assemblies (such as aircraft engines and machine tools), as well as photocopiers (see Kerr and Ryan (2001) about photocopier remanufacturing at Fuji Xerox Australia). A typical example of remanufacturing is the case of the diesel engine covered by Sutherland, Adler, Haapala and Kumar (2008), in which engine subsystems are disassembled, cleaned and inspected. The component (core) that cannot be refurbished is recycled or remanufactured using welding, machining, and other salvage operations. In their study, the authors concentrate on several cores, such as engine blocks, cylinder heads, crankshafts, connecting rods, and pistons. Finally, during the last step of the remanufacturing process, all the components are reassembled, and the finished product obtained is like a new diesel engine. Our modeling approach could be applied to many industrial cases, such as those mentioned in this section. It can also be applied in situations of a hybrid manufacturing/remanufacturing system, where remanufacturing machine can be subject to failures and deteriorates over time due to the return of heterogeneous quality used products. Taking into account these aspects in optimization models can be very useful for practical managerial decisions concerning the company’s production policies, and allow a better understanding of the real behavior of the closed-loop production system.

The integration of reverse logistics into the basic production environment began to take flight with the work of Fleischmann *et al.* (1997). The authors proposed an overview of quantitative approaches for production planning and inventory management. They divided the field of reverse logistics into three main areas, namely, distribution planning, inventory management, and production planning. In the literature, optimization models in deterministic and stochastic contexts for supply chains with a returned products recovery system have been proposed, with special attention paid to production planning and inventory management problems without failures and deteriorations. In particular, in Dobos (2003) a model for the optimization of production in a deterministic closed-loop environment is proposed. Stochastic optimization models for a closed-loop manufacturing/remanufacturing system can be found in the works of Kiesmüller and Scherer (2003), Inderfurth (2004), Oscar and Silva (2011), Corum *et al.* (2014), and Giri and Sharma (2016). In these works concerning the closed-loop supply chain, the system is considered reliable, and the dynamics of production facilities are set aside. In other words, at any time, the system can produce at its maximum production capacity if so required by the control policy. However, to make this assumption more realistic in control, the deterioration process and the dynamics of production facilities that affect their availabilities should be incorporated in the mathematical models.

In Kenné *et al.* (2012) the authors proposed a stochastic dynamic control model to optimize the global performance of the closed-loop supply chain in presence of two unreliable machines working in manufacturing and remanufacturing modes, respectively. The return of used products from market is considered homogeneous and infinite. Recently, Polotski *et al.* (2017) followed the methodology proposed in Kenné *et al.* (2012), and studied a hybrid system that consists of one facility and necessitates setup to switch between manufacturing and remanufacturing modes. The authors explored the situation in which the production process is stochastic in nature (due to machine failures), and the return is bounded. In Kenné *et al.* (2012) and Polotski *et al.* (2017), the optimality conditions were developed in the form of HJB equations, allowing a resolution of the optimal control problem based on stochastic dynamic programming and numerical methods. However, the problem of deterioration of machines and their replacement option by a new one were completely neglected.

In the context of deterioration, Kouedeu *et al.* (2014) developed the stochastic optimization model for a hybrid manufacturing/remanufacturing system consisting of two machines which produce one part type. The stochastic nature of the system is due to random failures and repairs by integrating the gradual deterioration on the manufacturing machine along the production process (i.e., its failure rate depends on its production rate and corrective maintenance activities restore the machine as good as new). In their model, the authors assumed that the return of the used products is homogeneous and bounded. The results obtained clearly showed that the structure of the optimal control production policy is characterized by several different threshold parameters due to the fact that the manufacturing machine degrades with its productivity speed. Their work represents the first attempt that considered the deterioration phenomena on the main manufacturing machine in the context of the closed-loop supply chains. The hybrid system considered in this paper differs from Kouedeu *et al.* (2014) as it considers both deterioration process and replacement of the remanufacturing machine. In most industrial cases, there is a need to take into account the existence of the heterogeneous used products. This additional dimensionality may nevertheless increase the complexity of the control problem of the hybrid system and its mathematical modeling.

We limit our analysis by the systems with bounded return, where the production in remanufacturing mode depends on the number of used products available in recoverable inventory. Considering the system in which the combined machines are required not for changing the product to be manufactured, but rather to take into account the heterogeneous nature of the recoverable supply and its negative effects. The main subject of our research is to investigate a production planning and replacement control problem for an unreliable hybrid closed-loop system under the deterioration of the remanufacturing machine with bounded returns. No research has considered the deterioration effect on the remanufacturing machine, and addressed the question related to the effect of the presence of this deterioration on the joint control of production and replacement. We pay particular attention to the deterioration caused by the return of heterogeneous used products that give rise to imperfect repairs. Therefore, through the domain of deteriorating systems, it would be interesting, in order to provide a useful framework for more realistic industrial situations, to control the case of a hybrid manufacturing/remanufacturing system consisting of at least two machines, taking into consideration the progressive deterioration occurring throughout the remanufacturing process. In this context, a replacement activity can be conducted on the remanufacturing machine to increase the production capacity of the system. This research deals with this problem and shows the coordination between the two machines and the need for a hybrid system using simultaneous control.

The specific problem of production optimization considered in this work implies a certain complexity in terms of modeling and approach. Hence, we propose a methodology with an extension of the states (not considered before in the literature) to solve this type of problem. We determine the optimal production rate of the manufacturing machine and the optimal production rate and replacement policy of the remanufacturing machine simultaneously. Since repair activities are imperfect in the case of the remanufacturing machine, we use its number of failures to indicate its history. The semi-Markov model is used to describe the dynamics of the system under deterioration. We propose a new mathematical modeling approach to solve a class of semi-Markov problems. This approach is based on the extension of the number of states to model the dynamics of the manufacturing and remanufacturing machines as a Markov decision process. This extension allows us to apply the theoretically supported techniques developed for the stochastic optimization of Markovian systems and to ensure the optimality of the solutions obtained. The optimal control policies are determined to satisfy a customer demand and minimize the total cost, which includes the costs of production, inventory holding, backlog, repair and replacement, over an infinite planning horizon. The solution obtained can be applied to real business case studies in order to help companies to make new decisions regarding the use of production resources. It can also be useful in evaluation and improvement processes for existing decisions in manufacturing/remanufacturing systems. The practical implication of the proposed model is examined in this paper through a numerical example and a sensitivity analysis.

5.3 Manufacturing/remanufacturing system

Before we formulate the problem, we define the notations and assumptions to be used throughout this article in the next two subsections.

5.3.1 Notations

$x_1(t)$	stock level of manufactured and remanufactured products at time t
$x_2(t)$	stock level of returned products at time t
$n(t)$	number of failures of the remanufacturing machine at time t
d	demand rate of customers (products/time unit)
r	proportion of returns in the recoverable inventory
$u_1(t)$	production rate of the manufacturing machine (products/time unit)
$u_2(t)$	production rate of the remanufacturing machine (products/time unit)
$w_1(.)$	control variable for the replacement of the remanufacturing machine in mode 1
$w_2(.)$	control variable for the replacement of the remanufacturing machine in mode 2
u_{max}^1	maximal production rate of the manufacturing machine (products/time unit)
u_{max}^2	maximal production rate of the remanufacturing machine (products/time unit)
$q_{\alpha\beta}(.)$	transition rate from mode α to mode β
$Q(.)$	transition rate matrix
ρ	discount rate
$h(.)$	inventory/backlog cost function
$c(.)$	manufacturing/remanufacturing cost function
$k(.)$	remanufacturing repair cost function
$G(.)$	instantaneous cost function
$J(.)$	expected discounted cost function

$v(.)$ value function

c_1^+ inventory cost for serviceable products ($/product/time unit)

c_2^+ inventory cost for returned products ($/product/time unit)

c_1^- backlog cost for serviceable products ($/missing product/time unit)

c_M manufacturing cost ($/product)

c_R remanufacturing cost ($/product)

c_r^1 repair cost rate for manufacturing machine ($/time unit)

c_r^2 repair cost rate for remanufacturing machine ($/time unit)

c_0 replacement cost for remanufacturing machine ($)

5.3.2 Assumptions

The mathematical model in this analysis is based on the following assumptions:

1) Raw materials for manufacturing are always available and in unlimited quantities.
2) Raw materials for remanufacturing are limited by the returned quantities and the number of parts available in the recoverable inventory.
3) Customer demand of finished products is known and subject to a constant rate over time.
4) The return rate depends on the current demand rate values, even if it the products being returned are at the end of their useful lives.
5) The maximal production rate of each machine is known.
6) The corrective maintenance activity on the remanufacturing machine is imperfect and characterized by the fact that the repair time increases progressively with the number of failures, while the repair rate of the manufacturing machine is constant (see Wang and Zhang (2006, 2009) and Dehayem-Nodem *et al.* (2011)).
7) The quality of remanufactured products is not different from that of manufactured products. The remanufactured product is sold in the same market and at the same price as the manufactured product. This assumption is consistent, and is used by several authors

(see Yang, Wee, Chung and Ho (2010), Kenné *et al.* (2012), Maiti and Giri (2015), and Giri and Sharma (2016)). However, this assumption cannot be generalized for all cases in practice, and it should be assumed that we have different markets with different prices.

5.3.3 Problem statement

The hybrid manufacturing system under consideration is composed of two machines, M_1 and M_2, used for manufacturing and remanufacturing, respectively, and subject to random breakdowns and repairs. We assume that the two machines produce the same type of product. Two inventories are defined; the serviceable inventory, to store the final product, and the recoverable inventory, to store returned end-of-life (EOL) products. The serviceable inventory is built from both manufacturing and remanufacturing activities. The recoverable inventory is operated to hold returned products for future remanufacturing processes. We also consider that the remanufacturing machine M_2 undergoes a progressive deterioration that manifests itself by the repair time increasing with the number of failures. The failure and repair rates of the manufacturing machine (denoted as M_1) are assumed constant. Initially, machine M_2 can satisfy the demand, but due to the combined effect of availability fluctuations and deterioration, it is unable to fulfill long-term product demand. However, even the help obtained from using the limit capacity of M_1 cannot resolve the issue because the closed-loop system reaches a certain level of deterioration, and involves a higher repair cost for M_2. Additionally, replacement activities are available, and can be conducted on machine M_2 to reduce the effects of deterioration by restoring its parameters to initial conditions. The situation is illustrated in figure 5.1. The system behavior is characterized by a hybrid state comprised of both continuous state variables, $x_1(t)$ and $x_2(t)$, and one discrete state variable $\xi(t)$. The continuous components $x_1(t)$ and $x_2(t)$ represent, respectively, the stock levels of the serviceable inventory and recoverable inventory at time t.

The dynamic behavior of the stock levels can be described by the following two-dimensional system of ordinary differential equations:

$$\dot{x}_1(t) = u_1(t) + u_2(t) - d, \quad x_1(0) = x_{10}, \quad t \geq 0 \tag{5.1}$$

$$\dot{x}_2(t) = rd - u_2(t), \quad x_2(0) = x_{20}, \quad t \geq 0 \tag{5.2}$$

where x_{10} and x_{20} are the given initial stock levels of the serviceable inventory and returned products, respectively.

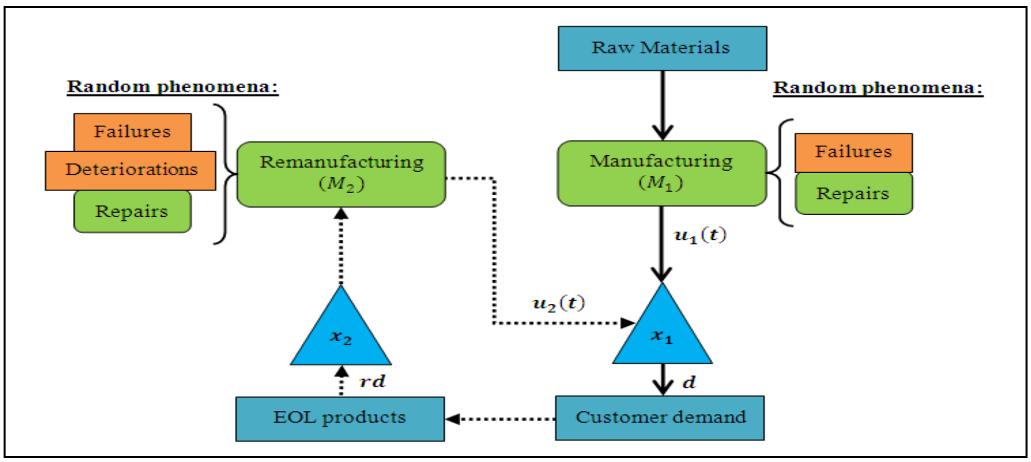


Figure 5.1 Hybrid manufacturing/remanufacturing system

We assume that the production rates $u_1(t)$ and $u_2(t)$ must satisfy the capacity constraints as follows:

$$0 \leq u_1(t) \leq u_{max}^1, \quad t \geq 0 \tag{5.3}$$

$$0 \leq u_2(t) \leq u_{max}^2, \quad t \geq 0 \tag{5.4}$$

where u_{max}^1 and u_{max}^2 are the maximal production rates of the manufacturing and remanufacturing machines, respectively, and are considered as positive given constants. We assume that the maximal production rate $u_{max}^2 > u_{max}^1$ and the demand rate $u_{max}^1 < d \leq u_{max}^2$. So, the machine M_1 has a limited capacity and the machine M_2 is not able to satisfy the customer demand in the long term due to the effect of deterioration after each repair.

The number of parts available in the recoverable inventory cannot be negative, and so we must therefore impose a non-negativity constraint on the inventory:

$$x_2(t) \geq 0, \text{ for all } t \geq 0 \tag{5.5}$$

The mode of the manufacturing machine M_1 can be classified either as under repair, denoted by 0, or as operational, denoted by 1. The mode of the remanufacturing machine M_2 is under repair, denoted by 0; as operational, denoted by 1, and under replacement, denoted by 2. The random variables $\xi_1(t)$ and $\xi_2(t)$ describe the state of machines M_1 and M_2 with values in $B_1 = \{0, 1\}$ and $B_2 = \{0, 1, 2\}$, respectively. The mode of the machines at time t is given by the random vector $\xi(t) = \big(\xi_1(t), \xi_2(t)\big)$ taking values in $B = B_1 \times B_2 = \{(1,1), (0,1), (1,0), (0,0), (1,2), (0,2)\} \equiv \{1, 2, 3, 4, 5, 6\}$, and is defined in Table 5.1.

Table 5.1 Manufacturing/remanufacturing transition modes

Mode	Machines state	Description
1	(1, 1)	Manufacturing machine is operational and remanufacturing machine is operational
2	(0, 1)	Manufacturing machine is under repair and remanufacturing machine is operational
3	(1, 0)	Manufacturing machine is operational and remanufacturing machine is under repair
4	(0, 0)	Manufacturing machine is under repair and remanufacturing machine is under repair
5	(1, 2)	Manufacturing machine is operational and remanufacturing machine is under replacement
6	(0, 2)	Manufacturing machine is under repair and remanufacturing machine is under replacement

Given that the maintenance activities of the remanufacturing machine depend on the history of repairs, it is evident that the dynamics of the hybrid manufacturing/remanufacturing system can naturally be modeled as a non-Markov decision process in a continuous-time discrete state $\{\xi(t) \geq 0\}$, with values of $\xi(t)$ in B. The transition diagram describing the dynamics of the considered hybrid system is illustrated in figure 5.2. Essentially, to improve the performance of the hybrid manufacturing/remanufacturing system during its life cycle, we control the production rates and the replacement rates in operational modes, simultaneously.

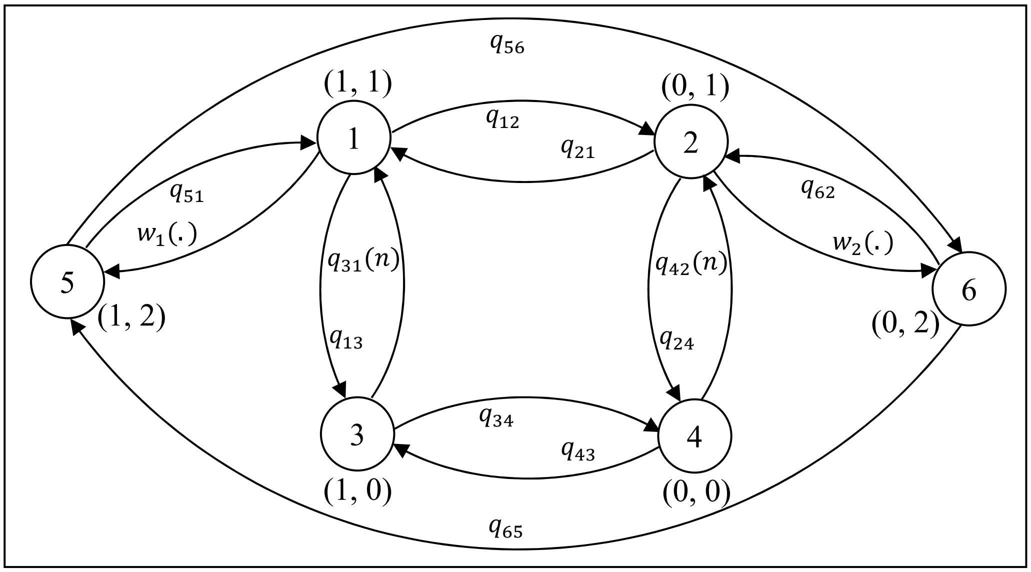


Figure 5.2 States transition diagram of the considered hybrid system

In failure modes, we do not have any control over the production, and we do not perform any replacement. We define the variables $w_1(.)$ and $w_2(.)$ as decision variables, continuous in the domains $[w_{min}^1, w_{max}^1]$ and $[w_{min}^2, w_{max}^2]$, respectively, which allow us to control the transitions to the replacement of the remanufacturing machine. w_{min}^1, w_{min}^2, w_{max}^1 and w_{max}^2 denote the minimum and maximum replacement rates, respectively. Let us define the transition rates $q_{15}(.) = w_1(.)$ and $q_{26}(.) = w_2(.)$ as functions of $w_1(.)$ and $w_2(.)$. The inverse of the replacement rates $q_{15}(.)$ and $q_{26}(.)$ are the mean time between the decision to perform the replacement of machine M_2 and the effective switch from the operational mode 1 to the replacement mode 5 and from the operational mode 2 to the replacement mode 6, respectively. Both transition rates $q_{15}(.)$ and $q_{26}(.)$ are controlled before machine M_2 is sent for replacement because of the different states of machine M_1 in modes 1 and 2. The transition rate $q_{15}(.)$ may allow machine M_2 to be sent for replacement with the operational state of machine M_1, while the transition rate $q_{26}(.)$ allows machine M_2 to be sent for replacement with the failed state of machine M_1. The replacements are allowed for all values within $[w^1_{min}, w^1_{max}]$ and $[w^2_{min}, w^2_{max}]$. When there are no actual replacements, machine M_2 remains operational, and at any failure instant, a repair action is considered. The transition rates q_{51} and q_{62} are equal and constant from the replacement mode to the operational mode of the remanufacturing machine M_2. The modes of the remanufacturing machine change from repair to operational with the transition rates $q_{31}(n)$ and $q_{42}(n)$, which are decreasing functions of the number of failures n. For the manufacturing machine M_1, the transition rates from the operational mode to the failure mode and vice versa are constant. Because the occurrences of breakdowns and repairs of both machines are independent, we can set $q_{12} = q_{34} = q_{56}$ and $q_{21} = q_{43} = q_{65}$. All other transition rates are equal to zero.

In practice, the two machines often have different characteristics, and manufacturing and remanufacturing are run as separate activities. The fact that the remanufacturing machine receives heterogeneous parts that are difficult to transform complicates its repair. Such a machine contains several components made of various materials. However, most components are deteriorative practically due to ageing and accumulated wearing. Upon a failure, the faulty component is repaired. At the same time, all other components of the machine continue to degrade, which will lead to further failures. These failures will affect several components, which will require a longer repair time. Wang and Zhang (2006, 2009) and Dehayem-Nodem *et al.* (2011) supported our point of view by using the age of the machine or the number of failures as an indicator to define the level of deterioration. Such a machine requires a considerable amount of repair time. Moreover, the consecutive repair times of the machine after failure will become longer and longer, and can increase with the age of the machine or the number of failures they have undergone. Although in general, the effect of deterioration in the case of imperfect repair leads to a semi-Markov model. In our paper, this complication on the remanufacturing machine can be modeled by a repair time, which progressively increases with the number of failures. For the manufacturing machine, it receives homogeneous raw materials, and for that reason, we will focus only on the deterioration constraints faced by remanufacturing machine. We will in fact assume that the manufacturing machine is not affected by this deterioration. We use the model of the repair transition rate as given in Dehayem-Nodem *et al.* (2011) to extend the concept of deterioration to the remanufacturing machine, which defines a function of the number of failures as follows:

$$q_{31}(n) = q_{42}(n) = q_0 + q_1\left(1 - \left(\frac{n-1}{N}\right)^{\theta}\right) \tag{5.6}$$

where q_0, q_1 and θ are given parameters, estimated from an analysis of historical data on the remanufacturing machine and N is the maximum number of failures. The inverse of $q_{31}(n)$ represents the mean time to repair $MTTR2(n)$, which increases with the number of failures n.

Due to the deterioration effect caused by imperfect repairs, the optimization problems for the hybrid production systems become very complex to solve. While the optimal solution of the semi-Markov model, taking into account the notion of memory with the number of failures, remains computationally difficult to obtain, we have to validate it in order to verify the optimum conditions. The main reason for this is that the stochastic optimal control theory using viscosity solutions has been more widely verified for homogeneous Markov models, but not for semi-Markov models. We propose a new mathematical modeling approach for the underlying class of semi-Markov problems and a verification of all conditions in order to guarantee the existence of viscosity solutions and the convergence of numerical methods used to find these solutions. This new approach is based on the extension of the state space of the system (operational, repair and replacement) in order to obtain an equivalent Markov decision model. The reader is referred to Annex V for more details on the proposed state extension approach.

The replacement cost is fixed at c_0, the repair cost for remanufacturing machine $k(n)$ varies with the number of failures n, and the manufacturing and remanufacturing production costs are fixed at c_M and c_R, respectively. Let $G(.)$ be the cost rate defined as follows:

$$G(\gamma, x_1, x_2, n, u_1, u_2, w_1, w_2) = h(x_1, x_2) + c(u_1, u_2) + w(\gamma, n) \tag{5.7}$$

where $h(x_1, x_2) = c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^-$. The constants c_1^+, c_2^+ and c_1^- are costs incurred per unit of produced parts, and are used to penalize the serviceable, recoverable inventories and backlog, respectively. The holding and backlog costs are such that $c_1^- > c_1^+ > c_2^+ > 0$. $x_i^+ = \max(0, x_i)$, $i = 1, 2$ and $x_1^- = \max(0, -x_1)$. Let $c(u_1, u_2) = c_M(u_1) + c_R(u_2) = c_M u_1 + c_R u_2$ be a linear cost function related to manufacturing and remanufacturing rates u_1 and u_2. We assume that $c_M > c_R > 0$ are costs incurred per unit of produced parts and are used to penalize the manufacturing and remanufacturing rates, respectively. $w(\gamma, n)$ is a manufacturing/remanufacturing function of repair and replacement cost rates of new process γ (see Annex V) and number of failures n, and is defined as follows:

$$w(\gamma, n) = c_0.\left(q_{51}.\mathrm{Ind}\{\gamma(t) = (5, n)\} + q_{62}.\mathrm{Ind}\{\gamma(t) = (6, n)\}\right)$$
$$+c_r^2.\left(\mathrm{Ind}\{\gamma(t) = (3, n)\} + \mathrm{Ind}\{\gamma(t) = (4, n)\}\right)$$
$$+c_r^1.\left(\mathrm{Ind}\{\gamma(t) = (2, n)\} + \mathrm{Ind}\{\gamma(t) = (4, n)\} + \mathrm{Ind}\{\gamma(t) = (6, n)\}\right)$$

where the indicator function of any function $\Theta(.)$ is defined as follows:

$$\mathrm{Ind}\{\Theta(.)\} = \begin{cases} 1 & \text{if } \Theta(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

The repair cost is an increasing function of repair time, and is given by: $k(n) = c_r^2.MTTR2(n)$, with $n \geq 0$ and $k(0) = 0$. It should be recalled that the repair cost rate is constant and fixed at c_r^2 and that the mean time to repair $MTTR2(n)$ is an increasing function of the number of failures n. In addition, the repair cost rate of machine M_1 denoted by c_r^1 is constant.

Let $\chi = (-\infty, +\infty) \times [0, +\infty) \subseteq \Re^2$ denote the state domain on inventories. Let $\Gamma(x_2(t), \gamma(t))$ denote the following control set; it depends on Markov process $\gamma(t)$ with respect to the state constraint $x_2(t) \in \chi$ (see Sethi *et al.* (2005)), including $u_1(.), u_2(.), w_1(.)$ and $w_2(.)$, and is given by:

$$\Gamma(x_2(t), \gamma(t)) = \begin{Bmatrix} (u_1(.), u_2(.), w_1(.), w_2(.)) \in \Re^4, 0 \leq u_1(.) \leq u^1_{max}, \\ 0 \leq u_2(.) \leq k \text{ with } k = \begin{cases} u^2_{max} & \text{if } x_2(t) > 0 \\ rd & \text{if } x_2(t) = 0 \end{cases}, \\ w^1_{min} \leq w_1(.) \leq w^1_{max}, w^2_{min} \leq w_2(.) \leq w^2_{max} \end{Bmatrix} \tag{5.8}$$

Let $\mathcal{F}_t$ denote the σ-algebra generated by the random process $\gamma(t)$, i.e., $\mathcal{F}_t = \sigma\{\gamma(s)\colon 0 \leq s \leq t\}$. We now define the concept of admissible controls.

Definition 5.1: A control process (production and replacement rates) $\pi(.) = (u_1(.), u_2(.), w_1(.), w_2(.))$ is said to be admissible with respect to the state constraint $x_2(t) \in \chi$ if:

(i) $\pi(.)$ is adapted to the filtration $\{\mathcal{F}_t\}$ for all $t \geq 0$ with respect to the σ-algebra;

(ii) $\pi(t) \in \Gamma(x_2(t), \gamma(t))$ for all $t \geq 0$; and

(iii) the solution $x_2(t) = x_2(0) + \int_0^t [rd - u_2(s)]ds \in \chi$ for all $t \geq 0$.

We use $\mathcal{A}$ to denote the set of all admissible controls with respect to the state constraint $x_2(t) \in \chi$. If we can adequately control the decision variables, we can improve the hybrid system performances in terms of productivity and availability, and intervene in a timely manner with the acquisition of a new machine. In this case, the objective is to find the four decision variables in $\mathcal{A}$, namely, the production rates $u_1(.)$ and $u_2(.)$ and the replacement variables $w_1(.)$ and $w_2(.)$, that minimize, for each initial state condition (α, x_1, x_2, n), the following expected discounted cost $J(.)$ given by:

$$J(\alpha, x_1, x_2, n, u_1, u_2, w_1, w_2) = \tag{5.9}$$

$$E\left\{\int_0^\infty e^{-\rho t} G(.)dt \,|\, \gamma(0) = \alpha, x_1(0) = x_1, x_2(0) = x_2, n(0) = n\right\}$$

where ρ is the discount rate.

The value function of the discounted cost problem is defined as follows:

$$v(\alpha, x_1, x_2, n) = \inf_{(u_1(.), u_2(.), w_1(.), w_2(.)) \in \mathcal{A}} J(\alpha, x_1, x_2, n, u_1, u_2, w_1, w_2) \qquad \forall \alpha \in B \subset \Omega \tag{5.10}$$

To address the feasibility of the hybrid system, as usual, we introduce the limiting probabilities $\pi_{i,n}$ of mode i, $i = 1, \dots, 6$ and at failure n that are known to be the steady state solutions of the forward Kolmogorov equations $\dot{\pi}_n(t) = \pi_n(t)Q(t)$. Therefore, we have:

$$\pi_n(.)Q(.) = 0 \tag{5.11}$$

with:

$$\sum_{i=1}^{6} \pi_{i,n} = 1 \tag{5.12}$$

where $\pi_n(.) = (\pi_{1,n}, \pi_{2,n}, \pi_{3,n}, \pi_{4,n}, \pi_{5,n}, \pi_{6,n})$ and $Q(.)$ is a $(6p + 4) \times (6p + 4)$ transition rates matrix defined by expression (V-4) in Annex V.

The hybrid system is considered feasible if:

$$(\pi_{1,n} + \pi_{3,n} + \pi_{5,n})u_{max}^1 + \min\{(\pi_{1,n} + \pi_{2,n})u_{max}^2, rd\} \geq d \tag{5.13}$$

This formula generalizes the condition used in Kenné *et al.* (2012). It can take into consideration the limit of used products that return from market, which represents a return rate rd with a proportion $r = [0\%, 100\%]$. In the case of an infinite return (more than $r = 100\%$), the used products can be managed by the condition (5.13), and the disposal option can then be required. The main reason is associated with the fact that remanufacturing or holding all used products could significantly raise serviceable and recoverable inventories, and, as result is the increasing of holding inventories costs (the disposal of the used products is out of scope of our paper). The derivation of equation (5.13) is presented in Annex VI.

The optimality conditions in the form of HJB equations for the hybrid stochastic optimal control problem with a state constraint are presented in the next section, and a numerical example is also provided below to illustrate the structure of the joint control policy.

5.4 Optimality conditions and numerical example

In this section, we develop the optimality conditions using the optimal control theory based on stochastic dynamic programming. The optimal control policy $(u_1^*(.)\, , u_2^*(.), w_1^*(.), w_2^*(.))$ denotes a minimizer over $\mathcal{A}$ on the right hand side of equation (5.9). This policy corresponds to the value function $v(.)$ given by equation (5.10) by showing that such a function should satisfy the set of partial differential equations known as the Hamilton-Jacobi-Bellman equations (HJB). The properties of the value function are justified by Theorem 3.1 of Sethi *et al.* (2005); the authors indicate that it is very convenient to write the HJB equation in terms of so-called directional derivatives (HJBDD), when dealing with control problems with state constraints, such as in flowshops and jobshops. The sequence of HJBDD equations corresponding to the proposed manufacturing/remanufacturing optimal control problem with the possibility of machine replacement can now be written as follows:

$$\rho v(\alpha, x_1, x_2, n) = \min_{(u_1, u_2, w_1, w_2) \in \mathcal{A}} \left\{ \begin{array}{c} G(.) + (u_1 + u_2 - d) \dfrac{\partial}{\partial x_1} v(\alpha, x_1, x_2, n) \\ +(rd - u_2) \dfrac{\partial}{\partial x_2} v(\alpha, x_1, x_2, n) \\ + \displaystyle\sum_{\beta} q_{\alpha\beta}(.) v\big(\beta, x_1, x_2, \varphi_n(\gamma, n)\big) \end{array} \right\} \tag{5.14}$$

where $\varphi_n(\gamma, n)$ defines a reset function that brings the number of failures to zero after replacing the remanufacturing machine and increments the number of failures after an imperfect repair. We can write this function as follows:

$$\varphi_n(\gamma, n) = \begin{cases} n+1 & \text{if } \begin{cases} \gamma(\tau^+) = (1, n) \text{ and } \gamma(\tau^-) = (3, n) \\ \text{or} \\ \gamma(\tau^+) = (2, n) \text{ and } \gamma(\tau^-) = (4, n) \end{cases} \\ 0 & \text{if } \begin{cases} \gamma(\tau^+) = (1, 0) \text{ and } \gamma(\tau^-) = (5, n) \\ \text{or} \\ \gamma(\tau^+) = (2, 0) \text{ and } \gamma(\tau^-) = (6, n) \end{cases} \\ n & \text{otherwise} \end{cases} \quad \text{where } \gamma(t) = \alpha \in \Omega \tag{5.15}$$

and $\frac{\partial v(.)}{\partial x_1}$ and $\frac{\partial v(.)}{\partial x_2}$ are the first-order partial derivatives of the value function $v(.)$. It is important to note that according to the first condition in equation (5.15), the HJB equations for different n are actually pair-wise coupled, since $\rho v(\alpha, x_1, x_2, n)$ depends on $v(\beta, x_1, x_2, n+1)$ according to the right hand side of equation (5.14). When the value function that represents the total cost is available, the optimal control policies $\left(u_1^*(.), u_2^*(.), w_1^*(.), w_2^*(.)\right)$ can be obtained to minimize the expected discounted cost given by equation (5.9). The properties of the value function that satisfy the HJB equations for the case of a multiple-machine, multiple-product manufacturing system can be found in Kenné *et al.* (2003). Based on this particular case, we provide our extension by addressing the problems of manufacturing and remanufacturing planning with state constraints. Considering that the second derivatives of $c_M(u_1)$ and $c_R(u_2)$ are strictly positive, the optimal production rates $\left(u_1^*(.), u_2^*(.)\right)$ under the state constraint on x_2 are computed as follows:

$$\left(u_1^*(.), u_2^*(.)\right) = \tag{5.16}$$

$$\arg \min_{(u_1,u_2)\in\mathcal{A}} \left\{u_1\left(\frac{\partial}{\partial x_1} v(.) + c_M\right) + u_2\left(\frac{\partial}{\partial x_1} v(.) - \frac{\partial}{\partial x_2} v(.) + c_R\right)\right\}$$

when $c_M(u_1) = c_M u_1$ and $c_R(u_2) = c_R u_2$ for some constants $c_M > 0$ and $c_R > 0$.

Unfortunately, the HJB equations are often analytically intractable. There is, however, a way to carry out numerical calculations within the framework of the stochastic formulation without having to solve the HJB equations analytically. Numerical methods will be used to approximate a solution of the HJB equation (5.14) and determine the optimal control policies. For more details, see Annex VII.

We provide a numerical example to solve the discrete version of HJB equation (5.14). A finite grid is denoted by $G_{x_1 x_2 n}^h$ and is needed to define the computational domain of the state variables (x_1, x_2) and the number of failures n, with $h = (h_{x_1}, h_{x_2})$, such that:

$$G_{x_1 x_2 n}^h = \{(x_1, x_2, n): -5 \le x_1 \le 30, 0 \le x_2 \le 20, 1 \le n \le 20\} \tag{5.17}$$

where h_{x_1} and h_{x_2} are the finite difference intervals of the state variables x_1 and x_2, respectively. Table 5.2 summarizes the parameters used in this numerical example. For the chosen parameters, the hybrid system can be considered feasible, over an infinite horizon and reaches a steady state, if the whole capacity constraint given by the equation (5.13) is verified.

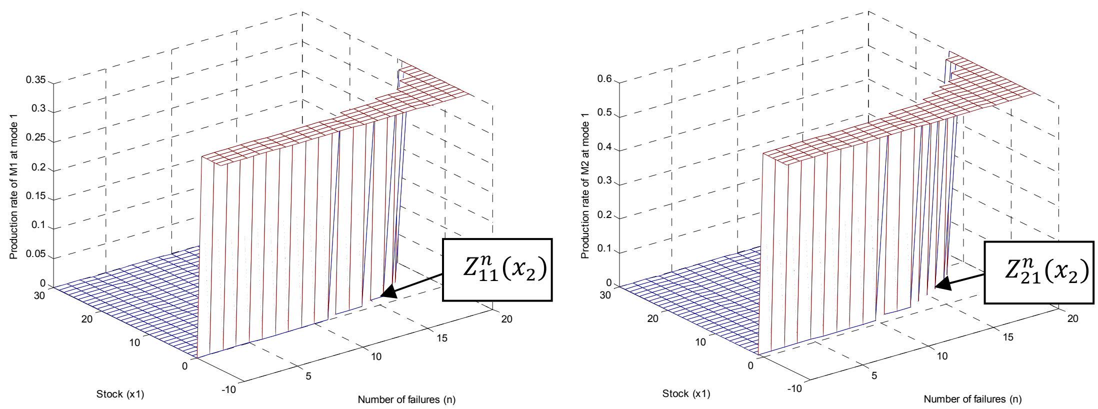
Table 5.2 Parameters of the numerical example

Parameter	c_1^+	c_2^+	c_1^-	c_r^1	c_r^2	c_0	c_M	c_R
Value	8	2	150	150	250	20000	100	50
Parameter	d	u_{max}^1	u_{max}^2	r	ρ	θ	q_0	q_1
Value	0.4	0.35	0.6	0.5	0.001	2	0.01	0.14
Parameter	q_{12}	q_{21}	q_{13}	w_{max}^1	w_{min}^1	q_{51}	N	(h_{x_1}, h_{x_2})
Value	80^{-1}	15^{-1}	100^{-1}	50^{-1}	10^{-5}	10^{-1}	20	(1,1)

with manufacturing/remanufacturing transition rates:

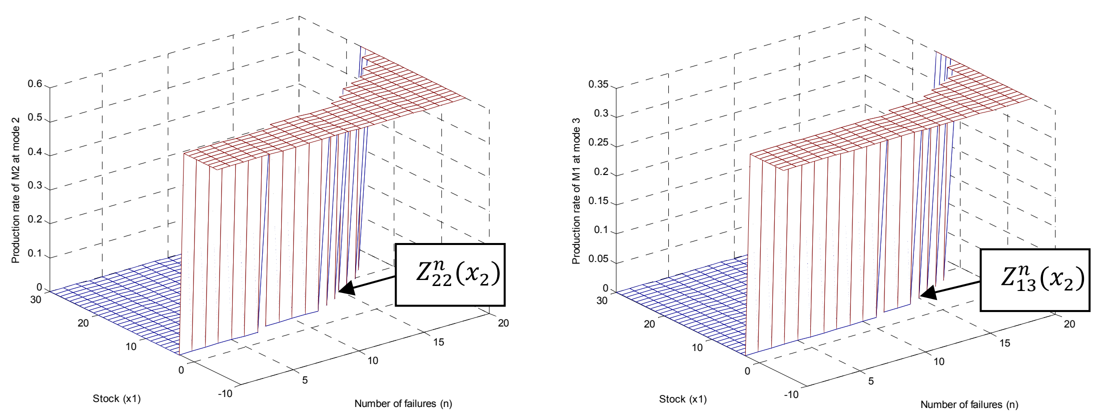
$$q_{34} = q_{56} = q_{12};\; q_{43} = q_{65} = q_{21};\; q_{24} = q_{13};\; w_{max}^2 = w_{max}^1;\; w_{min}^2 = w_{min}^1;\; q_{62} = q_{51}.$$

We proceed by first examining the obtained production policies of the manufacturing and the remanufacturing machines $u_1^*(.)$ and $u_2^*(.)$ in operational modes 1, 2, 3 and 5, which indicate the optimal production rates versus the stock level x_1 and the number of failures n at fixed x_2 (i.e., $x_2 = 5$). No production takes place in modes 4 and 6 because both machines M_1 and M_2 are down (mode 4) or under repair and replacement (mode 6). The results obtained in this paper show that the optimal control policy is characterized by multiple critical thresholds with machines M_1 and M_2 working in harmony. The threshold levels denoted by $Z_{1\alpha}^n(x_2)$ and $Z_{2\alpha}^n(x_2)$ for optimal production rates are all defined relative to the stock level x_1 for each number of failures n and stock level x_2 (with $\alpha \in \{1, 2, 3, 5\}$), as illustrated in figures 5.3-5.5. We observe that in each mode, the required number of parts in the serviceable inventory increases progressively with the number of failures. Indeed, when machine M_2 is relatively new, the threshold levels remain low, but once the deterioration effect on machine M_2 becomes significant, the threshold levels increase as the number of failures increases. This is mainly due to the fact that the returned used products are heterogeneous in nature, and this quality of products clearly reflects the deterioration effect of repair activities on the remanufacturing machine M_2, which are generally imperfect (i.e., the repair time increases with the number of failures).



a) Production policy of M_1 in mode 1   b) Production policy of M_2 in mode 1

Figure 5.3 Production rates of manufacturing/remanufacturing machines in mode 1



a) Production policy of M_2 in mode 2   b) Production policy of M_1 in mode 3

Figure 5.4 Production rates of manufacturing/remanufacturing machines in modes 2 and 3

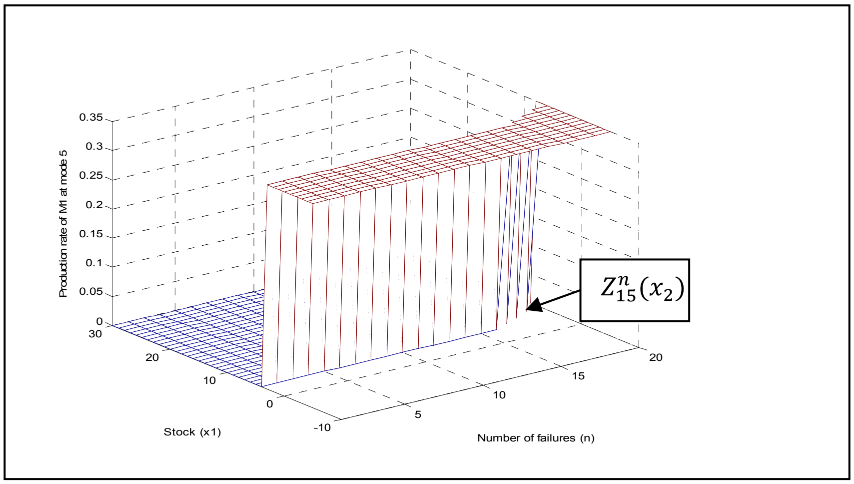
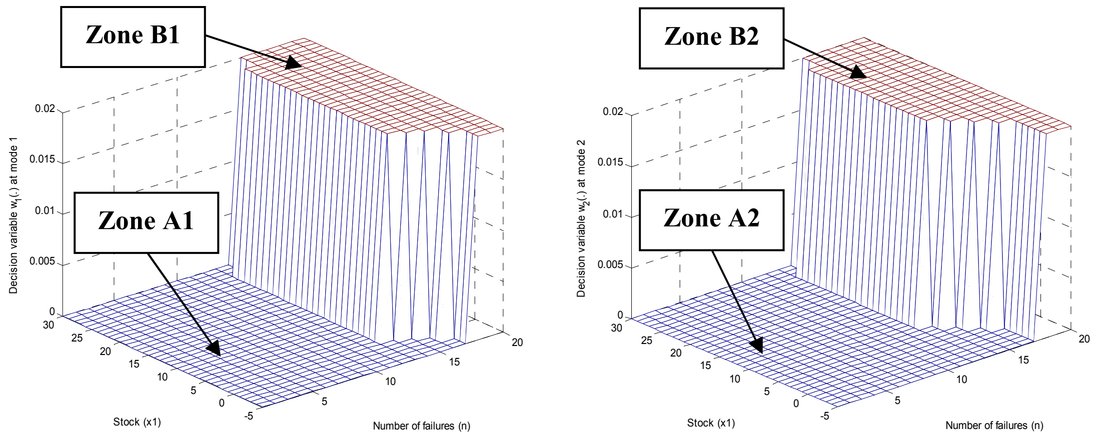


Figure 5.5 Production rate of manufacturing machine in mode 5

The replacement policies $w_1(.)$ and $w_2(.)$ of machine M_2 are only employed in operational modes 1 and 2. These control variables determine the optimal conditions for the remanufacturing machine to be replaced or repaired upon failure. We observe from figure 5.6 that a replacement is only conducted when the level of deterioration on machine M_2 reaches a significant level; even with the help of machine M_1, the replacement cost is justified and the machine M_2 is sent automatically for replacement maintenance to restore it to initial conditions. The pattern of the replacement policies divides the plane (x_1, n) at fixed x_2 into two regions, described as follows:

1. Zones A1 and A2 in modes 1 and 2, respectively: the replacement policies do not recommend the replacement of the remanufacturing machine because when machine M_2 is new, it can support a certain deterioration level, and with the support of machine M_1, it is more profitable to continue operating the same machine to satisfy the demand.

2. Zones B1 and B2 in modes 1 and 2, respectively: the recommendation is to perform replacement activities because the deterioration effect on the remanufacturing machine due to imperfect repairs justifies the cost of this type of maintenance.



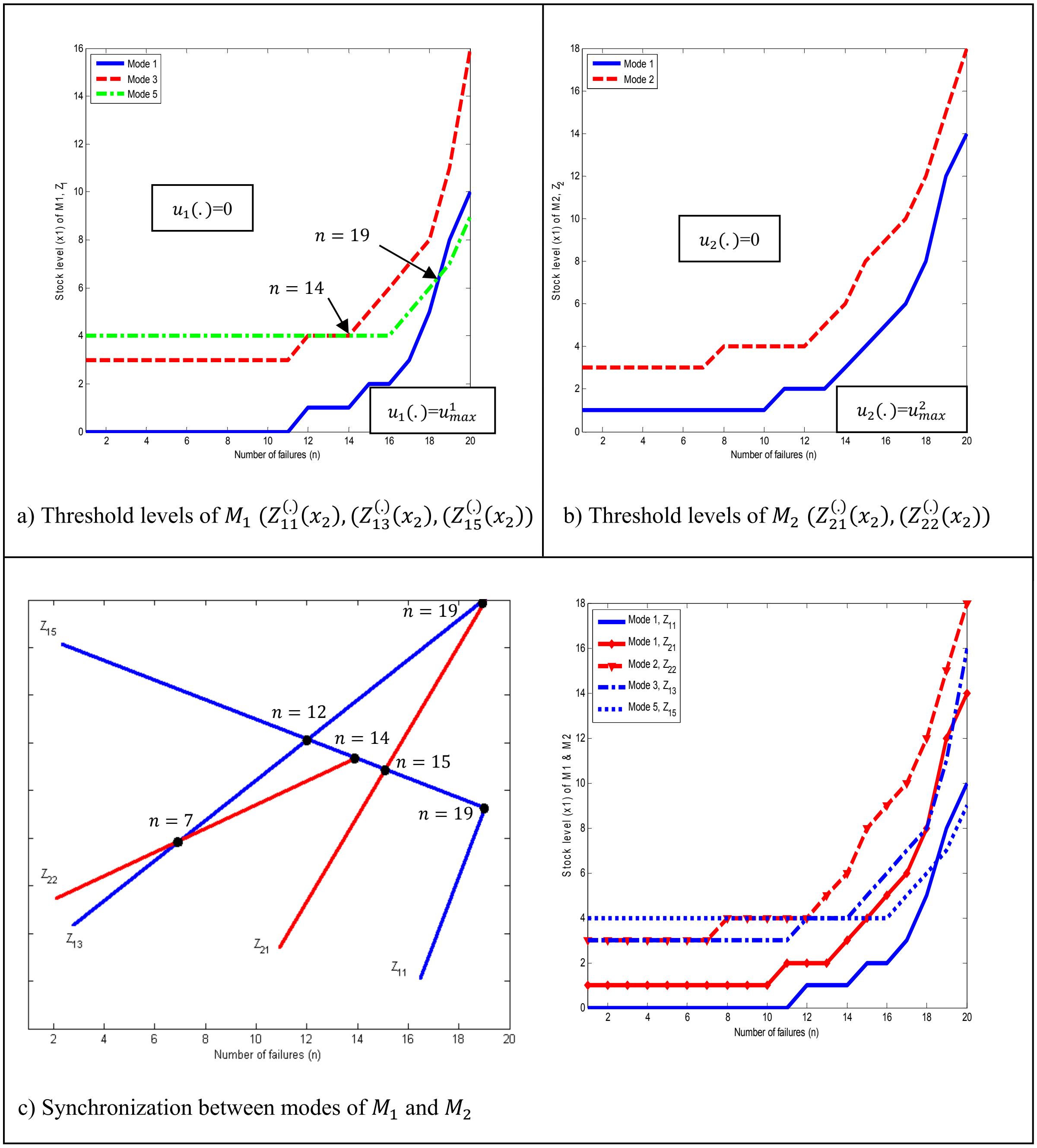
a) Replacement policy of M_2 in mode 1 b) Replacement policy of M_2 in mode 2

Figure 5.6 Replacement policies of remanufacturing machine in modes 1 and 2

To better illustrate the optimal production and replacement policies, we need to represent all trace functions that delimit the optimal zone to facilitate their characterizations, as presented in figures 5.7-5.8. We define the threshold levels $Z_{1\alpha}^{(.)}(x_2)$ and $Z_{2\alpha}^{(.)}(x_2)$ that determine, for each number of failures n and mode α at fixed x_2, the boundaries of the optimal production policies. Then we denote $D_{\alpha}^{(.)}(x_2)$ as the replacement trace of the remanufacturing machine in mode α for each number of failures n and for stock level x_2. The main observation in figures 5.7a-b is the required threshold levels of the manufacturing machine M_1 in modes 1, 3 and 5, and the remanufacturing machine M_2 in modes 1 and 2, to provide a necessary protection for the hybrid system against shortages during future breakdowns. The higher values of the remanufacturing threshold level, versus those of the manufacturing threshold level, are mainly due to system parameters such as:

1) the considered mean time to repair of machine M_2 (becomes significant when the number of failures increases) is greater than the mean time to repair of machine M_1 $\left(\left(q_{31}(n)\right)^{-1} > (q_{21})^{-1}\right)$;
2) the percentage of return rate is significant;
3) the production cost of remanufacturing is less expensive, and

4) the deterioration effect that pushes machine M_2 to produce at its maximum production rate for extended periods.



a) Threshold levels of M_1 $(Z_{11}^{(.)}(x_2), (Z_{13}^{(.)}(x_2), (Z_{15}^{(.)}(x_2))$

b) Threshold levels of M_2 $(Z_{21}^{(.)}(x_2), (Z_{22}^{(.)}(x_2))$

c) Synchronization between modes of M_1 and M_2

Figure 5.7 Threshold levels of M_1 and M_2

We can observe from figure 5.7a, in modes 1 and 3 of machine M_1, that the threshold level $Z_{13}^{(.)}(x_2) > Z_{11}^{(.)}(x_2)$. Something similar is observed in figure 5.7b in modes 1 and 2 of machine M_2, where we note that the threshold level $Z_{22}^{(.)}(x_2) > Z_{21}^{(.)}(x_2)$. Additionally, we can observe from figure 5.7a, that the threshold level $Z_{15}^{(.)}(x_2)$ of machine M_1 in mode 5 is greater than its threshold level $Z_{13}^{(.)}(x_2)$, given that the mean time to replacement of machine M_2 is greater than the mean time to repair in the first initial failures (n). This threshold level $Z_{15}^{(.)}(x_2)$ can be maintained at high level until it intersects with $Z_{13}^{(.)}(x_2)$ at $n = 14$ and $Z_{11}^{(.)}(x_2)$ at $n = 19$. In this case, we can say that the modes are interrelated, and we enter a zone where no difference exists between modes 3 and 5, and modes 1 and 5 for machine M_1. In other words, modes 3 and 5 coincide when the mean time to repair of machine M_2 is equal to its mean time to replacement. Then, modes 1 and 5 coincide again when machine M_2 enters a replacement zone, a zone where machine M_2 is considered as good as new. Based on the previous results of figures 5.7a-b, we clarify how the control policy obtained operates simultaneously in our system, in order to help satisfy customer demand, while keeping operating costs as low as possible to encourage profitability. Additionally, we can see from figure 5.7c that the threshold $Z_{15}^{(.)}(x_2)$ intersects with the thresholds $Z_{13}^{(.)}(x_2)$, $Z_{22}^{(.)}(x_2)$, $Z_{21}^{(.)}(x_2)$ and $Z_{11}^{(.)}(x_2)$ at $n = 12, 14, 15$ and 19, respectively, and also that the threshold $Z_{13}^{(.)}(x_2)$ intersects with the thresholds $Z_{22}^{(.)}(x_2)$ and $Z_{21}^{(.)}(x_2)$ at $n = 7$ and 19, respectively. Looking at figure 5.7c for the behavior of the hybrid system with both machines, it can clearly be concluded that the optimal control provides the accurate synchronization possible between production by manufacturing machine and production by remanufacturing machine in the different modes of the hybrid process.

We complement the results analysis of the numerical example with figure 5.8. We observe that the replacement trace of the remanufacturing machine is limited by the production boundary for each mode $\alpha \in \{1, 2\}$, given that the stock level is always restricted by the production threshold $Z_{2\alpha}^{(.)}(x_2)$. Due to the interrelation between the manufacturing, remanufacturing and replacement policies, only a section of the replacement zone is enabled, defining a feasible zone where the hybrid manufacturing/remanufacturing system resides. The intersection between the production threshold $Z_{2\alpha}^{(.)}(x_2)$ of zone Aα and the replacement trace $D_{\alpha}^{(.)}(x_2)$ of zone Bα defines the feasible zone Cα, (i.e., in figure 5.8, the intersections between Z_{21} and D_1 in mode 1 and between Z_{22} and D_2 in mode 2 for each number of failures n at fixed x_2 define C1 and C2, respectively). This feasible zone optimally recommends the number of failures and the necessary stock levels to replace the remanufacturing machine, taking into account random phenomena such as breakdowns and repairs, the deterioration effect on the machines and the reverse logistics environment, with its constraints. Finally, we can observe from the sub-feasible zone ® in figure 5.8 that when the number of failures is very large, a replacement is more often recommended. This is true even in backlog situations, due to the repair cost, which increases with the number of failures, and becomes very high when compared to replacement and backlog costs.

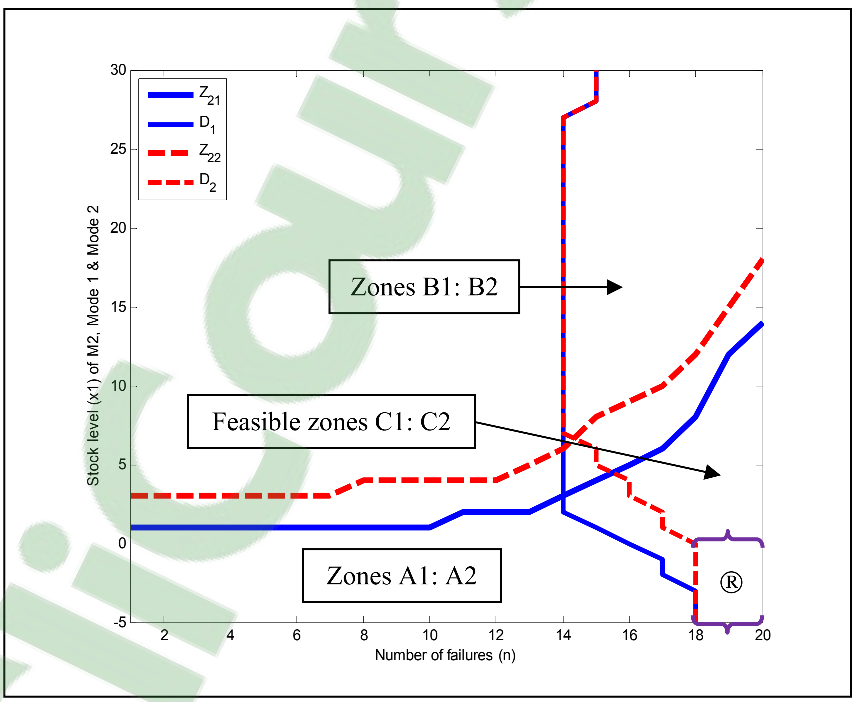


Figure 5.8 Replacement traces of remanufacturing machine in modes 1 and 2

5.5 Structure of optimal policies

The optimal production control policy obtained for the hybrid manufacturing/remanufacturing system is an extension of the hedging point policy, given that it respects the structure of the optimal solution presented in Akella and Kumar (1986). However, in our case, we take into account the deterioration effects on the remanufacturing machine due to imperfect repairs, as well as the limited return of products in reverse logistics. From the results obtained, and based on the illustration in figures 5.7-5.8, the production control policy is characterized by a vector of two parameters $Z = (Z_{1\alpha}^{(.)}(x_2),$ $Z_{2\alpha}^{(.)}(x_2))$ the hedging surface policy, and $Z_{1\alpha}^{(.)}(x_2)$ and $Z_{2\alpha}^{(.)}(x_2)$ are the hedging surfaces. As defined in figure 5.9, at a fixed number of failures n, $Z_{1\alpha}^{(.)}(x_2) \equiv Z_{1\alpha}^{n}(x_2)$ and $Z_{2\alpha}^{(.)}(x_2) \equiv Z_{2\alpha}^{n}(x_2)$ are constant, the hedging surface policy becomes an extension of the so-called hedging point policy, and $Z_{1\alpha}^{n}(x_2)$ and $Z_{2\alpha}^{n}(x_2)$ are the hedging points. In this case, we are interested in the functions $Z_{1\alpha}^{n}(x_2)$ and $Z_{2\alpha}^{n}(x_2)$ which give the optimal production threshold levels in serviceable inventory in mode α and each number of failures n at fixed stock level x_2 of returned products.

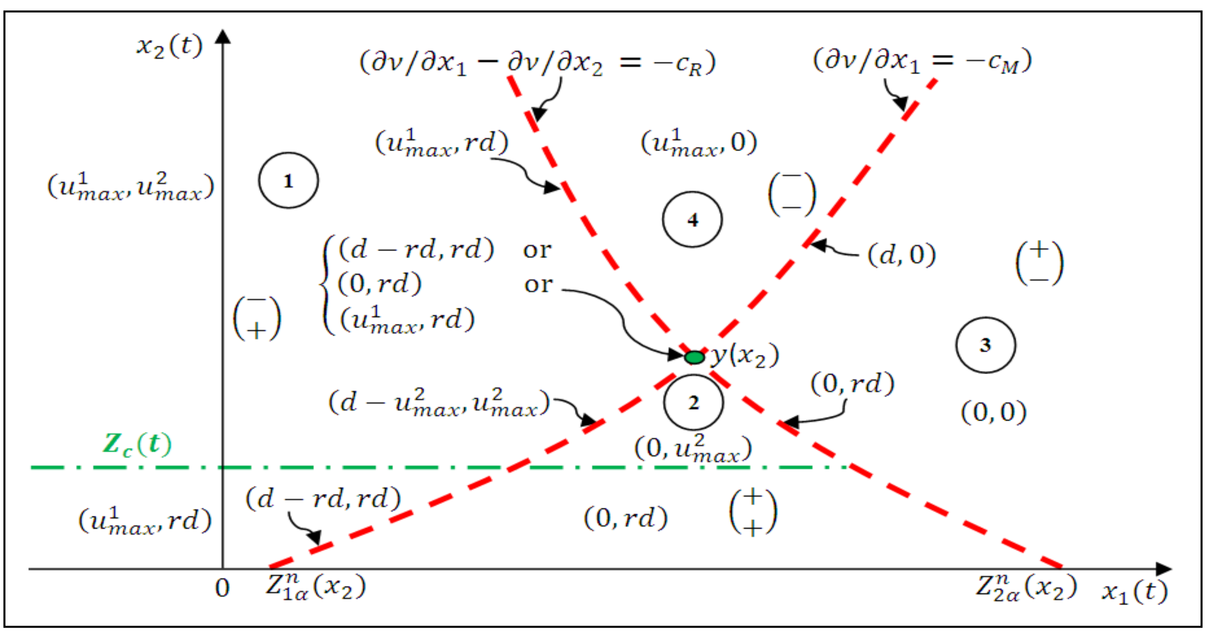


Figure 5.9 Manufacturing/remanufacturing control policy

A problem with constraints on the state ($x_2 \geq 0$ in our case) is difficult to solve analytically; addressing it numerically, we have to determine the optimal solution $u_2(.)$ on the state limit $x_2 = 0$ (it cannot be set at u_{max}^2 since we end up with negative x_2 that has no meaning). To address this problem, we propose a sort of regularization introducing a layer from $x_2 = 0$ to $x_2 = Z_c(t)$, with $Z_c(t)$ delimiting a zone of recoverable inventory where the remanufacturing machine M_2 can produce at its maximal production rate u_{max}^2 (when $x_2 \geq Z_c(t)$). When $x_2 < Z_c(t)$, machine M_2 is starving and cannot produce at its maximal production rate u_{max}^2, and the number of parts which can be remanufactured is limited by the number of parts available in the recoverable inventory. This value can be determined numerically using equation (5.2) of the dynamics of the stock level $x_2(t)$.

We can identify that the computational domain of the production is divided into four different zones (regions 1-4), as illustrated in figure 5.9. Below, we address the general case with all possible areas when the threshold level trace $Z_{1\alpha}^n(x_2)$ intersects with the threshold level trace $Z_{2\alpha}^n(x_2)$ on the plane (x_1, x_2). The optimal production control policy can be described by the following set of rules:

1. The production rate of each machine is set to its maximum value when the current stock level $x_1(t)$ is under the safety stock level described by the threshold level values ($Z_{1\alpha}^n(x_2)$ for manufacturing machine M_1, and $Z_{2\alpha}^n(x_2)$ for remanufacturing machine M_2), on the condition that the stock level $x_2(t)$ is greater than or equal to the lower bound $Z_c(t)$ in order to be able to produce with u_{max}^2 while respecting the state constraint $x_2(t) = 0$. This is because the production rate of the remanufacturing machine M_2 is limited by the number of parts available in the recoverable inventory and the returned quantity. If the stock level $x_2(t)$ is less than the level of recoverable inventory $Z_c(t)$, the production rate of M_2 will be downgraded to the return rate rd despite the optimal control chosen for the production at u_{max}^2.
2. The production rate of the manufacturing machine M_1 is set to $d - u_2^*(.)$ when the current stock level for serviceable products $x_1(t)$ is equal to the threshold level value

$Z_{1\alpha}^{n}(x_2)$, with $u_2^*(.)$ taking one of three values (u_{max}^2, rd or 0). The production rate of the remanufacturing machine M_2 is set to u_{max}^2 if the current stock level for serviceable products $x_1(t)$ is less than the threshold level value $Z_{2\alpha}^{n}(x_2)$ and the stock level $x_2(t)$ is greater than or equal to the value of $Z_c(t)$; if the stock level $x_1(t)$ is equal to $Z_{2\alpha}^{n}(x_2)$, the production rate $u_2^*(.)$ is set to the return rate rd. Finally, it is set to zero when the stock level $x_1(t)$ is greater than $Z_{2\alpha}^{n}(x_2)$.

3. The production rates of both the manufacturing and remanufacturing machines are set to zero when the current stock level of serviceable products $x_1(t)$ is greater than $Z_{1\alpha}^{n}(x_2)$ and $Z_{2\alpha}^{n}(x_2)$.

Thus, the production control policies $u_1^*(.)$ and $u_2^*(.)$ for manufacturing and remanufacturing are defined as below:

For the case where the return rd is large, we have $u_{max}^1 > d - rd$, hence the production policy of machine M_1 is:

$$u_1^*(\alpha, x_1, x_2, n) = \begin{cases} u_{max}^1 & \text{if } x_1(t) < Z_{1\alpha}^{n}(x_2) \\ d - rd & \text{if } x_1(t) = Z_{1\alpha}^{n}(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 = 0) \\ 0 & \text{if } x_1(t) > Z_{1\alpha}^{n}(x_2) \end{cases} \tag{5.18}$$

For the case where the return rd is small, we have $u_{max}^1 \leq d - rd$, hence the production policy of machine M_1 is therefore:

$$u_1^*(\alpha, x_1, x_2, n) = \begin{cases} u_{max}^1 & \text{if } x_1(t) \leq Z_{1\alpha}^{n}(x_2) \\ 0 & \text{otherwise} \end{cases} \tag{5.19}$$

For the case where rd is large, we have $rd > d - u_{max}^1$, hence the production policy of machine M_2 is:

$$u_2^*(\alpha, x_1, x_2, n) = \tag{5.20}$$

$$\begin{cases} u_{max}^2 & \text{if } x_1(t) < Z_{2\alpha}^{n}(x_2) \text{ and } x_2(t) \geq Z_c(t) \\ d & \text{if } x_1(t) > Z_{1\alpha}^{n}(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 < 0) \\ rd & \text{if } x_1(t) = Z_{2\alpha}^{n}(x_2) \text{ and } \dot{x}_2 = 0 \\ d - u_{max}^1 & \text{if } x_1(t) < Z_{1\alpha}^{n}(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 > 0) \\ 0 & \text{if } x_1(t) > Z_{2\alpha}^{n}(x_2) \end{cases}$$

For the case where rd is small, we have $rd < d - u_{max}^1$, and the production policy of machine M_2 is required, as follows:

$$u_2^*(\alpha, x_1, x_2, n) = \tag{5.21}$$

$$\begin{cases} u_{max}^2 & \text{if } x_1(t) < Z_{2\alpha}^n(x_2) \text{ and } x_2(t) \geq Z_c(t) \\ d & \text{if } x_1(t) > Z_{1\alpha}^n(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 < 0) \\ d - u_{max}^1 & \text{if } x_1(t) < Z_{1\alpha}^n(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 < 0) \\ rd & \text{if } x_1(t) = Z_{2\alpha}^n(x_2) \text{ and } \dot{x}_2 = 0 \\ 0 & \text{if } x_1(t) > Z_{2\alpha}^n(x_2) \end{cases}$$

For the case where $rd = d - u_{max}^1$, the production policy of machine M_2 is defined as follows:

$$u_2^*(\alpha, x_1, x_2, n) = \tag{5.22}$$

$$\begin{cases} u_{max}^2 & \text{if } x_1(t) < Z_{2\alpha}^n(x_2) \text{ and } x_2(t) \geq Z_c(t) \\ d & \text{if } x_1(t) > Z_{1\alpha}^n(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 < 0) \\ rd & \text{if } (x_1(t) = Z_{2\alpha}^n(x_2) \text{ and } \dot{x}_2 = 0) \text{ or} \\ & \quad (x_1(t) < Z_{1\alpha}^n(x_2) \text{ and } (\dot{x}_1 = 0 \;\&\; \dot{x}_2 = 0)) \\ 0 & \text{if } x_1(t) > Z_{2\alpha}^n(x_2) \end{cases}$$

The replacement policies $w_1^*(.)$ and $w_2^*(.)$ for remanufacturing, including two possible modes of manufacturing, have a bang-bang structure, with the value $w_{max}^1 = 0.02$ if a replacement is undertaken, and $w_{min}^1 = 10^{-5} \approx 0$ otherwise. Physically, when the value of w_{min}^1 is close to zero (i.e., 10^{-5}), the delay before replacement is very big (but finite) and the option to replace the remanufacturing machine is not recommended. The optimal control policies can be denoted as follows:

If the manufacturing machine is operational (mode 1):

$$w_1{}^*(1, x_1, x_2, n) = \begin{cases} w_{max}^1 & \text{if } \big(x_1(t), x_2(t), n(t)\big)_1 \in \text{zone C1} \\ w_{min}^1 & \text{otherwise} \end{cases} \tag{5.23}$$

If the manufacturing machine is under repair (mode 2):

$$w_2{}^*(2, x_1, x_2, n) = \begin{cases} w_{max}^2 & \text{if } \big(x_1(t), x_2(t), n(t)\big)_2 \in \text{zone C2} \\ w_{min}^2 & \text{otherwise} \end{cases} \tag{5.24}$$

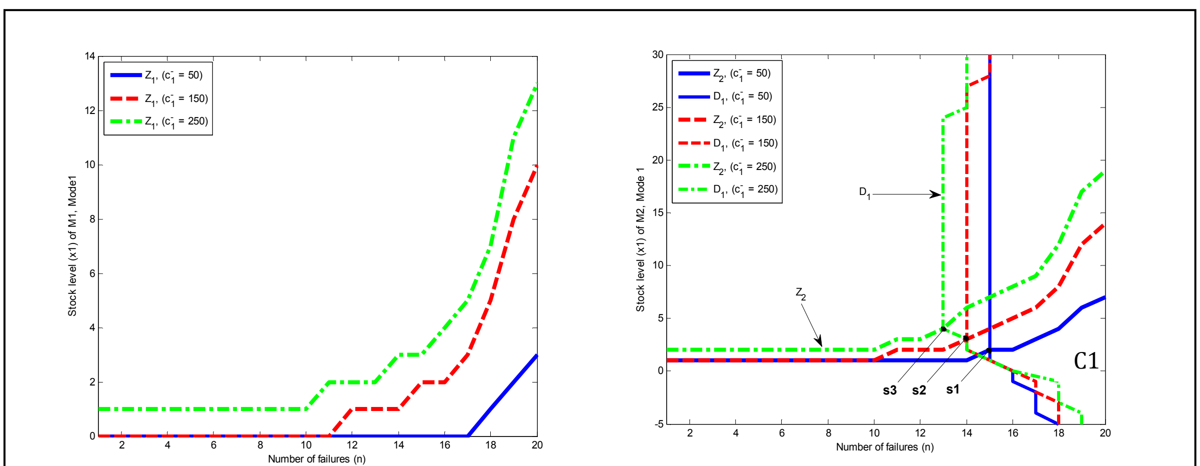
where $\big(x_1(t), x_2(t), n(t)\big)_1$ and $\big(x_1(t), x_2(t), n(t)\big)_2$ are the parameters of zones $\mathrm{C1}$ and $\mathrm{C2}$, respectively, with $\mathrm{C1} = \mathrm{A1} \cap \mathrm{B1}$ and $\mathrm{C2} = \mathrm{A2} \cap \mathrm{B2}$, as defined in figure 5.8.

In the next section, we will confirm the structure of the control policies obtained through a sensitivity analysis, thus illustrating the usefulness of the proposed approach.

5.6 Sensitivity analysis

We analyze the evolution of the hedging points for the number of failures which changes throughout its entire domain, as is typical for long-term behavior and planning level decision making. We show the evolution of the hedging points over a number of failures for a fixed level of recoverable inventory. The present section provides further evidence of the usefulness of the control policy obtained. The proposed approach is validated below through a sensitivity analysis to illustrate the contribution of the hybrid policy and to confirm its structure. This sensitivity analysis is performed based on variations of several parameters, such as the backlog and inventory costs for serviceable products, the intensity of repair of machine M_2 and the availability of machine M_1. Furthermore, to be more realistic, we analyze the effect of other parameters by considering a return rate of the used products that are different from the one used in the basic case. The production rates of machines M_1 and M_2 and the replacement policy of machine M_2 are illustrated in the operational mode for a given x_2 (i.e., mode 1 and $x_2 = 5$). The intersection point when the trace $D_1^{(.)}(x_2)$ crosses the production threshold $Z_{21}^{(.)}(x_2)$ denotes the replacement point of the remanufacturing machine, where it is recommended for the first time. In the following figures, the recommended replacement points $\mathrm{s1}, \mathrm{s2}$ and s3 are highlighted, and correspond, to small, middle and high values, respectively, when varying each parameter of the model.

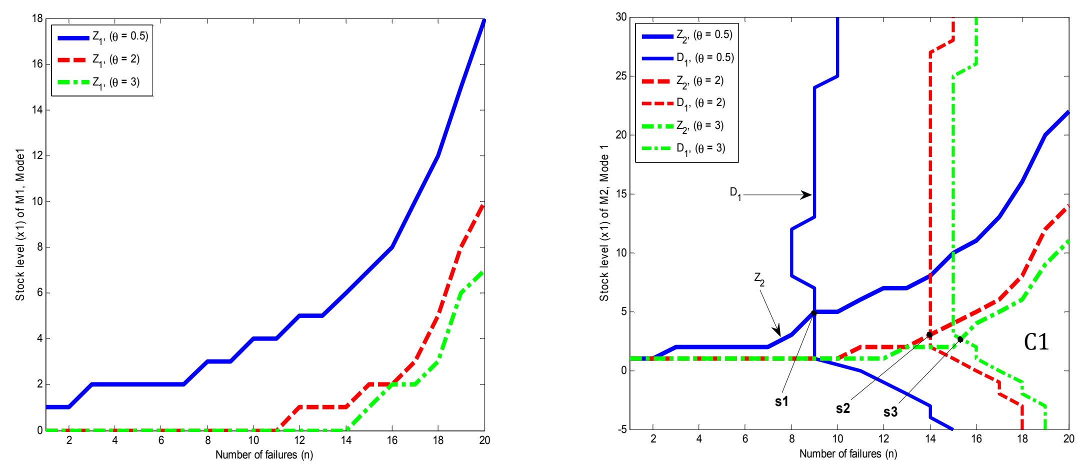
The variation of the backlog cost c_1^-, considerably affects the optimal production thresholds of the manufacturing and remanufacturing machines, as presented in figure 5.10, where three different cost values $c_1^- = 50, 150 \text{ and } 250$ are analyzed. The results presented in figure 5.10 show that the production thresholds $Z_{11}^{(.)}(x_2)$ and $Z_{21}^{(.)}(x_2)$ increase as the backlog cost increases. Because the backlog is more penalized with a higher backlog cost, the control policy suggests producing more parts to protect the system against product shortages caused by breakdowns and the deterioration effect of imperfect repairs. It is therefore difficult to support this situation for a long time. For this reason, the replacement of the remanufacturing machine M_2 is suggested for earlier: $n(s1) > n(s2) > n(s3)$. Consequently, this increases the feasible zone $\mathrm{C1}$. Moreover, a sensitivity analysis was conducted regarding to the variation of the serviceable inventory cost (c_1^+). We noticed that the effect of the serviceable inventory cost (c_1^+) on the manufacturing and remanufacturing control policies and the replacement feasible zone $\mathrm{C1}$ is the opposite of that of the backlog cost (c_1^-) with the exception of the replacement policy. Below, we will analyze the repair intensity parameter, which affects the deterioration with number of failures mechanism related to the remanufacturing machine (characterized by the mean time to repair, which increases with the number of failures).



a) Threshold of M_1 b) Threshold of and trace of M_2

Figure 5.10 Variation of the backlog cost c_1^- in mode 1: Effect on M_1 and M_2

Following our analysis, we note that the repair intensity θ influences the production and replacement policies significantly, as can be seen in figure 5.11. We analyze three different cases, with values $\theta = 0.5, 2, \text{and } 3$. According to equation (5.6), when θ increases, the repair time decreases, and the remanufacturing machine M_2 will be more available.



a) Threshold of M_1 b) Threshold and trace of M_2

Figure 5.11 Variation of θ in mode 1: Effect on M_1 and M_2

The results of figure 5.11 show this change clearly. This is what is observed machine M_2 produces fewer parts at its maximum production rate u^2_{max} and that the threshold level $Z^{(.)}_{21}(x_2)$ will decrease at that rate as well. In parallel, the manufacturing machine M_1 should also follow machine M_2 by decreasing the threshold level $Z^{(.)}_{11}(x_2)$. Thus, the trace $D^{(.)}_{1}(x_2)$ decreases, and as a result, a replacement is recommended, but for later: $n(s1) < n(s2) < n(s3)$. We notice that the feasible zone C1 to replace machine M_2 decreases when M_2 is more available. We can conclude from this case that the effect of the variation of deterioration on machine M_2 is reflected in both the production and replacement policies. A sensitivity analysis was conducted on the variation of the availability of the manufacturing machine, and we noticed that the effect of the availability of the manufacturing machine on the control policies (manufacturing, remanufacturing and replacement) and the replacement feasible zone C1 is the same of that of the repair intensity (θ).

To complete the sensitivity analysis, we discuss the variation of an important parameter of reverse logistics, which is the proportion of return of used products r to the recoverable inventory. We would first like to draw the reader's attention to consider the fixed number of failures as is typical for short-term behavior and operational level decision making. Figure 5.12 illustrates how our hybrid policy adapts to the changes in serviceable and recoverable inventories; in other words, how the hybrid manufacturing/remanufacturing system works locally for a given number of failures and with the inventory levels gradually changing. This is most appropriate for industrial applications involving short-term decision making. Figure 5.12 illustrates the joint production policy for a hybrid system in different modes (1, 2 and 3) before replacement, for four different values ($x_2 = 1, 5, 10$ and 15) with different proportions of return $r = 50\%$ and 90% for $n = 13$. Here, the threshold levels $Z_{1\alpha}^{n}(.)$ and $Z_{2\alpha}^{n}(.)$ for machines M_1 and M_2, respectively, are defined for a given number of failures n in mode α at stock level x_2.

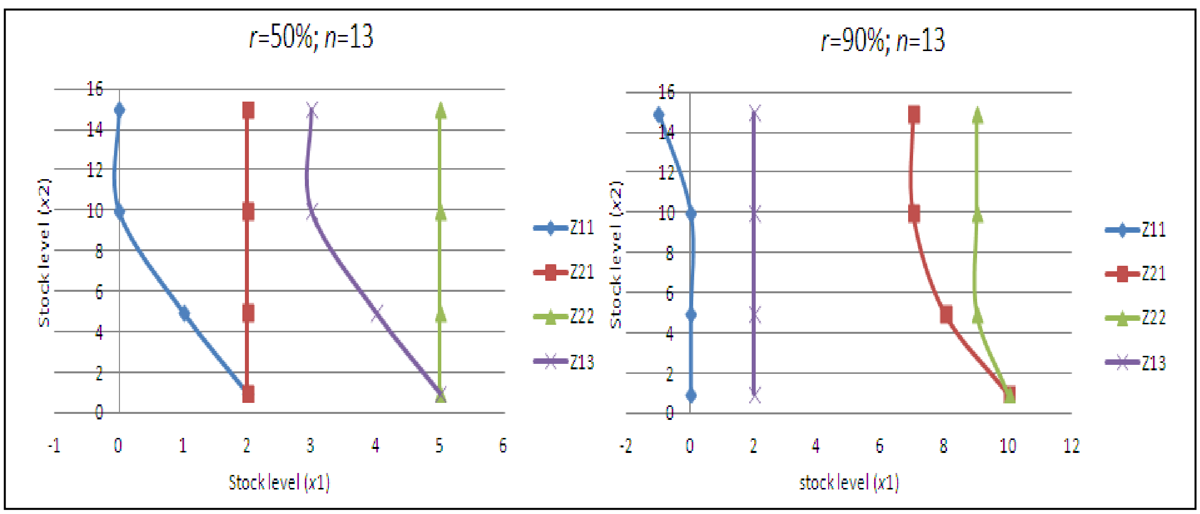
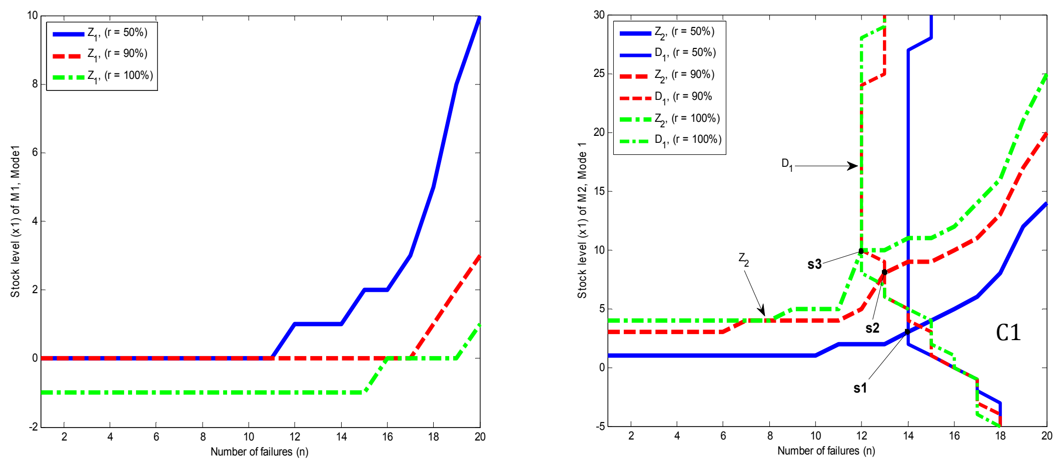


Figure 5.12 Variation of proportion of return r at a fixed number of failures: Effect on M_1 and M_2

- In mode 1, it can be seen that regardless of the proportions of return $r = 50\%$ or 90%, the behavior of machine M_1 may actually depend on x_2. When the stock level x_2 decreases, machine M_1 should increase its production with the threshold $Z_{11}^n(.)$ on x_1. This is logical given the coordination for production with machine M_2 to satisfy the demand rate in mode 1. However, the production of machine M_2 at the threshold $Z_{21}^n(.)$ on x_1 is not affected by the value of x_2 when the proportion of return is set to a lower value $r = 50\%$. Only when the proportion of return is high is the behavior of machine M_2 depending on x_2 visible. As indicated in figure 5.10a, if the proportion of return is increased to $r = 90\%$, comparing the value of threshold $Z_{21}^n(.)$ with the value of stock level x_2, it decreases when the stock level x_2 increases, and conversely, it increases when the stock level x_2 decreases. A few reasons can be advanced to help clarify such a behavior. The first is that when more used products are returned, and if there are some advantages (like a low production cost for remanufacturing) to remanufacturing more products, the stock level x_2 automatically decreases and the threshold $Z_{21}^n(.)$ on x_1 increases. The second reason is that when more used products are returned, and we can only remanufacture fewer products (due to the progressive deterioration effect on remanufacturing machine M_2), the availability of machine M_2 decreases, its repair cost becomes significant, and in this case, it loses its production advantages versus machine M_1. As a result, the stock level x_2 increases and the threshold $Z_{21}^n(.)$ on x_1 decreases.
- In mode 2, the behavior of machine M_2 does not change with variations of x_2 when the proportion of return is set to $r = 50\%$. The threshold $Z_{22}^n(.)$ remains constant regardless of the value of x_2. However, if the proportion of return is increased to $r = 90\%$, the threshold $Z_{22}^n(.)$ will depend on x_2. The reasons for this are the same as the previous ones explained in mode 1. Comparing the policy of machine M_2 in modes 1 and 2, it is natural that the threshold $Z_{22}^n(.)$ is greater than $Z_{21}^n(.)$ because machine M_2 is the only one which is operational in mode 2, and it must accumulate more parts in order to meet customer demand in case of failure and of its deterioration.
- In mode 3, we note that the behavior of machine M_1, represented by the threshold $Z_{13}^n(.)$, only changes with variations of x_2 when the proportion of return is set to $r = 50\%$, but remains constant with $r = 90\%$. The reason for this is that when more used products are

returned, machine M_2 increases the production to its maximum capacity for a long time with thresholds $Z_{21}^n(.)$ and $Z_{22}^n(.)$ in modes 1 and 2, respectively. Thus, it is not necessary for threshold $Z_{13}^n(.)$ of machine M_1 to change with variations of x_2. However, if the return of the used products is set to the lower value $r = 50\%$, thresholds $Z_{21}^n(.)$ and $Z_{22}^n(.)$ will have small values. In this case, when the stock level x_2 decreases, the machine M_1 increases its production to its maximum capacity with threshold $Z_{13}^n(.)$ on x_1. Naturally, threshold $Z_{13}^n(.)$ of machine M_1 in mode 3 is greater than its threshold $Z_{11}^n(.)$ in mode 1. This is explained by the fact that machine M_1 is working alone in mode 3, and must produce more parts for the serviceable inventory.

In order to get a complete idea of the sensitivity of the return proportion r on the control policies, we analyze three different values $r = 50\%, 90\%$ and 100% throughout the entire domain of the number of failures (long-term behavior), considering the fact that the maximal production rate $u_{max}^2 \geq rd$. As presented in figure 5.13, when the proportion of return r increases, the threshold values $Z_{11}^{(.)}(x_2)$ of M_1 decreases and $Z_{21}^{(.)}(x_2)$ of M_2 increases. This is because even though it is advantageous to produce more parts with machine M_2 than machine M_1 (production costs $c_R < c_M$), reducing recoverable inventory costs requires that more products in the serviceable inventory be obtained from the remanufacturing machine. The accumulation of return leads to increase in these costs as compared to other costs in the model. Thus, machine M_2 should produce parts at its maximum production capacity, depending on the return rate limit rd and the number of available parts in the recoverable inventory x_2. For machine M_1, it is available, given its capacity limit, to help machine M_2, and should produce additional parts as needed. When many parts are returned to the recoverable inventory, we note a significant increase in the trace $D_1^{(.)}(x_2)$ of machine M_2 for $r = 100\%$ than its trace for $r = 90\%$ and 50%. Machine M_2 suffers the severe effects of its progressive deterioration (less available after a certain number of failures) and reverse logistics constraints, with more parts available in the recoverable inventory, leading to an increase in recoverable inventory costs. In this case, machine M_2 cannot be kept for a longer period, and it will unavoidably be replaced early, which is why the replacement of the remanufacturing machine M_2 is more recommended: $n(s1) > n(s2) > n(s3)$. So, the feasible zone C1 will grow.



a) Threshold of M_1 b) Threshold and trace of M_2

Figure 5.13 Variation of proportion of return r in mode 1: Effect on M_1 and M_2

5.7 Discussions and policies implementation

In this sensitivity analysis, it can be seen that the structure of the control policy for the considered hybrid system is maintained. Regardless of the different situations considered, the control policy is able to provide the optimal solution both in the short term and in the long term. This policy in reverse logistics is a modified hedging point policy due to the presence of the mechanism of deterioration with the number of failures, the limit in the number of returned products, and the non-negativity state constraint on recoverable inventory. In these short- and long-term analyses, we note a similarity between the results obtained for a hybrid control policy and the structure discussed in section 5.5, and illustrated in figure 5.9. The manufacturing, remanufacturing and replacement policies are defined simultaneously by the production thresholds and replacement traces, and their intersection points determine the feasible zones with the recommended stock levels and number of failures with respect to the remanufacturing machine replacement.

If the difference between the manufacturing cost c_M and the remanufacturing cost c_R becomes smaller, the structure of the joint control policy still the same, given that c_R is always less than c_M. So, it is more advantageous to produce with machine M_2 than with machine M_1; the machine M_2 must produce at its maximum production capacity depending on retuned parts and available parts in recoverable inventory, and the machine M_1 is available to help in order to satisfy the customer demand. However, the only change when the difference between c_M and c_R becomes smaller is on the total cost. There is no influence on the optimum in term of threshold. The main reason is due to the fact that the deterioration is only related to imperfect repairs (due to heterogeneous return) and it is not related to the aging process in which the mechanism of the age is related to the production rate. According to equation (5.7) for the cost function, the threshold is influenced by the inventories and backlog costs c_1^+, c_2^+ and c_1^- and the repair intensity θ that affects the repair time. A sensitivity analysis by changing the cost c_R illustrated in figure 5.14 confirms our analysis.

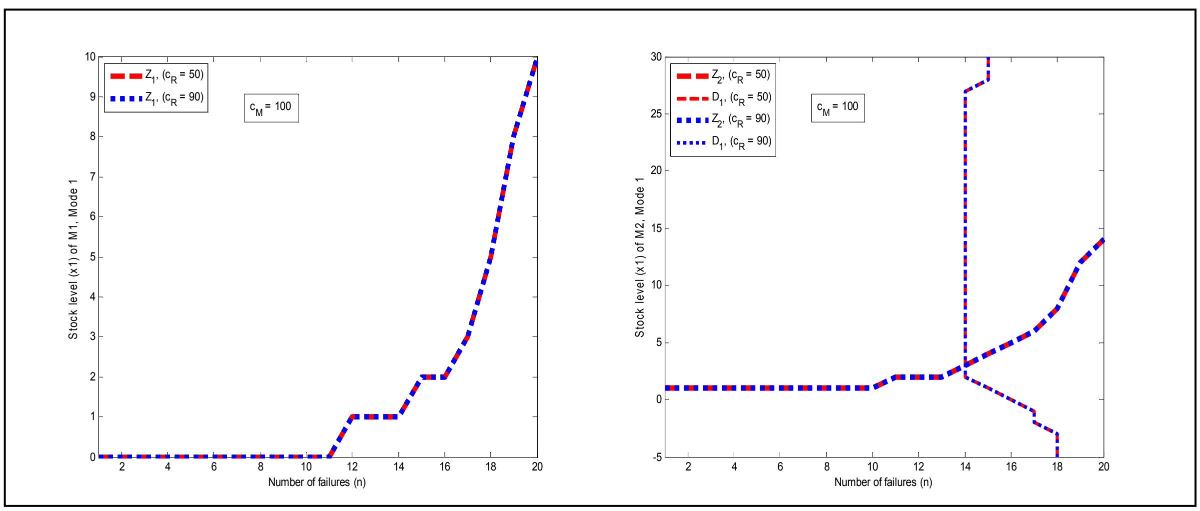


Figure 5.14 Variation of remanufacturing cost c_R in mode 1: Effect on M_1 and M_2

Figure 5.15 illustrates the implementation of the joint control policy over time in terms of $u_1(.,t), u_2(.,t), w_1(.,t)$ and its influence on serviceable and recoverable inventory levels $x_1(t)$ and $x_2(t)$. This illustration is obtained from the basic case; we should recall that at each failure of the remanufacturing machine, the threshold levels increase due to the increasing repair times with the number of failures. Based on the figure 5.15, we can see how the optimal stock levels and the appropriate moment to perform the replacement are determined. The numerical computation to solve the optimality conditions should be done off-line. However, to find the numerical solution, the impact of each iteration in terms of CPU-time and memory usage is relatively high for a large scale of the state space and numerical scheme dimensions. The implementation of this joint policy can be proposed in the form of a decision tool to facilitate the task to the manager. Ultimately, the obtained policies have a direct managerial implication, namely the manager can control the system with our obtained results to adequately improve the performance of the hybrid system, and make any necessary adjustments in its closed-loop system.

The two machines M_1 and M_2 are governed by the obtained modified hedging point policy, which has different threshold levels. The two threshold levels, denoted $Z_{21}^{(.)}$ and $Z_{22}^{(.)}$, are reached by machine M_2 with the support of machine M_1 which has three threshold levels, denoted $Z_{11}^{(.)}$, $Z_{13}^{(.)}$ and $Z_{15}^{(.)}$ (these levels of machine M_1 in modes 1, 3 and 5 were explained previously in figure 5.7a. Additionally, the threshold $Z_{22}^{(.)}$ of machine M_2 is obviously greater as compared to its threshold $Z_{21}^{(.)}$ when it is operating alone. The random availability of machine M_2 with the deterioration process makes it impossible for the system to provide the necessary stock protection to hedge against future breakdowns. Machine M_1 is an available option to support the hybrid system in satisfying the demand rate. However, with the production capacity limit in place using machine M_1 and the higher cost involved in the imperfect repair of machine M_2, the replacement can also be a complementary option to achieve such a goal.

According to figures 5.15a-c, the replacement policy $w_1(.,t)$ and the production control policies $u_1(.,t)$ and $u_2(.,t)$, with the limit state on the recoverable inventory x_2 ($x_2(t) \geq 0$), are illustrated for different scenarios over time, such as: machine M_1 has failed or is under repair, machine M_2 has failed, is under repair or is under replacement. We refer to figure 5.9 and equation (5.13) for more information about the production decisions on machines M_1 and M_2. We can also see in figures 5.15d-e that the behaviors of serviceable inventory $x_1(t)$ and recoverable inventory $x_2(t)$ can be divided into twelve zones, described as follows:

- When $x_1(t) < Z_{21}^{(.)}$: the serviceable stock level $x_1(t)$ is under the threshold level $Z_{21}^{(.)}$. We have:
  - Zones 1 and 4: both machines must produce according to their maximum production rates $(u_1, u_2) = (u_{max}^1, u_{max}^2)$. The stock $x_1(t)$ increases with: $u_{max}^1 + u_{max}^2 - d$ (figure 5.15d) and the stock $x_2(t)$ decreases with: $rd - u_{max}^2$ (figure 5.15e).
  - Zones 3 and 11: machine M_2 is down ($u_2 = 0$) and machine M_1 must produce at its maximum production rate $u_1 = u_{max}^1$ but it cannot reach the demand rate by itself because its production rate $u_{max}^1 < d$. The stock $x_1(t)$ decreases with: $u_{max}^1 - d$ (figure 5.15d) and the stock $x_2(t)$ increases with: rd (figure 5.15e).
  - Zone 12: if the replacement occurs at w_{max}^1 while respecting the feasibility zone of replacement C1, machine M_2 is considered not available ($u_2 = 0$) and machine M_1 must produce at its maximum production rate $u_1 = u_{max}^1$. The stock $x_1(t)$ decreases with: $u_{max}^1 - d$ (figure 5.15d) and the stock $x_2(t)$ increases with: rd (figure 5.15e).
- When $Z_{21}^{(.)} \leq x_1(t) < Z_{22}^{(.)}$: the serviceable stock level $x_1(t)$ is between the threshold levels $Z_{21}^{(.)}$ and $Z_{22}^{(.)}$. We have:
  - Zones 2, 5, 9, and 10: the production of both machines must respond to the demand rate $(u_1, u_2) = (u_{max}^1, d - u_{max}^1)$ or $(d - rd, rd)$. The stock $x_1(t)$ remains constant with: $d - d$ (figure 5.15d) and the stock $x_2(t)$ increases with: $rd - (d - u_{max}^1)$ or remains constant with: $rd - rd$ (figure 5.15e).
  - Zone 6: machine M_1 is down ($u_1 = 0$) and machine M_2 must produce according to its maximum production rate $u_2 = u_{max}^2$. The stock $x_1(t)$ increases with: $u_{max}^2 - d$ (figure 5.15d) and the stock $x_2(t)$ decreases with: $rd - u_{max}^2$ (figure 5.15e).

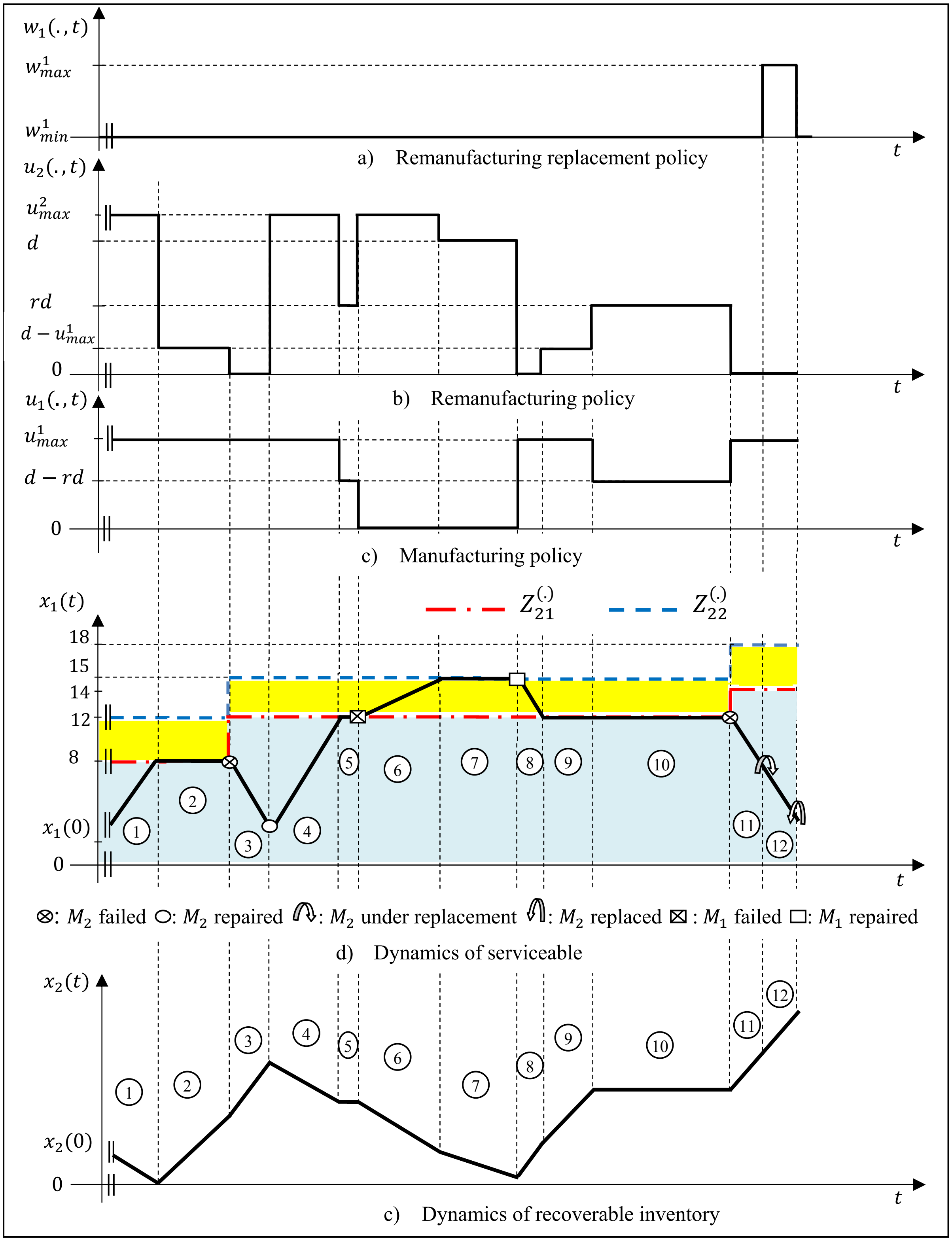


Figure 5.15 Implementation of the joint control policy

- Zone 8: machine M_1 must produce at its maximum production rate $u_1 = u_{max}^1$, while machine M_2 must reduce its production rate to $u_2 = 0$. The stock $x_1(t)$ decreases with: $u_{max}^1 - d$ (figure 5.15d) and the stock $x_2(t)$ increases with: rd (figure 5.15e).

- When $x_1(t) \geq Z_{22}^{(.)}$: the serviceable stock level $x_1(t)$ reaches the threshold level $Z_{22}^{(.)}$. We have:
  - Zone 7: because machine M_1 is down ($u_1 = 0$), machine M_2 must track the demand rate $u_2 = d$. The stock $x_1(t)$ remains constant with: $d - d$ (figure 5.15d) and the stock $x_2(t)$ decreases with: $rd - d$ (figure 5.15e).

For the hybrid system considered here, the central machine M_2 is represented by two operational modes (modes 1 and 2, depending on the machine M_1 mode), with two corresponding threshold levels. Machine M_1 is also unreliable, and when it is operational (in mode 1, 3 or 5), it produces at its maximum production rate under the thresholds levels $Z_{21}^{(.)}$ and $Z_{22}^{(.)}$, depending on the mode of machine M_2. The machine M_2 cannot produce at u_{max}^2 when $x_2(t)$ is low (i.e., $x_2(t) = 0$), and must reduce its production rate to the return rate rd (state constraint on the stock level $x_2(t)$). This policy is appropriate for operation-level decision making with respect to the behavior of the system over the short term; it works in the context of the current number of failures, and allows the management of the necessary stock level x_1 as a protection against random failures and deteriorations. When the number of failures increases, the threshold levels for each machine in each mode increases progressively, as can be seen in figure 5.7. This behavior is particularly affected by the deterioration of machine M_2, and is discussed in the sensitivity analysis.

5.8 Conclusion

This work investigates the production planning and replacement problem for the case of an unreliable hybrid closed-loop manufacturing/remanufacturing system in a deterioration context with bounded return of used products. Such a stochastic control problem is quite complex due to memory related to the dynamics of the system and when dealing with state constraints. We proposed a new mathematical model based on the extension of the state space of the system to determine the joint optimal control policy (production and replacement strategies). We demonstrated that despite the increase of the state space and numerical scheme dimensions, the problem remains tractable and its solution can be obtained numerically. We showed that the structure of the control policy is characterized by multiple critical thresholds, and that the coordination between the manufacturing and remanufacturing operations is considered. We illustrated the proposed approach using a numerical example and sensitivity analysis in order to examine its implication in practice. From the results of this article, we can see that our work seems very useful for hybrid closed-loop systems that experience imperfect repairs due the return of heterogeneous used products. This helps us to extend the study to more complex industrial situations.

CHAPITRE 6

ARTICLE 5: STOCHASTIC OPTIMAL CONTROL OF RANDOM QUALITY DETERIORATING HYBRID MANUFACTURING/REMANUFACTURING SYSTEMS

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Résumé

Dans cet article, nous analysons le problème de contrôle simultané de planification de la production et du remplacement pour un système hybride détériorant dans un contexte de logistique inverse en boucle fermée. Le système est composé de machines de fabrication et de rectification/refabrication non fiables dans lesquelles un type de pièce est fabriqué pour satisfaire une demande donnée. Particulièrement, la détérioration de la machine de fabrication, causée par le processus de vieillissement affecte de manière aléatoire sa disponibilité et la qualité des pièces qu'elle produit. Les pièces défectueuses produites par la machine de rectification/refabrication peuvent l'endommager. En raison de l'effet de la détérioration, le système n'est pas en mesure de répondre à la demande de produits à long terme et un remplacement de la machine de fabrication peut être effectué afin d'augmenter la capacité de production du système hybride. L'objectif principal de cette étude est de déterminer le plan de production optimale, pour la fabrication, la rectification et la refabrication, ainsi que la stratégie de remplacement de la machine de fabrication en minimisant le coût total sur un horizon de planification infini. Les conditions d'optimum sont développées sous la forme d'équations de Hamilton-Jacobi-Bellman (HJB) de second ordre afin de capturer les effets de la détérioration de la qualité aléatoire et des pannes et réparations aléatoires de machines pour lesquelles des équations d'HJB de premier ordre ont été développées avec succès dans la littérature. Nous adoptons des méthodes numériques pour résoudre les équations d'optimalité et un exemple numérique est présenté pour illustrer l'approche proposée. Finalement, une analyse de sensibilité est envisagée afin de confirmer la structure de la politique de commande conjointe obtenue.

Abstract

In this paper, we investigate the simultaneous production planning and replacement control problem for a deteriorating hybrid system in closed-loop reverse logistics. The system is composed of unreliable manufacturing and recovery machines in which one part type is produced to satisfy a given demand. In particular, the deterioration of the manufacturing machine, which is caused by the aging process, randomly affects its availability and the quality of the parts it produces. Defective parts produced by the recovery machine may cause it to fail. Thanks to the deterioration effect, the system is unable to fulfill long-term product demand, and the manufacturing machine can be replaced in order to increase the production capacity of the hybrid system. The main objective of this study is to determine the optimal production plan, in terms of manufacturing and recovery, as well as the replacement strategy, for the manufacturing machine, minimizing the total cost over an infinite planning horizon. The optimality conditions are developed in the form of second-order Hamilton-Jacobi-Bellman (HJB) equations in order to capture the effects of random quality deterioration and of random machine failures and repairs for which first-order HJB equations have been successfully developed in the literature. We adopt numerical methods to solve the optimality equations, and a numerical example is presented to illustrate the proposed approach. Finally, a sensitivity analysis is considered in order to confirm the structure of the joint control policy obtained.

Keywords: Deteriorating manufacturing system, Random quality, Production and quality failures, Minimal repairs, Remanufacturing/remediation, Replacement policies.

6.1 Introduction

As environmental regulations and market pressures have attracted increasing attention over the last few decades, the manufacturing system design quality and production strategies have become crucial factors in preserving the company's market position. The focus of logistics has shifted to the recovery system, taking back products after usage by clients, as well as defective items produced internally by the manufacturing activity, in order to return them to a like-new condition. In today's market, many products are recovered, and quite often, the customer cannot differentiate between new and recovered products. Such recovery can reduce the negative impact raw material extraction has on the environment, and additionally, it can reduce costs, energy and waste. Product recovery management is thus slated to become an important business activity, with the potential to result in the highest possible recovery of economic and ecological values as waste quantities are reduced or even eliminated altogether (see Huang *et al.* (2013) and Maiti and Giri (2015)).

Remanufacturing and remediation activities have the ability to recover degraded components and return products back into service. An example is the case of Cooper and Allwood (2012), who mention that reusing nondestructive steel and aluminum components in end-of-life products (EOLP) without melting can be highly effective, as it avoids the high energy costs associated with recycling through melting by preserving the microstructure and geometry of existing components. Both remanufacturing and remediation are widely used in automotive and heavy equipment systems, and specifically, on engine and fuel system components. An application in a diesel engine production system using machining, welding and other salvage operations for remanufacturing activities is mentioned in Sutherland *et al.* (2008), while Kerr and Ryan (2001) mention such an application in the photocopier remanufacturing process at Fuji Xerox Australia.

Product quality and production planning are critical issues facing modern organizations. Typically, hybrid production systems are complex in nature due to the fact that they face random issues such as breakdowns, repairs, production process deterioration, and the deterioration in the quality of parts produced. This complexity is reflected in the control challenges of the associated manufacturing systems we propose to address in this research. We look to model and illustrate the production planning problem of a random quality deteriorating manufacturing system in a closed-loop. In such a system, ordinary production activities, as well as those involved in remanufacturing returned EOLP and remediating defective items, are options that can be used to satisfy a demand rate. In this problem, the effects of deterioration affect the capacity of production processes, leading to increased lead times and customer dissatisfaction. These effects are countered by a replacement that restores the hybrid production system to initial conditions. Furthermore, in a context of deterioration, controlling production and replacement can improve the hybrid system performance in closed-loop reverse logistics in terms of productivity and availability.

Many contributions to production and maintenance planning problems in manufacturing/remanufacturing systems addressed in the literature can be divided into two major classes. In the first class, we find contributions relying on the closed-loop reverse logistics process without deterioration. Such models for manufacturing and remanufacturing activities can be found in Kiesmüller and Scherer (2003), Kenné *et al.* (2012), Polotski *et al.* (2015) and Giri and Sharma (2016). In this class of systems, the aging process and its effects on the machines are not considered, and the authors falling under it develop optimization models applied in closed loops for production planning problems. The second class comprises contributions based on closed-loop reverse logistics with deterioration. This deterioration has previously been used in the context of either manufacturing or remanufacturing activities, and the optimal control problems are investigated to determine the parameters which have to be optimized. The manufacturing case is illustrated by Colledani and Tolio (2011) and Kouedeu *et al.* (2015), covering the effects of deterioration on the system availability. The consideration of the combined effects on the availability of the manufacturing system and on quality was proposed by Kim and Gershwin (2005) for the quality failures and by Rivera-Gomez *et al.* (2013b) for the quality of parts produced. Remanufacturing was investigated by Jiang *et al.* (2016), who represented reliability through the failure rate of remanufacturing operations, with the associated failures being a function of the quality of returned EOLP. In other words, depending on the type and level of damage of parts, a variety of operations are employed, namely, material addition (welding, thermal spraying), material removal (machining, laser cutting), or surface treatment (heat treatment, anodizing). Attempts to extend the deterioration models to the closed-loop context face new basic challenges. Kouedeu *et al.* (2014) analyzed a hybrid manufacturing/remanufacturing system consisting of two machines, taking into account the gradual deterioration of the manufacturing machine during the production process. Similarly, an additional recovery system option is presented in Huang *et al.* (2013), where the quality deterioration of defective items produced by manufacturing and remanufacturing activities can be improved using remediation activities. However, their consideration is limited to a constant defective rate during the entire lifetime of the system. This is a restrictive assumption in the context of production systems, since the deterioration phenomenon on the machine could randomly affect the quality of the parts produced, as indicated in Kim and Gershwin (2005); it could also affect the availability of the system, as mentioned in Colledani and Tolio (2011) and Jiang *et al.* (2016).

The main problem with the above-mentioned research works is that none of them considers the simultaneous effect of deterioration on quality and availability in the context of a closed-loop reverse logistics system. Moreover, the interaction of quality and reliability on control policies is not addressed, and remains an important open issue, especially where its repercussion can be eliminated through replacement activities. Since quality and availability deterioration aspects are normally observed in practical situations, it will be very useful to take them into account in optimization models for managerial decisions. Many authors have contributed to the production planning and maintenance policies in the field of systems with deteriorations, but these have only been limited to systems in forward or backward configurations. This work addresses this drawback. It intends to contribute in this regard by extending deterioration models to the entire hybrid system to create a connection with failure and quality.

The main contribution of this paper consists in the joint optimization of the production and replacement control problem for an unreliable hybrid closed-loop system under deterioration, uncertainties and reverse logistics constraints. The hybrid system is composed of two machines that produce one part type. The machines are subject to random issues such as breakdowns and repairs. Deterioration affects the quality of the parts produced by the manufacturing machine and the availability of both machines, such that the recovery machine is affected by the aging process of the manufacturing machine. Following a replacement activity, the parameters of the hybrid system are restored to initial conditions. Given that the manufacturing machine deteriorates with age when it is in operation, and that the repair process is minimal, the deterioration effects naturally lead to a system with memory. Classical Markovian models are therefore not appropriate for describing the dynamics of the hybrid system. The manufacturing, recovery and replacement policies are determined in order to minimize the total incurred cost over an infinite planning horizon.

The paper is organized as follows. In Section 6.2, we present the industrial context of the problem under study. The model notations, the system dynamics and the control problem are presented in Section 6.3. The optimal control problem statement is also described in detail in Section 6.3. A numerical example is given in Section 6.4 to illustrate the proposed approach. The structure of the obtained joint control policy is confirmed in Section 6.5 through a sensitivity analysis. An example of implementation results is addressed in Section 6.6. Finally, the paper is concluded in Section 6.7.

6.2 Industrial context

The model presented in this paper can be suitable for many industries having manufacturing system context with reverse logistics, and characterized by a random deterioration that has severe effects on the availability of production processes and the quality of parts produced. The deterioration phenomenon is present in machinery and mechanical assemblies, including in automobiles, aircraft engines, and machine tools. Examples of such systems include machine tools (i.e., machining centers, grinders, milling) typically comprised of a number of components which stochastically deteriorate over time, as stated in Rivera-Gomez *et al.* (2016). This phenomenon could also randomly affect the product quality, as demonstrated in Kim and Gershwin (2005), for the automotive sector, in which case the machine stops producing good parts and starts producing defective parts due to a failure such as a sudden tool damage. The occurrence of such failures depends on the characteristics of common cause variations, such as raw material defects or when the operation uses a new technology that is difficult to control. Nevertheless, as has been observed in the machining industry, failures caused by defective products may have a significant impact on the production system reliability. In drilling processes, material properties (taken as the product quality) of the incoming work piece have a significant impact on the wear and breakage rate of the drill (see Chen and Jin (2005)). The effect of defective products has also been experienced in the brick industry. In this context, bricks are produced through four basic processes: mixing ground clay with water, forming the clay into the desired shape, drying the molded materials, and firing the bricks in the tunnel kiln (Brick Industry Association, 2006). The problem is that about 10% of bricks produced are usually defective (Hamer and Karius, 2002). During the firing process, poor quality products can be broken, and their accumulation within the kiln may cause it to fail.

Deterioration effects are very common industrial phenomena, as evidenced in the contexts above, and many production systems operate in the presence of such effects. In this paper, we attempt to incorporate not only the aging process, but also the failures caused by defective products, in the optimization model for an unreliable hybrid closed-loop system. The control policies to be obtained are easily adaptable to industrial requirements, and can be very useful for the practical managerial decisions of a company in improving the performance of the hybrid system in terms of productivity and availability. Below, we formulate the corresponding optimization model in a stochastic environment and develop appropriate techniques for its solution.

6.3 Hybrid production system with reverse logistics

In this section, we present the notations used throughout this article, the system dynamics, the control problem, and the optimal control problem statement under study.

6.3.1 Notations

The following notations are used throughout the paper:

$x_1(t)$	stock level of manufactured, remedied and remanufactured products at time t
$x_2(t)$	stock level of defective and returned products at time t
$a_1(t)$	age of the manufacturing machine at time t
$\xi(t)$	stochastic process for the hybrid system at time t
τ	jump time of $\xi(t)$
$\tilde{\beta}(.)$	random rate of defectives for manufacturing activity
p	proportion of defective remedied items
d	demand rate of customers (products/time unit)
r	proportion of return of EOLP to the recoverable inventory
c_1^+	inventory cost for serviceable products (\$/product/time unit)
c_2^+	inventory cost for defective and returned products (\$/product/time unit)
c_1^-	backlog cost for serviceable products (\$/missing product/time unit)
c_{P_1}	manufacturing cost (\$/product)
c_{P_2}	remediation/remanufacturing cost (\$/product)
c_{IP}	inspection cost (\$/product)
c_d	disposal cost (\$/product)
c_{r1}	repair cost for manufacturing machine (\$/time unit)

c_{r2} repair cost for recovery machine ($/time unit)

c_0 replacement cost for manufacturing machine ($)

$q_{\alpha\alpha'}(.)$ transition rate from mode α to mode α'

$G(.)$ instantaneous cost function

$J(.)$ expected discounted cost function

$v(.)$ value function

ρ discount rate

$u_1(t)$ production rate of the manufacturing machine (products/time unit)

$u_2(t)$ production rate of the recovery machine (products/time unit)

u_{max}^1 maximum production rate of the manufacturing machine (products/time unit)

u_{max}^2 maximum production rate of the recovery machine (products/time unit)

$w_\alpha(.)$ replacement rate for the manufacturing machine in mode α

6.3.2 Description of the system dynamics

The considered hybrid production system consists of two unreliable machines, denoted M_1 and M_2, mounted in closed-loop reverse logistics and producing a single type of product. As presented in the block diagram of figure 6.1, machine M_1 is used for manufacturing activities with original raw materials and machine M_2 is used for remediation and remanufacturing activities to recover returned products. Machine M_1 used for manufacturing activities deteriorates when it is in operation, and its deterioration affects its availability and the quality of the parts it produces. This can be attributed to the fact that the failure rate and the defective rate increase progressively with the age of the machine M_1. This age is considered as a dependent measurable function of the number of manufactured parts. The manufactured products are fully inspected. The perfect manufactured products are stored in the serviceable inventory $x_1(t)$ in order to attend the demand, whereas defectives are transferred to the recovery inventory $x_2(t)$, and will then be sent for remediation activities. However, it can be seen that the defective parts produced by the manufacturing activity may cause a failure of the machine M_2 (quality failure). This flow of defective parts progressively decreases the availability of the machine, specially manifested through an increase in its failure rate. In other words, the aging of the machine M_1 will cause more rejections, which will inflict more damage on machine M_2. The EOLP returned after being used by customers are also transferred to the recovery inventory $x_2(t)$ for remanufacturing activities. For the system under study, the remediation and remanufacturing activities are carried out on the same machine M_2 with different production rates, but in terms of policy, we talk about the control of the average rate, also called the weighted average rate.

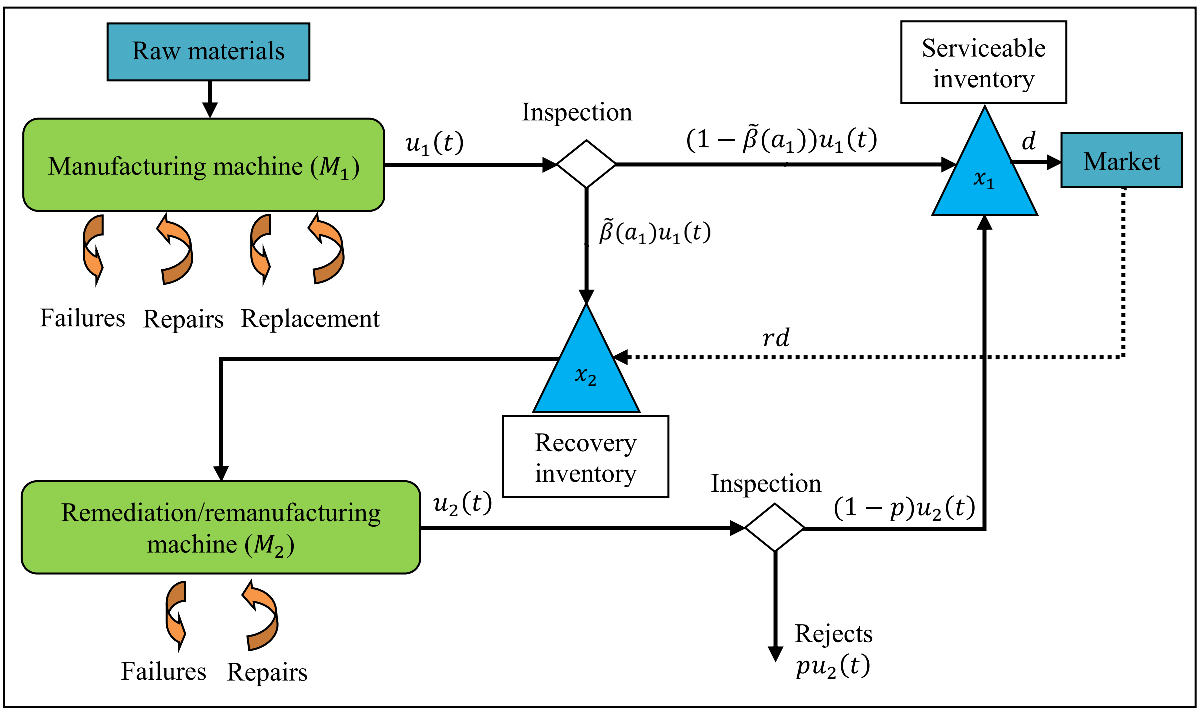

Figure 6.1 Hybrid production system

Given that the remediation activity is considered imperfect, the remedied products are inspected. Products with unacceptable quality are disposed of, while the remainders are then be sent along with the remanufactured products to the serviceable inventory $x_1(t)$ to meet demand. When the machine M_1 or M_2 is down, repair activities are employed, and are considered minimal on machine M_1 (as-bad-as-old). Given the severe effects of the deterioration phenomenon, the manufacturing machine M_1 may not satisfy the long-term demand, even with the help of the machine M_2. With the deterioration in availability and quality, the constraints of reverse logistics (i.e., the limits of returned quantities and production capacity, which depend on the number of parts available in recovery inventory $x_2(t)$), the hybrid system reaches a certain deterioration level and is unable to fulfill the demand for finished goods. Additionally, the replacement option can be conducted on the manufacturing machine M_1 to solve such a problem. This option will completely eliminate all the effects of deterioration and restore the parameters of the hybrid system to initial conditions.

Other features and properties for the system shown in figure 6.1 are: a) the demand rate is constant; b) the return process is deterministic, and is represented by a percentage of the demand rate; c) the rate of defectives of the manufacturing machine is directly related to its age, with a mixture of a dynamic component and a diffusion-type stochastic process; d) finished goods produced by the manufacturing, the remediation and the remanufacturing activities have the same quality, and can thus not be differentiated by customers; e) rejects of defective parts by the recovery machine are discarded, since a second remediation is also not possible, and f) a second remanufacturing is not allowed for returned EOLP from the market, which have already been remanufactured. The main reason these products are discarded is related to the inappropriate quality of products used for remanufacturing activities.

6.3.3 Formulation of the control problem

Let us formulate in this section an optimal hybrid production strategy for both machines and find the appropriate moment to replace the machine M_1. We take into account random phenomena, such as machine breakdowns, repairs and deterioration processes, as well as aspects of deterioration in the quality of manufactured products, which affect the hybrid production system in closed loops.

We shall begin the formulation of the control problem by presenting the continuous components of the hybrid system. We define the aging of the manufacturing machine at time t as an increasing function of its production rate with the following differential equation:

$$\dot{a}_1(t) = ku_1(t), \quad t \geq 0 \quad \text{and } a_1(0) = a_{10}, a_1(T^+) = a_1(T^-) \text{ and } a_1(T) = 0 \tag{6.1}$$

where k and a_{10} are the given positive constant and initial age. The random variables T^+, T^- and T are the last restart times of the manufacturing machine following repair, production and replacement activities, respectively.

We consider that the dynamic behavior of the stock levels evolves according to the following two-dimensional system of differential equations:

$$\dot{x}_1(t) = (1 - \tilde{\beta}(a_1))u_1(t) + (1-p)u_2(t) - d, \quad x_1(0) = x_{10}, \quad t \geq 0 \tag{6.2}$$

$$\dot{x}_2(t) = rd + \tilde{\beta}(a_1)u_1(t) - u_2(t), \quad x_2(0) = x_{20}, \quad t \geq 0 \tag{6.3}$$

where x_{10} and x_{20} refer to the initial stock levels of the serviceable inventory and returned products, respectively, $\tilde{\beta}(.)$ represents the rate of defectives as a function of the age a_1, and p is the proportion at which defective items produced by the remediation machine will be rejected.

We wish to carry out a control of the noise by adding it around the dynamic average to ensure consistency with previous work, in order to model a more realistic defective rate trajectory in the manufacturing system. In this paper, this rate is modeled with a mixture of deterministic and random components. Considering the age as a time-type variable, the defective rate model can be written as:

$$\tilde{\beta}(a_1(t)) = \mu_\beta(a_1(t)) + Z_\beta(a_1(t)) \tag{6.4}$$

where $\mu_\beta(a_1(t))$ is a dynamic and known average defective rate, and $Z_\beta(a_1(t))$ denotes the random varying portion which is constructed as an output of the shaping filter excited by the white noise. As the defective rate $\tilde{\beta}(.)$ of our system over an infinite horizon is assumed to be a stochastic process in nature with finite variance, the Ornstein-Uhlenbeck process become a perfect candidate to model the trajectory in which we are interested. This is appropriate for the following two reasons:

1. The Ornstein-Uhlenbeck process is the only nontrivial process that satisfies the so-called stationary, Gaussian and Markovian mathematical properties, up to allowing linear transformations of the space and time variables. These interesting properties are clearly natural candidates for generalizing the deterministic defective rate. The stationary property is used in a wide sense with respect to finite means and variances, while the Gaussian property for its part is definitely useful in manufacturing problems, and we have such powerful tools to study it. The Markovian property implies the independence between random variables (independent increments).
2. The Ornstein-Uhlenbeck process has Gaussian distributions characterized by great analytical simplicity, which allows the building of simple models on which computations can be carried out. The characteristics of this stochastic process make it a good noise given that it is able to generate random variations around the dynamic average of the defective rate without changing its main trajectory. The stochastic equation of such a random portion can be expressed with the following Itô stochastic differential equation:

$$dZ_\beta(a_1(t)) = -bZ_\beta\big(a_1(t)\big)da_1(t) + \sigma(a_1)dW(a_1(t)), \qquad Z_\beta(0) = Z_\beta^0 = 0 \tag{6.5}$$

where Z_β^0 is the initial random variable of the defective rate (this value cannot be negative, and therefore a zero value is imposed); b is a drift coefficient, and a known parameter; $\sigma(a_1)$ is a diffusion coefficient, and is assumed to be an increasing function of the age, while $dW(a_1(t))$ is the differential form of the standard Brownian motion. Additionally, when the age becomes very large ($a_1 \to \infty$), the portion $Z_\beta\big(a_1(t)\big)$ converges to a stochastic process which is approximately normally distributed with zero mean and a $\frac{\sigma^2(a_1)}{2b}$ variance that changes with the age $\left(\text{i.e.}, Z_\beta(a_1(t)) \sim \mathcal{N}(0, \frac{\sigma^2(a_1)}{2b})\right)$. We refer the reader to Ross (2003) and Chiarella *et al.* (2015) for more details on diffusion-type stochastic processes.

The portion $Z_\beta(a_1(t))$ can similarly be written mathematically as follows:

$$Z_\beta(a_1(t)) = \sigma_\beta(a_1)\ v_\beta(a_1(t)) \tag{6.6}$$

where the stochastic process $v_\beta(a_1(t))$ is stationary and normally distributed $\sim \mathcal{N}(0,1)$, and $\sigma_\beta(a_1)$ is the standard deviation of the defective rate, with $\sigma_\beta^2(a_1) = \frac{\sigma^2(a_1)}{2b}$.

The dynamic averages of the defective rate and diffusion coefficient can be modeled with the following increasing functions of the age:

$$\mu_\beta(a_1(t)) = \beta_0 + \beta_1(1 - e^{-(k_1\theta a_1(t)^3)}) \tag{6.7}$$

$$\sigma(a_1(t)) = \sigma_0 + \sigma_1(1 - e^{-(k_2\theta_s a_1(t)^3)}) \tag{6.8}$$

where θ and θ_s are the adjustment parameters for the desired trajectories of the rate of defectives and diffusion coefficient, respectively. β_0, β_1, k_1, σ_0, σ_1 and k_2 are given constants, and can be obtained from historical data on the machine during the manufacturing activity (Lam *et al.*, 2004).

Further, $u_1(t)$ and $u_2(t)$ are the production rates of machines M_1 and M_2, respectively, with:

$$0 \le u_1(t) \le u_{max}^1, \quad t \ge 0 \tag{6.9}$$

$$0 \le u_2(t) \le u_{max}^2, \quad t \ge 0 \tag{6.10}$$

where u_{max}^1 and u_{max}^2 are the maximum production rates of machines M_1 and M_2, respectively.

Using equations (6.4) and (6.6), the stochastic state differential equations during short intervals $(\delta t, \delta W)$ can be written as follows:

$$\begin{bmatrix} \delta x_1(t) \\ \delta x_2(t) \\ \delta a_1(t) \end{bmatrix} = \begin{bmatrix} \left(1-\mu_\beta(a_1(t))\right)u_1(t)+(1-p)u_2(t)-d & -\sigma_\beta(a_1)k^{-1} \\ rd+\mu_\beta(a_1(t))u_1(t)-u_2(t) & \sigma_\beta(a_1)k^{-1} \\ ku_1(t) & 0 \end{bmatrix} \begin{bmatrix} \delta t \\ \delta W \end{bmatrix} \tag{6.11}$$

with $\delta W = v_\beta\big(a_1(t)\big)\delta a_1(t)$.

Since the number of remedied and remanufactured parts is limited by the number of parts available in recoverable inventory, we must therefore impose a non-negativity constraint on this inventory:

$$x_2(t) \geq 0, \text{ for all } t \geq 0 \tag{6.12}$$

Let the random variables $\xi_1(t)$ and $\xi_2(t)$ describe the state of machines M_1 and M_2 with values in $B_1 = \{0, 1, 2\}$ and $B_2 = \{0, 1\}$, respectively. The state of machine M_1 is classified as under repair $\xi_1(t) = 0$, as operational $\xi_1(t) = 1$, and as under replacement $\xi_1(t) = 2$. The state of machine M_2 is under repair $\xi_2(t) = 0$, and operational $\xi_2(t) = 1$. Given that the hybrid system deteriorates with age while machine M_1 is in operation, and given fact that repair activities are considered minimal, this leads to a memory problem. In this situation, the dynamics of the hybrid system could be described by a continuous time semi-Markov process $\xi(t) = \big(\xi_1(t), \xi_2(t)\big)$ with values in $B = B_1 \times B_2 = \{(1,1), (1,0), (0,1), (0,0), (2,1), (2,0)\} \equiv \{1,2,3,4,5,6\}$. The stochastic process $\xi(t)$ implies a transition rates matrix $Q(.)$ such that $Q(.) = [q_{\alpha\alpha'}(.)]$, where $q_{\alpha\alpha'}(.)$ indicates the transition rate from α to α' with $\alpha, \alpha' \in B$, and verifies some conditions. For more details on these conditions, we refer the reader to Sethi *et al.* (2005). The transition diagram describing the considered hybrid system is presented in figure 6.2.

We use the model of the age-dependent failure transition rate as given in Rivera-Gomez *et al.* (2013b) and Xiang (2013) for the manufacturing machine M_1, which is assumed to be continuous, bounded and an increasing function of the age a_1, as indicated in the following expression:

$$q_{13}\big(a_1(t)\big) = q_{24}\big(a_1(t)\big) = q_0 + q_1\big(1 - e^{-(k_3 a_1(t)^3)}\big) \tag{6.13}$$

where the parameters q_0, q_1 and k_3 are given constants.

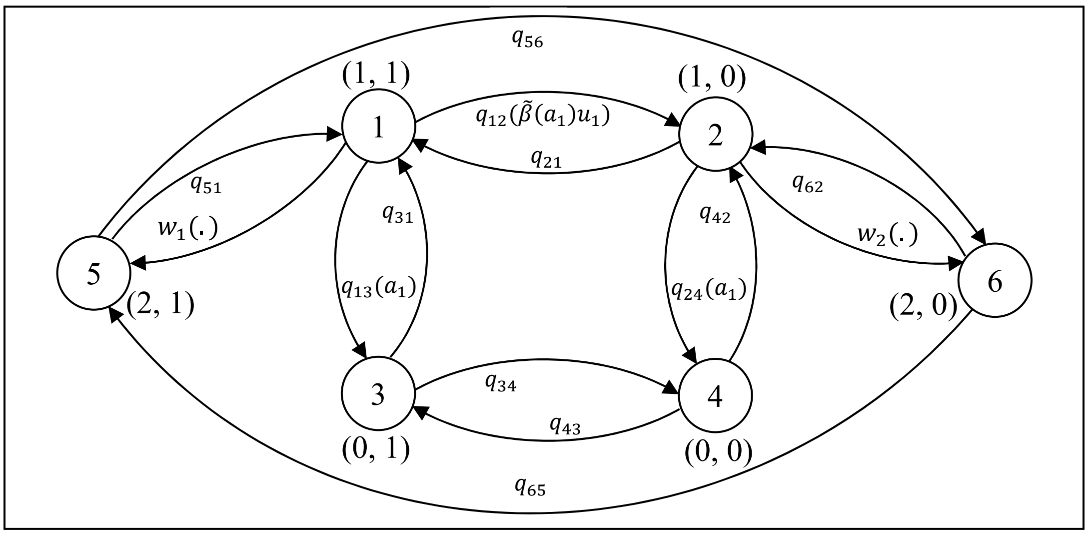

Figure 6.2 States transition diagram of the considered hybrid system

We therefore extend the concept of production and quality failures to the remediation/remanufacturing machine M_2, based on the relationship between the failure and the quality of the produced parts (see Kim and Gershwin (2005) for more details). In our case, we propose the increasing function given by equation (6.14), which defines the total failure rate of machine M_2 as a function of the flow of defective parts:

$$q_{12}(.) = q_2^c + q_2'\big(\tilde{\beta}\big(a_1(t)\big)u_1\big) = q_2^c + q_2\left(1 - e^{-\left(k_4\left(\tilde{\beta}(a_1(t))u_1\right)^3\right)}\right) \tag{6.14}$$

where the parameters q_2 and k_4 are given constants, the production component q_2^c of the failure rate $q_{12}(.)$ is considered constant, and is a function of the remanufacturing of perfect parts returned from the market, while the quality component $q_2'\big(\tilde{\beta}\big(a_1(t)\big)u_1\big)$ is the proportion of defective parts that cause the failure of machine M_2, and is a function of the remediation of defective parts produced by machine M_1. Thus, the component

$q_2'(\tilde{\beta}(a_1(t))u_1)$ increases progressively when machine M_1 is operational (its production rate $u_1 \neq 0$), it is equal to 0 when the production rate $u_1 = 0$ (i.e., machine M_1 is either down or available without production taking place) or when machine M_1 is replaced (the age a_1 is reset to zero). The randomness of the failure rate $q_{12}(.)$ is mainly due to two kinds of perturbations. In the first of these perturbations, machine M_1 has different production rates (i.e., $u_1 = 0, d, u_{max}^1$), whereas the second one is related to the random quality of the parts produced. Finally, to adequately incorporate the behavior of equation (6.14) in the control strategies, we need to employ a memory process resulting in a semi-Markov model. In figure 6.3, we present as an illustration the trajectories of the failure rate of machine M_2 (where we use $q_2^c = 0.02, q_2 = 0.1, k_4 = 5.10^{-6}$ and $0 \leq a_1 \leq 100$) in the three possible cases of u_1: produce at the maximum production rate $u_1 = 0.8$ when there is a random defective rate, produce at the demand rate $u_1 = 0.5$ for the deterministic defective rate $\mu_\beta(a_1)$, and when there is no production $u_1 = 0$. The other parameters used for this simulation relate to the rate of defectives (with $\beta_0 = 10^{-4}, \beta_1 = 0.3, k_1 = 15.10^{-6}, \theta = 0.6, b = 0.4, \sigma_0 = 10^{-4}$, $\sigma_1 = 0.23, k_2 = 15.10^{-6}$ and $\theta_s = 0.6$). As can be seen in figure 6.3, the perturbations become more severe by increasing the diffusion coefficient $\sigma(.)$ and less so by increasing the drift coefficient b.

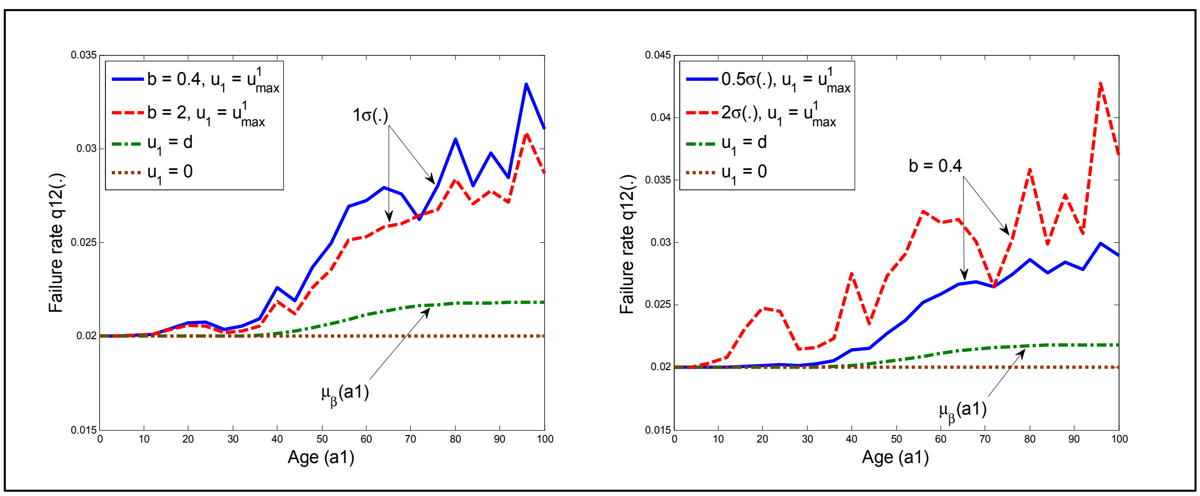

Figure 6.3 Failure rate of machine M_2 with effects of b and $\sigma(.)$

The transitions rates $w_1(.)$ and $w_2(.)$ used in modes 1 and 2 to control the replacement of the manufacturing machine are considered as decision variables, continuous in the domains $[w_{min}^1, w_{max}^1]$ and $[w_{min}^2, w_{max}^2]$, respectively. The values w_{min}^1, w_{min}^2, w_{max}^1 and w_{max}^2 denote the minimum and maximum replacement rates, respectively. The following transition rates are considered constant: $q_{34} = q_{56} = q_2^c$, $q_{31} = q_{42}$, $q_{51} = q_{62}$ and $q_{21} = q_{43} = q_{65}$, while all other transition rates are equal to zero.

6.3.4 Optimal control problem statement

The considered cost is composed of inventories, backlog, production, inspection, defectives, repair and replacement costs. Let $G(.)$ be the cost rate defined as follows:

$$G(\alpha, x_1, x_2, a_1, u_1, u_2, w_1, w_2) = h(x_1, x_2) + c(u_1, u_2) + c^\alpha \tag{6.15}$$

with:

$$h(x_1, x_2) = c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^-$$

$$c(u_1, u_2) = (c_{P_1} + c_{IP})u_1 + (c_{P_2} + c_{IP} + c_d p)u_2$$

$$x_i^+ = \max(0, x_i), i = 1, 2$$

$$x_1^- = \max(0, -x_1)$$

$$c^\alpha = c_{r1}.(\text{Ind}\{\xi(t) = 3\} + \text{Ind}\{\xi(t) = 4\}) + c_0.(q_{51}\text{Ind}\{\xi(t) = 5\} + q_{62}\text{Ind}\{\xi(t) = 6\})$$
$$+c_{r2}.(\text{Ind}\{\xi(t) = 2\} + \text{Ind}\{\xi(t) = 4\} + \text{Ind}\{\xi(t) = 6\})$$

where c_1^+, c_2^+ and c_1^- are the serviceable unit, recovery inventories and backlog costs, respectively, such that $c_1^- > c_1^+ > c_2^+ > 0$; c_{P_1} is the unit production cost of the manufacturing activity, while c_{P_2} refers to the remediation/remanufacturing activity unit cost, and is assumed to be much lower ($0 < c_{P_2} < c_{P_1}$); c_{IP} is the unit inspection cost, and c_d is the unit disposal cost of defective parts. In addition, c_{r1} and c_{r2} define the repair cost rates of machines M_1 and M_2, respectively; and c_0 denotes the replacement cost of machine M_1. The indicator function $\text{Ind}\{.\}$ is defined as follows:

$$\mathrm{Ind}\{\Theta(.)\} = \begin{cases} 1 & \text{if } \Theta(.) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

The set of feasible control policies $\Gamma(x_2, \alpha)$, including the decision variables (u_1, u_2, w_1, w_2), depends on the stochastic process $\xi(t)$ and the state constraint $x_2 \geq 0 \in \chi$ (see Sethi *et al.* (2005) for more details about the control problems with state constraints), and is given by:

$$\Gamma(x_2, \alpha) = \left\{ \begin{array}{c} (u_1(.), u_2(.), w_1(.), w_2(.)) \in \Re^4, 0 \leq u_1(.) \leq u_{max}^1, \\ 0 \leq u_2(.) \leq k_L \text{ with } k_L = \begin{cases} u_{max}^2 & \text{if } x_2(t) > 0 \\ rd + \tilde{\beta}\big(a_1(t)\big)u_1(.) & \text{if } x_2(t) = 0 \end{cases}, \\ w_{min}^1 \leq w_1(.) \leq w_{max}^1, w_{min}^2 \leq w_2(.) \leq w_{max}^2 \end{array} \right\} \tag{6.16}$$

where $\xi(t) = \alpha$, $\chi = (-\infty, +\infty) \times [0, +\infty) \subseteq \Re^2$ denote the state domain on inventories.

Our objective is to find in $\Gamma(x_2, \alpha)$ the optimal control policy $(u_1^*, u_2^*, w_1^*, w_2^*)$ which minimizes for each initial state condition (α, x_1, x_2, a_1), the following expected discounted cost $J(.)$ given by:

$$J(\alpha, x_1, x_2, a_1, u_1, u_2, w_1, w_2) = \tag{6.17}$$

$$E\left[\int_0^{\infty} e^{-\rho t} G(.) dt \,|\, \xi(0) = \alpha, x_1(0) = x_1, x_2(0) = x_2, a_1(0) = a_1\right]$$

where ρ is the discount rate and $E[.\,|\alpha, x_1, x_2, a_1]$ is the conditional expectation operator. The optimal policies of the planning problem are obtained by searching for the value function given by:

$$v(\alpha, x_1, x_2, a_1) = \inf_{(u_1, u_2, w_1, w_2) \in \Gamma(x_2, \alpha)} J(\alpha, x_1, x_2, a_1, u_1, u_2, w_1, w_2) \qquad \forall \alpha \in B \tag{6.18}$$

The value function $v(.)$ is shown to be continuously differentiable, and such a function provides the viscosity solution to the HJB equations. The properties of the value function leading to the first-order HJB equations are justified by Theorem 3.1 in Sethi *et al.* (2005). Such equations describe the optimality conditions for the stochastic control. In this paper, we talk about the Hamilton-Jacobi-Bellman equation in terms of a directional derivative (HJBDD) when dealing with a control problem with a state constraint. Given the random variation of quality issues, the heuristic derivation of the optimality conditions will be carried out via the rules of stochastic calculus introduced by Itô. More details about the development of Itô form can be found in Chiarella *et al.* (2015). We finally get the HJBDD equations associated with our stochastic control problem in the second-order Itô form:

$$\rho v(\alpha, x_1, x_2, a_1) = \min_{(u_1,u_2,w_1,w_2)\in\Gamma(x_2,\alpha)} \left\{ \begin{array}{c} G(.) + f_{1\alpha}\dfrac{\partial v}{\partial x_1} + f_{2\alpha}\dfrac{\partial v}{\partial x_2} + f_{3\alpha}\dfrac{\partial v}{\partial a_1} \\ +\dfrac{1}{2}g_{1\alpha}^2 f_{3\alpha}\dfrac{\partial^2 v}{\partial x_1^2} + \dfrac{1}{2}g_{2\alpha}^2 f_{3\alpha}\dfrac{\partial^2 v}{\partial x_2^2} \\ +g_{1\alpha}g_{2\alpha}f_{3\alpha}\dfrac{\partial^2 v}{\partial x_1 \partial x_2} \\ +\sum_{\alpha'} q_{\alpha\alpha'}(.)v\left(\alpha', x_1, x_2, \varphi_{a_1}(\xi, a_1)\right) \end{array} \right\} \tag{6.19}$$

where $f_{1\alpha} = \left(1 - \mu_\beta(a_1)\right)u_1 + (1-p)u_2 - d$, $f_{2\alpha} = rd + \mu_\beta(a_1)u_1 - u_2$, $f_{3\alpha} = ku_1$, $g_{1\alpha} = -\sigma_\beta(a_1)k^{-1}$ and $g_{2\alpha} = \sigma_\beta(a_1)k^{-1}$. The term $\varphi_{a_1}(\xi, a_1)$ denotes the reset function that brings the age of the manufacturing machine to zero after a replacement and the system to as-bad-as-old conditions after a minimal repair activity. We define this reset function at a jump time τ for the process ξ as follows:

$$\varphi_{a_1}(\xi, a_1) = \begin{cases} 0 & \text{if } \xi(\tau^+) = 1 \text{ and } \xi(\tau^-) = 5 \\ & \quad\text{or} \\ & \text{if } \xi(\tau^+) = 2 \text{ and } \xi(\tau^-) = 6 \\ a_1(\tau^-) & \text{otherwise} \end{cases} \tag{6.20}$$

Unfortunately, an analytical solution of equation (6.19) is almost impossible. However, there is a way to carry out an approximation of the solution using numerical methods. More specifically, Boukas and Haurie (1990) showed that such a problem in the context of production planning can be solved by the Kushner's method (Kushner and Dupuis, 1992). Based on the works of Boukas and Haurie (1990), Kouedeu *et al.* (2015) and references therein, the numerical methods (based on the finite difference approximations and policy improvement technique) will be used to solve the second-order optimality conditions for the proposed stochastic optimal control problem. The finite difference approximations of the value function and its first-order and second-order partial derivatives are presented in Annex VIII.

In the next section, we provide a numerical example to illustrate the structure of the hybrid control policies.

6.4 Numerical example

We present a numerical example to solve the discrete version of HJBDD equations given by equation (6.19). A finite grid denoted by $G^h_{x_1 x_2 a_1}$ is needed to define the computational domain of the state variables (x_1, x_2, a_1), with $h = (h_{x_1}, h_{x_2}, h_{a_1})$, such that:

$$G^h_{x_1 x_2 a_1} = \left\{\left(x_1^i, x_2^j, a_1^\ell\right): -5 \le x_1^i \le 35, 0 \le x_2^j \le 20, 0 \le a_1^\ell \le 100\right\} \tag{6.21}$$

where h_{x_1}, h_{x_2} and h_{a_1} are the finite difference intervals of the state variables x_1, x_2 and a_1, respectively, with $x_1^i = -5 + ih_{x_1}$ for $i = 0, 1, \dots N_{x_1}^{h_{x_1}}$; $x_2^j = jh_{x_2}$ for $j = 0, 1, \dots N_{x_2}^{h_{x_2}}$ and $a_1^\ell = \ell h_{a_1}$ for $\ell = 0, 1, \dots N_{a_1}^{h_{a_1}}$, with $N_{x_1}^{h_{x_1}} = \text{card}\left[G_{x_1}^{h_{x_1}}\right]$, $N_{x_2}^{h_{x_2}} = \text{card}\left[G_{x_2}^{h_{x_2}}\right]$ and $N_{a_1}^{h_{a_1}} = \text{card}\left[G_{a_1}^{h_{a_1}}\right]$.

Given that the remediation activity is considered imperfect, historical production data is the source used to determine the proper value of the proportion of defective remedied items p. The hybrid system will be able to satisfy the demand rate for the chosen parameters presented in Table 6.1 if the feasible condition given by the following equation is verified:

$$(\pi_1(a_1) + \pi_2(a_1))\left(1 - \tilde{\beta}(a_1)\right) u_{max}^1 + (1 - p) \tag{6.22}$$

$$\min\left\{(\pi_1(a_1) + \pi_3(a_1) + \pi_5(a_1)) u_{max}^2\, , rd + (\pi_1(a_1) + \pi_2(a_1))\tilde{\beta}(a_1) u_1\right\} \ge d$$

where $\pi_i(a_1)$, $i = 1, \dots, 6$ is the limiting probability of the hybrid system at mode i and at age a_1, and is computed as follows:

$$\pi(.)Q(.) = 0 \text{ and } \sum_{i=1}^{6} \pi_i(.) = 1 \tag{6.23}$$

where $\pi(.) = (\pi_1(.), \pi_2(.), \pi_3(.), \pi_4(.), \pi_5(.), \pi_6(.))$ and $Q(.)$ is the corresponding 6×6 transition rates matrix. The following Table 6.1 summarizes the parameters used in this paper.

Table 6.1 Parameters of the numerical example

Parameter	c_1^+	c_2^+	c_1^-	c_{r1}	c_{r2}	c_0	c_{P_1}	c_{P_2}
Unit	($/product/ time unit)	($/ product/ time unit)	($/ missing product/ time unit)	($/ time unit)	($/ time unit)	($/ replacement)	($/ product)	($/ product)
Value	5	1	200	20	40	4000	100	50
Parameter	c_{IP}	c_d	p	ρ	u_{max}^1	u_{max}^2	d	r
Unit	($/ product)	($/ product)			(products/ time unit)	(products/ time unit)	(products /time unit)	
Value	3	10	0.3	0.01	0.8	0.47	0.5	0.5
Parameter	k	β_0	β_1	k_1	θ	b	σ_0	σ_1
Value	0.5	10^{-4}	0.3	15.10^{-6}	0.6	0.4	10^{-4}	0.23
Parameter	k_2	θ_s	q_0	q_1	k_3	q_{31}	$w_{max}^{1,2}$	$w_{min}^{1,2}$
Value	15.10^{-6}	0.6	10^{-4}	0.01	5.10^{-6}	0.067	1	10^{-5}
Parameter	q_{51}	q_2^c	q_2	k_4	q_{21}	h_{x_1}	h_{x_2}	h_{a_1}
Value	0.1	0.02	0.1	5.10^{-6}	20^{-1}	2	2	4

with $q_{42} = q_{31}$, $q_{62} = q_{51}$ and $q_{43} = q_{65} = q_{21}$.

The production of the machine M_2 is limited by u_{max}^2 and the state constraint problem ($x_2 = 0$). It should be noted that if $x_2 > 0$, machine M_2 can produce at u_{max}^2, the condition

$(\pi_1(a_1)+\pi_3(a_1)+\pi_5(a_1))u_{max}^2-(\pi_1(a_1)+\pi_2(a_1))\tilde{\beta}(a_1)u_1 \leq rd$ is verified, and equation (6.22) takes the following form:

$$\left(\pi_1(a_1)+\pi_2(a_1)\right)\left(1-\tilde{\beta}(a_1)\right)u_{max}^1 \\ +(1-p)(\pi_1(a_1)+\pi_3(a_1)+\pi_5(a_1))u_{max}^2 \geq d \tag{6.24}$$

Otherwise $x_2=0$, machine M_2 cannot produce at u_{max}^2, and equation (6.22) takes the following form:

$$\left(\pi_1(a_1)+\pi_2(a_1)\right)\left(1-\tilde{\beta}(a_1)\right)u_{max}^1 \\ +(1-p)\left(rd+(\pi_1(a_1)+\pi_2(a_1))\tilde{\beta}(a_1)u_1\right) \geq d \tag{6.25}$$

We will now present the manufacturing, recovery (in the context of remediation and remanufacturing activities) and replacement policies. We will also clearly illustrate the joint control policy that optimally controls the hybrid system of interest.

6.4.1 Manufacturing and recovery policies

The optimal production policies of the manufacturing and recovery (remediation and remanufacturing) machines $u_1^*(.,x_2,.)$ and $u_2^*(.,x_2,.)$, illustrated in the figures 6.4a-b, indicate the production rates, for each stock level x_1, for each age a_1, and for a given stock level x_2 (only for illustration and representation purposes in three dimensions, we use $x_2=2$). The production takes place only in modes 1, 2, 3 and 5; hence, there is no production in modes 4 and 6 given that both machines are unavailable (i.e., under repair or replacement for machine M_1, and under repair for machine M_2). The structure of the production rates for both machines in all possible operational modes (i.e., modes 1, 2, 3 and 5) is similar to the one presented in figure 6.4a for the manufacturing machine at mode 1, but with a different level. The production control policy for each mode is an extension of the so-called production threshold form, as in Rivera-Gomez *et al.* (2016). Such an extension is developed in the context of deterioration of a closed loop for reverse logistics with a state constraints problem. We use the boundaries of the optimal production policies presented in figure 6.4b to facilitate their characterizations. The threshold levels denoted by $Z_{11}^{(.)}(x_2)$, $Z_{21}^{(.)}(x_2)$, $Z_{12}^{(.)}(x_2)$, $Z_{23}^{(.)}(x_2)$ and $Z_{25}^{(.)}(x_2)$ define the optimal production rates relative to the stock level x_1 for each age a_1 and for a given stock level x_2. As can be observed in figure 6.4b, because of the aging process and minimal repair activities on the manufacturing machine M_1, the threshold levels of machines M_1 and M_2 in each mode increase progressively when the deterioration effect becomes significant. So, the required number of parts is determined in the serviceable inventory to provide the necessary protection against backlogs caused by the deterioration effect, in order to meet customer demand.

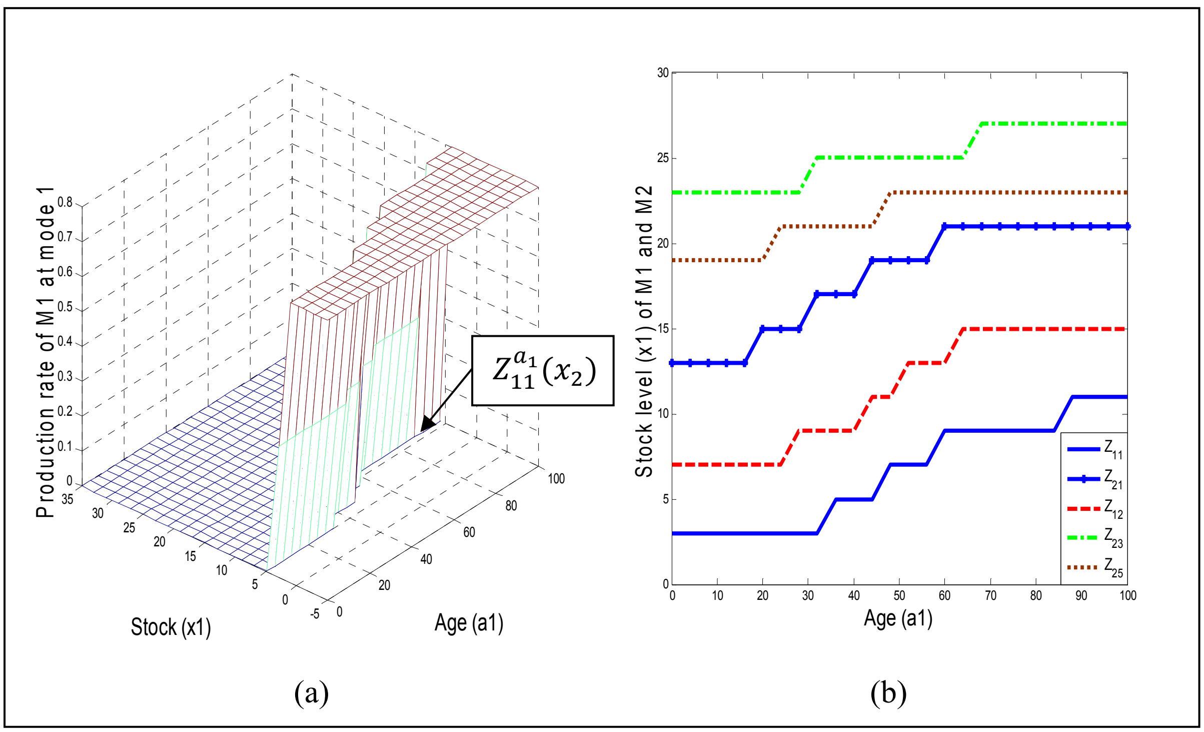

Figure 6.4 Manufacturing and recovery policies in different modes

Based on the results presented in figures 6.4a-b, the optimal production policies in a mode α can be defined as follows:

For the manufacturing machine M_1 ($\alpha \in \{1, 2\}$):

$$u_1^*(\alpha, x_1, x_2, a_1) =$$

$$\begin{cases} u_{max}^1 & \text{if } x_1(t) < Z_{1\alpha}^{a_1}(x_2) \\ \dfrac{d}{\left(1-\mu_\beta(a_1)\right)} & \text{if } \left(x_1(t) > Z_{2\alpha}^{a_1}(x_2) \text{ and } (f_{1\alpha} = 0 \,\&\, f_{2\alpha} \neq 0)\right) \\ \dfrac{(u_{max}^2 - rd)}{\mu_\beta(a_1)} & \text{if } \left(\left(x_1(t) < Z_{2\alpha}^{a_1}(x_2) \,\&\, x_2(t) > 0\right) \text{ and } (f_{1\alpha} \neq 0 \,\&\, f_{2\alpha} = 0)\right) \\ \dfrac{(d-(1-p)rd)}{\left(1-p\mu_\beta(a_1)\right)} & \text{if } \left(x_1(t) = Z_{1\alpha}^{a_1}(x_2) \text{ and } (f_{1\alpha} = 0 \,\&\, f_{2\alpha} = 0)\right) \\ \dfrac{(d-(1-p)u_{max}^2)}{\left(1-\mu_\beta(a_1)\right)} & \text{if } \left(\left(x_1(t) < Z_{2\alpha}^{a_1}(x_2) \,\&\, x_2(t) > 0\right) \text{ and } (f_{1\alpha} = 0 \,\&\, f_{2\alpha} \neq 0)\right) \\ 0 & \text{if } x_1(t) > Z_{1\alpha}^{a_1}(x_2) \end{cases} \tag{6.26}$$

For the case of the recovery machine M_2 ($\alpha \in \{1, 3, 5\}$):

$$u_2^*(\alpha, x_1, x_2, a_1) = \tag{6.27}$$

$$\begin{cases} u_{max}^2 & \text{if } \left(x_1(t) < Z_{2\alpha}^{a_1}(x_2) \,\&\, x_2(t) > 0\right) \\ \dfrac{\left(\mu_\beta(a_1)d + (1-\mu_\beta(a_1))rd\right)}{\left(1-p\mu_\beta(a_1)\right)} & \text{if } \left(x_1(t) = Z_{2\alpha}^{a_1}(x_2) \text{ and } (f_{1\alpha} = 0 \,\&\, f_{2\alpha} = 0)\right) \\ rd & \text{if } \left(x_1(t) > Z_{1\alpha}^{a_1}(x_2) \text{ and } (f_{1\alpha} \neq 0 \,\&\, f_{2\alpha} = 0)\right) \\ 0 & \text{if } x_1(t) > Z_{2\alpha}^{a_1}(x_2) \end{cases}$$

6.4.2 Manufacturing replacement policies

The obtained optimal replacement policies of the manufacturing machine $w_1^*(.,x_2,.)$ and $w_2^*(.,x_2,.)$ in modes 1 and 2 are presented in figures 6.5a-b for each stock level x_1, for each age a_1, and in the case of a given stock level x_2 (i.e., $x_2 = 2$). They show that when the deterioration of the manufacturing machine reaches a certain age level, the production capacity of the manufacturing and recovery machines is so diminished that the replacement activity is needed. This activity allows the elimination of the effects of deterioration and restores the parameters of the hybrid system to initial conditions. These parameters are the age, failure rate and defective rate, for the manufacturing machine, and the failure rate, for recovery machine. The pattern of the optimal replacement policies divides the plane (x_1, a_1) at given x_2 into two zones, such that the replacements are assumed to be continuously allowed for all possible values of the decision variables $w_1(.)$ and $w_2(.)$ within $[w_{min}^1, w_{max}^1]$ and $[w_{min}^2, w_{max}^2]$, respectively. Mainly according to the age of the manufacturing machine, the structure of the replacement policy switches between its minimum and maximum values (bang-bang control policy).

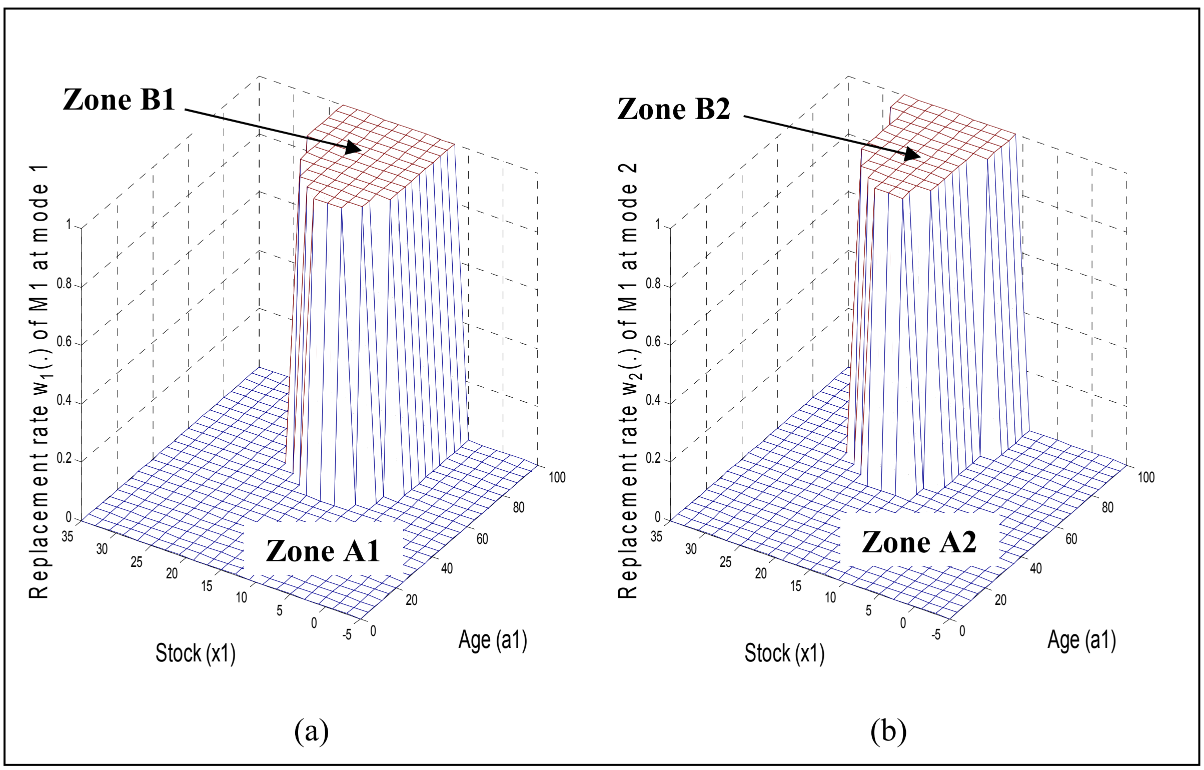

Figure 6.5 Manufacturing replacement policies with two different recovery machine states

These two zones are described as follows:

1. In zones A1 and A2, the replacement is not recommended. There are several reasons for this. The manufacturing machine that affects the entire hybrid system is still new and the

deterioration effect is low, the stock level x_1 is very high or we do not have the necessary stock level x_1 to support the backlog during periods of non-production. So, it is more profitable to continue production with the same machine. Hence, the decision variables $w_1(.)$ and $w_2(.)$ are set to their minimum values (no replacement).

2. In zones B1 and B2, it is recommended to replace the manufacturing machine, given that its age and the stock x_1 have reached such a level that the deterioration effect justifies the cost of performing this type of intervention. In this case, the decision variables $w_1(.)$ and $w_2(.)$ are set to their maximum values.

The replacement actions should be performed at the rate $w_1^*(.)$ with the operational state of machine M_2, or at the rate $w_2^*(.)$ with the failed state of machine M_2, given by the following equations:

$$w_1^*(1, x_1, x_2, a_1) = \begin{cases} w_1^{max} & \text{if } \big(x_1(t), x_2(t), a_1(t)\big) \in \text{zone B1} \\ w_1^{min} & \text{otherwise} \end{cases} \tag{6.28}$$

$$w_2^*(2, x_1, x_2, a_1) = \begin{cases} w_2^{max} & \text{if } \big(x_1(t), x_2(t), a_1(t)\big) \in \text{zone B2} \\ w_2^{max} & \text{otherwise} \end{cases} \tag{6.29}$$

To complement the results analysis of the numerical example, we illustrate the joint control policy using the interaction between production and replacement boundaries.

6.4.3 Joint control policy

In this section, we illustrate the obtained joint control policy for the hybrid system in figure 6.6. Since such policies are inter-related, we define simultaneously the boundaries of the optimal manufacturing, recovery and replacement strategies presented in figures 6.4-6.5 to facilitate their analysis. Each boundary of a policy, or its threshold level, delimits the optimal zone. As can be seen in figure 6.6, we will calculate, for all modes given by the proposed approach, the common production threshold of $(Z_{11}(.), Z_{21}(.), Z_{12}(.), Z_{23}(.), Z_{25}(.))$, denoted $Z(.)$, by using the weighted average method. The following formula can be used to find the threshold $Z(.)$:

$$Z(.) = \pi_1(.)(p_1 Z_{11}(.) + p_2 Z_{21}(.)) + \pi_2(.) Z_{12}(.) + \pi_3(.) Z_{23}(.) + \pi_5(.) Z_{25}(.) \qquad (6.30)$$

where $\pi_i(.)$, $i = 1, \dots, 5$ is the limiting probability of the hybrid system at mode i, and with the weights $p_1 = \frac{u_{max}^1}{(u_{max}^1 + u_{max}^2)}$ and $p_2 = \frac{u_{max}^2}{(u_{max}^1 + u_{max}^2)}$.

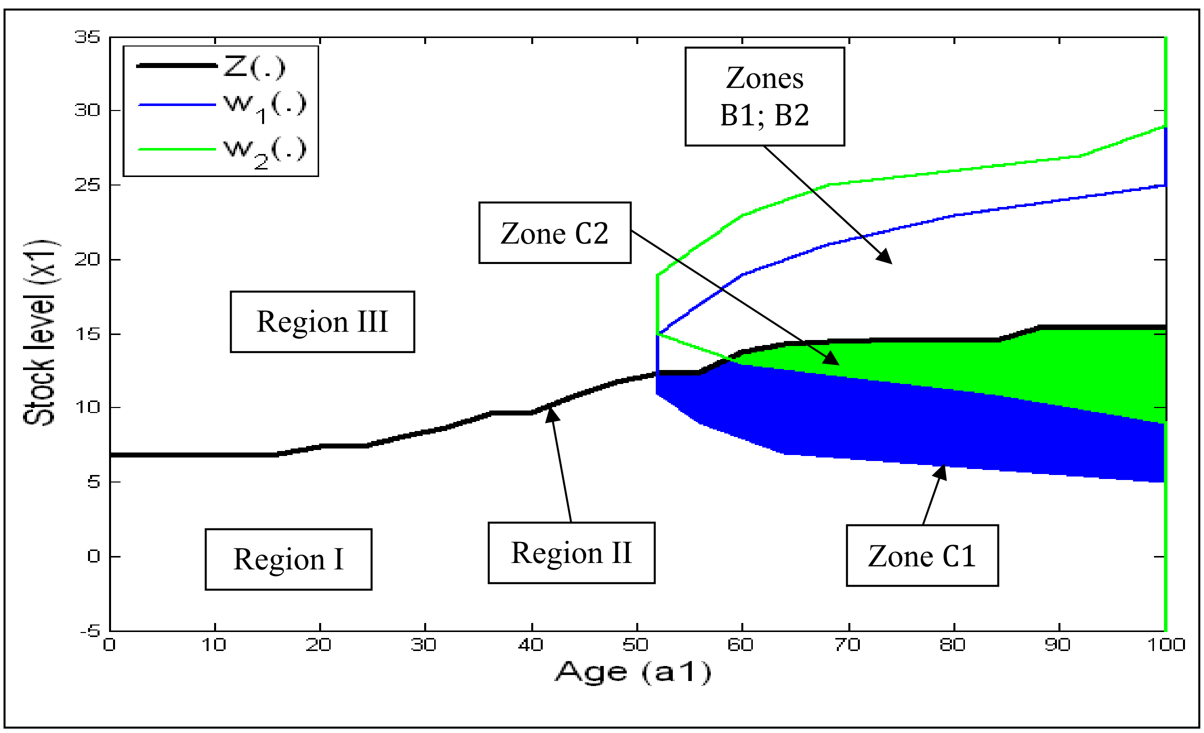

Figure 6.6 Joint control policy

From the obtained result (figure 6.6), the computational domain is divided into three regions where the optimal production control policy is described by the following set of rules:

1. In region I, each machine's production rate is set to its maximum value when the current stock level is under the threshold value $Z(.)$. In addition, machine M_2 is only able to produce with u_{max}^2 when the stock level $x_2(t)$ is greater than 0.

2. In region II, the production rate of the manufacturing machine is set to $\frac{(d-(1-p)u_2)}{(1-\mu_\beta(a_1))}$ and the production rate of recovery machine is set to $\frac{\left(\mu_\beta(a_1)d+\left(1-\mu_\beta(a_1)\right)rd\right)}{\left(1-p\mu_\beta(a_1)\right)}$ when the current stock level of serviceable products is equal to $Z(.)$. Thus, to satisfy the demand rate d, we must take into account the defective rate and the proportion of defective remedied items in the decisions.
3. In region III, the production rates of both machines are set to zero when the current stock level of serviceable products is greater than $Z(.)$.

Thus, the production control policies $u_1^*(.)$ and $u_2^*(.)$ for manufacturing and recovery are given by:

$$u_1^*(.) = \begin{cases} u_{max}^1 & \text{if } x_1(t) < Z(.) \\ \dfrac{\left(d-(1-p)u_2^*(.)\right)}{\left(1-\mu_\beta(a_1)\right)} & \text{if } x_1(t) = Z(.) \\ 0 & \text{if } x_1(t) > Z(.) \end{cases} \tag{6.31}$$

and

$$u_2^*(.) = \begin{cases} u_{max}^2 & \text{if } x_1(t) < Z(.)\ \&\ x_2(t) > 0 \\ \dfrac{\left(\mu_\beta(a_1)d+(1-\mu_\beta(a_1))rd\right)}{\left(1-p\mu_\beta(a_1)\right)} & \text{if } x_1(t) = Z(.) \\ 0 & \text{if } x_1(t) > Z(.) \end{cases} \tag{6.32}$$

As we can see from figure 6.6, since the sock level of the manufacturing and recovery machines is always limited by the production threshold $Z(.)$, only a section of the replacement zone is active. This implies a reduction in zones B1 and B2, defining the feasible replacement zones C1 and C2, where the hybrid production system resides. The main observation from figure 6.6 is that the feasible zone C2 is smaller than the feasible zone C1. This is due to the fact that in zone C1, both machines are operational, while in zone C2, the manufacturing machine is operational, and the recovery machine is under repair. In this case, the replacement of the manufacturing machine will therefore be delayed in zone C2, because we do not have the necessary stock x_1 to further recommend the replacement when the recovery machine is also unavailable. Furthermore, from a practical perspective, the replacement process is much more expensive than repairs in terms of time and cost. Hence, we will concentrate on the feasible zone C1, which is the more dominant, such that the zone C2 is included in zone C1 ($\text{C1} \cup \text{C2} = \text{C1}$). With the obtained joint control policy, the property of the feasible zone determines the optimal stock and age levels (x_1^*, a_1^*) when the manufacturing machine should be replaced in order to satisfy the product demand without delay, while keeping operating costs as low as possible, to encourage profitability.

To ascertain the validity of the obtained results, it will be shown that the structure of the obtained joint control policy moves as predicted when the system parameters are varied in a given direction. This is described in the next section through a sensitivity analysis.

6.5 Sensitivity analysis

To understand the effects of the system parameter variations on the control policies, sensitivity analyses are needed. For illustration purposes, we first concentrate our efforts on the backlog cost and the serviceable inventory cost. Next, we analyze the effect of other parameters as well, including: the diffusion coefficient and the drift coefficient for the random portion of defective rate, the proportion of defective items after remediation, and the proportion of returned EOLP for remanufacturing. In the following analysis, we will illustrate the control policy for the common production of manufacturing and recovery threshold $Z(.)$ characterized by the replacement zone of manufacturing machine C1.

6.5.1 Effect of backlog cost variation

Analyzing the results presented in figure 6.7 with two different backlog cost values, $c_1^- = \$150$ and $\$350$/missing product/time unit, we notice that the common production threshold for manufacturing and recovery machines M_1 and M_2 increases from 14 to 18.25 serviceable parts (i.e., at age $a = 100$) as the backlog cost increases, mainly due to the fact that the system must react to keep more stock for protection against product shortages. To reach this level, the machines M_1 and M_2 must produce more parts at their maximum production rates. Given that the deterioration effect depends on the production rate of machine M_1, this machine should deteriorate more rapidly. This deterioration severely affects the failure rate of machine M_2 due to the increase in the number of defective parts produced by M_1, and which will be remedied by M_2. It means that the replacement of machine M_1 needs to be more recommended when the backlog cost increases. Consequently, its replacement zone C1 moves to the left, and covers a larger area on the plane (x_1, a_1). A sensitivity analysis was conducted with respect to the serviceable inventory cost (c_1^+), and we observed that the effect of the serviceable inventory cost (c_1^+) on the replacement zone C1 is the inverse of that of the backlog cost (c_1^-).

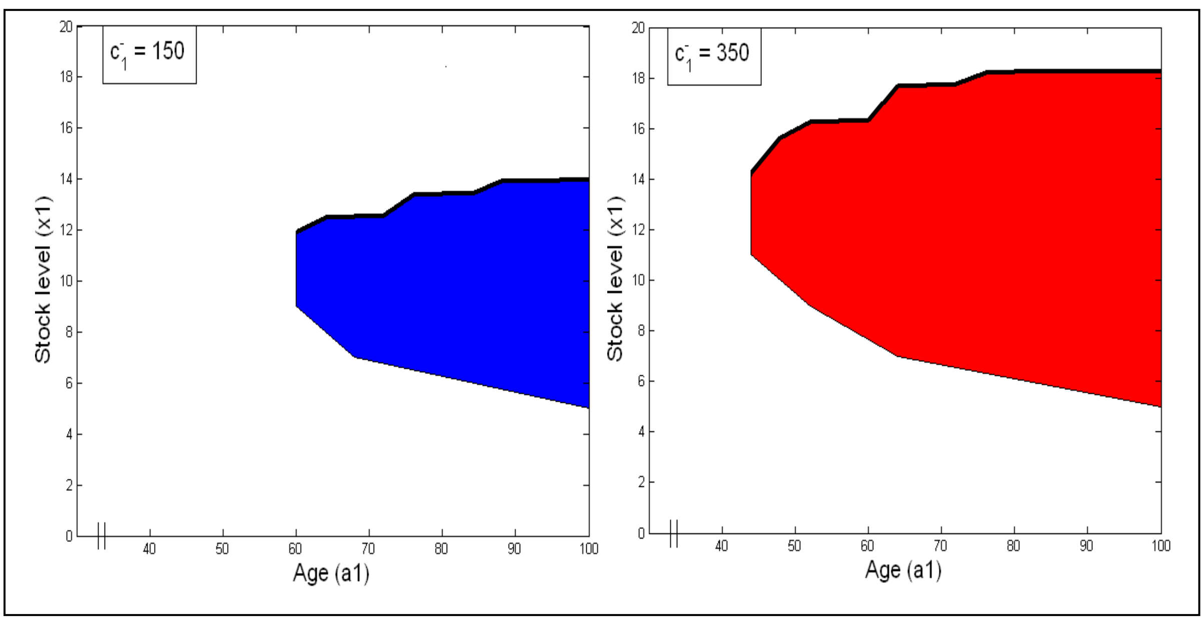

Figure 6.7 Influence of the backlog cost c_1^- on zone C1

6.5.2 Effect of the diffusion coefficient variation

The effect of variation of the diffusion coefficient $\sigma(.)$ of equation (6.8) is directly reflected in the standard deviation $\sigma_\beta(.)$ of the defective rate. This scenario analyzes the replacement zone C1 according to two different diffusion coefficient values, $0.5\sigma(.)$ and $1.5\sigma(.)$. As presented in figure 6.8, when the diffusion coefficient $\sigma(.)$ increases, the disturbances are more severe, and therefore, the risk of shortages is greater. To avoid such a situation, machines M_1 and M_2 must spend much more time working simultaneously at their maximum production rates, causing an increase in their common production threshold. Thus, machine M_1 deteriorates more rapidly, the failure rate of machine M_2 increases, and replacing machine M_1 earlier is recommended in order to restore the hybrid system to initial conditions and continue to ensure demand is met. As a result, the feasible replacement zone C1 increases considerably and moves to the left, covering a larger area on the plane (x_1, a_1).

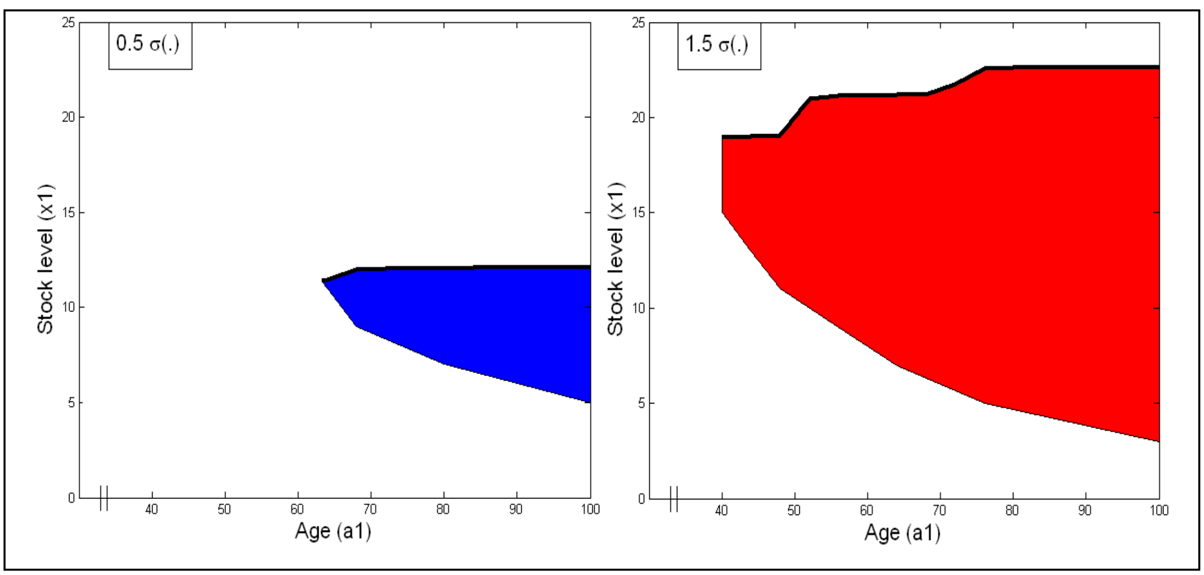

Figure 6.8 Influence of the diffusion coefficient $\sigma(.)$ on zone C1

A sensitivity analysis was also conducted to examine the variation of the drift coefficient b. We observed that the effect of the drift coefficient b on the replacement zone C1 is the opposite of that of the diffusion coefficient $\sigma(.)$, given that the perturbations are less severe by increasing b and more severe by increasing $\sigma(.)$.

6.5.3 Effect of variation of the proportion of defective remedied items

We will now highlight the replacement zone C1 under the effect of variation of the proportion of defective items after remediation. Figure 6.9 illustrates this zone with the values $p = 0.1$ and 0.6. Given that the remediation performed by machine M_2 is considered imperfect, whenever the proportion of defective remedied items after inspection increases, the number of perfect remedied products stored in the serviceable inventory should decrease. This will clearly increase the demand rate by an amount of $\frac{1}{(1-p)}$. Consequently, the common production threshold $Z(.)$ will increase, and therefore, machines M_1 and M_2 must produce more parts at their maximum production rates for an extended period. Nonetheless, this leads to a considerable increase in the effect of deterioration of the manufacturing machine. This effect will cause more failures and defectives, and thus the replacement of machine M_1 will be more recommended when the proportion of defective remedied items p increases. Consequently, the feasible replacement zone C1 moves to the left, covering a larger area on the plane (x_1, a_1).

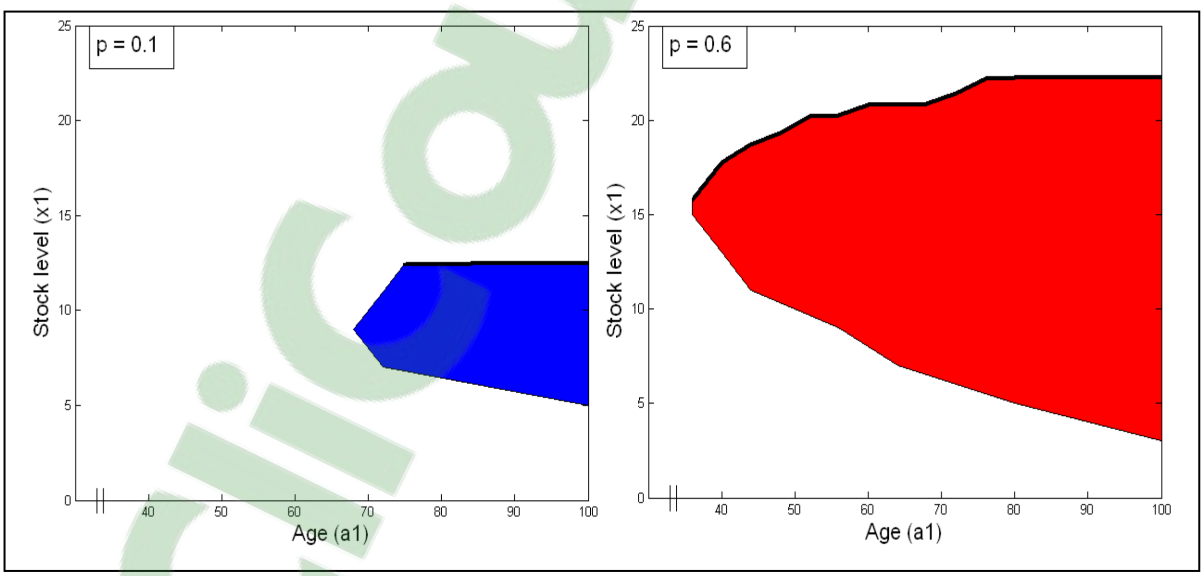

Figure 6.9 Influence of the proportion of defective remedied items p on zone C1

6.5.4 Effect of the proportion of return of EOLP variation

As a matter of interest, we complement the analysis with the variation of the proportion of return of EOLP r used by the remanufacturing machine. We analyze two different cases with values $r = 25\%$ and 75% in the case of a given stock level x_2 (i.e., $x_2 = 2$) as presented in figure 6.10. When the proportion of return r increases, we have a system in which the overall maximum production rate of machines M_1 and M_2 increases, and this causes a decrease in the common production threshold $Z(.)$ of machines M_1 and M_2. Consequently, machines M_1 and M_2 must produce parts for a shorter period at their maximum production rates to reach this threshold level. Given the aging process of machine M_1 as a result of the number of parts it produces, it deteriorates more slowly, and has a smaller effect on machine M_2 when the latter engages more in the remanufacturing of EOLP returns. In this situation, the replacement option for machine M_1 should be required for later. Hence, the feasible replacement zone C1 moves to the right by covering a lower area on the plane (x_1, a_1) when the EOLP returns increase.

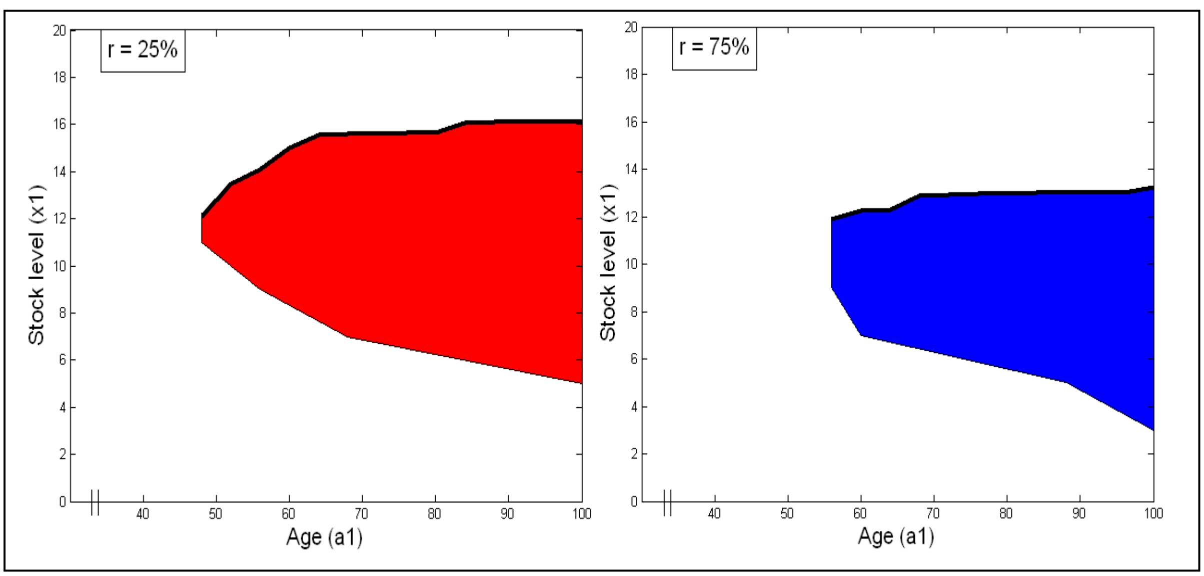

Figure 6.10 Variation of the proportion of return r on zone C1

Through this sensitivity analysis, we have clearly shown that the results obtained make sense, and that the structure of the obtained control policies is always maintained. The following

Table 6.2 provides a summary of the effects of variation on the optimal control policy parameters.

Table 6.2 Summary of the sensitivity analysis of the joint control policy

Parameters	Threshold $Z(.)$	Zone C1
c_1^- (serviceable backlog cost) ↑	↑ increases	↑ increases, ← moves to left
c_1^+ (serviceable inventory cost) ↑	↓ decreases	↓ decreases, → moves to right
$\sigma(.)$ (quality diffusion coefficient) ↑	↑ increases	↑ increases, ← moves to left
b (quality drift coefficient) ↑	↓ decreases	↓ decreases, → moves to right
p (proportion of defective remedied items) ↑	↑ increases	↑ increases, ← moves to left
r (proportion of return of EOLP) ↑	↓ decreases	↓ decreases, → moves to right

The boundaries of the threshold $Z(.)$ and replacement zone C1 move in the appropriate directions with respect to the variation of the parameters. The direction ↑ illustrates an increase, ↓ illustrates a decrease, → illustrates a movement to the right side of the grid, and the direction ← illustrates a movement to the left side of the grid.

6.6 Joint control policy implementation

To facilitate the implementation of the proposed joint control policy in practice, we illustrate a logic chart in figure 6.11. This illustration shows the actions that should be taken by the manager to control production and replacement processes. Managerial implications in business practice for the obtained policies require complete information about the state of the hybrid production system to implement the results obtained. This is information about the inventory positions, the age of the manufacturing machine, the failure and defective rates of the manufacturing machine with its age, and the failure rate of the recovery machine with defective parts. It should be continuously observed and updated to make any necessary adjustments in the hybrid closed-loop system.

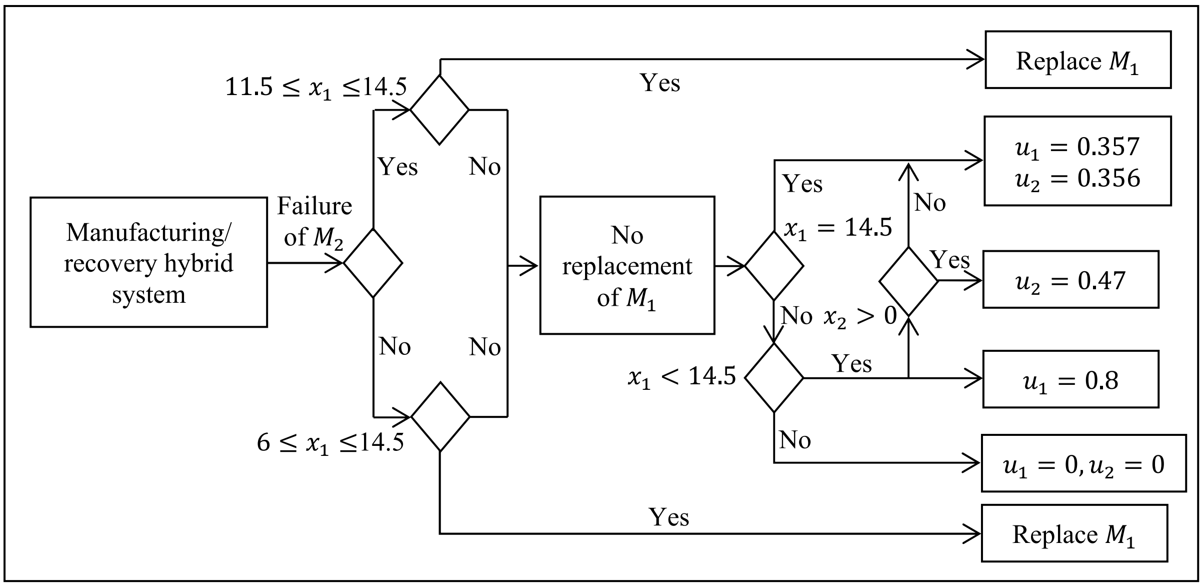

Figure 6.11 Implementation of the logic chart

We give an example in figure 6.11 to illustrate the joint control policy, in the case of a given stock level x_2 (i.e., $x_2 = 2$), and for three different points (P_1 to P_3) located on the grid (x_1, a_1) in figure 6.12. For instance, between the points P_1 and P_3 in zone C_1, the current stock level is within the interval $6 \leq x_1 \leq 14.5$, and the age is $a_1 = 80$, the common production threshold is $Z(.) = 14.5$, and the replacement of machine M_1 will be conducted at $w_1(.) = 1$. Between the points P_2 and P_3 in zone C_2, the current stock level is within the interval $11.5 \leq x_1 \leq 14.5$, the common production threshold is also $Z(.) = 14.5$, and the replacement of machine M_1 will be conducted at $w_2(.) = 1$. Otherwise, there is no replacement, and $w_1(.) = w_2(.) = 10^{-5}$. Hence, the production policy will be defined by three control rules, where the production rates (u_1, u_2) are set to different values depending on the current stock level x_1 with respect to the threshold value $Z(.)$, and the number of parts available in recovery inventory x_2. More specially, these three rules given by equations (6.31)-(6.32) state that:

1. If the current stock level is under the threshold value 14.5, then each production rate is set to its maximum value ($u_1 = 0.8, u_2 = 0.47$).

2. Once the current stock level is equal to the threshold value 14.5, both production rates are set to intermediate values (with an average rate of defectives $\mu_\beta(a_1 = 80) = 0.297$) to reach the demand rate ($u_1 = 0.356, u_2 = 0.357$).
3. If the current stock level is larger than the threshold value 14.5, then each production rate is set to zero ($u_1 = 0, u_2 = 0$).

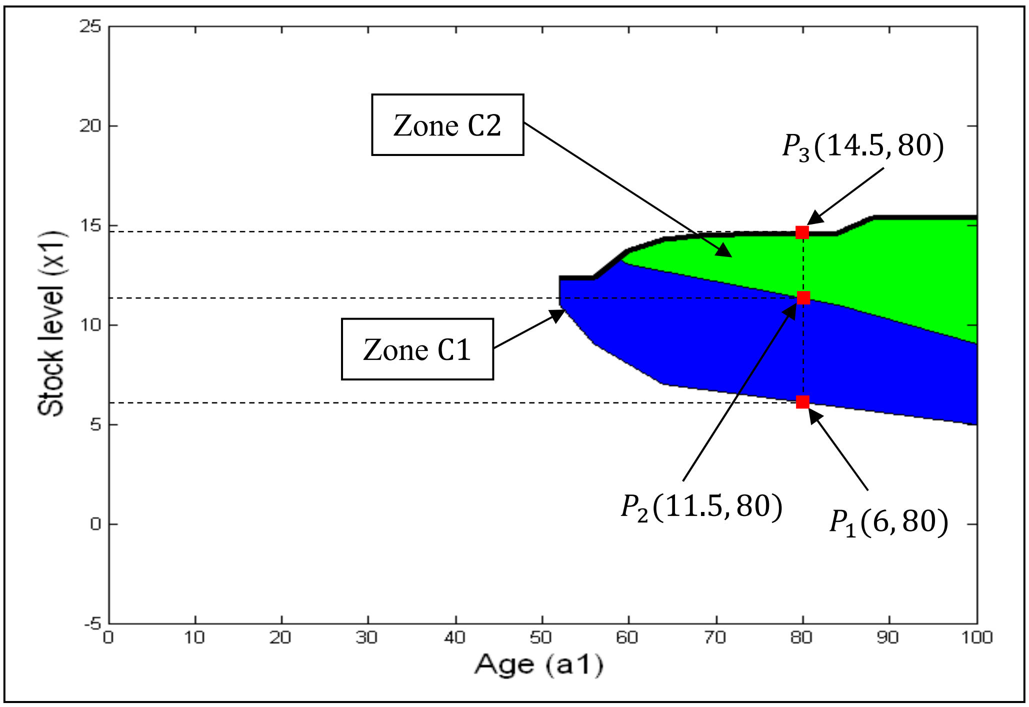

Figure 6.12 Implementation of the joint control policy

In light of the present discussion, and based on the illustration in figure 6.11, our policy proposes two steps: one for deciding on the common production threshold to select the optimal production rates for manufacturing and recovery machines, and the other to indicate the appropriate moment to adequately perform a replacement on the manufacturing machine. The numerical computation to solve the second-order optimality conditions should be done off-line. However, the optimization procedure in terms of CPU time and memory usage is relatively high due to the large scale of numerical scheme dimensions.

6.7 Conclusion

In this paper, we develop a stochastic optimization production planning and replacement problem model for an unreliable deteriorating hybrid closed-loop system with random quality parts produced. We extend the concept of deterioration with the age to create a connection with the rate of defectives and quality failure. We formulate the stochastic optimal control problem using dynamic programming. Optimality conditions are developed in the form of a second-order approximation of Hamilton-Jacobi-Bellman equations in terms of a directional derivative (HJBDD) in order to capture the random variation of the quality issues present, while dealing with state constraints. Despite the complexity of the optimality conditions, the problem remains tractable, and numerical methods are used to obtain the optimal production and replacement policies. A numerical example is considered to illustrate the proposed approach, and a sensitivity analysis is conducted to confirm the structure of the obtained control policies. By implementing such policies, companies will be able to adjust their production and replacement planning considering random quality deterioration such that the total incurred cost can also be minimized over an infinite planning horizon. To probe into other industrial problems with more realistic perspectives, further issues may be studied in the context where the EOLP returns contain defective items.

CONCLUSION GÉNÉRALE

La présente thèse a apporté une contribution scientifique importante en proposant des modèles mathématiques de commande optimale stochastique qui intègrent les aspects reliés aux processus de demande, de retour et de qualité aléatoires dans un contexte de systèmes de production hybride en boucle fermée, avec des machines non fiables et sujettes aux détériorations.

Dans le chapitre 2, nous avons traité un problème de planification de la production et du contrôle de remplacement d'un système manufacturier constitué d'une machine produisant un type de pièce. Ce système était soumis à des pannes et réparations aléatoires. L'effet de la détérioration sur la machine causé par le processus de vieillissement et de réparation minimale affectait progressivement sa disponibilité. Le processus de vieillissement se traduisait par l'âge que prend la machine dépendamment de la vitesse de production. Puisque l'âge de la machine n'était pas restauré à zéro suite aux réparations minimales, un processus de décision semi-Markovien a été utilisé pour décrire sa dynamique. Le problème d'optimisation a été résolu par des méthodes numériques à travers des équations d'Hamilton-Jacobi-Bellman (HJB) afin d'obtenir une loi de commande qui donne les politiques optimales de production et de remplacement. Pour illustrer l'utilité de nos résultats, une analyse de sensibilité a été effectuée pour valider la structure des politiques de commande obtenue.

Dans le chapitre 3, nous avons intégré dans la modélisation mathématique les aspects aléatoires de la demande des clients et de la qualité des pièces défectueuses dans le système manufacturier étudié au chapitre 2. L'effet combiné du processus de vieillissement de la machine et des réparations minimales a été observé dans la disponibilité de la machine et dans la qualité des pièces produites, où le taux de défaillance et le taux de rejets dépendaient de l'âge de la machine. Un problème de planification de la production et de contrôle de remplacement dans un contexte de détérioration et de présence des sources d'incertitude a été considéré. L'intégration des comportements aléatoires de la demande et de la qualité nous a amené à proposer une nouvelle approche de modélisation par l'extension des équations d'HJB classique de premier ordre à la forme Itô de second ordre. Cette forme de second ordre nous a permis de capturer les effets de la variabilité des phénomènes aléatoires. Des méthodes numériques ont été utilisées pour obtenir les politiques optimales de production et de remplacement. Les résultats obtenus ont montré que les politiques de commande sont de type à seuil critique. Une analyse de sensibilité a été faite pour illustrer l'utilité de l'approche proposée.

Au chapitre 4, nous avons ajouté à la machine de fabrication de base une autre machine montée en logistique inverse afin de traiter les activités de refabrication des produits retournés en fin de leur cycle de vie. Les machines étaient non-identiques, non fiables et produisaient un seul type de produit. Les phénomènes aléatoires sont les pannes et réparations des machines, la demande du client et le retour en fin de vie. Les variables de décision étaient les taux de production de la machine de fabrication et de la machine de refabrication. Le système a été modélisé par les chaînes de Markov homogènes et la solution du problème a été obtenue par la résolution numérique des équations d'HJB de second ordre. Un exemple numérique a été fourni pour démontrer que nous pouvons appliquer avec succès les méthodes numériques de résolution des équations d'HJB de premier ordre pour résoudre les équations d'HJB de second ordre.

Dans le chapitre 5, nous avons établi qu'il était possible d'intégrer la détérioration des machines dans le cadre d'un système hybride composé de deux (2) machines fabrication/refabrication en boucle fermée. Les machines étaient soumises à des pannes et réparations aléatoires et produisant un seul type de produit. Cette détérioration affectait la machine de refabrication à cause de la nature hétérogène des produits retournés en impliquant une réparation imparfaite sur cette machine. Cette réparation imparfaite était représentée par un temps de réparation qui devenait plus long au fur et à mesure que le nombre de pannes augmentait. À long terme, le système hybride ne pouvait pas satisfaire la demande des clients, et une option alternative était de remplacer la machine de refabrication afin de restaurer ses paramètres aux conditions initiales. Dans ce contexte, un problème de planification de la production et de remplacement a été formulé. L'objectif était de déterminer les stratégies optimales pour contrôler les taux de production des deux (2) machines et le taux de remplacement de la machine de refabrication. Compte tenu de la prise en compte de l'historique de réparation, le système était modélisé par un processus semi-Markovien. Pour résoudre ce problème, nous avons proposé une nouvelle approche de modélisation mathématique qui a permis de traiter une classe de problèmes semi-Markoviens en tant que Markoviens par l'extension de l'espace d'état. Nous avons démontré que, malgré cette augmentation de l'espace d'état et la dimension du schéma numérique, la solution du problème pouvait être obtenue numériquement. Nous avons montré que la structure de la politique de commande conjointe est caractérisée par des multiples seuils critiques et que la coordination entre les opérations de fabrication et de refabrication était bien justifiée. Nous avons illustré l'approche proposée en utilisant un exemple numérique et une analyse de sensibilité afin d'examiner son implication dans la pratique.

Dans le chapitre 6 de cette thèse, nous avons étendu le modèle de détérioration pour tenir compte de l'effet simultané de la détérioration sur les deux (2) machines et son impact sur les politiques de commande dans un contexte de logistique inverse. L'effet de détérioration sur la première machine de fabrication causé par les processus de vieillissement et de réparation minimale affectait de manière aléatoire la disponibilité de la machine et la qualité des pièces produites. Son effet sur la deuxième machine était généré par le traitement du flux des pièces défectueuses provenant de la machine de fabrication. La deuxième machine traitait les activités de rectification des produits défectueux de la première machine et de refabrication des produits récupérés en fin de vie. Afin de continuer à satisfaire la demande des clients à long terme, le remplacement de la machine de fabrication était nécessaire pour la restauration du système hybride aux conditions initiales. L'objectif était de déterminer des stratégies optimales de fabrication, de rectification et de refabrication, ainsi que la stratégie de remplacement tout en minimisant le coût total sur un horizon de planification infini. Avec cet effet de détérioration, le système a été modélisé une fois de plus par un processus semi-Markovien pour tenir compte de la notion de mémoire dans la prise de décisions. Nous avons formulé le problème de commande optimale stochastique en tant que problème de programmation dynamique stochastique. Les conditions d'optimum ont été développées sous la forme d'une approximation des équations d'Hamilton-Jacobi-Bellman en termes de dérivée directionnelle (HJBDD) de second ordre en raison du caractère aléatoire qui résidait au niveau de la qualité des pièces défectueuses, mais aussi parce qu'il s'agit d'un problème avec contrainte sur l'état. Le problème a été résolu par des méthodes numériques. La structure de la politique de commande conjointe obtenue était de type zones critiques. Une analyse de sensibilité a été menée pour confirmer la robustesse et l'efficacité de la structure des politiques obtenues.

Les travaux de recherche menés dans cette thèse ont proposé plusieurs modèles mathématiques pour surmonter la complexité du problème de commande optimale stochastique des systèmes de production avec logistique en boucle fermée. Les modèles développés permettent de fournir des lois de commande plus appropriées à la réalité en production manufacturière. Ils permettent ainsi de déterminer les stratégies de fabrication, de rectification, de refabrication et de remplacement pour gérer efficacement les produits récupérés du marché, les produits défectueux et les stocks en satisfaisant la demande en permanence. Ce travail a confirmé qu'en intégrant progressivement les perturbations et les détériorations aléatoires dans un système manufacturier, les performances en termes de productivité et de fiabilité sont améliorées.

TRAVAUX FUTURS

Ces contributions constituent une base solide pouvant s'ouvrir sur d'autres voies de recherche. Les modèles utilisés peuvent être étendus à des systèmes plus complexes aussi bien du point de vue structure que de la taille.

- Utiliser la structure des politiques de commande obtenue dans cette thèse pour étendre le champ d'application à d'autres classes de problèmes d'optimisation en systèmes manufacturiers hybrides plus larges, impliquant plusieurs machines de fabrication, plusieurs machines de refabrication/rectification ou plusieurs types de produits. La simulation pourrait être intégrée à une approche d'optimisation pour résoudre ces types de problèmes. Cependant, le recours au calcul parallèle est une voie à considérer pour gagner en performance dans le but de réduire le temps nécessaire à la résolution de problèmes.

- Autre idée à développer, explorer les perturbations aléatoires au niveau de la demande des clients combinés de processus de type-diffusion et de processus de Poisson (Bensoussan *et al.,* 2005). Dans le cadre de cette thèse, nous avons limité son application à des cas où uniquement des processus de type-diffusion sont présents. Cette extension doit donc tenir compte à la fois des sauts et des variations aléatoires continues de la demande. Elle permet de rendre les lois de commande plus efficaces et qui se rapprochent le plus de la réalité en production manufacturière. Par ailleurs, nous pensons que la théorie du contrôle implusionnel peut être une alternative intéressante pour passer d'un problème de la programmation dynamique à un système d'inéquations quasi-variationnelles afin de pouvoir obtenir la politique optimale.

- Il serait aussi intéressant de généraliser notre approche de modélisation mathématique d'un système manufacturier hybride en boucle fermée en présence des problèmes de détérioration de plusieurs machines, ainsi que l'effet combiné de la prise en compte de la demande et du retour aléatoires. Par ailleurs, la modélisation de ces aspects rend le problème d'optimisation très complexe en raison de la complexité en termes du caractère aléatoire et de la dimension du problème. Néanmoins, pour obtenir les politiques de commande optimale, une étude plus approfondie que celle employée dans cette thèse serait requise. Dans ce contexte, un dilemme subsiste entre l'approximation du problème par un ensemble de sous-problèmes plus faciles à résoudre et la qualité de la solution obtenue.

- Un autre élément motivant se situe par rapport à l'intégration de la détérioration liée à l'âge avec les effets du changement technologique à long terme sur les décisions de remplacement (Hartman et Tan, 2014). Sous le progrès technologique, les gestionnaires peuvent être encouragés à remplacer les machines avant la fin de leur vie économique afin de profiter de la technologie la plus avancée et ainsi bénéficier d'avantages supplémentaires en effectuant les mêmes fonctions à un coût réduit.

- Un dernier élément serait lié à la qualité du retour des produits en fin de vie. Nous pensons que cette piste reste à explorer pour raffiner nos résultats, notamment, la prise en compte des produits récupérables qui respectent les normes pour les activités de refabrication/rectification. Ces produits sont passés par l'étape d'inspection, ils peuvent être défectueux ou de nature hétérogène. L'objectif général de l'utilisation des processus de refabrication/rectification est de redonner une nouvelle vie à ces produits. Ce traitement a pour effet d'augmenter progressivement le taux de panne de la machine. Cette piste nous pousserait alors à proposer un plan d'échantillonnage d'acceptation pour effectuer le contrôle de qualité sur les produits retournés après inspection afin de limiter le processus de détérioration sur la machine qui les traite.

ANNEXE I

APPENDIX 2.A. NOTATION

$c(.)$	production cost function
c_r	repair cost rate of the machine
c_u	production cost rate
c_0	replacement cost of the machine
c^+	inventory cost
c^-	backlog cost
d	demand rate of customers
$G(.)$	instantaneous cost function
$J(.)$	expected discounted cost function
$Q(.)$	transition rate matrix
$q_{\alpha\beta}(.)$	transition rate from mode α to mode β
u_{max}	maximum production rate of the manufacturing system
$u(t)$	production rate of the manufacturing system at time t
$x(t)$	stock level at time t
$v(.)$	value function
$\xi(t)$	stochastic process of the system at time t
ρ	discount rate

ANNEXE II

APPENDIX 2.B. OPTIMALITY CONDITIONS AND NUMERICAL APPROACH

The value function $v(\alpha, x, a)$ defined in equation (2.18) represents the value of the total cost function described by equation (2.16), and provides the viscosity solution that satisfies the HJB equations (also called optimality conditions). These conditions are necessary and sufficient for an optimum. If we can solve the HJB equations to obtain the value function $v(\alpha, x, a)$, then we can derive the optimal controls (u, w) that achieve the minimum cost. Bellman's optimality principle requires that the optimal decision in a step be independent of the choice of optimal decisions of the previous steps. We assume that it is possible to determine the optimal trajectory in the time interval $[t, \infty]$. If $v(.,t)$ represents the optimal cost-to-go function at time t, then equation (2.18) takes the form of equation-A (II-1).

We apply the principle of dynamic programming on the optimal control problem when the system is in the $\alpha(t), x(t)$ and $a(t)$ states at time t. We affirm that if the control is optimal in the time interval $[t, \infty]$, with the initial conditions $\alpha(t), x(t)$ and $a(t)$, then it is also optimal in the time interval $[t + \delta t, \infty]$, with initial conditions $\alpha(t + \delta t), x(t + \delta t)$ and $a(t + \delta t)$ at time $t + \delta t > t$. The value function $v(.)$ between t and $t + \delta t$ is replaced by its expression from equation-A (II-2). The integral in the interval $[t + \delta t, \infty]$ is the value function $v(\alpha(t + \delta t), x(t + \delta t), a(t + \delta t), t + \delta t)$, the discount factor is transformed as $e^{-\rho(s-t)}|s = t + \delta t) = e^{-\rho \delta t}$. In this case, the value function $v(.)$ can be represented by equation-A (II-3). Since $G(.)$ is a continuous function and treated as constant in the interval $t \leq s \leq t + \delta t$, the discount factor over δt is $e^{-\rho \delta t} = 1 - \rho \delta t + o(\delta t)$ and its integral in the time interval $t \leq s \leq t + \delta t$ is given by equation-A (II-4).

Assuming that $v(.)$ is differentiable, we can apply the Taylor series expansion and using the conditional expectation operation $\tilde{E}$ (e.g. for any function $H(\alpha), \tilde{E}\{H(\alpha(t + \delta t))\} = E\{H(\alpha(t + \delta t))|\alpha(t)\}$), we obtain the equation-A (II-5).

Now, we expand the conditional expectation $\tilde{E}\{H(\alpha(t+\delta t))\} = H(\alpha(t)) + \sum_j H(j)\lambda_{j\alpha(t)}\,\delta t + o(\delta t)$, where the term $o(\delta t)$ denotes the rest in Taylor expansion and is negligible as compared to δt. After standard transformations, we find that the function $v(.,t)$ satisfies the equations of dynamic programming so-called Hamilton-Jacobi-Bellman equations, and represented by equation-A (II-6). By considering the stationary regime $v(\alpha, x, a, t) \rightarrow v(\alpha, x, a)$ as $t \rightarrow \infty$, and $\frac{\partial v}{\partial t} \rightarrow 0$, we finally get the HJB equations in the form (II-7).

We now indicate how to implement Kushner's approach to solve the HJB equations (II-7) numerically. This numerical method is based on the finite difference approximations and policy improvement technique, and is described in Kushner and Dupuis (1992) as well as also in Yan and Zhang (1997). It consists in using an approximation for the gradient of the value function based on the numerical scheme of finite differences. Let h_x and h_a denote the length of the finite difference intervals of the state variables x and a. Hence, the value function $v(\alpha, x, a)$ is approximated by $v^h(\alpha, x, a)$ and the partial derivatives of the first-order $\frac{\partial v(\alpha,x,a)}{\partial x}$ and $\frac{\partial v(\alpha,x,a)}{\partial a}$ are described in equations-A (II-8) and (II-9). The HJB equations (II-7) can be written in the form of (II-10), where $q_{\alpha\alpha} = -\sum_{\alpha\neq\beta} q_{\alpha\beta}$ and $\Gamma^h(\alpha)$ is the numerical control grid. The discrete dynamic programming equations (II-10) for the three modes of the system, give the following three equations (II-11) to (II-13).

The next lemma shows that the discrete approximation of the value function $v^h(\alpha, x, a)$ converges to an exact viscosity-type $v(\alpha, x, a)$ solution when the sizes of the grid h_x and h_a associated with state variables x and a tend to zero.

Lemma 2.3. Let $v^h(\alpha, x, a)$ denotes a solution to HJB equations (II-11) to (II-13). Assume that there exist positive constants C and $K > 0$ such that:
If $0 \leq v^h(\alpha, x, a) \leq C(1 + |z|^K)$. Then, $\lim_{h\rightarrow 0} v^h(\alpha, x, a) = v(\alpha, x, a)$.

The proof of *Lemma 2.3* is similar to that of Theorem 3 in Yan and Zhang (1997) when replacing x by $z = (x, a)$ and $\nu(\alpha, x)$ by $\nu(\alpha, z)$; Hence, we shall not repeat it here.

Finally, some boundary conditions must be imposed when the states are at the border of the finite domain G_{xa} in order to solve the HJB equations (II-11) to (II-13) numerically. The effect of this approximation of the boundary conditions on the solution of the original control problem will be negligible (see (Boukas and Haurie, 1990)).

$$\nu(\alpha(t), x(t), a(t), t) = \tag{II-1}$$

$$\min_{\substack{u(s),w(s)\\ t\le s\le\infty}} E\left\{\int_t^{\infty} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), w(s), s)ds \middle| \alpha(t), x(t), a(t)\right\}$$

$$\nu(\alpha(t), x(t), a(t), t) = \tag{II-2}$$

$$\min_{\substack{u(s),w(s)\\ t\le s\le\infty}} E\left\{\begin{matrix}\int_t^{t+\delta t} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), w(s), s)ds \\ +\int_{t+\delta t}^{\infty} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), w(s), s)ds\end{matrix}\middle| \alpha(t), x(t), a(t)\right\}$$

$$\nu(\alpha(t), x(t), a(t), t) = \tag{II-3}$$

$$\min_{\substack{u(s),w(s)\\ t\le s\le t+\delta t}} E\left\{\begin{matrix}\int_t^{t+\delta t} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), w(s), s)ds \\ +e^{-\rho\delta t}\nu(\alpha(t+\delta t), x(t+\delta t), a(t+\delta t), t+\delta t)\end{matrix}\middle| \alpha(t), x(t), a(t)\right\}$$

$$\begin{aligned}\int_t^{t+\delta t} e^{-\rho(s-t)} ds &= \frac{-1}{\rho}\left(e^{-\rho(t+\delta t-t)} - e^{-\rho(t-t)}\right) \\ &= \frac{-1}{\rho}\left(e^{-\rho\delta t} - 1\right) = \frac{-1}{\rho}(1 - \rho\delta t - 1) + o(\delta t) \\ &= \delta t + o(\delta t)\end{aligned} \tag{II-4}$$

$$\nu(\alpha(t),x(t),a(t),t) = \tag{II-5}$$

$$\min_{u(t),w(t)} \tilde{E}\left\{\begin{matrix} G\big(\alpha(t),x(t),a(t),u(t),w(t)\big)\delta t + (1-\rho\delta t) \\ [\nu(\alpha(t+\delta t),x(t),a(t),t) + \nu_t(.)\delta t + \nu_x(.)\delta x + \nu_a(.)\delta a] + o(\delta t) \end{matrix}\right\}$$

$$\rho\nu(\alpha,x,a,t) - \frac{\partial}{\partial t}\nu(\alpha,x,a,t) = \tag{II-6}$$

$$\min_{u(t),w(t)} \left\{\begin{matrix} G(\alpha,x,a,u,w) + \dot{x}\dfrac{\partial}{\partial x}\nu(\alpha,x,a,t) + \dot{a}\dfrac{\partial}{\partial a}\nu(\alpha,x,a,t) \\ +\displaystyle\sum_{\beta} q_{\alpha\beta}(.)\nu\big(\beta,x,\varphi_a(\xi,a)\big) \end{matrix}\right\}$$

$$\rho\nu(\alpha,x,a) = \tag{II-7}$$

$$\min_{(u,w)\in\Gamma(\alpha)} \left\{\begin{matrix} G(\alpha,x,a,u,w) + \dot{x}\dfrac{\partial}{\partial x}\nu(\alpha,x,a) + \dot{a}\dfrac{\partial}{\partial a}\nu(\alpha,x,a) \\ +\displaystyle\sum_{\beta} q_{\alpha\beta}(.)\nu\big(\beta,x,\varphi_a(\xi,a)\big) \end{matrix}\right\}$$

$$\frac{\partial}{\partial x}\nu(\alpha,x,a) = \begin{cases} \dfrac{1}{h_x}\Big(\nu^h(\alpha,x+h_x,a) - \nu^h(\alpha,x,a)\Big) & \text{if } (u-d) \geq 0 \\ \dfrac{1}{h_x}\Big(\nu^h(\alpha,x,a) - \nu^h(\alpha,x-h_x,a)\Big) & \text{otherwise} \end{cases} \tag{II-8}$$

$$\frac{\partial}{\partial a}\nu(\alpha,x,a) = \frac{1}{h_a}\Big(\nu^h(\alpha,x,a+h_a) - \nu^h(\alpha,x,a)\Big) \tag{II-9}$$

$$\nu^h(\alpha,x,a) = \tag{II-10}$$

$$\min_{(u,w)\in\Gamma^h(\alpha)} \left[\frac{\begin{matrix} G(\alpha,x,a,u,w) + \dfrac{|u-d|}{h_x}\begin{bmatrix} \nu^h(\alpha,x+h_x,a)\mathrm{Ind}\{u-d\geq 0\} \\ +\nu^h(\alpha,x-h_x,a)\mathrm{Ind}\{u-d<0\} \end{bmatrix} \\ +\dfrac{ku}{h_a}\nu^h(\alpha,x,a+h_a) + \sum_{\beta\neq\alpha} q_{\alpha\beta}(.)\nu^h\big(\beta,x,\varphi_a(\xi,a)\big) \end{matrix}}{\left(\rho + \dfrac{|u-d|}{h_x} + \dfrac{ku}{h_a} + |q_{\alpha\alpha}|\right)}\right]$$

- Mode 1: machine is operational

$$v^h(1,x,a) = \tag{II-11}$$

$$\min_{(u,w)\in\Gamma^h(1)} \left[\frac{c^+x^+ + c^-x^- + c_u u + \frac{|u-d|}{h_x}\begin{bmatrix} v^h(1,x+h_x,a)\mathrm{Ind}\{u-d\geq 0\} \\ +v^h(1,x-h_x,a)\mathrm{Ind}\{u-d<0\}\end{bmatrix} + \frac{ku}{h_a}v^h(1,x,a+h_a) + q_{12}(a)v^h(2,x,a) + qwv^h(3,x,0)}{\left(\rho + \frac{|u-d|}{h_x} + \frac{ku}{h_a} + q_{12}(a) + qw\right)} \right]$$

- Mode 2: machine is under repair

$$v^h(2,x,a) = \tag{II-12}$$

$$\min_{(u,w)\in\Gamma^h(2)} \left[\frac{c^+x^+ + c^-x^- + c_r + \frac{d}{h_x}v^h(2,x-h_x,a) + q_{21}v^h(1,x,a)}{\left(\rho + \frac{d}{h_x} + q_{21}\right)} \right]$$

- Mode 3: machine is under replacement

$$v^h(3,x,a) = \tag{II-13}$$

$$\min_{(u,w)\in\Gamma^h(3)} \left[\frac{c^+x^+ + c^-x^- + c_0 q_{31} + \frac{d}{h_x}v^h(3,x-h_x,a) + q_{31}v^h(1,x,a)}{\left(\rho + \frac{d}{h_x} + q_{31}\right)} \right]$$

ANNEXE III

APPENDIX 3.A. NUMERICAL APPROACH

In this section, we develop the numerical approach for solving the HJB equations, since arriving at a closed-form solution of such a production control problem would be difficult and often impossible to obtain. Therefore, a numerical method is a viable alternative. A numerical method initially introduced by Kushner and Dupuis (1992), and successfully implemented by Boukas and Haurie (1990) in the context of production planning, is used to solve the optimality conditions for the proposed stochastic control problem. This method is based on the finite difference approximations and policy improvement technique to determine an approximation of the value function, and an optimal control policy that achieves the minimum cost can be obtained as a solution of the HJB equations. The value function $\nu(\alpha, x, a)$ is approximated by $\nu^h(\alpha, x, a)$ and the first-order and second-order partial derivatives of the value function $\frac{\partial \nu(\alpha,x,a)}{\partial a}$, $\frac{\partial \nu(\alpha,x,a)}{\partial x}$ and $\frac{\partial^2 \nu(\alpha,x,a)}{\partial x^2}$ are described as follows:

$$\frac{\partial}{\partial a}\nu(\alpha, x, a) = \frac{1}{h_a}\Big(\nu^h(\alpha, x, a + h_a) - \nu^h(\alpha, x, a)\Big) \tag{III-1}$$

$$\frac{\partial}{\partial x}\nu(\alpha, x, a) = \begin{cases} \frac{1}{h_x}\Big(\nu^h(\alpha, x + h_x, a) - \nu^h(\alpha, x, a)\Big) & \text{if } f_1 \geq 0 \\ \frac{1}{h_x}\Big(\nu^h(\alpha, x, a) - \nu^h(\alpha, x - h_x, a)\Big) & \text{otherwise} \end{cases} \tag{III-2}$$

$$\frac{\partial^2}{\partial x^2}\nu(\alpha, x, a) = \frac{1}{h_x^2}\Big(\nu^h(\alpha, x + h_x, a) + \nu^h(\alpha, x - h_x, a) - 2\nu^h(\alpha, x, a)\Big) \tag{III-3}$$

where h_x and h_a denote the length of the finite difference intervals associated with state variables x and a.

The following three equations are the discrete dynamic programming equations obtained:

- Mode 1 (machine is operational):

$$v^h(1,x,a) = \tag{III-4}$$

$$\min_{(u,w)\in\Gamma^h(1)}\left[\begin{array}{c}\left(\begin{array}{c} c^+x^+ + c^-x^- + c_u u + \\ \dfrac{\left|(1-\mu_\beta(a))u-\mu_D\right|}{h_x}\begin{bmatrix} v^h(1,x+h_x,a)\mathrm{Ind}\{f_1\geq 0\} \\ +v^h(1,x-h_x,a)\mathrm{Ind}\{f_1<0\}\end{bmatrix} \\ +\dfrac{ku}{h_a}v^h(1,x,a+h_a)+ \\ \dfrac{\left(\sigma_D^2+\sigma_\beta^2(a)k^{-1}u\right)}{2h_x^2}\left(v^h(1,x+h_x,a)-v^h(1,x-h_x,a)\right) \\ +q_{12}(a)v^h(2,x,a)+qwv^h(3,x,0)\end{array}\right) \\ \times\left(\begin{array}{c}\rho+\dfrac{\left|(1-\mu_\beta(a))u-\mu_D\right|}{h_x}+\dfrac{ku}{h_a}+\dfrac{\left(\sigma_D^2+\sigma_\beta^2(a)k^{-1}u\right)}{h_x^2} \\ +q_{12}(a)+qw\end{array}\right)^{-1}\end{array}\right]$$

- Mode 2 (machine is under repair):

$$v^h(2,x,a) = \min_{(u,w)\in\Gamma^h(2)}\left[\begin{array}{c}\left(\begin{array}{c} c^+x^+ + c^-x^- + c_r + \dfrac{\mu_D}{h_x}v^h(2,x-h_x,a) \\ +\dfrac{\sigma_D^2}{2h_x^2}\left(v^h(2,x+h_x,a)-v^h(2,x-h_x,a)\right) \\ +q_{21}v^h(1,x,a)\end{array}\right) \\ \times\left(\rho+\dfrac{\mu_D}{h_x}+\dfrac{\sigma_D^2}{h_x^2}+q_{21}\right)^{-1}\end{array}\right] \tag{III-5}$$

- Mode 3 (machine is under replacement):

$$v^h(3,x,a) = \min_{(u,w)\in\Gamma^h(3)}\left[\begin{array}{c}\left(\begin{array}{c} c^+x^+ + c^-x^- + c_0q_{31} + \dfrac{\mu_D}{h_x}v^h(3,x-h_x,a) \\ +\dfrac{\sigma_D^2}{2h_x^2}\left(v^h(3,x+h_x,a)-v^h(3,x-h_x,a)\right) \\ +q_{31}v^h(1,x,0)\end{array}\right) \\ \times\left(\rho+\dfrac{\mu_D}{h_x}+\dfrac{\sigma_D^2}{h_x^2}+q_{31}\right)^{-1}\end{array}\right] \tag{III-6}$$

where $\Gamma^h(\alpha)$ is the numerical control grid (a singleton for $\alpha = 0$).

ANNEXE IV

APPENDIX 3.B. MILSTEIN NUMERICAL SCHEME

In this section, we consider the process $X_t, t \geq 0$ is defined by the following stochastic differential equation:

$$dX_t = b(X_t)dt + B(X_t)dW_t \tag{IV-1}$$

with initial condition $X_0 = x_0$, where $b(X_t)$ the drift coefficient, $B(X_t)$ the diffusion coefficient, and $W_t, t \geq 0$ the standard Wiener process, and it can be equivalently defined as Gaussian process with independent increments for which $W_0 = 0$ (with probability 1), such that the mean, variance and covariance of a Wiener process satisfy: $E\{W_t\} = 0$, $Var\{W_t - W_s\} = t - s$ and $C(s,t) = \min\{s,t\}$ for all $0 < s < t$ the process is not wide-sense stationary.

In Itô calculus, the Milstein method is a method to approximate numerical solution of a stochastic differential equation (IV-1). It is an improvement of the Euler-Maruyama method in order to obtain a higher order of numerical convergence, and

$$X_{t+h} \approx X_t + b(X_t)h + \int_t^{t+h} B(X_t + B(X_t)(W_s - W_t))dW_s$$

$$X_{t+h} \approx X_t + b(X_t)h + \int_t^{t+h} \big(B(X_t) + \dot{B}(X_t)B(X_t)(W_s - W_t)\big)dW_s$$

$$X_{t+h} \approx X_t + b(X_t)h + B(X_t)(W_{t+h} - W_t) + \dot{B}(X_t)B(X_t)\int_t^{t+h} (W_s - W_t)dW_s \tag{IV-2}$$

Finally, we obtain the scheme:

$$X_{t+h} \approx X_t + b(X_t)h + B(X_t)(W_{t+h} - W_t) + \frac{\dot{B}(X_t)B(X_t)((W_{t+h} - W_t)^2 - h)}{2} \tag{IV-3}$$

where $\dot{B}$ denotes the derivative of $B(X_t)$ with respect to X_t, h is the length of the finite difference interval and $W_{t+h} - W_t$ are the i.i.d. variables of normal distribution with expected value zero and h variance. Thus, for the numerical schemes of stochastic portion of demand and defectives rates, we set $b(Z_D(t)) = -b_1 Z_D(t)$, $B(Z_D(t)) = \sigma_1$, $b\left(Z_\beta(a(t))\right) = -b_2 Z_\beta\big(a(t)\big)$ and $B\left(Z_\beta(a(t))\right) = \sigma_2(a)$. The Milstein scheme is a scheme of order and dimension 1. However, in dimension greater than one, it involves stochastic integrals of type $\int_t^{t+h}(W_s - W_t)dW_s'$, where W and W' are two independent Brownian motions.

ANNEXE V

APPENDIX 5.A. APPROACH DESCRIPTION: FROM SEMI-MARKOVIAN TO MARKOVIAN

A Semi-Markov Decision Process (SMDP) is used to describe the dynamics of a machine facing deterioration and imperfect repairs. The system is modeled by three states (operational, repair and replacement), and the memory property is taken into account since the history of breakdowns and repairs affects the system evolution from one intervention to another. We can assume that the value p is the maximum number of failures the remanufacturing machine has experienced ($n = 0,1, \dots, p$). If the number of failures $n = 0$, it means the machine is new and has zero failures. In the proposed approach, we proceed with an extension of the state space, first increasing the number of states from 3 to $3p + 2$ to describe the remanufacturing machine modes, and then doubling them (to $6p + 4$) to include 2 possible modes of the manufacturing machine. As shown on the left side of figure-A V-1, when the manufacturing M_1 is operational (in modes 1-3-5), for a remanufacturing machine, there are $p + 1$ states in operation after i failures, indexed as $(1, i)$ in mode 1 with $i = 1, \dots, p + 1$; there are other p states at the j-th failure, indexed as $(3, j)$ in mode 3 with $j = 2p + 3, \dots, 3p + 2$, and $p + 1$ states at the m-th replacement, indexed as $(5, m)$ in mode 5 with $m = 4p + 3, \dots, 5p + 3$. Similarly, in the situation when the manufacturing M_1 is at failure (in modes 2-4-6) on the right side of figure-A V-1, for the remanufacturing machine, there are $p + 1$ states in operation, indexed as $(2, i)$ in mode 2 with $i = p + 2, \dots, 2p + 2$; there are p states at failure, indexed as $(4, j)$ in mode 4 with $j = 3p + 3, \dots, 4p + 2$, and $p + 1$ states at replacement, indexed as $(6, m)$ in mode 6 with $m = 5p + 4, \dots, 6p + 4$.

The total number of states is $6p + 4$ and figure 5.2 should be deemed to contain several layers under each mode actually shown. These layers, with several sub-modes and corresponding transitions, are shown in figure-A V-1. For example, mode 1 in figure 5.2 contains $p + 1$ sub-modes and mode 3 contains p sub-modes, as can be seen in figure-A V-1. A similar state space extension was used in Hu and Xiang (1995) to study a non-exponentially distributed failure repair flow. It should be recalled that the mode of the hybrid system at time t is given by the finite-state Markov process $\gamma(t) \in \Omega = \{1, 2, \dots, 6p + 4\}$, and is characterized by the matrix $Q(w_1, w_2) = \left[q_{\alpha\beta}(.)\right]$ of real numbers, $\alpha, \beta \in B \subset \Omega$.

with

$$\sum_{\beta} q_{\alpha\beta}(.) = 0, \qquad \forall\, \alpha, \beta \in B \tag{V-1}$$

and if $\alpha = \beta$

$$q_{\alpha\alpha}(.) = -\sum_{\alpha \neq \beta} q_{\alpha\beta}(.) \tag{V-2}$$

The transition probabilities of the manufacturing/remanufacturing system from mode α to mode β after time t are given by:

$$P[\gamma(t + \delta t) = \beta | \gamma(t) = \alpha, .] = q_{\alpha\beta}(.)\delta t + o(., \delta t) \tag{V-3}$$

$$P[\gamma(t + \delta t) = \alpha | \gamma(t) = \alpha, .] = 1 + q_{\alpha\alpha}(.)\delta t + o(., \delta t)$$

where $o(., \delta t)$ is a quantity such that:

$$\lim_{t \to 0} \frac{o(., \delta t)}{\delta t} \text{ for all } \alpha, \beta \in B \subset \Omega$$

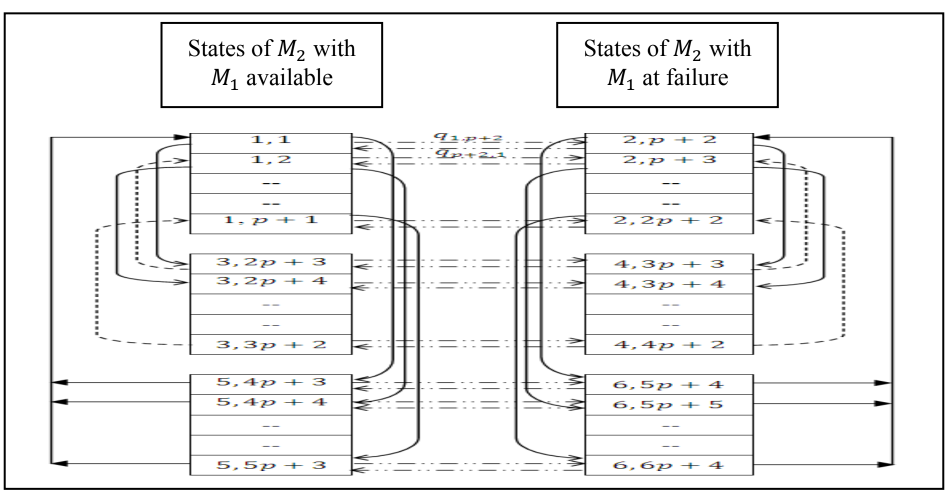

Figure-A V-1 States transition diagram of homogeneous Markov process (hybrid system)

The corresponding $(6p+4)\times(6p+4)$ transition matrix $Q(.)$ for the considered hybrid system is given by:

$$\left[\begin{array}{cccc|cccc|ccc|ccc|ccc|ccc}
q_{1,1} & 0 & \cdots & 0 & q_{1,p+2} & 0 & \cdots & 0 & q_{1,2p+3} & \cdots & 0 & 0 & \cdots & 0 & q_{1,4p+3} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & q_{p+1,p+1} & 0 & 0 & \cdots & q_{p+1,2p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & q_{p+1,5p+3} & 0 & \cdots & 0 \\
\hline
q_{p+2,1} & 0 & \cdots & 0 & q_{p+2,p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 & q_{p+2,3p+3} & \cdots & 0 & 0 & \cdots & 0 & q_{p+2,5p+4} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & q_{2p+2,p+1} & 0 & 0 & \cdots & q_{2p+2,2p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & q_{2p+2,6p+4} \\
\hline
0 & q_{2p+3,2} & \cdots & 0 & 0 & 0 & \cdots & 0 & q_{2p+3,2p+3} & \cdots & 0 & q_{2p+3,3p+3} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & q_{3p+2,p+1} & 0 & 0 & \cdots & 0 & 0 & \cdots & q_{3p+2,3p+2} & 0 & \cdots & q_{3p+2,4p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\hline
0 & 0 & \cdots & 0 & 0 & q_{3p+3,p+3} & \cdots & 0 & q_{3p+3,2p+3} & \cdots & 0 & q_{3p+3,3p+3} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & q_{4p+2,2p+2} & 0 & \cdots & q_{4p+2,3p+2} & 0 & \cdots & q_{4p+2,4p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\hline
q_{4p+3,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & q_{4p+3,4p+3} & \cdots & 0 & q_{4p+3,5p+4} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
q_{5p+3,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & q_{5p+3,5p+3} & 0 & \cdots & q_{5p+3,6p+4} \\
\hline
0 & 0 & \cdots & 0 & q_{5p+4,p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & q_{5p+4,4p+3} & \cdots & 0 & q_{5p+4,5p+4} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & q_{6p+4,p+2} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & q_{6p+4,5p+3} & 0 & \cdots & q_{6p+4,6p+4}
\end{array}\right] \tag{V-4}$$

Let us describe the set of matrix blocks T_{ij} of $Q(.)$ such that $Q(.) = [T_{ij}]$, with $i,j = 1,\dots,6$. We then have:

- First line block of Q:

$T_{11}(p+1,p+1) = \mathrm{diag}(q_{1,1},\dots,q_{p+1,p+1})$
$T_{12}(p+1,p+1) = \mathrm{diag}(q_{1,p+2},\dots,q_{p+1,2p+2})$
$T_{13}(p+1,p) = \mathrm{diag}(q_{1,2p+3},\dots,q_{p,3p+2}) + 0_{p+1,j}$, with $j = 2p+3,\dots,3p+2$
$T_{14}(p+1,p) = 0_{i,j}$, with $i = 1,\dots,p+1$ and $j = 3p+3,\dots,4p+2$
$T_{15}(p+1,p+1) = \mathrm{diag}(q_{1,4p+3},\dots,q_{p+1,5p+3})$
$T_{16}(p+1,p+1) = 0_{i,j}$, with $i = 1,\dots,p+1$ and $j = 5p+4,\dots,6p+4$

- Second line block of Q:

$T_{21}(p+1,p+1) = \mathrm{diag}(q_{p+2,1},\dots,q_{2p+2,p+1})$
$T_{22}(p+1,p+1) = \mathrm{diag}(q_{p+2,p+2},\dots,q_{2p+2,2p+2})$
$T_{23}(p+1,p) = 0_{i,j}$, with $i = p+2,\dots,2p+2$ and $j = 2p+3,\dots,3p+2$
$T_{24}(p+1,p) = \mathrm{diag}(q_{p+2,3p+3},\dots,q_{2p+1,4p+2}) + 0_{2p+2,j}$, with $j = 3p+3,\dots,4p+2$
$T_{25}(p+1,p+1) = 0_{i,j}$, with $i = p+2,\dots,2p+2$ and $j = 4p+3,\dots,5p+3$
$T_{26}(p+1,p+1) = \mathrm{diag}(q_{p+2,5p+4},\dots,q_{2p+2,6p+4})$

- Third line block of Q:

$T_{31}(p, p+1) = \text{diag}(q_{2p+3,2}, \dots, q_{3p+2,p+1}) + 0_{i,1}$, with $i = 2p+3, \dots, 3p+2$
$T_{32}(p, p+1) = 0_{i,j}$, with $i = 2p+3, \dots, 3p+2$ and $j = p+2, \dots, 2p+2$
$T_{33}(p, p) = \text{diag}(q_{2p+3,2p+3}, \dots, q_{3p+2,3p+2})$
$T_{34}(p, p) = \text{diag}(q_{2p+3,3p+3}, \dots, q_{3p+2,4p+2})$
$T_{35}(p, p+1) = 0_{i,j}$, with $i = 2p+3, \dots, 3p+2$ and $j = 4p+3, \dots, 5p+3$
$T_{36}(p, p+1) = 0_{i,j}$, with $i = 2p+3, \dots, 3p+2$ and $j = 5p+4, \dots, 6p+4$

- Fourth line block of Q:

$T_{41}(p, p+1) = 0_{i,j}$, with $i = 3p+3, \dots, 4p+2$ and $j = 1, \dots, p+1$
$T_{42}(p, p+1) = \text{diag}(q_{3p+3,p+3}, \dots, q_{4p+2,2p+2}) + 0_{i,p+2}$, with $i = 3p+3, \dots, 4p+2$
$T_{43}(p, p) = \text{diag}(q_{3p+3,2p+3}, \dots, q_{4p+2,3p+2})$
$T_{44}(p, p) = \text{diag}(q_{3p+3,3p+3}, \dots, q_{4p+2,4p+2})$
$T_{45}(p, p+1) = 0_{i,j}$, with $i = 3p+3, \dots, 4p+2$ and $j = 4p+3, \dots, 5p+3$
$T_{46}(p, p+1) = 0_{i,j}$, with $i = 3p+3, \dots, 4p+2$ and $j = 5p+4, \dots, 6p+4$

- Fifth line block of Q:

$T_{51}(p+1, p+1) = (q_{4p+3,1}, \dots, q_{5p+3,1}) + 0_{i,j}$, with $i = 4p+3, \dots, 5p+3$ and
$j = 2, \dots, p+1$
$T_{52}(p+1, p+1) = 0_{i,j}$, with $i = 4p+3, \dots, 5p+3$ and $j = p+2, \dots, 2p+2$
$T_{53}(p+1, p) = 0_{i,j}$, with $i = 4p+3, \dots, 5p+3$ and $j = 2p+3, \dots, 3p+2$
$T_{54}(p+1, p) = 0_{i,j}$, with $i = 4p+3, \dots, 5p+3$ and $j = 3p+3, \dots, 4p+2$
$T_{55}(p+1, p+1) = \text{diag}(q_{4p+3,4p+3}, \dots, q_{5p+3,5p+3})$
$T_{56}(p+1, p+1) = \text{diag}(q_{4p+3,5p+4}, \dots, q_{5p+3,6p+4})$

- Sixth line block of Q:

$T_{61}(p+1, p+1) = 0_{i,j}$, with $i = 5p+4, \dots, 6p+4$ and $j = 1, \dots, p+1$
$T_{62}(p+1, p+1) = (q_{5p+4,p+2}, \dots, q_{6p+4,p+2}) + 0_{i,j}$, with $i = 5p+4, \dots, 6p+4$ and
$j = p+3, \dots, 2p+2$
$T_{63}(p+1, p) = 0_{i,j}$, with $i = 5p+4, \dots, 6p+4$ and $j = 2p+3, \dots, 3p+2$
$T_{64}(p+1, p) = 0_{i,j}$, with $i = 5p+4, \dots, 6p+4$ and $j = 3p+3, \dots, 4p+2$
$T_{65}(p+1, p+1) = \text{diag}(q_{5p+4,4p+3}, \dots, q_{6p+4,5p+3})$
$T_{66}(p+1, p+1) = \text{diag}(q_{5p+4,5p+4}, \dots, q_{6p+4,6p+4})$

ANNEXE VI

APPENDIX 5.B. DERIVATION OF FEASIBILITY CONDITION

The feasibility condition holds that on average, the production level exceeds the demand level, with the production policies aimed at the maximization of the production level, and the term average meaning that the probability of being in a particular state is taken into account. Therefore, in our case, the feasibility condition can be written as follows:

$$\max_{u_1,u_2} \{\pi_{1,n}(u_1 + u_2) + \pi_{2,n}\, u_2 + (\pi_{3,n}+\pi_{5,n})u_1\} \geq d \tag{VI-1}$$

The maximum over u_1 is attained for $u_1 = u_{max}^1$; as for u_2, it is not so straightforward since we have to take into account equation (5.2) of $\dot{x}_2(t)$ and its state constraint (5.5). Due to these constraints, we have:

$$(\pi_{1,n} + \pi_{2,n})u_2 \ \leq rd \tag{VI-2}$$

Condition (VI-2) means that on average, the remanufacturing M_2 cannot produce more than has been returned. Therefore, maximization over u_2 is limited by both u_{max}^2 and condition (VI-2).

Finally, the maximization in (VI-1) results in the expression:

$$\pi_{1,n} + \pi_{3,n} + \pi_{5,n})u_{max}^1 + \left(\pi_{1,n} + \pi_{2,n}\right) \min \{ u_{max}^2 \,, \frac{rd}{\pi_{1,n} + \pi_{2,n}} \} \geq d \tag{VI-3}$$

which is equivalent to equation (5.13).

It should be noted that if $u_{max}^2 \leq \frac{rd}{\pi_{1,n}+\pi_{2,n}}$, condition (VI-3), due to equation (5.12), takes the form:

$$(\pi_{1,n} + \pi_{3,n} + \pi_{5,n})u_{max}^1 + \left(\pi_{1,n} + \pi_{2,n}\right) u_{max}^2 \geq d \tag{VI-4}$$

This inequality is identical to the one used in Kenné *et al.* (2012). However, in this case, the inequality (VI-2) will lead to infinite growth of the remanufacturing stock x_2, which cannot be bounded even if a maximal remanufacturing rate u_{max}^2 is used. From this perspective, the system parameters should rather be chosen to satisfy the condition $u_{max}^2 > \frac{rd}{\pi_{1,n}+\pi_{2,n}}$, which in that case (VI-3), leads to a simpler condition:

$$(\pi_{1,n} + \pi_{3,n} + \pi_{5,n})u_{max}^1 \geq d(1-r) \qquad \text{otherwise} \tag{VI-5}$$

ANNEXE VII

APPENDIX 5.C. NUMERICAL APPROACH

In this section, we now implement the Kushner approach to numerically solve the HJB equations given by equation (5.14). This approach is based on the finite difference approximations and policy improvement technique, and is described in Kushner and Dupuis (1992). It consists in using an approximation for the gradient of the value function based on the numerical scheme of finite differences. Let h_{x_1} and h_{x_2} denote the length of the finite difference interval of the state variables x_1 and x_2. The state space of the number of failures n is discretized according to a length equal to 1. Hence, the value function $v(\alpha, x_1, x_2, n)$ is approximated by $v^h(\alpha, x_1, x_2, n)$ and the first partial derivatives of the first order $\frac{\partial v(\alpha,x_1,x_2,n)}{\partial x_1}$ and $\frac{\partial v(\alpha,x_1,x_2,n)}{\partial x_2}$ are described as follows:

$$v_{x_1}(\alpha, x_1, x_2, n) = \tag{VII-1}$$

$$\begin{cases} \dfrac{1}{h_{x_1}}\Big(v^h\big(\alpha, x_1 + h_{x_1}, x_2, n\big) - v^h(\alpha, x_1, x_2, n)\Big) & \text{if } (u_{1\alpha} + u_{2\alpha} - d) \geq 0 \\ \dfrac{1}{h_{x_1}}\Big(v^h(\alpha, x_1, x_2, n) - v^h\big(\alpha, x_1 - h_{x_1}, x_2, n\big)\Big) & \text{otherwise} \end{cases}$$

and

$$v_{x_2}(\alpha, x_1, x_2, n) = \tag{VII-2}$$

$$\begin{cases} \dfrac{1}{h_{x_2}}\Big(v^h\big(\alpha, x_1, x_2 + h_{x_2}, n\big) - v^h(\alpha, x_1, x_2, n)\Big) & \text{if } (rd - u_{2\alpha}) \geq 0 \\ \dfrac{1}{h_{x_2}}\Big(v^h(\alpha, x_1, x_2, n) - v^h\big(\alpha, x_1, x_2 - h_{x_2}, n\big)\Big) & \text{otherwise} \end{cases}$$

The discrete dynamic programming equation of equation (5.14) gives the following six equations-A (VII-3)-(VII-8):

- Mode 1: M_1 is operational and M_2 is operational

$$\nu^h(1,x_1,x_2,n) = \left(\rho + \frac{|u_{11}+u_{21}-d|}{h_{x_1}} + \frac{|rd - u_{21}|}{h_{x_2}} - q_{11}\right)^{-1} * \tag{VII-3}$$

$$\left\{\begin{array}{c} c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^- + c_M u_{11} + c_R u_{21} \\ + \dfrac{|u_{11}+u_{21}-d|}{h_{x_1}} \begin{pmatrix} \nu^h\left(1, x_1 + h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{11}+u_{21}-d) \geq 0\} \\ + \nu^h\left(1, x_1 - h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{11}+u_{21}-d) < 0\} \end{pmatrix} \\ + \dfrac{|rd - u_{21}|}{h_{x_2}} \begin{pmatrix} \nu^h\left(1, x_1, x_2 + h_{x_2}, n\right) \mathrm{Ind}\{(rd - u_{21}) \geq 0\} \\ + \nu^h\left(1, x_1, x_2 - h_{x_2}, n\right) \mathrm{Ind}\{(rd - u_{21}) < 0\} \end{pmatrix} \\ + q_{12} \nu^h(2, x_1, x_2, n) + q_{13} \nu^h(3, x_1, x_2, n+1) + w_1(.) \nu^h(5, x_1, x_2, 0) \end{array}\right\}$$

- Mode 2: M_1 is under repair and M_2 is operational

$$\nu^h(2,x_1,x_2,n) = \left(\rho + \frac{|u_{22}-d|}{h_{x_1}} + \frac{|rd - u_{22}|}{h_{x_2}} - q_{22}\right)^{-1} * \tag{VII-4}$$

$$\left\{\begin{array}{c} c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^- + c_R u_{22} + c_r^1 \\ + \dfrac{|u_{22}-d|}{h_{x_1}} \begin{pmatrix} \nu^h\left(2, x_1 + h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{22}-d) \geq 0\} \\ + \nu^h\left(2, x_1 - h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{22}-d) < 0\} \end{pmatrix} \\ + \dfrac{|rd - u_{22}|}{h_{x_2}} \begin{pmatrix} \nu^h\left(2, x_1, x_2 + h_{x_2}, n\right) \mathrm{Ind}\{(rd - u_{22}) \geq 0\} \\ + \nu^h\left(2, x_1, x_2 - h_{x_2}, n\right) \mathrm{Ind}\{(rd - u_{22}) < 0\} \end{pmatrix} \\ + q_{21} \nu^h(1, x_1, x_2, n) + q_{24} \nu^h(4, x_1, x_2, n+1) + w_2(.) \nu^h(6, x_1, x_2, 0) \end{array}\right\}$$

- Mode 3: M_1 is operational and M_2 is under repair

$$\nu^h(3,x_1,x_2,n) = \left(\rho + \frac{|u_{13}-d|}{h_{x_1}} + \frac{rd}{h_{x_2}} - q_{33}\right)^{-1} * \tag{VII-5}$$

$$\left\{\begin{array}{c} c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^- + c_M u_{13} + c_r^2 \\ + \dfrac{|u_{13}-d|}{h_{x_1}} \begin{pmatrix} \nu^h\left(3, x_1 + h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{13}-d) \geq 0\} \\ + \nu^h\left(3, x_1 - h_{x_1}, x_2, n\right) \mathrm{Ind}\{(u_{13}-d) < 0\} \end{pmatrix} \\ + \dfrac{rd}{h_{x_2}} \nu^h\left(3, x_1, x_2 + h_{x_2}, n\right) \\ + q_{31}(n) \nu^h(1, x_1, x_2, n) + q_{34} \nu^h(4, x_1, x_2, n) \end{array}\right\}$$

- Mode 4: M_1 is under repair and M_2 is under repair

$$v^h(4,x_1,x_2,n) = \left(\rho + \frac{d}{h_{x_1}} + \frac{rd}{h_{x_2}} - q_{44}\right)^{-1} * \tag{VII-6}$$

$$\left\{\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + c_r^1 + c_r^2 \\ +\dfrac{d}{h_{x_1}}v^h\left(4,x_1-h_{x_1},x_2,n\right) + \dfrac{rd}{h_{x_2}}v^h\left(4,x_1,x_2+h_{x_2},n\right) \\ +q_{42}(n)v^h(2,x_1,x_2,n) + q_{43}v^h(3,x_1,x_2,n) \end{array}\right\}$$

- Mode 5: M_1 is operational and M_2 is under replacement

$$v^h(5,x_1,x_2,n) = \left(\rho + \frac{|u_{15}-d|}{h_{x_1}} + \frac{rd}{h_{x_2}} - q_{55}\right)^{-1} * \tag{VII-7}$$

$$\left\{\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + c_M u_{15} + c_0 q_{51} \\ +\dfrac{|u_{15}-d|}{h_{x_1}}\begin{pmatrix} v^h\left(5,x_1+h_{x_1},x_2,n\right)\mathrm{Ind}\{(u_{15}-d)\geq 0\} \\ +v^h\left(5,x_1-h_{x_1},x_2,n\right)\mathrm{Ind}\{(u_{15}-d)<0\} \end{pmatrix} \\ +\dfrac{rd}{h_{x_2}}v^h\left(5,x_1,x_2+h_{x_2},n\right) \\ +q_{51}v^h(1,x_1,x_2,0) + q_{56}v^h(6,x_1,x_2,n) \end{array}\right\}$$

- Mode 6: M_1 is under repair and M_2 is under replacement

$$v^h(6,x_1,x_2,n) = \left(\rho + \frac{d}{h_{x_1}} + \frac{rd}{h_{x_2}} - q_{66}\right)^{-1} * \tag{VII-8}$$

$$\left\{\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + c_r^1 + c_0 q_{62} \\ +\dfrac{d}{h_{x_1}}v^h\left(6,x_1-h_{x_1},x_2,n\right) + \dfrac{rd}{h_{x_2}}v^h\left(6,x_1,x_2+h_{x_2},n\right) \\ +q_{62}v^h(2,x_1,x_2,0) + q_{65}v^h(5,x_1,x_2,n) \end{array}\right\}$$

The next theorem shows that the discrete approximation of the value function $v^h(\alpha,x_1,x_2,n)$ converges to an exact viscosity-type $v(\alpha,x_1,x_2,n)$ solution when the sizes of the grid h_{x_1} and h_{x_2} tend to zero.

Theorem 5.1

Let $v^h(\alpha, x_1, x_2, n)$ denote a solution to HJB equations (VII-3)-(VII-8). Assume that there are positive constants C and K such that:

If $0 \leq v^h(\alpha, x_1, x_2, n) \leq C(1 + |x|^K)$, then $\lim_{h \to 0} v^h(\alpha, x_1, x_2, n) = v(\alpha, x_1, x_2, n)$

Proof. This theorem can be proven similarly as in Yan and Zhang (1997) when replacing x by $z = (x_1, x_2)$ and $v(\alpha, x)$ by $v(\alpha, z, n)$. ■

ANNEXE VIII

APPENDIX 6.A. NUMERICAL APPROACH

In this section, numerical methods are used to approximate the solution of the HJBDD equations given by equation (6.19) for the proposed stochastic control problem. This method was initially introduced by Kushner and Dupuis (1992), and successfully implemented by Boukas and Haurie (1990) in the context of a production planning problem. It is based on the finite difference approximations and policy improvement technique, and it consists in using an approximation for the gradient of the value function based on the numerical scheme of finite differences. Let h_{x_1}, h_{x_2} and h_{a_1} denote the length of the finite difference interval of the state variables x_1, x_2 and a_1. Hence, the value function $v(\alpha, x_1, x_2, a_1)$ is approximated by $v^h(\alpha, x_1, x_2, a_1)$, and its first and second partial derivatives are described as follows:

The first-order partial derivatives:

$$\frac{\partial}{\partial x_1} v(\alpha, x_1, x_2, a_1) = \qquad \text{(VIII-1)}$$

$$\begin{cases} \frac{1}{h_{x_1}}\left(v^h\left(\alpha, x_1 + h_{x_1}, x_2, a_1\right) - v^h(\alpha, x_1, x_2, a_1)\right) & \text{if } f_{1\alpha} \geq 0 \\ \frac{1}{h_{x_1}}\left(v^h(\alpha, x_1, x_2, a_1) - v^h\left(\alpha, x_1 - h_{x_1}, x_2, a_1\right)\right) & \text{otherwise} \end{cases}$$

$$\frac{\partial}{\partial x_2} v(\alpha, x_1, x_2, a_1) = \qquad \text{(VIII-2)}$$

$$\begin{cases} \frac{1}{h_{x_2}}\left(v^h\left(\alpha, x_1, x_2 + h_{x_2}, a_1\right) - v^h(\alpha, x_1, x_2, a_1)\right) & \text{if } f_{2\alpha} \geq 0 \\ \frac{1}{h_{x_2}}\left(v^h(\alpha, x_1, x_2, a_1) - v^h\left(\alpha, x_1, x_2 - h_{x_2}, a_1\right)\right) & \text{otherwise} \end{cases}$$

$$\frac{\partial}{\partial a_1} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{a_1}}\left(v^h\left(\alpha, x_1, x_2, a_1 + h_{a_1}\right) - v^h(\alpha, x_1, x_2, a_1)\right) \qquad \text{(VIII-3)}$$

with: $f_{1\alpha} = \left(1 - \mu_\beta(a_1)\right) u_{1\alpha} + (1-p)u_{2\alpha} - d$, and $f_{2\alpha} = rd + \mu_\beta(a_1)u_{1\alpha} - u_{2\alpha}$.

The second-order partial derivatives:

$$\frac{\partial^2}{\partial x_1^2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_1}^2}\left(v^h\left(\alpha, x_1 + h_{x_1}, x_2, a_1\right) + v^h\left(\alpha, x_1 - h_{x_1}, x_2, a_1\right) - 2v^h(\alpha, x_1, x_2, a_1)\right) \tag{VIII-4}$$

$$\frac{\partial^2}{\partial x_2^2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_2}^2}\left(v^h\left(\alpha, x_1, x_2 + h_{x_2}, a_1\right) + v^h\left(\alpha, x_1, x_2 - h_{x_2}, a_1\right) - 2v^h(\alpha, x_1, x_2, a_1)\right) \tag{VIII-5}$$

With the crossed partial derivatives of the second order, there are four possible cases according to the signs of terms $f_{1\alpha}$ and $f_{2\alpha}$:

$$\frac{\partial^2}{\partial x_1 x_2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_1} h_{x_2}}\begin{pmatrix} v^h\left(., x_1 + h_{x_1}, x_2 + h_{x_2}, .\right) - v^h\left(., x_1 + h_{x_1}, x_2, .\right) \\ -v^h\left(., x_1, x_2 + h_{x_2}, .\right) + v^h(., x_1, x_2, .) \end{pmatrix} \tag{VIII-6}$$

$$\frac{\partial^2}{\partial x_1 x_2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_1} h_{x_2}}\begin{pmatrix} v^h\left(., x_1, x_2 + h_{x_2}, .\right) - v^h\left(., x_1 - h_{x_1}, x_2 + h_{x_2}, .\right) \\ +v^h\left(., x_1 - h_{x_1}, x_2, .\right) - v^h(., x_1, x_2, .) \end{pmatrix} \tag{VIII-7}$$

$$\frac{\partial^2}{\partial x_1 x_2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_1} h_{x_2}}\begin{pmatrix} v^h\left(., x_1 + h_{x_1}, x_2, .\right) - v^h\left(., x_1 + h_{x_1}, x_2 - h_{x_2}, .\right) \\ +v^h\left(., x_1, x_2 - h_{x_2}, .\right) - v^h(., x_1, x_2, .) \end{pmatrix} \tag{VIII-8}$$

$$\frac{\partial^2}{\partial x_1 x_2} v(\alpha, x_1, x_2, a_1) = \frac{1}{h_{x_1} h_{x_2}}\begin{pmatrix} v^h\left(., x_1 - h_{x_1}, x_2 - h_{x_2}, .\right) - v^h\left(., x_1, x_2 - h_{x_2}, .\right) \\ -v^h\left(., x_1 - h_{x_1}, x_2, .\right) + v^h(., x_1, x_2, .) \end{pmatrix} \tag{VIII-9}$$

The discrete dynamic programming equation (6.19) gives the following six equations-A (VIII-10)-(VIII-15):

- Mode 1: M_1 is operational and M_2 is operational

$v^h(1, x_1, x_2, a_1) =$

$$\min_{(u,w)\in\Gamma^h(x_2,1)} \left[\begin{array}{c} \left(\begin{array}{c} c_1^+ x_1^+ + c_2^+ x_2^+ + c_1^- x_1^- + (c_{P_1} + c_{IP})u_{11} + (c_{P_2} + c_{IP} + c_d p)u_{21} \\ + \dfrac{|f_{11}|}{h_{x_1}} \begin{bmatrix} v^h(1, x_1 + h_{x_1}, x_2, a_1)K_1^+ \\ + v^h(1, x_1 - h_{x_1}, x_2, a_1)K_1^- \end{bmatrix} + \dfrac{|f_{21}|}{h_{x_2}} \begin{bmatrix} v^h(1, x_1, x_2 + h_{x_2}, a_1)T_1^+ \\ + v^h(1, x_1, x_2 - h_{x_2}, a_1)T_1^- \end{bmatrix} \\ + \dfrac{f_{31}}{h_{a_1}} v^h(1, x_1, x_2, a_1 + h_{a_1}) \\ + f_{41} \left[\begin{array}{c} \dfrac{1}{2h_{x_1}^2}\left(v^h(1, x_1 + h_{x_1}, x_2, a_1) - v^h(1, x_1 - h_{x_1}, x_2, a_1)\right) \\ + \dfrac{1}{2h_{x_2}^2}\left(v^h(1, x_1, x_2 + h_{x_2}, a_1) - v^h(1, x_1, x_2 - h_{x_2}, a_1)\right) \\ - \dfrac{1}{h_{x_1}h_{x_2}} \left[\begin{array}{l} \left(\begin{array}{c} v^h(1, x_1 + h_{x_1}, x_2 + h_{x_2}, a_1) - \\ v^h(1, x_1 + h_{x_1}, x_2, a_1) - v^h(1, x_1, x_2 + h_{x_2}, a_1) \end{array} \right) K_1^+ T_1^+ \\ + \left(\begin{array}{c} -v^h(1, x_1 + h_{x_1}, x_2 - h_{x_2}, a_1) + \\ v^h(1, x_1 + h_{x_1}, x_2, a_1) + v^h(1, x_1, x_2 - h_{x_2}, a_1) \end{array} \right) K_1^+ T_1^- \\ + \left(\begin{array}{c} -v^h(1, x_1 - h_{x_1}, x_2 + h_{x_2}, a_1) + \\ v^h(1, x_1 - h_{x_1}, x_2, a_1) + v^h(1, x_1, x_2 + h_{x_2}, a_1) \end{array} \right) K_1^- T_1^+ \\ + \left(\begin{array}{c} v^h(1, x_1 - h_{x_1}, x_2 - h_{x_2}, a_1) - \\ v^h(1, x_1 - h_{x_1}, x_2, a_1) - v^h(1, x_1, x_2 - h_{x_2}, a_1) \end{array} \right) K_1^- T_1^- \end{array} \right] \end{array} \right] \\ + q_{12}\left(\tilde{\beta}(a_1)u_{11}\right)v^h(2, x_1, x_2, a_1) + q_{13}(a_1)v^h(3, x_1, x_2, a_1) + w_1 v^h(5, x_1, x_2, 0) \end{array} \right) \\ \cdot \left(\dfrac{1}{Q_1}\right) \end{array} \right]$$

(VIII-10)

here: $Q_1 =$

$$\left(\begin{array}{c} \rho + \dfrac{|f_{11}|}{h_{x_1}} + \dfrac{|f_{21}|}{h_{x_2}} + \dfrac{f_{31}}{h_{a_1}} + f_{41}\left[\dfrac{1}{h_{x_1}^2} + \dfrac{1}{h_{x_2}^2} + \dfrac{1}{h_{x_1}h_{x_2}}(K_1^+T_1^+ - K_1^+T_1^- - K_1^-T_1^+ + K_1^-T_1^-)\right] \\ + q_{12}\left(\tilde{\beta}(a_1)u_{11}\right) + q_{13}(a_1) + w_1 \end{array} \right)$$

where $f_{11} = \left(1-\mu_\beta(a_1)\right)u_{11} + (1-p)u_{21} - d,$ $f_{21} = rd + \mu_\beta(a_1)u_{11} - u_{21},$ $f_{31} = ku_{11},$ $f_{41} = \sigma_\beta^2(a_1)k^{-1}u_{11},$ $K_1^+ = \text{Ind}\{f_{11} \ge 0\},$ $K_1^- = \text{Ind}\{f_{11} < 0\},$ $T_1^+ = \text{Ind}\{f_{21} \ge 0\}$ and $T_1^- = \text{Ind}\{f_{21} < 0\}$.

- Mode 2: M_1 is operational and M_2 is under repair

$$\nu^h(2,x_1,x_2,a_1) =$$

$$\min_{(u,w)\in\Gamma^h(x_2,2)}\left[\left(\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + (c_{P_1}+c_{IP})u_{12} + c_{r2} \\ +\dfrac{|f_{12}|}{h_{x_1}}\begin{bmatrix} \nu^h(2,x_1+h_{x_1},x_2,a_1)K_2^+ \\ +\nu^h(2,x_1-h_{x_1},x_2,a_1)K_2^- \end{bmatrix} + \dfrac{f_{22}}{h_{x_2}}\nu^h(2,x_1,x_2+h_{x_2},a_1) \\ +\dfrac{f_{32}}{h_{a_1}}\nu^h(2,x_1,x_2,a_1+h_{a_1}) \\ +f_{42}\left[\begin{array}{c} \dfrac{1}{2h_{x_1}^2}\left(\nu^h(2,x_1+h_{x_1},x_2,a_1) - \nu^h(2,x_1-h_{x_1},x_2,a_1)\right) \\ +\dfrac{1}{2h_{x_2}^2}\left(\nu^h(2,x_1,x_2+h_{x_2},a_1) - \nu^h(2,x_1,x_2-h_{x_2},a_1)\right) \\ -\dfrac{1}{h_{x_1}h_{x_2}}\left[\begin{array}{c} \left(\begin{array}{c} \nu^h(2,x_1+h_{x_1},x_2+h_{x_2},a_1) - \\ \nu^h(2,x_1+h_{x_1},x_2,a_1) - \nu^h(2,x_1,x_2+h_{x_2},a_1) \end{array}\right)K_2^+T_2^+ \\ +\left(\begin{array}{c} -\nu^h(2,x_1-h_{x_1},x_2+h_{x_2},a_1) + \\ \nu^h(2,x_1-h_{x_1},x_2,a_1) + \nu^h(2,x_1,x_2+h_{x_2},a_1) \end{array}\right)K_2^-T_2^+ \end{array}\right] \end{array}\right] \\ +q_{21}\nu^h(1,x_1,x_2,a_1) + q_{24}(a_1)\nu^h(4,x_1,x_2,a_1) + w_2\nu^h(6,x_1,x_2,0) \end{array}\right) \cdot\left(\frac{1}{Q_2}\right)\right]$$

(VIII-11)

here: $$Q_2 = \left(\begin{array}{c} \rho + \frac{|f_{12}|}{h_{x_1}} + \frac{f_{22}}{h_{x_2}} + \frac{f_{32}}{h_{a_1}} + f_{42}\left[\frac{1}{h_{x_1}^2} + \frac{1}{h_{x_2}^2} + \frac{1}{h_{x_1}h_{x_2}}(K_2^+T_2^+ - K_2^-T_2^+)\right] \\ +q_{21} + q_{24}(a_1) + w_2 \end{array}\right)$$

where $f_{12} = \left(1-\mu_\beta(a_1)\right)u_{12} - d,$ $f_{22} = rd + \mu_\beta(a_1)u_{12},$ $f_{32} = ku_{12},$ $f_{42} = \sigma_\beta^2(a_1)k^{-1}u_{12},$ $K_2^+ = \text{Ind}\{f_{12} \ge 0\},$ $K_2^- = \text{Ind}\{f_{12} < 0\}$ and $T_2^+ = \text{Ind}\{f_{22} \ge 0\}$.

- Mode 3: M_1 is under repair and M_2 is operational

$$\nu^h(3,x_1,x_2,a_1) =$$

$$\min_{(u,w)\in\Gamma^h(x_2,3)}\left[\begin{array}{c}\left(\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + (c_{P_2} + c_{IP} + c_d p)u_{23} + c_{r1} \\ +\dfrac{|f_{13}|}{h_{x_1}}\begin{bmatrix} \nu^h\big(3,x_1+h_{x_1},x_2,a_1\big)K_3^+ \\ +\nu^h\big(3,x_1-h_{x_1},x_2,a_1\big)K_3^- \end{bmatrix} + \dfrac{|f_{23}|}{h_{x_2}}\begin{bmatrix} \nu^h\big(3,x_1,x_2+h_{x_2},a_1\big)T_3^+ \\ +\nu^h\big(3,x_1,x_2-h_{x_2},a_1\big)T_3^- \end{bmatrix} \\ +q_{31}\nu^h(1,x_1,x_2,a_1) + q_2^c\nu^h(4,x_1,x_2,a_1) \end{array}\right) \\ \cdot\left(\dfrac{1}{Q_3}\right)\end{array}\right]$$

(VIII-12)

here: $Q_3 = \left(\rho + \frac{|f_{13}|}{h_{x_1}} + \frac{|f_{23}|}{h_{x_2}} + q_{31} + q_2^c\right)$ where $f_{13} = (1-p)u_{23} - d$, $f_{23} = rd - u_{23}$, $K_3^+ = \text{Ind}\{f_{13} \geq 0\}$, $K_3^- = \text{Ind}\{f_{13} < 0\}$, $T_3^+ = \text{Ind}\{f_{23} \geq 0\}$ and $T_3^- = \text{Ind}\{f_{23} < 0\}$.

- Mode 4: M_1 is under repair and M_2 is under repair

$$\nu^h(4,x_1,x_2,a_1) = \tag{VIII-13}$$

$$\min_{(u,w)\in\Gamma^h(x_2,4)}\left[\begin{array}{c}\left(\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + c_{r1} + c_{r2} \\ +\dfrac{d}{h_{x_1}}\nu^h\big(4,x_1-h_{x_1},x_2,a_1\big) + \dfrac{rd}{h_{x_2}}\nu^h\big(4,x_1,x_2+h_{x_2},a_1\big) \\ +q_{42}\nu^h(2,x_1,x_2,a_1) + q_{43}\nu^h(3,x_1,x_2,a_1) \end{array}\right) \\ \cdot\left(\dfrac{1}{Q_4}\right)\end{array}\right]$$

here: $Q_4 = \left(\rho + \frac{d}{h_{x_1}} + \frac{rd}{h_{x_2}} + q_{42} + q_{43}\right)$.

- Mode 5: M_1 is under replacement and M_2 is operational

$$v^h(5,x_1,x_2,a_1) =$$

$$\min_{(u,w)\in\Gamma^h(x_2,5)}\left[\begin{array}{c}\left(\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + (c_{P_2} + c_{IP} + c_d p)u_{25} + c_0 q_{51} \\ +\dfrac{|f_{15}|}{h_{x_1}}\begin{bmatrix} v^h\left(5,x_1+h_{x_1},x_2,a_1\right)K_5^+ \\ +v^h\left(5,x_1-h_{x_1},x_2,a_1\right)K_5^- \end{bmatrix} + \dfrac{|f_{25}|}{h_{x_2}}\begin{bmatrix} v^h\left(5,x_1,x_2+h_{x_2},a_1\right)T_5^+ \\ +v^h\left(5,x_1,x_2-h_{x_2},a_1\right)T_5^- \end{bmatrix} \\ +q_{51}v^h(1,x_1,x_2,0) + q_2^c v^h(6,x_1,x_2,a_1) \end{array}\right) \\ \cdot\left(\dfrac{1}{Q_5}\right)\end{array}\right]$$

(VIII-14)

here: $Q_5 = \left(\rho + \frac{|f_{15}|}{h_{x_1}} + \frac{|f_{25}|}{h_{x_2}} + q_{51} + q_2^c\right)$ where $f_{15} = (1-p)u_{25} - d$, $f_{25} = rd - u_{25}$, $K_5^+ = \text{Ind}\{f_{15} \geq 0\}$, $K_5^- = \text{Ind}\{f_{15} < 0\}$, $T_5^+ = \text{Ind}\{f_{25} \geq 0\}$ and $T_5^- = \text{Ind}\{f_{25} < 0\}$.

- Mode 6: M_1 is under replacement and M_2 is under repair

$$v^h(6,x_1,x_2,a_1) = \qquad \text{(VIII-15)}$$

$$\min_{(u,w)\in\Gamma^h(x_2,6)}\left[\begin{array}{c}\left(\begin{array}{c} c_1^+x_1^+ + c_2^+x_2^+ + c_1^-x_1^- + c_0 q_{62} + c_{r2} \\ +\dfrac{d}{h_{x_1}}v^h\left(6,x_1-h_{x_1},x_2,a_1\right) + \dfrac{rd}{h_{x_2}}v^h\left(6,x_1,x_2+h_{x_2},a_1\right) \\ +q_{62}v^h(2,x_1,x_2,0) + q_{65}v^h(5,x_1,x_2,a_1) \end{array}\right) \\ \cdot\left(\dfrac{1}{Q_6}\right)\end{array}\right]$$

here: $Q_6 = \left(\rho + \frac{d}{h_{x_1}} + \frac{rd}{h_{x_2}} + q_{62} + q_{65}\right)$.

ANNEXE IX

ARTICLES DE CONFÉRENCES

- **S. Ouaret**, V. Polotski, J. P. Kenné, A. Gharbi, (2013) "*Manufacturing-remanufacturing failure-prone systems under random demands*". 7th IFAC Conference on Manufacturing Modelling, Management and Control, Saint-Petersburg, Russia, June 19-21, 2013.

- **S. Ouaret**, J. P. Kenné, A. Gharbi, V. Polotski, (2015) "*Joint production and replacement strategy for a quality deteriorating failure-prone manufacturing system*". 15th IFAC Symposium on Information Control Problems in Manufacturing, Ottawa, Canada, May 11-13, 2015.

- **S. Ouaret**, J. P. Kenné, A. Gharbi, (2016) "*Joint optimal production and replacement policy for a deteriorated closed-loop manufacturing system*". 11th International Conference on Modeling, Optimization and Simulation, Montreal, Canada, August 22-24, 2016.

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